Study, implementation and test of a solid propellant rocket motor

Report

Degree: Màster Universitari en Enginyeria Aeronàutica
Delivery date: 20-06-2019
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Call: spring 2019
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1. Abstract

This thesis has the intention to conduct a study and design of a solid rocket motor and its propellant, for a later implementation, construction and testing of it in a test stand.

The study and design of the motor will be carried out using as outlines the requirements set either by the legislation or the ones imposed by the objectives of the study. These requirements, together with a bibliographic search for the state of the art and the expressions needed to compute the parameters defining the motor, and the use of adequate software will define the expected performance of the motor.

The implementation and construction of the motor itself will be carried out by the thesis’ director, as he has the knowledge on how to operate the machinery to obtain the parts conforming the motor.

Finally, the testing of the motor will have to be done by using a test stand, the design and construction of which are the final master’s thesis of another student. During the test of the motor, the thrust and other parameters will be measured and compared with the calculated ones during the design phase.

Aquesta tesi té la intenció de dur a terme el disseny d’un motor de coet sòlid i del seu propergol, per una posterior implementació, construcció i prova del mateix en un banc d’assaig.

L’estudi i disseny del motor es durà a terme utilitzant com a marc els requeriments imposats tant per la legislació com pels imposats pels objectius de l’estudi. Aquests requeriments, juntament amb una cerca bibliogràfica de l’estat de l’art i de les expressions necessàries pel càlcul dels paràmetres que defineixen el motor, i l’ús de software adequat, definiran el rendiment del motor.

La implementació i construcció del motor en si mateix la durà a terme el director del treball, ja que ell té el coneixement necessari per operar la maquinària per obtenir les peces que formen el motor.

Finalment, la prova del motor s’haurà de fer mitjançant l’ús d’un banc de proves, el disseny i la construcció del qual són el treball final de màster d’un altre estudiant. Durant la prova del motor, l’empenta i altres paràmetres seran mesurats i comparats amb els calculats a la fase de disseny.
2. Scope

The main scope of this project is to find the expected values of the different parameters that govern the performance of a solid rocket motor using specialised software, and then compare these results with experimental data of the actual motor tested in a test stand. For this purpose, different studies will be carried out in order to take into account all the characteristics that the motor must comply with in order to assure its good performance, as well as its safety and that it doesn’t enter in conflict with any legal regulation.

2.1. Regulations

The laws and rules that applies to the object of this thesis have to be understood and taken into account as they can become design parameters.

- Search for the current regulations in Spain regarding amateur solid rocket motors and amateur rockets launches in general.
- Define the design parameters in order to comply with the regulation.

2.2. Structure

The structural integrity of the motor has to be assessed and proved to be capable to withstand the expected loads.

- Material and geometry decision.
- Loads estimation using referenced formulas and calculations.
- Validation of the proposed geometry.

2.3. Propulsion

The evaluation of the propulsion of the motor gives the performance parameters that are expected from it.

- Propellant characterisation using adequate calculations and useful software (ProPEP 3).
- Propulsion-related parameters computation using adequate calculations and useful software (SRM 2014 spreadsheet).

2.4. Manufacture

The manufacturing process of the elements that conform the motor has to be assessed, as they may impose some restrictions in the design.

- Decision of the necessary techniques to build the designed rocket motor casing.
- Decision of the necessary techniques to build the propellant grain.
- Decision of the necessary techniques to build and integrate the nozzle.
3. Requirements

The study has to have a clear set of requirements in order to guide it to its completion, and these requirements must be verifiable at the end of the project. Each requirement has a code assigned to it in order to be easily referred to during the study development.

- **Req.Fmax**
  *The motor shall have a maximum thrust greater than 507 N.*
  This requirement will be completed in two phases. During the design, the value computed of expected maximum thrust has to fulfil this requirement, and during the static test, the maximum thrust measured has to surpass the value set in this requirement.

- **Req.Itot**
  *The motor shall have a total impulse greater than 1871 N · s.*
  This requirement will be completed in two phases. During the design, the value computed of expected total impulse has to fulfil this requirement, and during the static test, the total impulse measured has to surpass the value set in this requirement.

- **Req.Safety**
  *The motor casing shall be designed to operate with a safety factor of 1.5 or greater.*
  The fulfilment of this requirement will be checked during the performance analysis of the motor, by checking that the maximum expected pressure doesn’t exceed the casing design pressure.

- **Req.Reuse**
  *The motor shall be reusable.*
  This requirement will be checked after the static test, when a post-firing check is conducted on the motor to see if its shape allows for reusability.

- **Req.PropMat**
  *The propellant chosen shall be cold-cast.*
  The fulfilment of this requirement will be checked in the propellant design phase, when choosing the ingredients and manufacturing techniques of the propellant.

- **Req.Diam**
  *The casing outer diameter shall be 100 mm.*
  This requirement will be taken into account when designing the casing and analysing its structural resistance.

- **Req.Thick**
  *The casing wall shall be 5 mm thick.*
  This requirement will be taken into account when designing the casing and analysing its structural resistance.

- **Req.Length**
  *The casing length shall be 1000 mm.*
  This requirement will be taken into account when designing the casing and analysing its structural resistance.

- **Req.Legis**
  *The designed motor shall comply with the norms and regulations that apply to it.*
This requirement will be fulfilled first by laying out the norms and regulations that affect its usage and then by checking that the parameters and miscellanea comply with them.
4. Background

The motor being designed in this thesis is an evolution of that currently being used by this thesis’ tutor in his model rocket. The main design aspects are kept the same, that is, a metallic casing and a cold-cast handmade propellant, while the dimensions are escalated in order to obtain a more powerful motor capable of achieving greater heights and carry more equipment in the rocket. However, this study will take a look at the different options for material and propellant just to have a clear overview of the problem at hand.

The main intent with this thesis is to conduct a study of the expected performance of this new motor and later on to check these previous calculations with experimental data obtained in a static test of the manufactured motor. This data will be acquired using a test stand made by a fellow student as his final master thesis. As of the delivery date of this report, the said test stand is not available, as the student has postponed his delivery to September.
5. State of the art

In order to get to a starting point in the project, it is needed to have knowledge of the technology available, as well as to base the ideas using as reference other similar projects. Here, the state of the art in each aspect concerning the design and manufacturing of an amateur solid rocket motor is analysed.

5.1. Structure – Motor casing

The motor casing is in charge of containing the very high pressure achieved during the operation of the motor (that is why it is also considered the combustion chamber of the rocket motor). The correct design of this element is crucial to prevent any explosive outcome of the use of the rocket and to be able to use it in more than one launch, as the construction of this element can be both expensive and hard. In terms of geometry there is not a lot of options to choose from: the casing is cylindrical, with the adequate thickness of material to prevent failure under the expected operating conditions with a safety value, and with the adequate diameter in order to house the designed propellant.

To choose the correct material of the casing many factors have to be taken into account. Nonetheless the three most typical ones used in this element are aluminium, steel and carbon fibre reinforced plastics.

The aluminium approach is one of the most used, as it is a cheap, light and easy to machine. The main negative aspect of this material is that it cannot bear quite well with high temperatures, such as the ones that the combustion of the propellant creates. This makes it necessary to take a lot of care regarding the thermal insulation of the inner wall of the casing.

The steel is a great choice for the casing, as it has a high specific strength and is able to bear with very high temperature much better than aluminium. However, it is heavier than the latter, meaning that the choice of this material may affect the potential of the overall rocket to get to a certain height, which in model rocketry is the main objective. Also it is not as easy to work with when manufacturing the casing as aluminium, requiring more time to machine to the desired shape.

The carbon fibre reinforced plastics (and similar, such as Kevlar) are a very attractive choice because of the outstanding strength-to-weight ratio that translates in a huge weight reduction of the overall propulsion system of the rocket, making it much more powerful and able to get to much higher altitudes. The main set-back of this design is the great difficulty of manufacture of the casing, as some specialised tools would be required, as well as thermal treating to cure the polymer matrix. The price is also a factor to take into account here, as the ceramic fibres are not cheap to come by compare to the metallic options exposed here.

5.2. Propulsion

The propulsive properties of the motor are given by two elements: the propellant and the nozzle. The first one is key, as the mixture of elements used and the manufacturing process have a key influence in the final behaviour of the motor. The nozzle design also has to be such that correctly expands the hot gases produced in the combustion chamber, while being feasible to manufacture and avoiding complications in the mating procedure with the casing.
5.2.1. Propellant

The propellant choice is maybe the aspect that can present the most choices from which to decide. There are many combination of chemical species that can form a propellant suitable for a solid rocket motor, and moreover, different kinds of propellants can be mixed in order to alter the properties of the motor and modify its behaviour in certain moments of the flight. The choice also can vary from manufacturing the propellant to buying commercial grains.

The simplest propellant, used only in very small rockets and mainly as an igniter is the classical black powder. The powder is compressed to a solid grain, but doesn't have much mechanical strength, meaning that under high pressures it can rupture and cause a catastrophic failure of both the propellant and the overall engine. That is why its use is limited to the smallest, less-powered motors.

On the other hand, the most advanced and powerful rockets use an ammonium perchlorate composite propellant. Using aluminium powder as fuel and adding a binder material in order to have mechanical resistance, this propellant is used widely in the aerospace sector as well as in amateur rocketry.

In between these two, there are many more combinations that each has its pros and cons, but they can be divided in two main groups: the ones that need heat to be manufactured (hot-cast) and the ones that can be fabricated at room temperature (cold-cast). The hot-cast propellants tend to have better properties but are not as safe when casting than the cold-cast ones.

5.2.2. Nozzle

The motor nozzle is in charge of ultimately convert all the pressure and heat of the gases created inside the motor casing by the combustion of the propellant to velocity and force acting upon the whole rocket. The geometry of the nozzle is key to ensure that the rocket is efficient, but the implementation of this element into the motor itself can be a challenge, as well as the manufacturing of it. That’s why the nozzle of the motor will be a trade-off between efficiency and manufacturing/implementation, meaning that the ideal shape for maximising efficiency will not be used in order to end up with a design that allows for the fabrication of the element with conventional tools and that allows with a relatively quick and safe way of implementation with the rest of the motor.

The bell nozzle is the one that can offer a better efficiency with a reduced length, being an interesting choice if the availability of space in the rocket is limited or there is some other constraint that can be tackled with this reduction in size. However, the geometry of this nozzle is hard to machine using conventional tools, such as a lathe, so it is not widely used in the amateur rocketry.

The conic nozzle is a much more feasible approach in terms of manufacturing. This makes it the most typical choice among the amateur rocketry enthusiasts.

Regarding the implementation of the nozzle with the casing, there are some options. It can be screwed to the bottom part of the casing, as a bottom cap and having the screws on the casing wall or on the nozzle. This approach makes the mating easy, as it consists only on screwing the nozzle in its place. However, these screws must withstand the high pressure created inside the casing when burning the propellant that will try to push the nozzle out of its place.
A different approach is to make the nozzle rest at the bottom of the casing, with no need of bolts or screws to keep it in place, as the pressure will maintain it well pressed to the bottom of the casing and will keep the seals in its place. The only disadvantage of this method is that the under-pressure created at propellant depletion may suck the nozzle into the casing, potentially damaging this hard-to-manufacture element.

Finally, another key design factor in the nozzle is the material selection. Given that this element has to withstand the high temperatures of the exhaust gases, there is little choice but metallic and ceramic materials. The high resistance to extreme temperatures of some minerals like graphite makes for a somewhat common choice to form the convergent section and throat of the nozzle, as in these sections the flow hasn’t expanded much and thus is still very hot. However, the high price of this material together with the dust it generates may sometimes deter the average hobbyist of its use although it is easier to machine thanks to its low hardness. The most usual approach is to make the nozzle in these two separate sections, one of a more heat resistant metal and the divergent part of a lighter and not so high heat resistant one. If a less thermal resistant metal is used there is risk of high erosion in the throat section, making it larger and thus lowering the overall performance of the rocket while it fires.

5.3. Manufacturing

The elements the manufacturing of which must be assessed are the casing, the nozzle and the propellant. As the geometries are very different in each element and the materials may as well vary, the techniques can be of different kind.

5.3.1. Casing

Being a fairly simple geometry the manufacture of this element is not quite challenging. Its tubed shape makes it even a possibility to just acquire a commercial size of tube of the chosen material without any need of manufacturing process.

When choosing a metallic material, the fabrication of the casing tube can be tackled by welding a metallic sheet with the correct thickness at its ends. This fabrication method requires the knowledge of basic welding techniques as well as the necessary equipment to give a flat metallic sheet the necessary curvature to achieve the final cylindrical shape. Another approach to the manufacture of a metallic casing can be by machining a solid cylinder to the desired final inner diameter using a lathe. This method is not the most efficient one as it requires a lot more material than the one needed for the final product, but on the other hand it has the advantage of not needing high-complexity tools or operations to obtain the casing cylinder. The most practical and extended way of fabricating the casing in the model rocketry community is to buy a commercial tube that has the required diameter and thickness, and cut it to the desired length. The only setback is that rarely will the commercial tube match the desired length, so there will be some material that will have to be lost, although not as much nor with so much complexity and work time as working from a solid tube.

If a composite material is chosen as the casing material there is once again the option of finding a commercial tube that matches the desired diameter and thickness of the design. If the fabrication is intended to be done, the most suiting approach would be winding the impregnated fibres of the reinforcement onto a mandrel of the desired diameter. This approach requires of the necessary machines to ensure a correct fabrication procedure, as well as a curing oven to harden the matrix and obtain the final
product. This method may be used by some members of the amateur rocketry community, but is not usual to find such a casing.

5.3.2. Nozzle

This element may be the one with the most complex geometry and thus the one which presents the highest challenge when it comes to the fabrication. Also it makes it even more challenging as the dimensions of this element tend to be relatively small and critical to the correct operation and performance of the overall motor.

The high complexity in the geometry coupled with the newer technologies making its way to the consumer market makes the option of the metallic 3D printing of this element a very attractive one. However, the technology is still in its early phases and can’t stand as good as more traditional techniques the erosion in the most demanding sections of the nozzle, putting at risk the potential performance of the motor. That is why this technique is still not widely spread in the amateur rocketry community.

When talking about the more traditional techniques, the lathe is the go-to option, as the nozzle itself is a revolution solid. For the smaller rockets however this option can present its challenges as there may be some dimensions, namely the throat diameter, that are too small for any tool to machine. It has to be taken into account also the different materials that are used in the nozzle, as different approaches have to be taken when machining different kinds of metals or other types of materials.

5.3.3. Propellant

There are two main parts in the propellant manufacturing: the ingredients mixing and binding, and the grain fabrication.

From the moment that the selection of ingredients is made, the mixing is determined. Some of the ingredients may require a hot mix and others a cold one. In the amateur rocketry community, the cold cast propellants are more popular, as the risk of a spontaneous ignition during the mixing is very low. This comes with the downside that this kind of propellants don’t have as much energy density as the hot cast ones, as no energy is added during the mixing. When working with hot mixtures, a lot of precautionary measures have to be taken, as hazardous fumes can be created and material can be ignited if the temperature gets too high. That’s why the most common mixing of the ingredients in the home-made rocket propellant is cold cast.

Regarding the fabrication of the grain, the usual approach is to build a mould with the desired outer diameter of the grain and the core geometry chosen for the motor. For each mixture the moulding technique can vary as a very energetic compression of the propellant for some mixtures can be enough to ignite it. However, the vast majority of propellants used in this model rocketry community are safe to compress into the desired shape. The key factor during this process is to prevent the creation of air bubbles inside the grain, as they could alter the chamber pressure instantaneously and cause an unexpected overpressure that could even lead to the casing failure.
6. Main alternatives and decision of the best one

6.1. Motor casing

6.1.1. Material

Choosing the ideal material for the casing is extremely important, as it will determine how much chamber pressure will be able to withstand and consequently how much performance can be gotten out of the motor. Therefore, the highest resistant materials are the ones best suited for this application. However, the weight plays also a huge role in the final performance of the overall rocket, as the final aim of the motor is not only to get a good performance and a high thrust, but to lift a rocket as fast and high as possible. The motor’s own weight plays against this objective, so the material chosen has to have good mechanical properties while at the same time be relatively light.

For the manufacturing challenges it presents, as well as its higher price, a casing made of composite material is quickly ruled out. In Figure 1 it is seen the equipment needed for winding a cylinder of carbon fibre. To that it should be added the software behind the control of the machine. The metallic option is thus embraced, but to choose one of the many alloys available in the market is quite a challenge. As it has been said, the main requirement for this material is a high specific mechanical resistance. That makes the aluminium alloys the most suited candidates for fulfilling the role of the casing material.

Among the aluminium alloys there are plenty of options, and if the criteria to choose the best regarding the specific strength is to be followed, the final option would end up being a high-grade expensive material. As this study is intended to show the approach to a challenge from a point of view of a standard hobbyist, the most expensive materials are discarded, and so the final choice comes to availability and reasonable price of the alloy. With that in mind, the casing material chosen has been the aluminium alloy:

**Al 6082 T5**

This alloy has very good mechanical properties, summarised in Table 1, and is available by the tutor of this project who is the one that will build the motor. Moreover, the advantage of the aluminium in comparison with a more resistant option such as steel is the ease of machining the material when building the motor. The only setback of the
Main alternatives and decision of the best one

The choice made is the variation on the mechanical properties of the material at high temperatures as the ones reached when the combustion of the propellant happens. This problem is solved by

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus</td>
<td>$E$</td>
<td>70</td>
<td>GPa</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$G$</td>
<td>26</td>
<td>GPa</td>
</tr>
<tr>
<td>Ultimate tensile strength</td>
<td>$\sigma_u$</td>
<td>300</td>
<td>MPa</td>
</tr>
<tr>
<td>Yield strength</td>
<td>$\sigma_y$</td>
<td>260</td>
<td>MPa</td>
</tr>
<tr>
<td>Ultimate shear strength</td>
<td>$\tau_u$</td>
<td>300</td>
<td>MPa</td>
</tr>
<tr>
<td>Yield shear strength</td>
<td>$\tau_y$</td>
<td>260</td>
<td>MPa</td>
</tr>
<tr>
<td>Elongation at break</td>
<td>$\varepsilon_{\text{break}}$</td>
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<td></td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>2.7</td>
<td>g/cm$^3$</td>
</tr>
</tbody>
</table>

*Table 1: Al 6082 T5 mechanical properties [2], [3].*

6.1.2. Geometry

There is not much room for improvisation when trying to design a simple but effective pressure vessel such as a combustion chamber for a motor rocket: a cylinder is the way to go. The only parameters to choose in this case are the outer diameter of the cylinder, its length and the thickness of the wall.

The diameter choice sizes in a way the burn time that the motor will have, as a higher diameter will allow for a greater thickness of the fuel and will in the end mean that the time it takes for the burning surface to consume the grain will be greater. This longer burn time translates to a greater total impulse of the motor, a parameter which defines the class of the motor.

The length of the casing defines the length of the combustion chamber, and thus the amount of grain segments that fit inside it. The more segments, the more propellant that is burning at a given time and so more thrust.

Finally, the thickness of the wall defines the structural resistance of the casing. A very thick wall will be able to withstand huge chamber pressure, but it comes to a cost. The more thickness, the more material it is used and that can make the motor too heavy.

As stated in chapter 4, this study is intended to design an upgrade from a motor that currently uses the tutor of this thesis. This motor has a diameter of 75 mm, with a length of 380 mm and a wall thickness of 2.5 mm. Given these starting parameters, and keeping in mind that the final goal is to obtain a motor that can deliver more thrust and more total impulse, the geometry finally chosen is the one complying with the requirements set regarding to the casing size, *Req.Diam, Req.Thick* and *Req.Length*.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Casing diameter</td>
<td>$\emptyset$</td>
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</tr>
<tr>
<td>Casing length</td>
<td>$L$</td>
<td>1000</td>
<td>mm</td>
</tr>
<tr>
<td>Casing wall thickness</td>
<td>$t$</td>
<td>5</td>
<td>mm</td>
</tr>
</tbody>
</table>

*Table 2: Casing geometry.*
6. Main alternatives and decision of the best one

6.2. Propellant

6.2.1. Composition

When choosing which ingredients will be used as propellant for the motor, some practical aspects have to be considered that can narrow down the search. If the only aim were to obtain the maximum power density or the maximum efficiency, the results would yield a combination that would require professional equipment to handle and controlled environment to ensure safety when fabricating the propellant. As the approach to this study is from an amateur point of view, the propellant’s highest requirement $\text{Req}_\text{PropMat}$ is followed. In this case, the performance potential of the propellant is a wanted asset but not the priority.

Looking at the cold mixed propellant, the ones that don’t need any heat input during its fabrication, the most widely used are based in the use of potassium nitrate as the oxidiser. There are commonly used formulas that use as fuel for the propellant sucrose (sugar) and other similar ingredients but these formulations need to be mixed and then molten using a heat source to cast it into the mould. As this has already been discarded, the aim is to find a fuel for the propellant that doesn’t need to be molten to adapt to the shape of the mould. That means that it has to be in a liquid or gel form and be able to harden without imputing heat into the system.

The solution to this problem is given by the epoxy resin. The use of this ingredient allows the mixture and casting at room temperature, minimising the risk of accidental ignition of the propellant during fabrication. However, the burn rate that is found from the mixture of potassium nitrate and epoxy in their own is too low to be able to produce any good amount of pressure in the chamber. For that, a catalyst is needed to improve the reaction speed and thus the burn rate of the propellant. This element provides an extra boost to the velocity at which the combustion occurs and allows for the motor to achieve a higher chamber pressure, enough to provide the required thrust.

In order to have a good combustion process, each ingredient’s contribution in the total mass of the propellant is defined.

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Formula</th>
<th>Function</th>
<th>% in mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potassium nitrate</td>
<td>$K\text{NO}_3$</td>
<td>Oxidiser</td>
<td>69</td>
</tr>
<tr>
<td>Iron oxide</td>
<td>$F_2\text{O}_3$</td>
<td>Catalyst</td>
<td>8</td>
</tr>
<tr>
<td>Epoxy</td>
<td>$C_{16}H_{24}O_4$</td>
<td>Fuel</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 3: Propellant composition.

6.2.2. Geometry and configuration

The shape of each grain will affect very much the pressure curve expected in the motor operation, as well as how they are stacked on top of one another. The core geometry of the grain will determine how the burning area progresses radially, so it has to be designed according to what is wanted. In this case, the aim is to obtain as a flat pressure curve as possible, while not making the manufacturing process of the grain complicated. That means that a configuration has to be found of the grain that allows for a relatively constant burning area without the need of creating an intricate mould for the grain.
6. Main alternatives and decision of the best one

The easiest shape to obtain is a cylindrical core, although this looks like an option that would not comply with the criteria of wanting a flat pressure curve. This is where the configuration of the grain enters into play. If not only the core surface of the grain, but also the top and bottom faces of each tubular grain are exposed, the increase in burning area in the core is counteracted by the decrease of burning are in the top and bottom faces, as well as by the decrease in overall height of the segment. This configuration, known as BATES (BAllistic Test and Evaluation System) configuration, allows for a more or less flat pressure curve in the chamber. In Figure 3 the configuration is schematically shown, with the inhibited outer surface coloured in red, while the inner core surface and each segment’s top and bottom surface are exposed by means of indenting a small slope that create a space between each segment so that the hot gases can access and start the combustion of these surfaces.

Regarding the size of each grain, it is limited by the diameter and length of the combustion chamber. As the casing has 100 mm of outer diameter, a 5 mm wall and a rubber liner that prevent the heat to get to the wall of about 1.5 mm of thickness, the outer
diameter of the grain is found to be $87 \, mm$. The initial core diameter has been chosen to be the same as the one that the motor from which this one is an upgrade, which is $19 \, mm$. That way, the thickness and thus the burn time is certainly increased in this new version of the motor. The mould height defines the segment length, which is $130 \, mm$. Finally, as the combustion chamber length is about $800 \, mm$, there will be 6 segments of propellant, giving a total length of the propellant of $780 \, mm$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter (initial)</td>
<td>$D_0$</td>
<td>87</td>
<td>$mm$</td>
</tr>
<tr>
<td>Core diameter (initial)</td>
<td>$d_0$</td>
<td>19</td>
<td>$mm$</td>
</tr>
<tr>
<td>Segment length (initial)</td>
<td>$L_0$</td>
<td>130</td>
<td>$mm$</td>
</tr>
<tr>
<td>Number of segments</td>
<td>$N$</td>
<td>6</td>
<td>–</td>
</tr>
</tbody>
</table>

*Table 4: Propellant geometry and configuration.*

### 6.3. Nozzle

The nozzle of the motor is perhaps the most complicated element to design, as it presents a relatively complex geometry compared to the rest of elements of the motor. Moreover, it has to bear with the high chamber pressure and the high temperature of the exhaust gases. And on top of that, it must present a good sealed union with the motor casing and as little erosion as possible as supersonic heated gas passes through its throat.

About the material to choose for the nozzle, there are two possible approaches: make all the nozzle of the same material or make the parts that enter in contact with the hottest gas with a heat resistant material, and the rest with a not so complex one. Making the nozzle of only one material has the advantage that the element can be made as one single piece, minimising the need for unions and seals. However, it also means that the totality of the nozzle will be dimensioned to withstand the throat conditions, which are the most aggressive as it is the section which dimension is most critical and can potentially affect more the performance of the rocket if it erodes and increases in diameter, and thus it will end up being a heavier and more expensive element compared to other approaches in its design. Usually, when this design is carried out, the material choice is carbon steel. Aluminium could also be used, but then the said erosion would most likely appear in the form of an increased throat diameter and an overall scraped nozzle that could not be re-used.

The alternative to this mono-material design is to have the convergent part of the nozzle and the throat section made of a material resistant to the high temperatures of the exhaust gases, while the final divergent section is made of a not so resistant material, as the flow in that section has already expanded and cooled down. This design is a more efficient approach economically and weight-wise, but adds complexity to the mating and seal of the two parts of the nozzle. The choice for the low temperature resistant material is easy as a light metallic material as aluminium can fulfil the requirements easily. Regarding the temperature resistant material, the carbon steel can be chosen, but there is an interesting alternative. There are some designs that this part is made out of graphite. This ceramic material has an extremely high resistance to high temperatures, without losing mechanical properties with the increase of temperature. The main challenge it presents – on top of its high price – is that the manufacturing process is quite messy, as graphite creates a high amount of dust while machining, which is electrically conductive and can cause problems in the machinery surrounding the working space. This dust can also be harmful to the worker that is manufacturing the piece, so it must
be collected with some vacuum device right as the dust appears. Also, this material presents chemical erosion, as the carbon in the graphite reacts with the exhaust oxygen to create $CO_2$. This means that the throat diameter is likely to increase a bit during motor operation.

Figure 4: Metallic nozzles with graphite in the convergent and throat sections. [6]

In order to simplify the design and to avoid adding seals and unions, which are weak points in the design, the decision is made to make the nozzle of a single material, that is, carbon steel. This nozzle will be hold in place by an aluminium cap screwed to the casing.

Figure 5: Final nozzle configuration, steel nozzle with aluminium cap.
6.4. Motor configuration

It is very important for the correct operation of the rocket to have an assembly of the different components that ensures its safety while at the same time providing an easy way to assemble and disassemble it for inspections or reparations after a successful burn. This concerns mainly the sealing of the top and bottom ends of the cylindrical casing. There are different main ways in which this problem is solved. The one that makes it easiest for the assembly and disassembly of the rocket tries to take advantage of the high pressure inside the combustion chamber in a way that the overpressure presses the top and bottom caps against the wall of the casing. This design is aimed to suppress the need for any fixing elements that may make the assembly tedious. However, when the propellant is fully consumed, the pressure inside the chamber can decrease under that of the ambient, and it could suck in the combustion chamber these elements that aren’t fixed, with the possibility that they may cause damage to the walls or themselves.

A different option is to make the ends of the cylinder threaded so that the caps can be directly screwed to the cylinder without the need of any extra element. This method allows for a more fixed union that the latter, but it will be working against the pressure inside the chamber. This means that the cap’s and the cylinder’s thread has to be tight and the union of both elements must be strong enough to prevent any unwinding of the cap due to the vibrations during the operation and the chamber pressure pushing the cap outwards.

Another approach at solving the design problem of sealing the ends of the casing is to have some screws through the casing wall that fix the caps in their place. This provides a very reliable union, as the screws are placed perpendicular to the acting force thus reducing the risk of unscrewing. Moreover, the fact that there are several bolts restraining the cap means that it is a failsafe system, as one screw failing to keep in place may not doom the entirety of the motor. As this system is the one that shows more safety potential, this approach is the one chosen to fix the top cap and nozzle at the bottom of the motor. These caps will have to include some O-rings that ensure the union has a proper seal and that the pressure won’t be able to escape through it.

*Figure 6: Example of motor configuration as the one chosen. [7]*
7. Development of the solution

7.1. Casing structural integrity

The first step in the development of the solution is to find the parameters at which the structural integrity of the metallic casing of the motor will not be at risk. The key parameter here is the maximum design pressure ($P_D$), that is, the maximum pressure that the casing can resist without experiencing permanent deformation with a safety factor. A couple more values of interest here are the burst factor ($B$), which depends on the geometry of the casing and is needed to compute the other parameter regarding the structural resistance of the casing, the ultimate pressure ($P_U$) that it can withstand without failing. While the previous pressure parameter may be acceptable to reach if the motor is intended to be used only once, the ultimate pressure must be avoided at all costs, as reaching this value of chamber pressure will lead to the casing failing and the whole motor exploding, putting the safety of anyone near the launch at risk.

7.1.1. Maximum design pressure, $P_D$

The design pressure for this casing will be defined as the chamber pressure that creates a stress equal to that of the yield strength of the material, with a margin of a safety factor. The design safety factor ($S_D$) of the calculation of the design pressure is usually advised to be between 1.5 and 2, depending on how well the properties of the material are known with certainty. As the intent of this project is to obtain a very powerful motor, the highest pressures are wanted in the chamber. The material supplier is considered reliable enough to be certain that the properties of the alloy are well known. So, in this case, and according to the requirement set Req.Safety, $S_D = 1.5$.

To compute the maximum design pressure, the formula that is used is that found in [8]. This is the case of a thin-walled, capped cylinder. It can be considered thin-walled as the inner radius of the casing is almost ten times greater than the thickness of the wall, and it is considered capped, although technically one of the ends is open, but as the nozzle chokes the flow and makes the pressure rise, for calculation purposes it is considered to be capped.

With these considerations, the formula found in the reference that expresses the stress $\sigma$ caused by a uniform inner pressure $P$ in a cylindrical pressure vessel with an inner radius $R$ and a wall thickness of $t$, with a design safety factor $S_D$, is:

$$\sigma = \frac{P \times R}{t} \times S_D$$

To find the design pressure $P_D$, the value of the stress is known, and equal to $\sigma_y$. The radius can be expressed as one half the diameter $D$. With these two considerations, and developing the equation, one can get to the equation that yields the value of the design pressure:

$$P_D = 2 \times \frac{t \times \sigma_y}{D \times S_D}$$

$$P_D = 2 \times \frac{5[mm] \times 260[MPa]}{100[mm] \times 1.5} = 17.33 \text{ MPa} = 173.3 \text{ bar}$$
Thus, the motor nozzle must be designed in order to obtain a maximum chamber pressure of 173.3 bar, or 2514 psi.

7.1.2. Ultimate pressure $P_U$

The design pressure found previously is the target maximum pressure for the combustion chamber of the motor. However, small deviations can be accepted, as a safety factor has been used and the value of maximum allowable stress has been set at the yield strength of the material.

Nonetheless, if the pressure inside the casing gets too high there is a risk of failure of the material itself. It is very important to know this value, as any pressure near it can make the casing fail and explode, risking the mission, the integrity of the rest of the rocket and most importantly putting at risk anyone standing near the launch site.

One would assume that the computation of the ultimate could be done using the same procedure as with the design pressure, but this is not the case. As this is a pressurised vessel, the mechanical resistance of the assembly varies when pressurised. This is shown in [9], where a burst factor $B$ is presented as an experimentally-obtained curve as a function of the ratio between the yield and ultimate pressure of the vessel’s material. This curve can be approximated as a polynomial function such as the following:

$$B = 9.5833 \left(\frac{\sigma_y}{\sigma_u}\right)^4 - 33.528 \left(\frac{\sigma_y}{\sigma_u}\right)^3 + 44.929 \left(\frac{\sigma_y}{\sigma_u}\right)^2 - 28.479 \left(\frac{\sigma_y}{\sigma_u}\right) + 8.6475$$

In this case, putting in the above equation the values of the material shown in Table 1, a burst factor $B = 1.293$ is obtained. This factor allows for the computation of the ultimate pressure as a function of the yield strength rather than with the ultimate tensile strength. This makes for a more accurate prediction, critical when dealing with such high pressures.

Once the burst factor of the casing has been obtained, the computation of the ultimate pressure can be carried out by multiplying the casing wall thickness $t$ times the yield strength $\sigma_y$ and divide it by the cylinder radius (or half the diameter, $D/2$), and all of that multiplied by the burst factor $B$.

$$P_U = 2 \times \frac{t \times \sigma_y}{D} \times B$$

$$P_U = 2 \times \frac{5[mm] \times 260[MPa]}{100[mm]} \times 1.293 = 33.618 MPa = 336.18 bar$$

So the chamber pressure at which the casing structure is predicted to fail is 336.18 bar (or 4875.88 psi), a value which must be avoided in the design of the motor.

The burst safety factor is a non-dimensional value that gives a quick glance of the design margin in the casing structural resistance. It consists simply in the ratio of the ultimate pressure vs the design pressure:

$$S_u = \frac{P_U}{P_D} = \frac{33.618 [MPa]}{17.33 [MPa]} = 1.94$$

This shows at a quick glance that the design pressure has a margin of about a 100%, that means that the design pressure has to be exceeded by that margin in order for the
casing to fail catastrophically, so the conclusion can be drawn that this design is a safe one.

7.1.3. Results summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design safety factor</td>
<td>( S_D )</td>
<td>1.5</td>
<td>—</td>
</tr>
<tr>
<td>Design pressure</td>
<td>( P_D )</td>
<td>17.33</td>
<td>MPa</td>
</tr>
<tr>
<td>Burst factor</td>
<td>( B )</td>
<td>1.293</td>
<td>—</td>
</tr>
<tr>
<td>Ultimate pressure</td>
<td>( P_U )</td>
<td>33.618</td>
<td>MPa</td>
</tr>
<tr>
<td>Burst safety factor</td>
<td>( S_U )</td>
<td>1.94</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 5: Motor casing design parameters.

7.2. Propellant and combustion characterisation

In order to be able to model the correct performance of the rocket motor it is important to know the thermodynamic parameters of the propellant being used. That means that the ideal density of the grain \( (\rho_P) \) must be obtained, as well as the molecular mass of the gaseous exhaust \( (M_P) \), its specific heat ratio \( (\gamma_P) \) and the expected chamber temperature \( (T_C) \). The ideal density can be computed easily, but the rest of parameters need a specialised software to be determined. There are two more parameters that are needed to model correctly the performance of the fuel: the burn rate coefficient \( a \) and the burn rate exponent \( n \) that describe the burn rate of the propellant as a function of the chamber pressure as the De Vielle’s law models:

\[
\frac{BR}{\alpha} = a \times \frac{P^\gamma}{c} \ [m/s]
\]

These two parameters need to be obtained by experimental testing of the propellant in a controlled pressure environment, measuring the burn rate for different values of chamber pressure and getting the value of these two parameters. As this experiment can’t be done by this thesis author, it has been taken from [10] where the actual experiment was made and the burn rate obtained. Evaluating the results from this reference with the ingredients proportion of the chosen mix, the values of these two parameters have been found to be \( a = 2.26 [mm/s] \) and \( n = 0.424 \).

7.2.1. Ideal grain density \( \rho_P \)

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Formula</th>
<th>Mass fraction</th>
<th>Density ( \left[ \frac{g}{cm^3} \right] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potassium nitrate</td>
<td>( KNO_3 )</td>
<td>0.69</td>
<td>2.109</td>
</tr>
<tr>
<td>Iron oxide</td>
<td>( Fe_2O_3 )</td>
<td>0.08</td>
<td>5.250</td>
</tr>
<tr>
<td>Epoxy</td>
<td>( C_{16}H_{24}O_4 )</td>
<td>0.23</td>
<td>1.096</td>
</tr>
</tbody>
</table>

Table 6: Propellant ingredients density. [11], [12]

The grain density is computed taking into account the fraction of each ingredient in the mixture. From its definition, the final formula can be computed, taking into account 1 unit of mass of propellant \( (M_P = 1) \) and the known mass fraction of each ingredient in the final mixture as seen in Table 3:

\[
\rho_P = \frac{M_P}{V_P} = \frac{f_1 + f_2 + f_3}{V_1 + V_2 + V_3} = \frac{1}{\frac{f_1}{\rho_1} + \frac{f_2}{\rho_2} + \frac{f_3}{\rho_3}}
\]

\[
\rho_P = \frac{1}{\frac{0.69}{2.109 \left[ \frac{g}{cm^3} \right]} + \frac{0.08}{5.250 \left[ \frac{g}{cm^3} \right]} + \frac{0.23}{1.096 \left[ \frac{g}{cm^3} \right]} = 1.811 \left[ \frac{g}{cm^3} \right]}
\]
The rest of the parameters are more difficult to obtain, as it must be taken into account all the by-products of the combustion process at the chamber pressure. For this, the software PROPEP 3 is used. This software used by the rocket model community uses a database of several ingredients used for all kind of solid propellants, and is capable of computing the properties of the gases generated in the combustion of the fuel to be able to later on compute the performance of the motor.

7.2.2. PROPEP 3 analysis

The software used to estimate the propellant parameters needed for the performance calculation relies on solving the chemical equilibrium of the reaction taking place in the combustion chamber.

The inputs required by the software to be able to carry out the calculations are:

- Propellant ingredients: the user must choose the ingredients that have been used in the fabrication of the propellant out of a list of components which are stored with its properties in a .daf file. Any new ingredient can be added to this database manually by the user.
- Mass fraction of each ingredient.
- Temperature of the ingredients.
- Chamber pressure: this parameter is yet to be computed, so as a first approach the design pressure is used. When the chamber pressure is computed, this value can be corrected, an iterative process that ultimately converges to a given value of chamber pressure.
- Exhaust pressure: in this case, an adapted nozzle is considered, so the exhaust pressure equals to the ambient pressure.

To compute the final outputs of the process, the program works under the following assumptions:

- The flow inside the chamber and through the nozzle is considered one-dimensional.
- The flow at the nozzle inlet has no velocity.
- The combustion is considered complete and the process adiabatic.
- The flow expansion in the nozzle is considered isentropic.
- Inside the combustion chamber, the mixing of the reactants and products is homogenous.
- The exhaust products follow the ideal gas law.
- The combustion happens at constant pressure.

With these assumptions, the program can run its calculations. First, it assumes the products of the combustion process from the reactants specified as propellant ingredients. Then it computes the number of moles of each product, which it does by solving an equation system made of the equation for mass balance, the equation for chemical equilibrium and the equation for energy balance.

The mass balance is simply the conservation of mass applied to the reaction. That means that the program sets the condition that the amount of atoms of each element in the reactants has to match the amount of the same element in the products.
7. Development of the solution

The chemical equilibrium then is solved in order to obtain the relative concentration of the constituents of the reaction. From values of the equilibrium constant $K_p$ that the program takes from the JANAF tables, it computes all the molar fractions of the products. However, it still lacks the knowledge of the actual temperature of the products, which define the value of the equilibrium constant.

This is solved by applying the energy balance equation, in a system which is considered adiabatic. That means that the enthalpy of the reactants must be equal to that of the products. The program starts with an assumption of combustion temperature, which uses to obtain the equilibrium constant from the tables. With this, the molar fractions of the products are found, and the energy conservation equation is applied. This equation yields a new value of chamber temperature, which is compared with the previous one. If the difference of these two values is greater than a given tolerance, the calculations are repeated using this new temperature value.

Once all the calculations have ended, the program produces a file detailing the final results. Among these outputs there are some that are relevant to the final performance of the motor:

- Combustion chamber temperature $T_C$.
- Specific heats ratio of the exhaust gases $\gamma$.
- Molecular weight of the combustion products $M_p$.

The program also gives the composition of the products, as well as more thermodynamic parameters that don’t have any impact in the posterior calculations but nonetheless are interesting to know:

![Figure 7: Results given by PROPEP 3.](image)

An interesting value to compute from the results obtained by the PROPEP 3 calculations is the fraction of condensed products, which determines the effective molecular weight of the products (taking only into account the gaseous products) and gives an idea of how much smoke can be expected to be seen exiting the nozzle.

This parameter is computed by taking the solid products (as seen in Table 3, the $K_2CO_3$ and the $FeO$) and converting their molar amount to their mass, and dividing it by the total mass of the calculations, that is, 100 g.
7. Development of the solution

\[ X = \frac{M_{K_2CO_3} \times \text{mol} \, K_2CO_3 + M_{FeO} \times \text{mol} \, FeO}{100 \, [g]} = \frac{138.205 \left( \frac{g}{\text{mol}} \right) \times 0.339[\text{mol}] + 71.844 \left( \frac{g}{\text{mol}} \right) \times 0.100}{100 \, [g]} = 0.541 \]

That means that more than the 50% in mass of the products of the combustion are in solid form. This has some consequences worth mentioning:

- The solid particles can’t expand in the nozzle and thus they don’t contribute in accelerating the flow, making the motor not as efficient as it could theoretically be.
- The thermal energy stored in the form of temperature in these particles isn’t transmitted to the flow, meaning that this energy is thrown away without the motor being able to take profit of it.
- The higher mass of the solid particles translates in a lower acceleration and a lower velocity inside the nozzle. This factor, combined with the friction with the exhaust flow makes the exhaust slow down a bit and further reduces the efficiency of the motor.

However, in this analysis this effect will be discarded. That way, the results obtained will be those of an ideal motor, meaning that the pressure values will be actually lower than the ones computed. This means that if the ideal design and calculations comply with the safety parameters imposed in the casing design, the real motor in operation will be even safer, as the pressure will not reach the ideal higher values.

7.2.3. Characteristic exhaust velocity \( C^* \)

There is only one parameter left to compute regarding the combustion of the propellant, and that is the characteristic exhaust velocity \( C^* \). This figure of merit gives an idea of the propellant efficiency when burning. A higher \( C^* \) means a greater combustion efficiency. This parameter aims to compare the ratio between the force that would create the chamber pressure applied solely in the throat diameter with the mass flow rate of the engine:

\[ C^* = \frac{P_e \times A_t}{m} \]

The throat and mass flow can be related via the continuity equation:

\[ \dot{m} = \rho_t \times v_t \times A_t \rightarrow \frac{A_t}{\dot{m}} = \frac{1}{\rho_t \times v_t} \]

Using the definition of the speed of sound (which is the velocity of the flow at the throat), the ideal gas law and the isentropic properties of the flow, the equation can be rearranged into:

\[ \frac{A_t}{\dot{m}} = \frac{1}{P_t \frac{R}{T_t} \times \sqrt{\gamma R T_t}} = \frac{\sqrt{T_t R}}{\sqrt{\gamma + 1}} \frac{R}{2} = \frac{\rho_t}{P_t} \frac{\gamma - 1}{\gamma} \sqrt{\frac{R}{T_t}} = \frac{1}{P_t} \times \sqrt{\frac{T_c R}{\gamma (\gamma + 1) \sqrt{\frac{R}{T_c}}}} \]

Combining this equation with the definition of \( C^* \), the chamber pressure term is eliminated and the equation yields:
7. Development of the solution

\[ C^* = \sqrt{\frac{T_c R}{\gamma (\frac{\gamma + 1}{2})^{\gamma-1}}} \]

Substituting the values found in the calculations and shown in Table 8 the value of the characteristic exhaust velocity is \( C^* = 944.016 \text{m/s} \).

7.2.4. Results summary

Here are detailed the results obtained from the process of characterising the combustion process and the propellant.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Chemical formula</th>
<th>Molar percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon monoxide</td>
<td>CO</td>
<td>28.224 %</td>
</tr>
<tr>
<td>Molecular hydrogen</td>
<td>H₂</td>
<td>26.416 %</td>
</tr>
<tr>
<td>Molecular nitrogen</td>
<td>N₂</td>
<td>12.549 %</td>
</tr>
<tr>
<td>Potassium carbonate (solid)</td>
<td>K₂CO₃</td>
<td>12.546 %</td>
</tr>
<tr>
<td>Water</td>
<td>H₂O</td>
<td>9.054 %</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>CO₂</td>
<td>7.148 %</td>
</tr>
<tr>
<td>Iron oxide II (solid)</td>
<td>FeO</td>
<td>3.685 %</td>
</tr>
<tr>
<td>Methane</td>
<td>CH₄</td>
<td>0.362 %</td>
</tr>
<tr>
<td>Ammonia</td>
<td>NH₃</td>
<td>0.011 %</td>
</tr>
<tr>
<td>Potassium hydroxide</td>
<td>KOH</td>
<td>0.006 %</td>
</tr>
<tr>
<td>Atomic potassium</td>
<td>K</td>
<td>&lt; 0.001 %</td>
</tr>
<tr>
<td>Hydrocyanic acid</td>
<td>CNH</td>
<td>&lt; 0.001 %</td>
</tr>
<tr>
<td>Potassium cyanide</td>
<td>KCN</td>
<td>&lt; 0.001 %</td>
</tr>
</tbody>
</table>

Table 7: Exhaust composition.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain density</td>
<td>ρₚ</td>
<td>1.811</td>
<td>g/cm³</td>
</tr>
<tr>
<td>Combustion temperature</td>
<td>Tₕ</td>
<td>1583.066</td>
<td>K</td>
</tr>
<tr>
<td>Specific heats ratio</td>
<td>γₚ</td>
<td>1.132</td>
<td>–</td>
</tr>
<tr>
<td>Molecular weight</td>
<td>Mₚ</td>
<td>36.629</td>
<td>g/mol</td>
</tr>
<tr>
<td>Effective molecular weight</td>
<td>Mₑff</td>
<td>43.668</td>
<td>mol</td>
</tr>
<tr>
<td>Specific gas constant</td>
<td>R</td>
<td>226.979</td>
<td>J/(kg · K)</td>
</tr>
<tr>
<td>Burn rate coefficient</td>
<td>a</td>
<td>2.26</td>
<td>mm/s</td>
</tr>
<tr>
<td>Burn rate exponent</td>
<td>n</td>
<td>0.424</td>
<td>–</td>
</tr>
<tr>
<td>Characteristic exhaust velocity</td>
<td>C⁺</td>
<td>944.016</td>
<td>m/s</td>
</tr>
</tbody>
</table>

Table 8: Propellant and combustion characterisation results.

7.3. Performance parameters computation

Once the casing structural resistance has been computed and the propellant has been characterised, it is time to obtain the different performance parameters that will define the motor’s performance.

7.3.1. \( K_N \), burning area and nozzle throat

This parameter must be fixed before the beginning of the calculations. Its definition is simply the ratio between the burning area of the propellant over the nozzle throat area.
7. Development of the solution

\[ K_N = \frac{A_b}{A_t} \]

The chamber pressure that is obtained in the motor will be greater if a much bigger \( K_N \) ratio is achieved, meaning that this parameter is the one to be modified to achieve a targeted design pressure. From its definition, it is clear that there are two different ways of modifying the \( K_N \):

- **Change the burning area.**

  If the surface of the grain that is exposed to the hot gases and thus considered to be burning changes, so will the \( K_N \). To control this variation, a complex geometry of the propellant core can be chosen in order to keep this value as constant as possible. However, this is a difficult achievement and is not the usual case in the model rocketry world. Usually a cylindrical core is chosen (as in this case), meaning that as the burning of the propellant progresses, the burning area does so and consequently the \( K_N \) increases. The BATES configuration of several grains with the top and bottom faces of each also exposed, as explained in chapter 6.2, mitigates this effect as the increase in burning area of the core is partially counteracted with the decreasing top and bottom area of each segment.

- **Change the throat area of the nozzle.**

  Given a certain propellant configuration, the only way to change the \( K_N \) is to change the throat of the nozzle. The narrower the throat, the greater the chamber pressure achieved will be. That allows for a fine tuning of the maximum value of \( K_N \) (and thus of the maximum chamber pressure) once the propellant configuration has been chosen.

Fixing the value of the \( K_N \) is one of the first steps to obtain the performance parameters of the motor. However, as the design pressure is determined by the material and geometry of the motor casing, the maximum value of \( K_N \) is already fixed, and given by the following equation:

\[ P_C = \left[K_N \times a \times \rho_p \times C^*\right]^{\frac{1}{1-n}} \rightarrow K_N = \frac{P_C^{1-n}}{a \times \rho_p \times C^*} \]

where:

- \( P_C \): chamber pressure
- \( a \): burn rate coefficient
- \( \rho_p \): grain density
- \( C^* \): characteristic exhaust velocity
- \( n \): burn rate exponent

Aiming for the maximum possible safely achievable chamber pressure, that is, the casing design pressure, the following result is obtained:

\[ K_{N,MAX} = \frac{(17.33 \times 10^6 [Pa])^{1-0.424}}{(2.26 \times 10^{-3}[m/s]) \times (1.811[kg/m^3]) \times (944.016[m/s])} = 3824.210 \]

The maximum value of \( K_N \) will be achieved at maximum burning area. Using the BATES configuration of the propellant grain, the expression that gives the burning area as a
function of the linear regression \( x \) of the propellant at any given time during the burn is the following:

\[
A_b = N \times \left[ \frac{1}{2} \pi (D_0^2 - (d_0 + 2x)^2) + \pi (L_0 - 2x)(d_0 + 2x) \right]
\]

To find the maximum burning area of the propellant, this expression has to be derived with respect to \( x \) and find the value of this variable that makes this derivative equal to zero.

\[
\frac{dA_b}{dx} = -2\pi N \times [6x + 2d_0 - L_0] \rightarrow \frac{dA_b}{dx} = 0 \rightarrow x_{A_{b,\text{MAX}}} = \frac{1}{6} \times (2d_0 - L_0)
\]

\[
A_{b,\text{MAX}} = N \times \left[ \frac{1}{2} \pi \left( D_0^2 - \left( \frac{L_0 + d_0}{3} \right)^2 \right) + \pi \left( \frac{2(L_0 - d_0)}{3} \right) \left( \frac{L_0 + d_0}{3} \right) \right]
\]

\[
A_{b,\text{MAX}} = 6 \times \left[ \frac{1}{2} \pi \left( 87^2 - \left( \frac{130 + 19}{3} \right)^2 \right) + \pi \left( \frac{2(130 - 19)}{3} \right) \left( \frac{130 + 19}{3} \right) \right]
\]

\[
= 1.411 \times 10^5 \text{ mm}^2
\]

Once the burning area corresponding to the maximum value of \( K_N \) has been found, the nozzle throat can be sized:

\[
A_t = \frac{A_{b,\text{MAX}}}{K_{N,\text{MAX}}} = \frac{1.411 \times 10^5 \text{ mm}^2}{3824.21} = 36.879 \text{ mm}^2 \rightarrow d_t = 6.85 \text{ mm}
\]

The throat diameter that achieves the maximum chamber pressure for which the casing has been designed is the one just found. However, for manufacturing reasons and to prevent the clogging of the throat as the solid products of the combustion go through it, the actual size of the throat has to be a little bit higher. The real diameter of the throat achieved with the available machinery is:

\[
d_t = 12.2 \text{ mm}
\]

The consequence of this change in the throat diameter is that the motor will not be able to achieve a maximum chamber pressure that matches the design pressure of the casing. From the performance point of view, this is bad as it leaves margin for creating more thrust with the same motor. However, this margin allows for a safer operation of the motor as the pressures involved don’t reach as high values and the structural integrity of the casing is further reassured.

As the maximum burning area remains unchanged by this modification, the final real maximum \( K_N \) found is:

\[
K_{N,\text{MAX}} = 1206.9
\]

The evolution of the KN throughout the operation of the motor can be found once the throat has been sized, using the following equation:

\[
K_N = \frac{A_b}{A_t} = \frac{N \times \left[ \frac{1}{2} \pi (D_0^2 - (d_0 + 2x)^2) + \pi (L_0 - 2x)(d_0 + 2x) \right]}{\frac{\pi}{4} d_t^2}
\]
Plotting the equation shown above in the interval ranging from no regression at all \((x = 0 \, mm)\) to propellant depletion \((x = \frac{d_0}{2} - \frac{d_a}{2} = 34 \, mm)\), the following curve is obtained:

As it is seen, a fairly symmetric \(K_N\) curve is obtained without extreme variations that other configurations of the grain could have achieved. This allows for a steadier and more regular thrust of the motor.

7.3.2. Chamber pressure \(P_C\)

The evolution of the chamber pressure inside the motor is very much related to the evolution of the \(K_N\) during the burn. As the propellant burns and the burning area changes, so will the chamber pressure. But the pressure build-up in the motor at ignition is not instantaneous, as well as at propellant burnout the motor doesn’t cut off immediately. So three different scenarios have to be analysed in the pressure evolution:

- **Start-up:**

  This transient phase is a very brief one, in which the heat and pressure created by the igniter should start the combustion of the propellant. It is very important then to have a correct ignition mechanism for the motor, as a too slow start-up may cause a deviation in the rocket’s intended path as it takes longer to put it up to speed, and a too fast and powerful ignition may cause an overpressure with catastrophic consequences.

  In order to compute how the pressure builds up in the first instants of the motor operation, an assumption is made that the igniter used to start the motor acts instantaneously and the calculations can be carried out as if all the propellant inside the combustion chamber had started to burn right at the ignition.

  The computation of the pressure evolution can be tackled using the conservation of mass principle, stating that the mass flow of the gases generated by the combustion of the propellant \(\left(\dot{m}_g\right)\) must equal the variation of mass stored in the chamber \(\left(\frac{dM_s}{dt}\right)\) and the mass flow of the gases that exit through the nozzle \(\left(\dot{m}_n\right)\):
7. Development of the solution

\[ m_g = \frac{dM_s}{dt} + m_n \]

The first term of the equation can be expressed as its definition. The mass flow of the gas generated is equal to the mass flow of the burner that is being burned. This is calculated by multiplying the propellant’s density \( \rho_p \) times the burning area \( A_b \) times the burning rate of the propellant \( BR \):

\[ m_g = \rho_p \times A_b \times BR \]

The variation of the stored mass of gas in the combustion chamber can be expressed as the variation of the instantaneous density of the gases in the chamber \( \rho_0 \) times the instantaneous volume that they fill \( V_0 \), that can be determined by the regression rate of the propellant:

\[ \frac{dM_s}{dt} = \frac{d(\rho_0 V_0)}{dt} = \rho_0 \times \frac{dV_0}{dt} + V_0 \times \frac{d\rho_0}{dt} = \rho_0 \times A_b \times BR + V_0 \times \frac{d\rho_0}{dt} \]

Using the assumption that the exhaust gases produced in the combustion chamber follow the ideal gas law and that the chamber temperature doesn’t change during the burn, the last term of the last equation can be rewritten to:

\[ \frac{d\rho_0}{dt} = \frac{d}{dt} \left( \frac{P_c}{R T_0} \right) = \frac{1}{R \times T_0} \frac{dP_c}{dt} \]

Finally, the mass flow of the exhaust that exits through the nozzle can be computed considering choked flow conditions at the nozzle throat, for which the equation is known:

\[ m_n = P_c \times A_t \times \sqrt{\frac{\gamma_p}{R \times T_0}} \times \left( \frac{2}{\gamma_p + 1} \right)^{\frac{\gamma_p+1}{2(\gamma_p-1)}} \]

Rearranging all the terms and taking into account the definition of burn rate as \( BR = a P_c^n \), the initial equation can be expressed as:

\[ \rho_p A_b a P_c^n = \rho_0 A_b a P_c^n + \frac{V_0}{R T_0} \frac{dP_c}{dt} + P_c A_t \sqrt{\frac{\gamma_p}{R T_0}} \left( \frac{2}{\gamma_p + 1} \right)^{\frac{\gamma_p+1}{2(\gamma_p-1)}} \]

\[ \frac{V_0}{R T_0} \frac{dP_c}{dt} = (\rho_p - \rho_0) A_b a P_c^n - P_c A_t \sqrt{\frac{\gamma_p}{R T_0}} \left( \frac{2}{\gamma_p + 1} \right)^{\frac{\gamma_p+1}{2(\gamma_p-1)}} \]

This equation defines the rate at which the chamber pressure increases during the motor operation, and it is very interesting to know the firsts instants of the burn, as this variation in time will be great. Once the motor reaches the steady state, the term dependant on time can be neglected.

- **Steady-state:**

The longest phase of the motor operation and the one that is based on the design calculations. During this phase, the value of the chamber pressure varies accordingly to the burning area evolution. Almost all the impulse of the motor is delivered during this phase.
To model the evolution of the chamber pressure during this stage of the burn, the equation found as a result of the start-up analysis can be re-used, in this case neglecting the derivative of the chamber pressure with respect to time as it is considered steady-state. Also, the term of the gases density is dropped from the equation as it is much lower than the grain density. With these changes, the equation yields:

\[
0 = \rho_p A_b a P_c^n - P_c A_c \sqrt{\frac{\gamma_p}{RT_0} \left( \frac{2}{\gamma_p + 1} \right)^{2(\gamma_p + 1)}} \rightarrow P_c = \left[ \frac{A_b}{A_t} \frac{a \rho_p}{\sqrt{\frac{R T_0}{\gamma_p (\gamma_p + 1)^{\gamma_p + 1}}}} \right]^{1/(1-n)}
\]

Using the definition of the \( K_N (K_N = \frac{A_b}{A_t}) \) parameter and of the characteristic exhaust speed \((C^* = \sqrt{\frac{RT_0}{\gamma_p (\gamma_p + 1)^{\gamma_p + 1}}})\), the equation is greatly simplified:

\[
P_c = [K_N \times \rho_p \times a \times C^*]^{1/(1-n)}
\]

With this equation, the value of the chamber pressure depends solely on the \( K_N \) parameter, as it has been discussed previously in this study.

- **Tail-off:**

The final part of the motor operation arrives when the propellant has been depleted. The exhaust gases remaining in the chamber at high pressure are expelled through the nozzle until the pressure inside the chamber matches the ambient one. This gives a fast but gradual descent in the chamber pressure that produces some residual thrust and impulse.

Using again the equation of the chamber pressure evolution in time, this time the parameter that can be neglected is the burning area, as there is no more propellant to burn. This yields the following ODE:

\[
\frac{V_0}{RT_0} \frac{dP_0}{dt} = -P_0 A_t \sqrt{\frac{\gamma_p}{RT_0} \left( \frac{2}{\gamma_p + 1} \right)^{2(\gamma_p + 1)}}
\]

This has an analytical solution, which is just an exponential equation as a function of time from burn-out \( t_{burn-out} \):

\[
P_c = P_{burn-out} \times e^{-\frac{RT_0 A_t}{V_0 C^*} t_{burn-out}}
\]

This shows an exponential decrease of the chamber pressure up to the pressure at burn-out, which matches the ambient one.

All this analysis also helps to obtain the expected burn time of the propellant, as well as the expected duration of the pressurisation of the chamber. The latter will be a little longer than the former, as when the burn ends there is still pressure inside the combustion chamber that has to undergo the tail-off.

Using the excel spreadsheet \textit{SRM2014} [13], these equations are solved and plotted in order to obtain the pressure curve of the motor and the final results regarding values of
maximum pressure and different times that may be of interest. In Figure 9 is shown the pressure curve while in Table 9 are displayed the main parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum pressure</td>
<td>( p_{\text{MAX}} )</td>
<td>13.88</td>
<td>MPa</td>
</tr>
<tr>
<td>Burn time</td>
<td>( t_{\text{burn}} )</td>
<td>5.252</td>
<td>s</td>
</tr>
<tr>
<td>Start-up time</td>
<td>( t_{\text{start-up}} )</td>
<td>0.125</td>
<td>s</td>
</tr>
<tr>
<td>Burn-out time</td>
<td>( t_{\text{burn-out}} )</td>
<td>0.426</td>
<td>s</td>
</tr>
</tbody>
</table>

Table 9: Pressure results summary.

Comparing the value of maximum pressure with the design pressure (Table 5) it is found that the motor will be operating at about an 80% of its potential. This complies with the requirement \textit{Req.Safety} as the design pressure has been determined using a 1.5 safety margin, and the maximum pressure found doesn’t even reach this value.

7.3.3. Motor thrust

Once the chamber pressure evolution has been computed, the thrust of the motor can be obtained. From the conservation of momentum equation, an expression can be obtained that expresses the force produced by the motor:

\[
F = \int P \, dA = \dot{m} \times v_e + (P_e - P_a) \times A_e
\]
7. Development of the solution

where:

- \( m \): propellant mass flow expelled through the nozzle.
- \( v_e \): velocity of the exhaust gases.
- \( P_e \): pressure of the exhaust gases at the nozzle exit.
- \( P_a \): ambient pressure.
- \( A_e \): area of the nozzle exit.

The aim is now to obtain an expression with known parameters that can relate the above equation with the chamber pressure. That way, from the curve shown in Figure 9 a thrust curve will be obtained and it will be possible to evaluate the total impulse of the motor.

The first term that will be evaluated will be the exhaust mass flow \( \dot{m} \). To determine this parameter, the conditions at the nozzle throat have to be found, as it is there where the mass flow is determined. There the mass flow can be expressed using the conservation of mass equation as:

\[
\dot{m} = \rho_t \times A_t \times v_t
\]

The value of the throat area \( A_t \) has already been computed in previous calculations. The value of the flow velocity at the throat \( v_t \) is known to be \( M = 1 \) at throat conditions. Finally, the value of the flow density at the throat \( \rho_t \) can be computed if the consideration that the exhaust gases follow the ideal gas law is accepted.

To compute the speed of sound at the throat \( a_t \) from its definition \( a = \sqrt{\gamma \times R \times T} \) it is clear that the throat temperature has to be obtained. The values of the specific heats ratio \( \gamma \) for the combustion products has already been found and can be seen in Table 8. The specific gas constant \( R \) is obtained by dividing the universal gas constant with the effective molar weight of the combustion products. For the temperature at the nozzle throat \( T_t \) an analysis of the flow inside of it has to be carried out.

- **Nozzle analysis**

The first step in this analysis is to define the assumptions made to simplify and be able to carry out all the calculations:

- Steady flow.
- One-dimensional flow along the nozzle longitudinal axis.
- Compressible flow.
- Exhaust gases follow ideal gas law.
- Flow is isentropic.
- Stagnation conditions at nozzle inlet.
- Matched nozzle (exit pressure matches ambient pressure).

From the energy conservation equation between any 2 points in the flow with all the above mentioned assumptions taken into account can be written as:

\[
\Delta h = c_p \Delta T = \frac{1}{2} (v_2^2 - v_1^2)
\]
That equation provides a relation between the temperature and the velocity in a way that yields the definition of stagnation temperature $T_0$ as:

$$ T_0 = T + \frac{v^2}{2c_p} $$

The isentropic flow assumption has as a consequence that throughout all the nozzle the stagnation temperature remains constant, as there is nothing adding or removing heat from the system. The ratio of temperatures can be rewritten, taking the definition of stagnation temperature, the definition of the speed of sound and the relations $c_p - c_v = R$ and $\frac{c_p}{c_v} = \gamma$ as:

$$ \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 $$

As the Mach number at the nozzle throat is known and equal to one, the flow temperature at the nozzle is defined as:

$$ T_t = \frac{T_0}{1 + \frac{\gamma - 1}{2} \frac{1}{2}} = \frac{T_0}{\frac{\gamma + 1}{2}} $$

That way now the flow velocity at the nozzle throat can be expressed using only known parameters:

$$ v_t = a_t = \sqrt{\gamma R T_0} \frac{T_0}{\gamma + 1 / 2} $$

The parameters $\gamma$ and $R$ have been obtained when computing the propellant characterisation and can be found in Table 8, as well as $T_0$ because this parameter is no other than the chamber temperature as it has been considered that the conditions at the nozzle inlet are the stagnation ones.

To find the flow density at the nozzle throat the isentropic relations between the thermodynamic properties have to be found. From the entropy equations the following relation can be found for an isentropic flow:

$$ \frac{P}{\rho^\gamma} = \text{constant} $$

From this relation and with the help of the ideal gas law ($P = \rho RT$), the equation can be developed to the relation between the temperature and density ratios:

$$ \frac{P}{\rho^\gamma} = \text{const} = \frac{P_0}{\rho_0^\gamma} = \frac{\rho_0 RT_0}{\rho_0^\gamma} = \rho_0 \gamma^{-1} RT_0 = \rho_0 \gamma^{-1} RT_0 \rightarrow \left( \frac{\rho_0}{\rho_t} \right)^{\gamma^{-1}} = \frac{T_0}{T_t} $$

As the objective is to write the flow density at the throat as a function of known parameters, using the definition of stagnation temperature and using once more the ideal gas law the final expression can be achieved:

$$ \left( \frac{\rho_0}{\rho_t} \right)^{\gamma^{-1}} = \frac{\gamma + 1}{2} \rightarrow \rho_t = \frac{\rho_0}{\left( \frac{\gamma + 1}{2} \right)^{\gamma^{-1}}} \rightarrow \rho_t = \frac{P_0}{RT_0 \left( \frac{\gamma + 1}{2} \right)^{\gamma^{-1}}} $$
With the nozzle throat flow velocity and density known and as a function of the chamber pressure, the mass flow through the nozzle can be written as:

$$\dot{m} = \frac{P_c}{RT_c} \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}} \times A_t \times \sqrt{\frac{\gamma R T_c}{\gamma + 1}}$$

$$\dot{m} = P_c \times A_t \times \frac{\gamma}{\sqrt{R \times T_c}} \times \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}}$$

To compute the exit velocity of the flow, starting from the energy conservation equation and operating with different equations already presented:

$$c_p(T_0 - T_e) = \frac{1}{2} v_e^2 \rightarrow v_e = \sqrt{2 c_p (T_0 - T_e)}$$

$$\begin{align*}
\left\{ \frac{c_p - c_v}{c_p} = R \rightarrow c_p = \frac{\gamma R}{\gamma - 1} \rightarrow v_e = \frac{2 \gamma R}{\gamma - 1} T_0 \left(1 - \frac{T_e}{T_0}\right) \right. \\
T_e = \left(\frac{P_e}{\rho_0}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{P_e}{RT_e} \frac{T_0}{P_0 T_0}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{P_e}{P_0}\right)^{\frac{1}{\gamma - 1}} \rightarrow v_e = \sqrt{\frac{2 \gamma R}{\gamma - 1} T_c \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{1}{\gamma - 1}}\right]} \end{align*}$$

As it can be seen, an expression has been found that relates directly the flow exit velocity with the chamber pressure, chamber temperature and exit pressure. If the nozzle is designed to be matched, the exit pressure of the flow will be the ambient pressure. That is the optimal design point for the nozzle. To achieve that, the exit area of the nozzle must be exactly the one that expands the flow to just the right pressure. To compute the area, the first step is to consider the mass conservation equation, for which the mass flow in the nozzle remains constant in the nozzle.

$$\dot{m} = \text{constant} = \rho_tA_t v_t = \rho_e A_e v_e \rightarrow A_t = \frac{A_e}{\rho_e \frac{v_e}{v_t}}$$

$$\frac{\rho_e}{\rho_t} = \left(\frac{P_e}{P_t}\right) = \left(\frac{P_e}{P_c} \frac{T_c}{T_e}\right)^{\frac{1}{\gamma - 1}} \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{P_e}{P_0}\right) \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}}$$

$$\frac{v_e}{v_t} = \frac{2 \gamma R T_c}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{1}{\gamma - 1}}\right] = \sqrt{\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{1}{\gamma - 1}}\right]}$$
In this case, with the parameters computed until now, in order for the nozzle to be able to expand the exhaust gases to the atmospheric pressure of 101325 Pa the exit area has to be:

\[ A_{e_{opt}} = 2113.27 \text{ mm}^2 \rightarrow d_{e_{opt}} = 51.87 \text{ mm} \]

After obtaining the expressions of the mass flow and exit velocity as a function of the chamber pressure, the general expression for the thrust can be written:

\[
F = P_c \times A_t \times \frac{Y}{R \times T_c} \times \left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma-1}} \times 2 \frac{\gamma R}{\gamma - 1} T_c \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{1}{\gamma}} \right] \\
+ \frac{(P_e - P_a) \times A_t}{ \left( \frac{Y + 1}{2} \right)^{\frac{1}{\gamma-1}} \left( \frac{P_e}{P_c} \right)^{\frac{1}{\gamma}} \left[ \frac{\gamma + 1}{\gamma - 1} \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{1}{\gamma}} \right] \right]}
\]

This general expression is the one used to make the graphs, as the nozzle will be able to match the atmospheric pressure exactly at the value of maximum pressure, which is the one used to compute the optimum exit area of the nozzle. In that case, the second term of the equation is null and it simplifies to:

\[
F_{opt} = P_c \times A_t \times \frac{Y}{R \times T_c} \times \left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma-1}} \times 2 \frac{\gamma R}{\gamma - 1} T_c \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{1}{\gamma}} \right]
\]

The excel spreadsheet SRM2014.xls [13] computes using the general equation the thrust curve from the pressure curve already presented. It is shown in Figure 10 how the thrust curve follows a similar profile as the pressure curve shown in Figure 9. The main thrust parameters are summarised in Table 10.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum thrust</td>
<td>(F_{\text{max}})</td>
<td>2286</td>
<td>N</td>
</tr>
<tr>
<td>Average thrust</td>
<td>(F_{\text{avg}})</td>
<td>1845</td>
<td>N</td>
</tr>
<tr>
<td>Thrust time</td>
<td>(t_{\text{thrust}})</td>
<td>5.678</td>
<td>s</td>
</tr>
</tbody>
</table>

Table 10: Thrust parameters.
7. Development of the solution

7.3.4. Total impulse

The motor’s total impulse is calculated integrating the thrust curve over time:

\[ I = \int_{t=0}^{t=\text{thrust}} F \, dt \]

Integrating the thrust general expression is not easy. The main parameter which is time-dependant is the chamber pressure. It varies with the burning area, which varies according to the De Vielle’s law of the burning rate, an expression which includes the same parameter of the chamber pressure that depends on it. That is why, having the thrust curve, a numerical integration is far easier than trying to analytically integrate the thrust expressed as a function of time.

To numerically integrate the thrust curve, the trapezoidal rule is implemented:

\[ I = \int_{t=0}^{t=\text{thrust}} F \, dt \approx \sum_{i=1}^{N} \frac{F_{i-1} + F_{i}}{2} \times \Delta t_{i} \]

Figure 10: Thrust curve of the motor.
For each instant $i$, the average thrust value between the previous instant $i-1$ and the actual is computed, and then multiplied by the interval between both instants. This calculation is then added to all the previous ones, and this procedure continues for all instants. After this is done, the total impulse found is:

$$I = 10477 \, N \cdot s$$

In the model rocketry community there is a classification for the rockets as a function of their motor’s total impulse. This classification is based on letters, ranging from A at the lowest end and AH at the highest (this one given to the Ares V rocket). From the classification found in [14], this motor classifies as a $N$ category rocket, surpassing the lower end of this classification position that is at 10240.01 $N \cdot s$. In order to operate these powerful rocket motor, a Level 3 certification is needed. This thesis’ tutor has this certification [15] and will be able to test and operate the motor without problem.

7.3.5. Results summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Klemmung parameter</td>
<td>$K_{N_{\text{max}}}$</td>
<td>1206.9</td>
<td>–</td>
</tr>
<tr>
<td>Nozzle throat area</td>
<td>$A_t$</td>
<td>116.90</td>
<td>$mm^2$</td>
</tr>
<tr>
<td>Nozzle throat diameter</td>
<td>$d_t$</td>
<td>12.2</td>
<td>$mm$</td>
</tr>
<tr>
<td>Maximum chamber pressure</td>
<td>$P_{c_{\text{max}}}$</td>
<td>13.88</td>
<td>$MPa$</td>
</tr>
<tr>
<td>Propellant burn time</td>
<td>$t_{\text{burn}}$</td>
<td>5.252</td>
<td>$s$</td>
</tr>
<tr>
<td>Propellant start-up time</td>
<td>$t_{\text{start-up}}$</td>
<td>0.125</td>
<td>$s$</td>
</tr>
<tr>
<td>Propellant burn-out time</td>
<td>$t_{\text{burn-out}}$</td>
<td>0.426</td>
<td>$s$</td>
</tr>
<tr>
<td>Optimum nozzle exit area</td>
<td>$A_{e_{\text{opt}}}$</td>
<td>2113.27</td>
<td>$mm^2$</td>
</tr>
<tr>
<td>Optimum nozzle exit diameter</td>
<td>$d_e$</td>
<td>51.87</td>
<td>$mm$</td>
</tr>
<tr>
<td>Maximum thrust</td>
<td>$F_{\text{max}}$</td>
<td>2286</td>
<td>$N$</td>
</tr>
<tr>
<td>Average thrust</td>
<td>$F_{\text{avg}}$</td>
<td>1845</td>
<td>$N$</td>
</tr>
<tr>
<td>Thrust time</td>
<td>$t_{\text{thrust}}$</td>
<td>5.678</td>
<td>$s$</td>
</tr>
<tr>
<td>Total impulse</td>
<td>$I$</td>
<td>10477</td>
<td>$N \cdot s$</td>
</tr>
</tbody>
</table>

Table 11: Performance analysis results.
8. Economical summary

The costs related to the development of this thesis can be divided in three different groups: the ones related to the labour of the engineer making the calculations and the worker in charge of developing the prototype designed; the ones related to the material, tools and services of the construction of the motor; the ones related to the energy consumed during the development of this thesis’ activities.

8.1. Labour costs

The work done by the engineer in charge of designing the motor is not free. The hourly price of the worker will be set at 15 €/hour when working on the computer during the design, organisation and miscellaneous development of the thesis, while the price of the worker working in the manufacture of the prototype will be set at 12 €/hour. The time invested in the design and documental phase of the thesis has been about 300 hours. Regarding the hours invested in the fabrication of the prototype, they have been used in the following activities:

- Nozzle machining: 6 hours.
- Nozzle cap machining: 4 hours.
- Top cap machining: 3 hours.
- Casing machining: 0.5 hours.
- Cap-fixing screws: 4 hours.

This gives a total of 17.5 hours of work. So the total cost of the labour has been:

\[
Labour\ cost = 300\ [hours] \times 15\ [€/h] + 17.5\ [hours] \times 12\ [€/h] = 4710\ [€]
\]

8.2. Materials, tools and services costs

The construction of the prototype designed in this thesis requires material that needs to be bought and tools to machine these materials into the final shape. If there are some complex operation that can’t be done with the available machinery or the available ability with said machinery, some services will have to be hired.

- The aluminium cost for the caps and the casing has amounted to 30 €.
- The steel cost for the nozzle has amounted to 8 €.
- The cost of the O-ring for the seals has amounted to 9 €.

This gives a total material cost of:

\[
Material\ cost = 48\ [€]
\]

8.3. Energy costs

The power that the computer used to develop this thesis uses is 135 W. As it has taken about 300 hours of work to complete the work on the computer, the energy consumed during this time is 40.5 kWh. In Spain, the average price of electricity is about 0.123 €/kWh [16]. That means that the cost of the energy used for the design and all the rest of the work done in the computer has been:

\[
Energy\ costs = 40.5\ [kWh] \times 0.123\ [€/kWh] = 4.98\ [€]
\]
8.4. Final costs

The project has involved several hours of work, both in the design phase and during the fabrication of the prototype. This amounts to almost all of the costs of this thesis. The costs regarding the material amounts to around 100 times less money, while the energy costs are even an order of magnitude less. All in all, the final cost of the thesis is here summarised:

<table>
<thead>
<tr>
<th>Concept</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour cost</td>
<td>4710 €</td>
</tr>
<tr>
<td>Material cost</td>
<td>48 €</td>
</tr>
<tr>
<td>Energy costs</td>
<td>4.98 €</td>
</tr>
<tr>
<td><strong>Total costs</strong></td>
<td><strong>4762.98 €</strong></td>
</tr>
</tbody>
</table>

*Table 12: Final costs of the thesis.*
9. Environmental impact

The impact on the environment that this thesis has caused is hard to determine. On one hand, there are the direct effects caused by the execution of the work, such as the energy consumed, which more or less can be estimated. On the other hand, there are the indirect effects, caused by all the elements that were not under the direct control of this thesis’ author, such as the ones derived from the transportation of the needed materials bought to make the prototype. Finally, it will be considered the impact that a test of the motor would make.

9.1. Direct impact

The total amount of hours worked on the computer yield an electrical power consumption that is offered by the city’s electrical network. This power has been generated by an unknown source, so the average value of CO\textsubscript{2} emissions per kWh consumed is going to be taken. From [17] the average emission of this gases per unit of power consumed in Catalonia is 321 g CO\textsubscript{2}/kWh.

\[
40.5 \text{ kWh} \times \frac{321 \text{ g CO}_2}{1 \text{ kWh}} = 13000.5 \text{ g CO}_2
\]

The lathe used for the fabrication of the pieces is the model FTX 1000x360-D of the brand Abratools, which has a power consumption of 1.5 kW [18]. Taking the hours that has been used to machine every component and using the same average of CO\textsubscript{2} emissions per kWh, the emissions due to the use of the lathe can be found:

\[
1.5 \text{ kW} \times 17.5 \text{ hours} \times \frac{321 \text{ g CO}_2}{1 \text{ kWh}} = 8426.25 \text{ g CO}_2
\]

9.2. Indirect impact

The production of the amount of metal needed for the construction of the motor has associated a given amount of carbon dioxide emissions.

It is estimated that the fabrication of aluminium has associated a CO\textsubscript{2} emission level equivalent to that of the consumption of 14 MWh of energy per metric tonne of material [19]. As in this project a total of 4.795 kg of aluminium has been used to form the casing and both caps, it can be computed the amount of emissions that this has caused:

\[
4.795 \text{ kg aluminium} \times \frac{14 \times 10^3 \text{ kWh}}{1000 \text{ kg aluminium}} \times \frac{0.321 \text{ kg CO}_2}{1 \text{ kWh}} = 21.55\text{kg CO}_2
\]

The emission levels of CO\textsubscript{2} linked to the fabrication of steel are of about 1.95 tonnes per metric tonne of material. Repeating the same procedure as before, taking the 1.163 kg of carbon steel used to make the nozzle:

\[
1.163 \text{ kg steel} \times \frac{1950 \text{ kg CO}_2}{1000 \text{ kg steel}} = 2.268 \text{ kg CO}_2
\]

A part from the emissions related to the manufacturing of the metals, there is also the environmental cost related to its transportation from its manufacturer to the workstation where they are machined.
9.3. Motor burn test impact

From the results shown in Figure 7 the moles of components expelled to the atmosphere by the engine for every 100 g of propellant is found. Taking the amount of moles of $CO_2$ created by the combustion and finding their mass, it is possible to extrapolate and find how much grams of this compound will be created when burning the propellant.

$$\frac{\text{moles } CO_2}{100 \text{ g propellant}} \times \frac{44.01 \text{ g } CO_2}{1 \text{ mole } CO_2} \times \frac{1.811 \text{ g propellant}}{1 \text{ cm}^3 \text{ propellant}} \times \left(\frac{\pi}{4} (8.7^2 - 1.9^2) \times 13 \times 6 \text{ cm}^3 \text{ propellant}\right) = 464.561 \text{ g } CO_2$$

Similarly, it can be also found the amount of other gases that are considered environmentally polluting, as the carbon monoxide or methane.

$$\frac{\text{moles } CO}{100 \text{ g propellant}} \times \frac{28.01 \text{ g } CO}{1 \text{ mole } CO} \times \frac{1.811 \text{ g propellant}}{1 \text{ cm}^3 \text{ propellant}} \times \left(\frac{\pi}{4} (8.7^2 - 1.9^2) \times 13 \times 6 \text{ cm}^3 \text{ propellant}\right) = 1872.566 \text{ g } CO$$

$$5.12 \times 10^{-3} \frac{\text{moles } CH_4}{100 \text{ g propellant}} \times \frac{16.04 \text{ g } CH_4}{1 \text{ mole } CH_4} \times \frac{1.811 \text{ g propellant}}{1 \text{ cm}^3 \text{ propellant}} \times \left(\frac{\pi}{4} (8.7^2 - 1.9^2) \times 13 \times 6 \text{ cm}^3 \text{ propellant}\right) = 6.567 \text{ g } CH_4$$
10. Social and safety considerations

It is important that this thesis takes into account all the safety issues that can appear during the construction and use of the motor that may affect the operator or a third person, as well as all the legislation that may affect the development of the activity that is intended to carry out with the motor, that is launching model rockets.

10.1. Construction

During the construction of the motor parts, all the precautions related with the use of the lathe have to be taken [20]. This includes:

- Protection glasses
- Entanglement hazards (jewellery, long hair, etc.) are removed or safely confined.
- Keep slipping/tripping hazards clear of the lathe’s working area
- The lathe should have an emergency stop button at reach of the operator.
- The power to the lathe should be off when putting/removing parts to machine.

During the preparation of the propellant, some precautions have to be taken.

- Wear latex gloves to prevent direct contact with the high adhesive epoxy resin with the skin.
- It is recommended the use of masks as the potassium nitrate dust can arise and be breathed in.

10.2. Legislation

In Spain there is no legislation that regulates the use and testing of these kind of motors. However, there are some rules that the hobbyist clubs follow, and in which this study is based. In this case, the norms set by the National Association of Rocketry (NAR) are the ones in which the club SpainRocketry bases their own. The main rules affecting this thesis are [21]:

- A high power rocket (HPR) motor will be fabricated to support the operation stresses and to conserve its structural integrity under extreme or known conditions.
- A rocket can’t have installed a motor or a combination of motors that yield a total impulse larger than 40960 \( N \cdot s \).
- Regarding ignition devices:
  - Use a remotely-controlled electrically-activated ignition system and have the switch go back to the “off” position when not pressed.
  - The ignition system must be expelled out of the motor after ignition.
  - The combination of the ignition system and the ignition switch must be designed to ensure the ignition at most after 3 seconds after the command is sent.
  - The installation of the ignition device must be done only when the rocket is in the launch pad or the motor in the test stand.

These rules lay a criterion for the structural design of the motor, its maximum impulse and some aspects regarding ignition. These rules, combined with the requirements, define some design parameters of the motor.
11. Organization and scheduling

The aim of this chapter is to show how the thesis has been chosen to be tackled, with the identification of the different work packages and its interdependencies, as well as the timing and scheduling of them all.

11.1. Work Breakdown Structure and tasks identification

Figure 11: Project work breakdown structure.

11.2. Brief tasks description

As seen in the WBS, the tasks can be divided in three main groups:

- Project organization
- Technical design
- Economic and legislative study
Each of the groups has several tasks associated to its activities. A brief description of these tasks is given in the following sections.

11.2.1. Project organization

- **Scheduling**: gives a time duration for every task, and places them in the calendar.
- **Communication**: task corresponding to all the communication between the thesis’ developer (student) and the tutor.
- **Quality assessment**: in charge of the coherence and uniformity of the work developed time apart.

11.2.2. Technical design

- **Structure**
  - Casing material selection: analysis of different candidates and choice of the most suitable material for the casing of the motor
  - Casing structural study: definition of the working parameters of the casing from the material choice and the proposed geometry.

- **Propulsion**
  - Propellant ingredient selection: study of the possible ingredients and choice of the final recipe for the propellant.
  - Propellant geometry: evaluation of the possible geometries of the grain and decision of the most suitable one for the intended purpose.
  - Nozzle: decision of the materials, geometry and implementation of the motor nozzle.

- **Performance**
  - Pressure: analysis of the pressure evolution in the motor combustion chamber during its operation.
  - Thrust: analysis of the thrust delivered by the motor during its operation.
  - Impulse: computation of the total impulse of the rocket and correspondent classification.

- **Manufacture**: study on the best techniques and the equipment they entail to fabricate the motor’s casing, nozzle and caps, as well as the propellant grain and the segments.

11.2.3. Economic and legislative study

- **Economic study**: evaluation of all the costs derived from the engineering work, the material and software used for the design and the material, tools and workforce needed for the prototype fabrication.
- **Legislative study**: research and understanding the legislation that regulates the activity intended for the motor.
11.3. Tasks interdependencies

Some of the tasks shown in the work breakdown structure require that they are completed prior to others, as their results may be needed to accomplish the task’s objective. While the project organization tasks and the economical and legislative tasks don’t have any need of precedent tasks being complete, the interdependencies of the technical design tasks are the ones analysed.

<table>
<thead>
<tr>
<th>Task Code</th>
<th>Preceding tasks code</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD-St.Std</td>
<td>TD-St.Mat</td>
</tr>
<tr>
<td>TD-Prop.Geom</td>
<td>TD-St.Std</td>
</tr>
<tr>
<td>TD-Prop.N</td>
<td>TD-Prop.Ing</td>
</tr>
<tr>
<td>TD-Prop.Geom</td>
<td></td>
</tr>
<tr>
<td>TD-P.P</td>
<td>TD-St.Std</td>
</tr>
<tr>
<td>TD-P.T</td>
<td>TD-P.P</td>
</tr>
<tr>
<td>TD-P.I</td>
<td>TD-P.T</td>
</tr>
<tr>
<td>TD-M</td>
<td>TD-St.Mat</td>
</tr>
<tr>
<td></td>
<td>TD-Prop.Ing</td>
</tr>
<tr>
<td></td>
<td>TD-Prop.Geom</td>
</tr>
<tr>
<td></td>
<td>TD-Prop.N</td>
</tr>
</tbody>
</table>

*Table 13: Task interdependencies.*

11.4. Schedule: Gantt chart

With the interdependencies clear, and reminding that the tasks in the project organization package and in the economical and legislative study one are completed as the technical design advances, it is only this last package that is represented in the Gantt chart.

It can be seen in the chart that the technical design of the motor lasts for almost all the duration of this thesis period, which started at mid-February and ends on June 20th. At the beginning of March, the design kicks off with the definition of some parameters with the thesis’ tutor and ends at the beginning of June in order to have time to review all the documentation and check for any mistakes, errors and incoherencies.
11. Organization and scheduling

Figure 12: Gantt chart of the technical design.
12. Conclusions

During the 6 years combined of bachelor’s and master’s degree it has been taught to us, the students, all the necessary knowledge about the working principles of rocket motors, be it of solid or liquid propellant and even electrically powered. The main purpose of this thesis has been to put into practice all this knowledge and convert it into a design of a real motor that can be constructed and tested. It has been a pity that there was not the possibility of testing the motor as the student in charge of the test stand design and construction has postponed the delivery of his master’s thesis to September.

The requirements set for this project have not all been verified. Taking a look at chapter 3, where the requirements are laid out, the following evaluation on requirement fulfilment is presented.

- **Req.\(F_{\text{max}}\)** and **Req.\(I_{\text{tot}}\)** have been fulfilled in the design phase. However, the impossibility to test the real motor in the test stand doesn’t allow for the total verification of these two requirements (although the design values allow for an optimist conclusion, as they exceed the minimum value set in this requirements by a lot).

- The requirement **Req.Safety** has been fulfilled, as the expected maximum chamber pressure doesn’t match the casing design pressure.

- The requirement **Req.Reuse** has not been fulfilled as the motor has not been used and thus its reusability cannot be assessed.

- Regarding the **Req.PropMat**, it has been fulfilled as the propellant chosen for the motor doesn’t require any heat source whatsoever during it fabrication.

- The requirements **Req.Diam** and **Req.Thick** have also been fulfilled, defining the casing’s diameter and thickness.

- The requirement **Req.Legis** has been partially fulfilled as the design parameters for sure stick to the regulations, but there is still the need to experimentally determine these parameters.

Although not all the requirements have been fully met, the design of the motor still stands as valid. A structural analysis of the casing has been carried out, finding its design pressure according to the safety margin set by the requirement **Req.Safety** and also its ultimate pressure. The propellant has been chosen and characterised, finding the necessary thermodynamic parameters that are used for the computation of the performance. This analysis of the propellant has been idealised and the effects of the solid particles in the exhaust have not been taken into account. Thus, a less efficiency and thrust is expected from the real motor. However, this assumption puts the design on the safe side by estimating a greater potential for the propellant to build up the chamber pressure. The performance analysis has found the pressure evolution in the combustion chamber as the propellant burns and depletes. The start-up phase as well as the burn-out have been considered too, giving the complete expected pressure curve of the motor. From there, the thrust curve has also been obtained and finally the total impulse of the motor. This puts the motor in an **\(N\)** category, considered a High Powered Rocket, and requires a special certification from the user in order to be operated. The design has also taken into consideration the manufacturing techniques needed for the development of a motor of identical characteristics.
As this project comes to an end, one can think about the possibilities that can appear as a follow-up to it. Of course, one extension of the work exposed in this thesis could be the fine-tuned calculation of the propellant performance, taking into account the solid particles created during the combustion that subtract performance from it. The other rather natural extension of this project is testing the motor and get experimentally the results computed in this design in order to later on compare them and hypothesize about the cause of eventual discrepancies. Another follow-up possibility of this project could be the design of a new rocket based around the motor here presented, using the results from this study to predict the performance of the overall rocket and later launch it and compare the real results to the ones predicted.

All in all, the project can be considered successful as it has defined the characteristics of a solid rocket motor that in the design complies with all the requirements set. It is left for future work to light it up and put it to the test.

Figure 13: Prototype of the designed motor and a 3D drawing of a section of it.
References


