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Efficient Multi-Attribute Auctions Considering Supply Disruption

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Although supply disruption is ubiquitous because of natural or man-made disasters, many firms still use the price-only reverse auction (only the cost is considered) to make purchase decisions. We first study the suppliers' equilibrium bidding strategies and the

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buyer's expected revenue under the first- and second-price price-only reverse auctions when the suppliers are unreliable and have private information on their costs and disruption probabilities. We show that the two auctions are equivalent and not efficient. Then we propose two easily implementable reverse auctions, namely the first-price and second-price format announced penalty reverse auction (APRA), and show that the "revenue equivalence principle" holds, i.e., the two auctions generate the same *ex ante* expected profit to the buyer. We further show that the two reverse auctions are efficient and "truth telling" is the suppliers' dominant strategy in the second-price format APRA. We conduct numerical studies to assess the impacts of some parameters on the bidding strategies, the buyer's profit and social profit.

Keywords: Supply risk management; mechanism design; reverse auction; game theory.

1. Introduction

Because of economic globalization, and increasing natural and man-made accidents, the supply failure risk has increased and the reliability of suppliers has become more uncertain (Chaturvedi and Martínez-de Albéniz, 2011; Simchi-Levi *et al.*, 2014). For example, Mattel experienced a considerable shortage of supply during a peak sale season, and it severely stroke its financial performance (Bapuji and Beamish, 2008). In fact, such disruption incidents were ubiquitous in the past decades, significantly impacting global supply chains and posting great challenges to supply chain risk management. Hendricks and Singhal (2003) reported that disruptions caused by the contracted suppliers might on average decrease about 12% shareholders' returns of the sourcing firms. In view of this, many firms take supply disruption risk into consideration when they make decisions on supplier selection, and supply disruption management has attracted considerable research attention (Sawik, 2014).

Recently more and more firms have used the reverse auction to procure goods or services (Santamaría, 2015). For example, Sun Microsystems and Hewlett-Packard (HP) have exploited specific online auctions to sell and buy products and materials worth hundreds of millions of dollars (Chen, 2014). Beall *et al.* (2003) reported that an average of 5% of total corporate spending was sourced by reverse auctions in 2003. Companies like Target, Dell, and General Electric are also said to use reverse auctions liberally. Pham *et al.* (2015) pointed out that the reverse auction is so popular because it has excellent potential to save purchase cost compared with traditional procurement approaches. Jap (2000) also found that the reverse auction can not only save the buyer's money but also increase the competitiveness of the supply base by forcing suppliers to bid for contracts.

While the reverse auction has been widely adopted in business and it performs excellently in saving procurement cost, the majority of the various forms of the reverse auction are conducted in the price-only regime without considering other attributes (Pham *et al.*, 2015; Beil and Wein, 2003). Price-only auctions have faced criticism since they may lead to some problems such as the lack of trust and long-term relationship between the buyer and suppliers (Smeltzer *et al.*, 2002; Pham *et al.*, 2015). Consequently, many experts have proposed to extend the

price-only reverse auction to the cases where other nonmonetary attributes such as product quality, lead-time, and payment schedule are considered, and studied the multi-attribute reverse auction.

The economics and operations management literature on the multi-attribute auction mainly focus on two streams: optimal mechanism and efficient mechanism (Xu and Huang, 2017). The first stream of works adopt score function to determine the winner of auctions and prove the utility equivalence between different auction formats (Che, 1993; Asker and Cantillon, 2008). The second stream of works focus on the design of efficient multi-attribute reverse auctions because efficient auctions can yield high revenue for the buyer in the long run (Parkes and Kalagnanam, 2005; Xu and Huang, 2017; Zhang *et al.*, 2019). These studies ignore the attribute of supply disruption risk, and do not study the optimality and efficiency of the mechanism simultaneously. To our best knowledge, Chaturvedi and Martínez-de Albéniz (2011) is the only paper that studies order allocation under the optimal procurement mechanism considering supply disruption risk. However, they do not study the efficiency of the mechanism.

Despite the ubiquity of supply disruption risk, a large number of firms ignore this factor and use the price-only auction to select supplier(s) when designing their procurement mechanisms. In this context, the buyer faces the following questions: (1) What is the supplier's equilibrium bidding strategy under the price-only reverse auction? (2) Which auction format should it adopt, the first-price reverse auction or the second-price reverse auction? (3) Is the price-only reverse auction efficient? (4) What is the efficient reverse auction?

At the same time, with growing globalization and increasing popularity of outsourcing, firms tend to purchase materials from and sell products to the global market. It is very difficult to obtain complete information about suppliers' costs and disruption probabilities. This asymmetric information may lead to significant damage to buyers (Yang *et al.*, 2009). Most of the literature on supply risk management assumes that the suppliers' costs and disruption probabilities are complete information. In this paper we assume that the suppliers' costs and supply disruption probabilities are their private information. That is, only suppliers themselves know their costs and supply disruption probabilities, and the buyer and other suppliers do not know this information.

In summary, we study the multi-attribute reverse auction design problem of a buyer facing a set of unreliable suppliers, whose production costs and supply disruption probabilities are private information. We show that the first- and second-price price-only auctions are equivalent, and they are not efficient. Then we propose two mechanisms, the first- and second-price *announced penalty reverse auctions*, and show that they are efficient and generate the same *ex ante* expected revenue, i.e., the "revenue equivalence principle" holds. The idea of the new mechanisms is simple and they are easy to implement in, that is, the buyer announces a penalty for the winner that cannot execute the contract.

Compared with the existing literature, our work makes the following contributions:

(1) We first propose a supplier selection mechanism under supply disruption risk for the case where the suppliers' costs and disruption probabilities are private information. The majority of studies on supplier selection under supply disruption assume that suppliers' costs and disruption probabilities are common knowledge.
(2) We characterize the equilibrium bidding strategies under the first- and second-price reverse auctions. We also show that the two auctions are revenue equivalent and they are not efficient. This means that the commonly used price-only reverse auctions in practice are problematic when the suppliers are unreliable.
(3) We propose two new efficient multi-attribute auctions that can help the buyer to jointly evaluate the suppliers' costs and supply reliabilities when selecting suppliers. Unlike the existing studies on multi-attribute auctions, which use a score function to select the winner, the new auctions select the supplier with the lowest bidding price as the winner. The auctioneer only needs to announce a penalty for the undelivered order. The implementation of the new auctions is easy compared with the existing multi-attribute auctions because it is very difficult to choose a proper score function for the latter.
(4) We show that the new auctions, i.e., the first- and the second-price APRAs, generate the same *ex ante* expected profits. This extends the "revenue equivalence principle" to multi-attribute auctions. Also, the two auctions are efficient and can yield high revenues to the buyer (Parkes and Kalagnanam, 2005; Xu and Huang, 2017). Moreover, the incentive compatibility holds for the second-price APRA, i.e., bidding the expected unit cost is the dominant strategy.

We organize the rest of the paper as follows: In Sec. 2 we give a concise review of the closely related literature. Then we introduce the assumptions and notations, and discuss the price-only reverse auction model in Sec. 3. We present the new auctions and analyze their properties in Sec. 4. We present the results of numerical studies and derive managerial insights from the analytical findings in Sec. 5. We conclude the paper and suggest topics for future research in Sec. 6.

2. Literature Review

In this paper we study the buyer's problem of procurement mechanism design facing suppliers whose production costs and supply disruption probabilities are private information. Two streams of literature are highly related to this work.

The first stream is on the reverse auction or procurement auction. Since the reverse auction has been adopted extensively as the main procurement mechanism in industries since the end of the last century (Smeltzer *et al.*, 2002), it has attracted

considerable research efforts and have produced a large body of literature. Jap (2002), Pham *et al.* (2015), and Teich *et al.* (2004) provided extensive reviews on this topic. We review in the following the studies closely related to our work.

Che (1993) extended the price-only reverse auction to consider quality as an attribute and used a score function to select the winner. He showed that the first-score, second-score, and second-preferred-offer auctions yield the same expected utility to the buyer when the buyer lacks commitment power. David *et al.* (2006) generalized Che's results to the case with more than two quality attributes, and showed that the buyer's expected payoff in the first-score, second-score, and English auctions differ only by a predefined constant. Budde and Minner (2014) considered a newsvendor-type retailer sourcing from a set of suppliers with private cost information. They compared combinations of different reverse auction formats (the first- and second-price) and risk sharing supply contracts (pull and push) under full contract compliance with a risk-neutral retailer and risk-averse suppliers. They showed that the first-price format is more preferred in most cases. Lorentziadis (2014) studied the bidding strategies when the buyer may default and fail to pay the winner the contractual amount. They showed that the *ex ante* expected payment for the first-price auction is lower than that for the second-price auction. Dang *et al.* (2015) studied the price-only reverse auction with different ending rules and value assumptions. They found that the first-price online auction with soft-close endings under private value is strategically equivalent to the English auction with a reserve price. Qian *et al.* (2018) examined the impacts of suppliers' anticipated regrets on their bidding strategies and the buyer's expected cost. They showed that the "revenue equivalence principle" does not hold when the winners' regrets differ from the losers' regrets.

Nishimura (2015) studied the optimal design of the scoring auction for public and private procurement. Asker and Cantillon (2008) studied the equilibrium behavior for the scoring auction and proved that any two scoring auctions that use the same quasi-linear scoring rule will generate the same expected utility for the buyer. Katok and Roth (2004) compared the performance of two forms of the Dutch auction using experiments and showed the descending-price auction performs well in a variety of environments. Beil and Wein (2003) proposed the multi-round open-ascending auction mechanism and showed that it is efficient to maximize the buyer's utility. Kostamis *et al.* (2009) compared the first-price open-bid and the first-price sealed-bid total-cost procurement auctions considering two attributes, i.e., price and fixed cost adjustment, and showed that open-bid is better than sealed-bid only when the expected information rent in the open-bid case is greater than that in the sealed-bid case, and vice versa. Nassiri-Mofakham *et al.* (2015) studied the bidding strategy for agents for the multi-attribute combinatorial double auction.

Chen *et al.* (2005) proposed an efficient mechanism for supply chain procurement incorporating the transport cost. Parkes and Kalagnanam (2005) studied on efficient iterative auctions for the single-unit procurement. Xu and Huang (2017) proposed three efficient auctions for the multi-unit procurement. Zhang *et al.* (2019)

proposed an optimal and efficient auction for the multi-unit transport procurement with private information on the suppliers' capacities. The above studies ignore the attribute of supply disruption risk. Xiang [2018] proposed a VCG-based efficient auction to the buyer to select suppliers among unreliable suppliers. We consider a much simpler format with easy-implementation in this paper.

The second stream of literature is on supply disruption risk management. As supply chain risk is very important for firms because of strategic outsourcing, globalization of markets, and increasing reliance on suppliers (Narasimhan and Talluri, 2009), many scholars have studied this problem and produced a large body of literature. Tang (2006), Narasimhan and Talluri (2009), Snyder *et al.* (2016), Tang and Musa (2011), and Fahimnia *et al.* (2015) provided extensive reviews on this topic. In the following we only review papers that are closely related to our study.

Supplier selection is an important problem when the buyer faces a set of suppliers with disruption risk. Sawik (2014) studied the supplier selection problem in a make-to-order system where the suppliers may suffer from disruption. He modeled the problem as a mix integer program. Sarkar and Mohapatra (2009) examined the optimal size of the supply base facing supply disruption. Federgruen and Yang (2014) considered the supplier selection problem over an infinite horizon with uncertain demand and supply, and characterized the structure of the optimal policy. Tomlin (2006), Babich *et al.* (2007), Chen and Guo (2014), Tang *et al.* (2014), Burke *et al.* (2009), Meena *et al.* (2011), and Hu *et al.* (2013) studied the effect of dual-sourcing or multi-sourcing on the mitigation of supply disruption risk. All these works assume that the buyer knows the suppliers' disruption probabilities.

There a lot of researches focusing on supply disruption risk management under asymmetric/incomplete information. Lim (2001) studied the cooperative contract design problem under incomplete information about product quality. Tomlin (2009) used the Bayesian model to examine the effects of information updating on dual-sourcing and single-sourcing strategies. Yang *et al.* (2012) studied the effects of asymmetric information and the correlation of disruption on the buyer's decisions, and concluded that the problem can be viewed as a choice between diversification and competition. Chen (2014) examined the influences of information asymmetry on the supplier's belief, the control of backup production, and the verifiability of supply disruption. Yang and Murthy (2014) investigated the supplier's and buyer's reactions to supply chain disruption under asymmetric information about the severity of disruption. Gurnani and Shi (2006) studied the first-time interaction between the supplier and the buyer when they have different estimates of the disruption risk. Gümüş *et al.* (2012) studied the value of price and quantity (P&Q) guarantee when the buyer seeks to procure from two suppliers (one is cheap and unreliable, while the other is expensive and reliable). Gurnani *et al.* (2012) analyzed the allocation problem under symmetric and asymmetric information, respectively. Chen *et al.* (2010) explored the decisions of dual-sourcing and inventory management of a manufacturer when facing unreliable supplies and proposed to use a Bayesian model to

dynamically update the information on the supply risk. Huang and Xu (2015) studied the optimal procurement contract design problem when the manufacturer or the supplier can initiate the enhancement of supply reliability. Yang *et al.* (2009) studied the information screening problem for the case where a manufacturer faces a reliable supplier and an unreliable supplier, and has incomplete information on the unreliable supplier's disruption probability. The above studies do not study the reverse auction design when considering the supply disruption.

To the best of our knowledge, Chaturvedi and Martínez-de Albéniz (2011) is the only paper that studies the optimal procurement allocation problem considering supply disruption risk. They designed the optimal mechanism that depends on the buyer's level of information about the suppliers' production costs and reliability. They showed that when the suppliers' reliabilities are known or unknown but independent, the optimal allocation is similar to that under full information. They also conducted numerical experiments to assess the benefits of the optimal mechanism in comparison with the traditional price-only auction that ignores supply risk.

We also consider the procurement mechanism design for the case where the buyer faces a set of suppliers with supply disruption risk and has incomplete information on the suppliers' production costs and disruption probabilities. But there are major differences between Chaturvedi and Martínez-de Albéniz (2011) and ours. They focused on order allocation under the optimal mechanism with different information structures. We study the revenue equivalence properties under different auction mechanisms and propose an efficient and easy-to-implement auction mechanism to help the buyer choose a supplier.

3. Assumptions and Price-Only Auction

Despite that supply disruption risk is ubiquitous, the price-only auction is commonly used in practice (Pham *et al.*, 2015). Chaturvedi and Martínez-de Albéniz (2011) used the price-only auction as the benchmark for comparison with their proposed mechanism. In this section, we study the equilibrium bidding strategies, the buyer's profit, and the efficiency of the first- and second-price price-only auctions with supply disruption risk.

Suppose that a buyer wants to buy one unit item from a set of N potential unreliable suppliers (the order is indivisible). The order may be undelivered by the selected supplier because of various potential disruption incidents that may occur during its production and/or transport such as loss of the product in transit, imperfect quality, natural disaster. Let supplier i's disruption probability be θ_i and unit production cost be c_i, which are private information (only known to the supplier itself). The buyer and the other suppliers only know that (c_i, θ_i) are random variables over $(0, \omega_c) \times (0, 1)$[a] (Chaturvedi and Martínez-de Albéniz, 2011), where ω_c

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[a]As we know, there are some situations that the costs and the disruption probabilities are dependent. Here, we assume that the suppliers' production cost is independent of their disruption
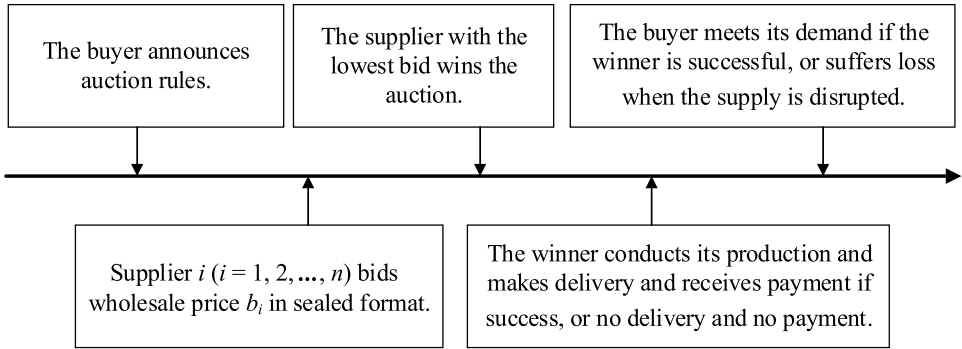
Fig. 1. The sequence of events.

is the upper bound of the production cost. We assume that suppliers are symmetric and independent, i.e., $c_i(\theta_i)$ $(i = 1, 2, \ldots, N)$ follows the same random variable $C(\Theta)$ with the distribution function $F_c(F_\theta)$ and density function $f_c(f_\theta)$. Also, we focus on the symmetric bidding strategies in this paper.

The sequence of events is as follows (Fig. 1): First, the buyer announces the auction rule. Second, suppliers submit bids of their wholesale prices in the sealed format and let supplier i's bid be b_i. Third, the supplier with the lowest bid wins and signs a contract with the buyer.[b] The payment is the winner's bid or the second lowest bid, which is determined by the first-price or the second-price auction format adopted by the buyer. Fourth, the winner produces the product and prepares to deliver it, which incurs a cost c_i. The contract is executed if no disruption happens. When disruption events happen, the winner is not able to execute the contract. Thus, there is no delivery and no payment, and the buyer suffers a loss l (Federgruen and Yang, 2014; Tomlin, 2006; Huang and Xu, 2015).

3.1. *First-price format*

In this subsection we study the suppliers' equilibrium bidding strategies and the buyer's *ex ante* expected profit for the first-price price-only auction. In this case, the wholesale price will be exactly the winner's bid. When supplier i has information (c_i, θ_i) and bids b_i, its expected payoff is

$$\Pi_i^{P_1}(b_i) = \begin{cases} (1-\theta_i)b_i - c_i & \text{if } b_i < \min_{j \neq i} b_j, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where P_1 indicates the first-price price-only reverse auction.

[b]If any tie bids happen, the order goes to one of the suppliers with same probability.

To simplify the analysis, we define $z_i = \frac{c_i}{1-\theta_i}$ as supplier i's virtual cost. The exact value of the virtual cost is only known to supplier i, and the buyer and other suppliers do not know it. But the buyer and the other suppliers know the distributions of c_i and θ_i, they can easily deduce that z_i follows the distribution function F_{Z_i}. Because $c_i(i = 1, \ldots, N)$ follows the same distribution F_C and $\theta_i(i = 1, \ldots, N)$ follows the same distribution F_θ, $z_i(i = 1, \ldots, N)$ follows the same distribution F_Z. Let the random variable Y_i denote the lowest value of the virtual cost of the other $N-1$ suppliers except supplier i. In other words, Y_i is the lowest order statistics of $Z_1, \ldots, Z_{i-1}, Z_{i+1}, \ldots, Z_N$. Let G denote the distribution function of Y_i, so, for any value of y, we have $G(y) = 1 - (1 - F_Z(y))^{N-1}$. Note that c_i is supplier i's unit production cost and $1 - \theta_i$ is supplier i's successful delivery probability, the virtual cost z_i represents supplier i's expected unit cost of successfully delivering the product to the buyer. It is increasing in the unit production cost c_i and decreasing in the successful delivery probability $1 - \theta_i$.

Proposition 1 gives supplier i's equilibrium bidding strategy in the first-price price-only auction.

Proposition 1. *Supplier i's equilibrium bidding strategy for the first-price price-only reverse auction is*

$$\beta^{P_1}(z_i) = \frac{1}{1 - G(z_i)} \int_{z_i}^{\infty} xg(x)dx. \tag{2}$$

Proof. Note that supplier i's payoff function can be rewritten as follows:

$$\Pi_i^{P_1}(b_i) = \begin{cases} (1 - \theta_i)(b_i - z_i) & \text{if } b_i < \min_{j \neq i} b_j, \\ 0 & \text{otherwise.} \end{cases}$$

Since the successful delivery probability $(1 - \theta_i)$ only affects the amount of the payoff when supplier i wins and does not affect the winning probability, supplier i's bidding strategy b_i will not change when θ_i and c_i change but z_i does not change. Thus, supplier i's equilibrium bidding strategy is a function of z_i, i.e., $b_i = \beta(z_i)$.

We first prove that $\beta(z)$ is an increasing function. Assume that the suppliers' virtual costs are $z_1, z_2, \ldots, z_N$. Consider supplier 1's equilibrium bidding strategy, i.e., $\beta(z_1)$. When supplier 1's virtual cost increases to z_1' ($z_1' > z_1$) while the other $N-1$ suppliers' virtual costs remain unchanged, its equilibrium bidding strategy is $\beta(z_1')$. We need to prove that $\beta(z_1') \geq \beta(z_1)$. Conversely, suppose that $\beta(z_1') < \beta(z_1)$. Let $\bar{b}_{-1} = \min_{i \neq 1}\{\beta(z_i)\}$. There are three cases:

(1) $\bar{b}_{-1} \leq \beta(z_1') < \beta(z_1)$, supplier 1 still loses the auction;
(2) $\beta(z_1') < \bar{b}_{-1} < \beta(z_1)$, supplier 1 will win the auction by bidding $\beta(z_1') < \beta(z_1)$. Note that $(1 - \theta_1)(\beta(z_1) - z_1) \geq 0$ in equilibrium, combining the fact that $\beta(z_1') < \beta(z_1)$ and $z_1' > z_1$, we can derive that $\beta(z_1') - z_1' < \beta(z_1) - z_1$, which means that there exists a positive probability that supplier 1 earns a negative profit;

(3) $\beta(z_1') < \beta(z_1) \leq \bar{b}_{-1}$, supplier 1 still wins the auction. However, it makes a lower profit by bidding $\beta(z_1') < \beta(z_1)$.

Then supplier 1 does not have the incentive to bid $\beta(z_1') < \beta(z_1)$.

Supplier i wins the auction if and only if its bid is the lowest, i.e., $b_i < \min_{j \neq i} \beta(z_j)$. This implies that $z_i = \beta^{-1}(b_i) < Y_i$. Then its expected payoff is

$$\Pi_i^{P_1}(b_i) = (1 - G(\beta^{-1}(b_i)))(1 - \theta_i)(b_i - z_i). \tag{3}$$

Taking the first derivative of Eq. (3) with respect to b_i yields the first-order condition

$$(1 - G(\beta^{-1}(b_i))) + \left(0 - \frac{g(\beta^{-1}(b_i))}{\beta'(\beta^{-1}(b_i))}\right)(b_i - z_i) = 0.$$

Substituting $\beta^{-1}(b_i) = z_i$ into the equation, we have

$$\beta'(z_i)(1 - G(z_i)) - g(z_i)(\beta(z_i) - z_i) = 0.$$

Then

$$\beta'(z_i)(G(z_i) - 1) + g(z_i)\beta(z_i) = g(z_i)z_i.$$

$$\frac{d}{dz_i}\beta(z_i)(G(z_i) - 1) = g(z_i)z_i.$$

Noting that $G(z_i) = 1$ when $z_i \to \infty$ and taking integration on both sides, we obtain

$$\beta(z_i) = \frac{1}{1 - G(z_i)} \int_{z_i}^{\infty} xg(x)dx. \tag{4}$$

Then

$$\beta(z_i) = \frac{1}{1 - G(z_i)} \left(\int_0^{\infty} xg(x)dx - \int_0^{z_i} xg(x)dx \right)$$

$$= \frac{1}{1 - G(z_i)} \left(E[Y_i] - z_iG(z_i) + \int_0^{z_i} G(x)dx \right)$$

$$= \frac{1}{1 - G(z_i)} \left(E[Y_i] - z_iG(z_i) + z_i - z_i + \int_0^{z_i} G(x)dx \right)$$

$$= z_i + \frac{1}{1 - G(z_i)} \left(E[Y_i] - \int_0^{z_i} 1dx + \int_0^{z_i} G(x)dx \right)$$

$$= z_i + \frac{E[Y_i] - \int_0^{z_i}(1 - G(x))dx}{1 - G(z_i)}.$$

In the following we shall show that if the other $N - 1$ suppliers follow the bidding strategy β, it is optimal for supplier i with the virtual cost z_i to bid $\beta(z_i)$.

As β is an increasing and continuous function, the bidder with the lowest virtual cost submits the lowest bid and wins the auction. It is never optimal for supplier i

to bid $b_i < \beta(0)$ since it will result in a negative payoff. Then we can assume that supplier i bids $b_i \geq \beta(0)$ when its virtual cost is z_i. Let z_i' satisfy $b_i = \beta(z_i')$. Then supplier i's expected payoff is

$$\Pi_i^{P_1}(b_i) = (1 - \theta_i)(1 - G(z_i'))(\beta(z_i') - z_i)$$

$$= (1 - \theta_i)(1 - G(z_i')) \left(z_i' + \frac{E[Y_i] - \int_0^{z_i'}(1 - G(x))dx}{1 - G(z_i')} - z_i \right)$$

$$= (1 - \theta_i)(1 - G(z_i')) \left(\frac{(1 - G(z_i'))(z_i' - z_i) + E[Y_i] - \int_0^{z_i'}(1 - G(x))dx}{1 - G(z_i')} \right)$$

$$= (1 - \theta_i) \left((1 - G(z_i'))(z_i' - z_i) + E[Y_i] - \int_0^{z_i}(1 - G(x))dx \right).$$

Then we could get

$$\Pi_i^{P_1}(\beta(z_i)) - \Pi_i^{P_1}(\beta(z_i')) = (1 - \theta_i) \left[G(z_i')(z_i' - z_i) - \int_{z_i}^{z_i'} G(x)dx \right]. \qquad (5)$$

We consider two cases: $z_i' \geq z_i$ and $z_i' < z_i$.

(1) $z_i' \geq z_i$. Since $G(\cdot)$ is a nondecreasing function, we have $G(z_i') \geq G(z_i)$ and

$$\int_{z_i}^{z_i'} G(x)dx \leq \int_{z_i}^{z_i'} G(z_i')dx$$

$$= G(z_i')(z_i' - z_i).$$

(2) $z_i' < z_i$. In this case, we have $G(z_i') < G(z_i)$ and

$$\int_{z_i}^{z_i'} G(x)dx = - \int_{z_i'}^{z_i} G(x)dx$$

$$\leq - \int_{z_i'}^{z_i} G(z_i')dx$$

$$= -G(z_i')(z_i - z_i')$$

$$= G(z_i')(z_i' - z_i).$$

For both cases, we have

$$\Pi_i^{P}(\beta(z_i)) - \Pi_i^{P}(\beta(z_i')) = (1 - \theta_i) \left[G(z_i')(z_i' - z_i) - \int_{z_i}^{z_i'} G(x)dx \right] \geq 0.$$

Thus, we proved the proposition. □

It is well known that the suppliers' biding strategies are determined by their production costs and the supplier with the lowest production cost will win the

auction in the first (second) price-only auction if we assume that the suppliers are reliable. Taking suppliers' supply disruption into consideration, Proposition 1 implies that the supply disruption probabilities also influence the suppliers' bidding strategies and the supplier with the lowest virtual cost, not unit production cost, will win the auction even though the buyer adopts the price-only auction. Then the supplier with a lower disruption probability will bid lower and has a higher probability to win the auction. Note that the results in Proposition 1 reduce to the results in the traditional price-only auction without considering supply disruption when all suppliers are reliable, i.e., $\theta_i = 0$ for $i = 1, 2, \ldots, N$. Then Proposition 1 generalizes the results in the traditional price-only auction assuming reliable supply to the case with supply disruption risk.

Now, we calculate the buyer's *ex ante* expected profit. From Proposition 1, the expected profit that the buyer can make from supplier i if it wins the auction is

$$\Pi_M^{P_1} = (1 - \theta_i)(r - w^{P_1}(c_i, \theta_i)) - \theta_i l, \tag{6}$$

where $w^{P_1}(c_i, \theta_i)$ denotes the wholesale price, which equals supplier i's bid, i.e., $w^{P_1}(c_i, \theta_i) = \beta(z_i) = \frac{1}{1 - G(z_i)} \int_{z_i}^{\infty} x g(x) dx$, and r is the unit revenue brought by the product, and l is the unit loss resulting from unsatisfied demand. The subscript M indicates the buyer. Note that supplier i's winning probability is $(1 - G(\frac{c_i}{1 - \theta_i}))$ and the buyer's *ex ante* expected profit is

$$E[\Pi_M^{P_1}] = N \int_0^1 \int_0^{\omega_c} \left(1 - G\left(\frac{c}{1 - \theta}\right)\right)$$

$$\times ((1 - \theta)(r - w^{P_1}(c, \theta)) - \theta l) f_c(c) f_\theta(\theta) dc d\theta. \tag{7}$$

3.2. Second-price format

In this subsection we study the suppliers' equilibrium bidding strategies and the buyer's expected profit for the second-price price-only auction. In this case, the lowest bid will win the auction and the wholesale price will be the second lowest bid.

When bidding b_i, the payoff of supplier i with (c_i, θ_i) is

$$\Pi_i^{P_2}(b_i) = \begin{cases} (1 - \theta_i)\bar{b}_{-i} - c_i & \text{if } b_i < \min_{j \neq i} b_j, \\ 0 & \text{otherwise}, \end{cases} \tag{8}$$

where $\bar{b}_{-i}$ is the second lowest bid, i.e., the lowest bid among the other $N - 1$ suppliers except i, and P_2 indicates the second-price price-only reverse auction. The following proposition characterizes the suppliers' equilibrium bidding strategies.

Proposition 2. *Supplier i's equilibrium bidding strategy in the second-price price-only reverse auction is given by*

$$\beta^{P_2}(c_i, \theta_i) = z_i = \frac{1}{1 - \theta_i}. \tag{9}$$

Proof. Consider supplier 1, say, and suppose that $y = \min_{j \neq 1} b_j$ is the second lowest bid. By bidding z_1, supplier 1 wins the auction if $z_1 < y$ and loses it otherwise.

Suppose, however, that supplier 1 bids an amount $z'' < z_1$. There are three cases:

(1) $z'' < z_1 < y$, supplier 1 still wins the auction and the wholesale price is still y. It cannot obtain more profit;

(2) $z'' < y < z_1$, supplier 1 will win the auction. However, by simple calculation, $y - z_1 < 0$, we can find that its profit is negative;

(3) $y < z'' < z_1$, supplier 1 still loses the auction and cannot obtain more profit.

Summarizing the above analysis, we see that supplier 1 will not bid less than z_1. On the other hand, if supplier 1 bids an amount $z'' > z_1$. There are also three cases:

(1) $z_1 < z'' < y$, supplier 1 still wins without obtaining more profit. However, its winning probability will decrease instead;

(2) $z_1 < y < z''$, supplier 1 will lose the auction and its profit decreases;

(3) $y < z_1 < z''$, supplier 1 still loses the auction and cannot obtain more profit.

The above means supplier will never bid larger than z_i. Thus, we have proved that supplier 1's dominant strategy is to bid z_i. $\qquad\square$

Proposition 2 implies that the dominant strategy for a supplier is to bid its virtual cost in the second-price reverse auction. As the virtual cost is the expected cost of successfully delivering one unit of the product to the buyer, the second-price reverse auction is incentive compatible in this context. This extends the results in the traditional price-only auction without considering supply disruption risk to the case where supply disruption is taken into consideration.

Note that the wholesale price is $w^{P_2}(c_i, \theta_i) = E[Y_i | z_i < Y_i]$ if supplier i is the winner in the second-price format. So the expected profit the buyer can make from supplier i, if it wins the auction, is

$$\Pi_M^{P_2} = (1 - \theta_i)(r - E[Y_i | z_i < Y_i]) - \theta_i l. \tag{10}$$

By similar arguments in Sec. 3.1, we obtain the buyer's *ex ante* expected profit as follows:

$$E[\Pi_M^{P_2}] = N \int_0^1 \int_0^{\omega_c} \left(1 - G\left(\frac{c}{1 - \theta}\right)\right)$$
$$\times \left((1 - \theta)(r - w^{P_2}(c, \theta)) - \theta l\right) f_c(c) f_\theta(\theta) dc d\theta. \tag{11}$$

3.3. *Performance analysis*

By comparing the *ex ante* expected profits in the first-price and second-price format price-only auctions, we have the following proposition. It states that the revenue equivalence principle holds in the price-only auction with supply disruption.

Proposition 3 (Revenue Equivalence). *In the price-only reverse auction with supply disruption, both the first- and second-price formats generate the same ex ante expected profit for the buyer.*

Proof. We prove the proposition by showing that $w^{P_1}(c, \theta) = w^{P_2}(c, \theta)$. From the analysis from Secs. 3.1 and 3.2, we know that the expected payment in the first-price auction is

$$w^{P_1}(c, \theta) = \frac{1}{1 - G(z_i)} \int_{z_i}^{\infty} xg(x)dx.$$

While the expected payment in the second-price auction is

$$w^{P_2}(c_i, \theta_i) = E[Y_i | z_i < Y_i]$$

$$= \frac{\int_{z_i}^{\infty} tg(t)dt}{\text{Pro}(Y_i > z_i)}$$

$$= \frac{\int_{z_i}^{\infty} xg(x)dx}{1 - G(z_i)}$$

$$= w^{P_1}(c, \theta).$$

Then the proposition is proved. □

As pointed out by Krishna (2009), the "revenue equivalence principle" is very important for a firm to choose the auction format. It is well known that the "revenue equivalence principle" holds if we do not consider supply disruption. Che (1993) and David *et al.* (2006) extended the principle to the case where quality is the nonprice attribute. Although they considered the multi-attribute reverse auction, they only study one uncertain parameter that is known privately to the suppliers. In other words, the suppliers' private information is one-dimensional. Proposition 3 extends the "revenue equivalence principle" to the case where suppliers' production costs and disruption probabilities are private information.

It is well known that the price-only auction is efficient, i.e., the supplier with the lowest cost is chosen as the winner when suppliers are reliable and the private information is their production costs. If we consider supply disruption risk, the private information owned by suppliers contains their supply disruption probabilities. Is the price-only auction still efficient for this case? This is an important question that needs to be answered. Note that a supplier is called the most efficient supplier among a set of suppliers if it generates the highest social welfare, i.e., the highest

Table 1. A simple example.

Supplier i	Production cost c_i	Disruption probability θ_i	Virtual cost z_i
1	0.79	0.2	0.9875
2	0.9	0.1	1
3	0.88	0.2	1.1

expected total profit of the supply chain consisting of the buyer and the supplier. A mechanism is called efficient if it selects the most efficient supplier as the winner (Xu and Huang, 2017).

Obviously, a supplier with the lowest production cost as well as the lowest disruption probability is the most efficient supplier by definition. However, if there is no such supplier in a set of suppliers, it is not obvious as to which supplier is the most efficient supplier between one that has a low cost but a high disruption probability and another that has a high cost but a low disruption probability.

The following example illustrates that the price-only auction may not be efficient for the case where there is potential supply disruption risk. Since the first- and second-price formats are equivalent, we consider only the second-price format. In this case, supplier i's equilibrium strategy is $\beta(z_i) = z_i = \frac{c_i}{1-\theta_i}$.

Example 1. Suppose that there are three suppliers with unit production costs and supply disruption probabilities given in Table 1. The unit revenue and loss are $r = 2$ and $l = 1$, respectively.

It is obvious that supplier 1 will win the auction. The buyer's profit is $1 \times [(1 - 0.2)(2 - 1) - 0.2] = 0.6$, the supplier's profit is $1 \times [(1 - 0.2)1 - 0.79] = 0.01$, and the total profit of the supply chain is 0.61.

However, if supplier 2 secures the contract, the buyer earns $1 \times [(1 - 0.1)(2 - 1.1) - 0.1] = 0.71$ and the supplier's profit is $1 \times [(1 - 0.1)1.1 - 0.88] = 0.11$. The total profit of the supply chain is $0.82 > 0.61$. This shows that the price-only auction cannot select the most efficient supplier as the winner, so it is not efficient.

The reason is that supplier 1, which has a low production cost but a high disruption probability, bids low and wins the contract because there is no penalty for the undelivered order in the price-only auction. In order to overcome the inefficiency of the price-only auction, we propose the *announced penalty reverse auction* (APRA) as a new auction mechanism in the following section.

4. Announced Penalty Reverse Auction

In this section we consider a new reverse auction mechanism, i.e., the APRA, and analyze its performance. As pointed out before, the supplier with a low production cost but a high disruption probability might bid low and win the auction because there is no penalty for the winner if it is disrupted and cannot deliver the product in

the price-only auction. This will result in inefficiency of the mechanism. A common and naive solution is to announce a penalty in advance if the winner fails to execute the contract. Based on this motivation, we propose the APRA to help the buyer to choose the most efficient supplier and manage supply disruption risk.

The APRA works as follows: First, the buyer announces a unit penalty p_0 for the undelivered order and the auction format (the first- or the second-price). Then, each supplier bids its wholesale price. Third, the supplier with the lowest bid will be selected as the winner and sign a contract with the buyer according to the first- or second-price format. Fourth, the winner delivers the product and receives the payment from the buyer if no disruption happens, or pays the penalty to the buyer otherwise. Fifth, the buyer meets its demand if the order is delivered successfully or suffers a demand loss if the supply disrupts and no product is delivered to the buyer.

We first analyze the suppliers' equilibrium bidding strategies (here we only study the symmetric bidding strategy), the buyer's *ex ante* expected profits in the first- and second-price auction formats, and the "revenue equivalence principle". Then we study how to choose the optimal penalty and discuss the efficiency of the mechanism.

4.1. *First-price format APRA*

In this subsection we study the suppliers' equilibrium bidding strategies and the buyer's expected profit for the first-price APRA. For this case, the wholesale price will be the winner's bid when the winner successfully delivers the order. Then for any given penalty p_0, supplier i's payoff function is

$$\Pi_i^{A_1,p_0}(b_i) = \begin{cases} (1-\theta_i)b_i - c_i - \theta_i p_0 & \text{if } b_i < \min_{j \neq i} b_j, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where the superscript A_1 indicates the first-price APRA.

For convenience, we define supplier i's virtual cost with penalty p_0 as $\hat{z}_{p_0,i} = \frac{c_i + \theta_i p_0}{1 - \theta_i}$ when the penalty for no delivery is p_0. Let the random variable $X_{p_0,i}$ denotes the lowest virtual cost except supplier i, i.e., its realization is $x_{p_0,i} = \min_{j \neq i} \frac{c_j + \theta_j p_0}{1 - \theta_j}$, with its distribution function H, and density function h. By similar arguments as previous section, H and h are common knowledge.

Similar to the virtual cost defined in Sec. 3.1, supplier i's virtual cost with penalty p_0, $\hat{z}_{p_0,i}$, represents the expected unit cost (including the penalty paid to the buyer) if it wins the auction. It is increasing in the unit production cost c_i, the penalty p_0, and the disruption probability θ_i.

The following proposition characterizes the suppliers' equilibrium bidding strategies.

Proposition 4. *Supplier i's equilibrium bidding strategy in the first-price ARPA is*

$$\beta^{A_1,p_0}(\hat{z}_{p_0,i}) = \frac{1}{1 - H(\hat{z}_{p_0,i})} \int_{\hat{z}_{p_0,i}}^{\infty} x h(x) dx. \tag{13}$$

Proof. We assume that the bidding strategy of supplier i in equilibrium is $\beta^{A_1,p_0}(c_i, \theta_i) = b_i$. Note that the expected payoff function of supplier i when its private information is (c_i, θ_i) can be rewritten as follows:

$$\Pi_i^{A_1,p_0}(b_i) = \begin{cases} (1-\theta_i)(b_i - \hat{z}_{p_0,i}) & \text{if } b_i < \min_{j \neq i} b_j, \\ 0 & \text{otherwise.} \end{cases}$$

By similar arguments to the proof of Proposition 1, we can derive that supplier i's equilibrium bidding strategy is

$$\beta^{A_1,p_0}(c_i, \theta_i) = b_i = \beta^{A_1,p_0}(\hat{z}_{p_0,i}) = \frac{1}{1 - H(\hat{z}_{p_0,i})} \int_{\hat{z}_{p_0,i}}^{\infty} x h(x) dx. \qquad \square$$

Proposition 4 means that supplier i's bidding policy depends on (c_i, θ_i) and p_0 through supplier i's virtual cost with penalty p_0, $\hat{z}_{p_0,i} = \frac{c_i + \theta_i p_0}{1 - \theta_i}$. This implies that two suppliers will bid the same wholesale price if they have the same virtual cost although they have different private information (c_i, θ_i).

Now, we turn to the buyer's problem. If the winner is supplier i, then the buyer's expected profit is

$$\Pi_M^{A_1,p_0} = (1 - \theta_i)(r - w^{A_1,p_0}(c_i, \theta_i)) - \theta_i(l - p_0). \qquad (14)$$

For the first-price format, we have $w^{A_1,p_0}(c_i, \theta_i) = \beta^{A_1,p_0}(\hat{z}_{p_0,i})$. Then the total expected profit of the buyer is

$$E[\Pi_M^{A_1,p_0}] = N \int_0^1 \int_0^{\omega_c} \left(1 - H\left(\frac{c + \theta p_0}{1 - \theta}\right)\right)$$

$$\times ((1 - \theta)(r - w^{A_1,p_0}(c, \theta)) - \theta(l - p_0)) f_c(c) f_\theta(\theta) dc d\theta. \qquad (15)$$

4.2. *Second-price format APRA*

In this subsection we discuss the suppliers' equilibrium bidding strategies and the buyer's expected profit in the second-price APRA. For this case, the lowest bid will win the auction but the wholesale price will be the second lowest bid. Then, for any given penalty p_0, supplier i's payoff function is

$$\Pi_i^{A_2,p_0}(b_i) = \begin{cases} (1 - \theta_i)\bar{b}_{-i} - c_i - \theta_i p_0 & \text{if } b_i < \min_{j \neq i} b_j, \\ 0 & \text{otherwise,} \end{cases} \qquad (16)$$

where $\bar{b}_{-i}$ denotes the lowest bid among the other $N - 1$ suppliers except i and A_2 indicates the second-price APAR. Proposition 5 characterizes the suppliers' equilibrium bidding strategies.

Proposition 5. *Supplier i's equilibrium bidding strategy in the second-price APRA is given by*

$$\beta^{A_2,p_0}(c_i, \theta_i) = \hat{z}_{p_0,i} = \frac{c_i + \theta_i p_0}{1 - \theta_i}. \qquad (17)$$

Proof. The proof is similar to that of Proposition 2. Here, we only sketch the procedure. Consider supplier 1, say, and suppose that $y = \min\{b_j \,|\, j \neq 1\}$ is the second lowest bid. By bidding $\hat{z}_{p_0,1}$, supplier 1 wins the auction if $\hat{z}_{p_0,1} < y$ and loses the auction otherwise. Suppose, however, that it bids another number $z'' < \hat{z}_{p_0,1}$. If $z'' < \hat{z}_{p_0,1} < y$, then it still wins the auction and makes a marginal profit $y - \hat{z}_{p_0,1}$ for every successful product delivery. If $z'' < y < \hat{z}_{p_0,1}$, it will win the auction but obtains a negative profit. If $y < z'' < \hat{z}_{p_0,1}$, it still loses the auction. Then we know that bidding less than $\hat{z}_{p_0,1}$ cannot generate more profit. Similarly, we can show that it is not profitable for supplier 1 to bid more than $\hat{z}_{p_0,1}$, either. Then supplier 1's dominant strategy is to bid $\hat{z}_{p_0,1}$. □

This proposition indicates that "telling the truth (bidding the expected unit cost)" is always a dominant strategy for all the suppliers, i.e., the second-price ARPA is "incentive compatible". Although the buyer cannot get the exact cost and reliability information of the suppliers, the buyer knows the suppliers' expected unit costs for successful product delivery.

If the winner is suppler i, the buyer's expected profit is

$$\Pi_M^{A_2,p_0} = (1 - \theta_i)\left(r - w^{A_2,p_0}(c_i, \theta_i)\right) - \theta_i(l - p_0). \tag{18}$$

For the second-price format, we have $w^{A_2,p_0}(c_i, \theta_i) = E[X_{p_0,i} \,|\, X_{p_0,i} \geq \hat{z}_{p_0,i}]$. Then the buyer's expected total profit is

$$E[\Pi_M^{A_2,p_0}] = N \int_0^1 \int_0^{\omega_c} \left(1 - H\left(\frac{c + \theta p_0}{1 - \theta}\right)\right)$$
$$\times \left((1 - \theta)(r - w^{A_2,p_0}(c, \theta)) - \theta(l - p_0)\right) f_c(c) f_\theta(\theta) dc d\theta. \tag{19}$$

4.3. *APRA performance analysis*

In this subsection we focus on comparing the first- and second-price formats, the impact of the penalty p_0 on the suppliers' bidding strategies, and the winner determination problem. We also discuss the efficiency of the mechanism.

Compared with the price-only auction, the APRA has a similar structure of the bidding strategies in equilibrium. In the price-only auction, z_i plays a key role in determining the winner of the auction and the probability of wining the auction, while $\hat{z}_{p_0,i}$ plays the same role in the APRA.

First, we show that "revenue equivalence principle" holds in the APRA.

Proposition 6. *The first- and second-price APRAs generate the same ex ante expected profit for the buyer for a given penalty p_0. In addition, the same supplier will be selected as the winner under these two mechanisms.*

Proof. Replacing z_i by $\hat{z}_{p_0,i}$ and by similar arguments to those used in the proof of Proposition 3, we can prove the proposition. □

Proposition 6 means that the first- and second-price APRAs generate the same *ex ante* expected profit to the buyer. Hence, the buyer can choose either auction format when it designs its procurement mechanism.

Now, we study the impact of the penalty p_0 on the suppliers' bidding strategies. Since the first- and second-price APRAs are equivalent, we consider only the second-price format.

Proposition 7. *Compared to the price-only auction,*

(1) *For any given positive penalty p_0, supplier i's bid in the APRA is larger than that in the price-only reverse auction.*
(2) *Suppliers' bids in the APRA increase in the penalty p_0.*
(3) *The expected bid that wins the auction (as well as the expected wholesale price paid by the buyer) in the APRA is greater than that in the price-only reverse auction.*

Proof. From Proposition 5, supplier i's biding strategy is $\beta^{A_2,p_0}(c_i, \theta_i) = \hat{z}_{p_0,i} = \frac{c_i + \theta_i p_0}{1-\theta_i}$. Since $0 < \theta_i < 1$, $\hat{z}_{p_0,i}$ is increasing in the penalty p_0. When $p_0 = 0$, the APRA reduces to the price-only reverse auction. Thus, the first two parts hold.

Suppose that supplier 1 (with bid $z_1 = \frac{c_1}{1-\theta_1}$) wins in the price-only auction and supplier k (with bid $\hat{z}_{p_0,k} = \frac{c_k + \theta_k p_0}{1-\theta_k}, 1 \le k \le N$) wins in the APRA. We have $z_1 = \frac{c_1}{1-\theta_1} < \frac{c_k}{1-\theta_k} \le \frac{c_k + \theta_k p_0}{1-\theta_k}$. The bid that wins the auction for the APRA is greater than that for the price-only reverse auction. Then the last part holds. $\square$

Next, we study the efficiency of the APRA.

Proposition 8. *For any given pair of suppliers j and k with $c_j < c_k$ and $\theta_j > \theta_k$, there exists a unique $\bar{p}_0$ such that $\beta^{A_2,p_0}(\hat{z}_{p_0,j}) > \beta^{A_2,p_0}(\hat{z}_{p_0,k})$ for all $p_0 > \bar{p}_0$.*

Proof. Also by Proposition 5, we have $\beta^{A_2,p_0}(c_i, \theta_i) = \hat{z}_{p_0,i}$. If we view $\hat{z}_{p_0,j}$ and $\hat{z}_{p_0,k}$ as two affine functions with respect to p_0, then their slopes are $\frac{\theta_j}{1-\theta_j}$ and $\frac{\theta_k}{1-\theta_k}$, respectively. Since $\theta_j > \theta_k$, we have $\frac{\theta_j}{1-\theta_j} > \frac{\theta_k}{1-\theta_k}$. Let $\bar{p}_0 = \frac{(1-\theta_k)c_j - (1-\theta_j)c_k}{\theta_k - \theta_j}$, then we have $\beta^{A_2,p_0}(\hat{z}_{p_0,j}) > \beta^{A_2,p_0}(\hat{z}_{p_0,k})$ for all $p_0 > \bar{p}_0$. $\square$

From Proposition 7, we know that the suppliers' bids and the wholesale price paid by the buyer in the APRA are greater than those in the price-only reverse auction. Moreover, the larger p_0 is, the larger the bids are and the more the buyer will pay. However, Proposition 8 shows that we can select a more reliable supplier by increasing the announced penalty p_0. On the other hand, the buyer has to pay more if it chooses a more reliable supplier. Then the buyer can set a proper p_0 to select the supplier with a proper production cost and supply reliability as the winner as it wished. The buyer can set a small p_0 if it prefers a low cost supplier or

set a large p_0 if it prefers a reliable supplier. The announced penalty p_0 plays a key role in balancing the cost and supply reliability.

Assumption 1. The suppliers' real production costs and disruption probabilities satisfy $c_1 \leq c_2 \leq \cdots \leq c_N$ and $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_N$.

Please note that if supplier i is dominated by supplier j (i.e., $c_j \leq c_i$ and $\theta_j \leq \theta_i$), supplier i cannot win the auction for any p_0 in our mechanism. Therefore, we could exclude all such suppliers that are dominated by some suppliers from the supplier pool. The rest suppliers satisfy Assumption 1, and for any p_0, the result will be the same when compared to original auction without exclusion. Because of this, we make this assumption to simply the analysis.

Proposition 9. *Suppose that Assumption 1 holds and let supplier k be the most efficient supplier*

(1) *If $k = 1$ or N, then there exists a unique p_0^* such that the APRA is efficient.*
(2) *If $1 < k < N$ and $0 \leq \frac{(1-\theta_k)c_i - (1-\theta_i)c_k}{\theta_k - \theta_i} \leq \frac{(1-\theta_k)c_j - (1-\theta_j)c_k}{\theta_k - \theta_j}$ for any given supplier $i(i < k)$ and supplier $j(j > k)$, then there exists $p_0^* > 0$ such that the APRA is efficient.*

Proof. For a given announced penalty p_0, when supplier m is selected, the total profit of the supply chain consisting of supplier m and the buyer is

$$S(p_0) = (1 - \theta_m)(r - w^A) - \theta_m(l - p_0) + ((1 - \theta_m)w^A - c_m) - \theta_m p_0$$

$$= (1 - \theta_m)r - c_m - \theta_m l. \tag{20}$$

Since supplier k is the most efficient supplier, for any supplier $m \neq k$, we have

$$(1 - \theta_k)r - c_k - \theta_k l \geq (1 - \theta_m)r - c_m - \theta_m l, \tag{21}$$

i.e.,

$$c_m - c_k \geq (r + l)(\theta_k - \theta_m). \tag{22}$$

Now, we prove that there exists a penalty p_0 such that the APRA is efficient. Note that if the APRA with p_0 is efficient, for any $m \neq k$, we must have

$$\frac{c_k + \theta_k p_0}{1 - \theta_k} \leq \frac{c_m + \theta_m p_0}{1 - \theta_m}. \tag{23}$$

Now, we discuss the choice of p_0. There are two cases:

(1) $i < k$. From Proposition 8, we have that supplier k dominates all the other suppliers for all $p_0 \geq \bar{p}_0 = \frac{(1-\theta_k)c_i - (1-\theta_i)c_k}{\theta_k - \theta_i}$;
(2) $j > k$. Also from Proposition 8, we have that supplier k dominates all the other suppliers for all $p_0 \leq \bar{p}_0' = \frac{(1-\theta_k)c_j - (1-\theta_j)c_k}{\theta_k - \theta_j}$.

Table 2. The suppliers' production cost and disruption probabilities.

Supplier	Production cost	Disruption probability
i	17.5	0.2
k	17.6	0.18
j	18.6	0.1
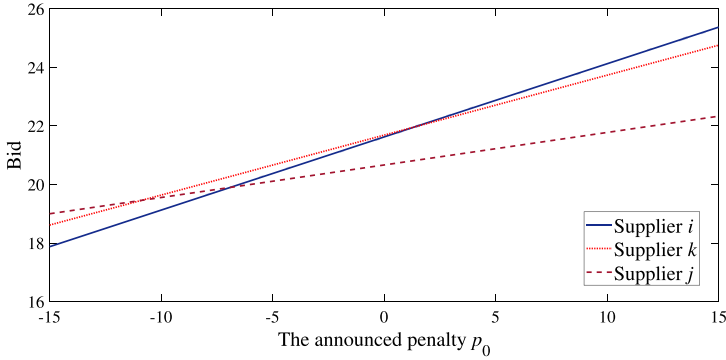
Fig. 2. The bidding strategies with respect to p_0.

Thus, the condition $0 \leq \bar{p}_0 \leq \bar{p}_0'$ ensures that we can choose p_0 $(\bar{p}_0 \leq p_0 \leq \bar{p}_0')$ such that the APRA is efficient. □

Example 2. Suppose that there are three suppliers, whose production costs and disruption probabilities are given by Table 2.

It is easy to check that supplier k is the most efficient. But the condition $0 \leq \frac{(1-\theta_k)c_i-(1-\theta_i)c_k}{\theta_k-\theta_i} \leq \frac{(1-\theta_k)c_j-(1-\theta_j)c_k}{\theta_k-\theta_j}$ does not hold. Figure 2 illustrates the suppliers' equilibrium strategies with respect to the change of p_0. From Fig. 2, we see that supplier k will never win in the APRA for any $p_0 \geq 0$. So the APRA is not efficient for any $p_0 \geq 0$.

5. Numerical Studies

In this section, we conducted numerical studies to explore the impacts of some parameters on the buyer's *ex ante* expected profit and the social welfare for the proposed auctions. We also investigate the impact of the pre-announced penalty on winner determination. We set the parameters as follows: $N = 5$, $r = 5$, $l = 2$, $\omega_c = 3$, and $p_0 = 4$. Random variable C and Θ are uniform distributed on $(0, \omega_c)$ and $(0, 1)$, respectively.

First, we study how the announced penalty p_0 affects the buyer's *ex ante* expected profit and the social welfare. Figure 3 shows the results. From Fig. 3, we see that the buyer's *ex ante* expected profit first increases in p_0 and then decreases
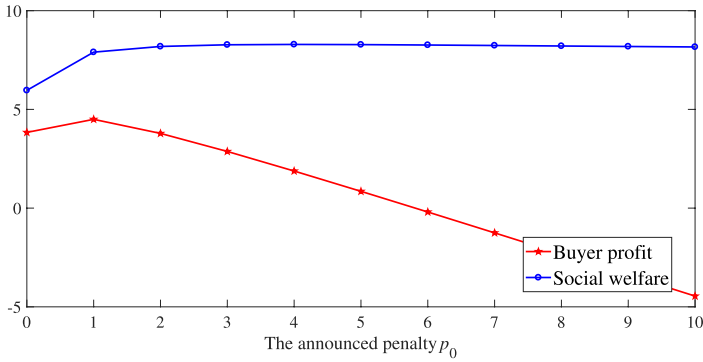
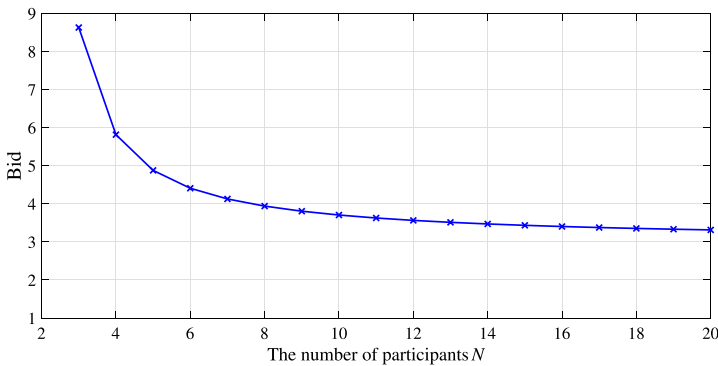Fig. 3. The impact of penalty on the buyer's expected profit and social welfare.

Fig. 4. The impact of number of suppliers on suppliers' bids.

in p_0, and the social welfare first increases in p_0 and then decreases slowly. This means that a proper announced penalty can help the buyer choose the most efficient supplier. However, if p_0 is too large, the buyer will pay more to the winner since a large p_0 will force suppliers to make large bids (Proposition 7). As shown in Table 4, we can easily calculate the optimal p_0 for the second-price APRA.

Next we study the impacts of the number of suppliers on the suppliers' bids, the buyer's expected profit, and the social welfare. As we know, competition intensity increases in the number of suppliers. Figures 4 and 5 show the results.

Figure 4 shows that the supplier's bid decreases quickly first and then changes slowly. Figure 5 shows that the buyer's *ex ante* expected profit is increasing rapidly at first and then slows down as the number of suppliers increases. This means that proper competition can bring profit to the buyer. So the buyer should maintain a relative large pool of potential suppliers, but need not keep a huge set of suppliers because keeping suppliers will result in cost but bring little marginal profit to the buyer. From Fig. 5, we also see that the social welfare increases in the number of
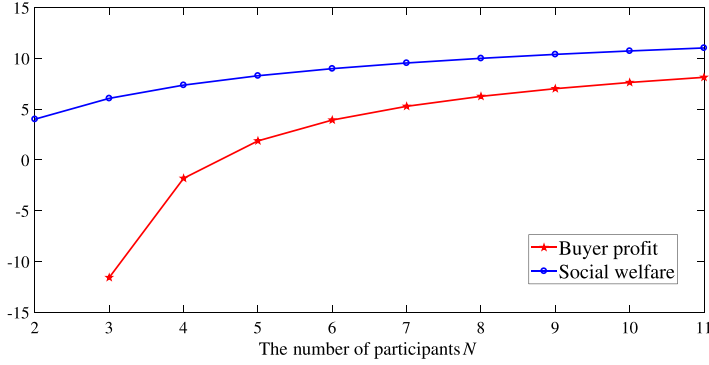
Fig. 5. The impact of number of suppliers on the buyer's expected profit and social welfare.

suppliers. Competition can also increase social welfare and increase social efficiency. In addition, the profit of the supplier, i.e., the gap between the social welfare/profit and the buyer's profit, decreases as the number of suppliers increases. This means that fierce competition can help the buyer extort suppliers' profit.

We next examine the determinant of the winner with regarding to the pre-announced penalty p_0. Consider five suppliers, whose unit production costs and disruption reliabilities are given in Table 3. Column 3 gives the social welfare when the supplier is the winner of the auction. Obviously, supplier 2 is the most efficient.

We only consider the second-price APRA for convenience since the first- and secondprice APRAs are equivalent. Table 4 shows the winner of the auction with respect to different penalties.

From Table 4, we could find that the APRA could select different suppliers as winners with different penalties. Combined with the results in Table 3, it shows that the most efficient supplier, Supplier 2, will win the auction only when

Table 3. Suppliers' information.

Supplier	Production cost	Disruption probability	Social welfare
1	2.81	0.031	1.973
2	1.85	0.141	2.163*
3	1.51	0.229	1.887
4	0.84	0.287	2.151
5	0.67	0.367	1.761

Note: *The most efficient supplier.

Table 4. Winner of the auction.

Penalty	$[0, 0.675)$	$(0.675, 4.092)$	$(4.0925.647)$*	$(5.647, \infty)$
Winner	Supplier 5	Supplier 4	Supplier 2	Supplier 1

Note: *The APRA achieves social efficiency.

$p_0 \in (4.092, 5.647)$, which means the social efficiency is achieved. When the penalty is small, the suppliers with low production costs (e.g., Supplier 5 and 4) are more competitive in the auction. On the other hand, when the penalty is large, the suppliers with low disruption probabilities (e.g., Supplier 2 and 1) will win the auction. As a result, the buyer can choose its preferred supplier by adjusting the pre-announced penalty p_0. The more reliable the buyer prefers, the larger p_0 should be set.

6. Conclusions

In this paper we study the procurement auction design problem considering supply disruption. We first consider the price-only auction in which the buyer ignores the supply disruption risk when it designs the procurement mechanism and there is no penalty when the winner cannot deliver the product. We characterize the suppliers' equilibrium bidding strategies and show that the first-price and second-price auctions generate the same *ex ante* expected profit to the buyer. That is, the well known "revenue equivalence principle" holds. We give an example to show that the price-only auction is not efficient. We then propose the APRA as a new and easy-to-implement auction mechanism. We show that the "revenue equivalence principle" holds for the APRA and it is efficient. We provide numerical results to show the impacts of the penalty parameter and the number of suppliers on the buyer's and social *ex ante* expected profits, as well as the suppliers' bids.

There are some research topics that can be further studied as follows: in this work we assume that the order is indivisible and only one supplier is selected as the winner. As supplier diversification is a commonly used method to mitigate supply risk, extending the results in this paper to the case where the order is divisible and more than one supplier can be selected as the winner is an interesting and challenging topic.

In this study suppliers only bid the wholesale price although we consider two attributes, i.e., the price and the supply reliability. We use a penalty to force the supplier with a low production cost but a high disruption risk not to bid too low. In this way, we can select an efficient supplier as the winner. We can consider another mechanism, e.g., the VCG-type auction. The suppliers bid their production costs and disruption probabilities, and the winner and wholesale prices are specified by the auction.

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