An iterative linear matrix inequality controller-design strategy for H_∞ structural vibration control

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ABSTRACT: In this paper, we present a novel Iterative Linear Matrix Inequality (ILMI) strategy for controller design that makes it possible to compute suboptimal H_∞ static output-feedback (SOF) controllers with high-performance characteristics. The obtained SOF controllers can be effective in reducing the vibrational response of multi-degree-of-freedom structures subjected to broad-band excitations. To demonstrate the effectiveness of the proposed methodology, a SOF controller is designed for the seismic protection of a multi-actuated five-story shear-frame structure with positive results.

1 INTRODUCTION

Static output-feedback (SOF) control strategies have shown a significant performance for structural vibration control of multi-degree-of-freedom (MDOF) systems. In this approach, a reduced set of sensors is used to obtain a convenient vector of measured outputs $\mathbf{y}(t)$, and the vector of control actions $\mathbf{u}(t)$ is computed by means of a simple matrix product in the form $\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t)$, where $\mathbf{K}$ is a constant matrix (Li and Adeli 2018). Designing a proper output-feedback control gain-matrix $\mathbf{K}$ raises challenging issues in theory and computation, which have attracted extensive research attention. An effective computational tool for this problem is the function `hinfstruct` included in the *Matlab Robust Control Toolbox* (Balas et al. 2018), which allows designing SOF controllers following an H_∞ approach. In recent works, we have been devising a Linear Matrix Inequality (LMI) controller-design strategy that has produced positive results in obtaining effective SOF controllers for the seismic protection of multistory buildings equipped with a system of distributed interstory actuators (Palacios-Quiñonero et al. 2016, Palacios-Quiñonero et al. 2017). This strategy includes a parameter-matrix $\mathbf{L}$ that can be arbitrarily selected in the controller design to enhance the system performance (Palacios-Quiñonero et al. 2014). In this paper, we present a novel Iterative Linear Matrix Inequality (ILMI) procedure that facilitates computing suitable values for the parameter-matrix $\mathbf{L}$ and, consequently, allows obtaining improved SOF controllers. To demonstrate the effectiveness of the proposed methodology in structural vibration control, an H_∞ SOF controller is designed for the seismic protection of a five-story building equipped with a distributed set of interstory actuators. As a reference, two additional controllers are also computed: (i) an H_∞ state-feedback controller with full-state information, and (ii) an H_∞ SOF controller obtained with the Matlab function `hinfstruct`. The content of the rest of the paper is as follows: In Section 2, the LMI design of H_∞ state-feedback and SOF controllers is summarized. In Section 3, the proposed ILMI design procedure is presented. In Section 4, the building model is introduced and the different controllers are computed. Also, the corresponding frequency and time responses are obtained and compared. Finally, some brief conclusions are provided in Section 5.

2 H_∞ CONTROLLERS DESIGN

2.1 *State-feedback controller*

Let us consider a dynamical system described by the state-space model

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{w}(t) \\ \mathbf{z}(t) = \mathbf{C}_z\mathbf{x}(t) + \mathbf{D}_z\mathbf{u}(t) \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state, $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the control input, $\mathbf{w}(t) \in \mathbb{R}^{n_w}$ is the exogenous disturbance, $\mathbf{z}(t) \in \mathbb{R}^{n_z}$ is the controlled output, and $\mathbf{A} \in \mathbb{R}^{n_x \times n_x}$, $\mathbf{B} \in \mathbb{R}^{n_x \times n_u}$, $\mathbf{E} \in \mathbb{R}^{n_x \times n_w}$, $\mathbf{C}_z \in \mathbb{R}^{n_z \times n_x}$ and $\mathbf{D}_z \in \mathbb{R}^{n_z \times n_u}$ are constant matrices. In the H_∞ controller-design approach, the performance of a state-feedback controller $\mathbf{u}(t) = \mathbf{G}\mathbf{x}(t)$ with state gain-matrix $\mathbf{G} \in \mathbb{R}^{n_u \times n_x}$ is evaluated by means of the H_∞ system-norm

$$\gamma(\mathbf{G}) = \sup_{\|\mathbf{w}\|_2 \neq 0} \frac{\|\mathbf{z}\|_2}{\|\mathbf{w}\|_2}, \tag{2}$$

where $\|\mathbf{f}\|_2 = \left[\int_0^\infty \mathbf{f}^T(t)\mathbf{f}(t)dt\right]^{1/2}$ is the usual continuous 2-norm. The γ-value in (2) can be computed in the frequency domain by solving the optimization problem

$$\gamma(\mathbf{G}) = \sup_{\omega \in \mathbb{R}} \left\{\sigma_{\max}\left[\mathbf{F}_G(\omega)\right]\right\}, \tag{3}$$

where ω is the frequency in Hz, $\sigma_{\max}[\cdot]$ denotes the maximum singular value, and $\mathbf{F}_G(\omega)$ is the Frequency Response Function (FRF)

$$\mathbf{F}_G(\omega) = \mathbf{C}_G(2\pi\omega j\mathbf{I}_{n_x} - \mathbf{A}_G)^{-1}\mathbf{E}, \tag{4}$$

with $j = \sqrt{-1}$, $\mathbf{A}_G = \mathbf{A} + \mathbf{B}\mathbf{G}$ and $\mathbf{C}_G = \mathbf{C}_z + \mathbf{D}_z\mathbf{G}$.

According to the Bounded Real Lemma (Boyd et al. 1994), an optimal H_∞ state-feedback controller $\mathbf{u}(t) = \widehat{\mathbf{G}}\mathbf{x}(t)$ can be obtained by solving the following optimization problem with Matrix Inequality (MI) constraints:

$$\mathcal{P}_s : \begin{cases} \text{minimize } \gamma_s \\ \text{subject to } \mathbf{X} > 0,\ \gamma_s > 0 \text{ and the MI in } (5) \end{cases}$$

$$\begin{bmatrix} \mathrm{sym}(\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{G}\mathbf{X}) & * & * \\ \mathbf{E}^T & -\gamma_s\mathbf{I}_{n_w} & * \\ \mathbf{C}_z\mathbf{X} + \mathbf{D}_z\mathbf{G}\mathbf{X} & [\mathbf{0}]_{n_z \times n_w} & -\gamma_s\mathbf{I}_{n_z} \end{bmatrix} < 0, \tag{5}$$

where $\mathbf{X} = \mathbf{X}^T$ and $\mathbf{G}$ are variable matrices, sym($\mathbf{M}$) denotes $\mathbf{M} + \mathbf{M}^T$, $[\mathbf{0}]_{m\times n}$ is a zero matrix of dimension $m \times n$, $\mathbf{I}_n$ is an identity matrix of dimension n, and * represents the transpose of the element in the symmetric position. By means of the substitution $\mathbf{G}\mathbf{X} = \mathbf{Y}$, $\mathcal{P}_s$ can be transformed into the following Linear Matrix Inequality (LMI) optimization problem:

$$\mathcal{P}_s' : \begin{cases} \text{minimize } \gamma_s \\ \text{subject to } \mathbf{X} > 0,\ \gamma_s > 0 \text{ and the LMI in } (6) \end{cases}$$

$$\begin{bmatrix} \mathrm{sym}(\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{Y}) & * & * \\ \mathbf{E}^T & -\gamma_s\mathbf{I}_{n_w} & * \\ \mathbf{C}_z\mathbf{X} + \mathbf{D}_z\mathbf{Y} & [\mathbf{0}]_{n_z \times n_w} & -\gamma_s\mathbf{I}_{n_z} \end{bmatrix} < 0, \tag{6}$$

where $\mathbf{X} = \mathbf{X}^T$ and $\mathbf{Y}$ are variable matrices. If the LMI optimization problem $\mathcal{P}_s'$ can be successfully solved, attaining an optimal value $\widehat{\gamma}_s$ for the matrices $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$, then the state-feedback control matrix $\widehat{\mathbf{G}} = \widehat{\mathbf{Y}}\widehat{\mathbf{X}}^{-1}$ defines an optimal H_∞ state-feedback controller with an asymptotically stable closed-loop matrix $\mathbf{A}_{\widehat{G}}$ and associated H_∞-norm $\gamma(\widehat{\mathbf{G}}) = \widehat{\gamma}_s$.

2.2 *Static output-feedback controller*

In the output-feedback approach, we assume that the state information $\mathbf{x}(t)$ cannot be directly accessed and we consider a vector of $n_y \leq n_x$ measured outputs $\mathbf{y}(t) = \mathbf{C}_y\mathbf{x}(t)$, which can be expressed as linear combinations of the states using the observed-output matrix $\mathbf{C}_y \in \mathbb{R}^{n_y \times n_x}$. In this case, we want to obtain a SOF controller

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t), \tag{7}$$

which allows computing the control actions from the measured-output information by means of a constant output gain-matrix $\mathbf{K} \in \mathbb{R}^{n_u \times n_y}$. The SOF controller in (7) defines the FRF function

$$\mathbf{F}_K(\omega) = \mathbf{C}_K(2\pi\omega j\mathbf{I}_{n_x} - \mathbf{A}_K)^{-1}\mathbf{E}, \tag{8}$$

with $\mathbf{A}_K = \mathbf{A} + \mathbf{B}\mathbf{K}\mathbf{C}_y$ and $\mathbf{C}_K = \mathbf{C}_z + \mathbf{D}_z\mathbf{K}\mathbf{C}_y$, and the corresponding H_∞-norm $\gamma(\mathbf{K})$ can be obtained by maximizing $\sigma_{\max}[\mathbf{F}_K(\omega)]$. If we proceed as in the previous section to design an optimal H_∞ SOF controller, we obtain the following optimization problem with MI constraints:

$$\mathcal{P}_o : \begin{cases} \text{minimize } \gamma_o \\ \text{subject to } \mathbf{X} > 0,\ \gamma_o > 0 \text{ and the MI in } (9) \end{cases}$$

$$\begin{bmatrix} \mathrm{sym}(\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{K}\mathbf{C}_y\mathbf{X}) & * & * \\ \mathbf{E}^T & -\gamma_o\mathbf{I}_{n_w} & * \\ \mathbf{C}_z\mathbf{X} + \mathbf{D}_z\mathbf{K}\mathbf{C}_y\mathbf{X} & [\mathbf{0}]_{n_z \times n_w} & -\gamma_o\mathbf{I}_{n_z} \end{bmatrix} < 0, \tag{9}$$

where $\mathbf{X} = \mathbf{X}^T$ and $\mathbf{K}$ are variable matrices. However, it should be noted that the observed-output matrix $\mathbf{C}_y$ is non-invertible for $n_y < n_x$ and, in this case, an effective LMI formulation cannot be directly derived by means of the substitution $\mathbf{K}\mathbf{C}_y\mathbf{X} = \mathbf{Y}$.

In the case that rank$(\mathbf{C}_y) = n_y < n_x$, positive results can be obtained by considering the LMI optimization problem $\mathcal{P}_s'$ with $\mathbf{Y} = \mathbf{K}\mathbf{C}_y\mathbf{X}$, and introducing the following transformations of variables (Rubió-Masssegú et al. 2013):

$$\mathbf{X} = \mathbf{Q}\mathbf{X}_Q\mathbf{Q}^T + \mathbf{R}\mathbf{X}_R\mathbf{R}^T, \quad \mathbf{Y} = \mathbf{Y}_R\mathbf{R}^T; \tag{10}$$

where $\mathbf{X}_Q \in \mathbb{R}^{(n_x - n_y)\times(n_x - n_y)}$ and $\mathbf{X}_R \in \mathbb{R}^{n_y \times n_y}$ are symmetric positive-definite matrices; $\mathbf{Y}_R \in \mathbb{R}^{n_u \times n_y}$ is a general matrix; $\mathbf{Q} \in \mathbb{R}^{n_x \times (n_x - n_y)}$ is a matrix with rank$(\mathbf{Q}) = n_x - n_y$ that satisfies $\mathbf{C}_y\mathbf{Q} = [\mathbf{0}]_{n_y \times (n_x - n_y)}$; and $\mathbf{R} \in \mathbb{R}^{n_x \times n_y}$ is a matrix of the form

$$\mathbf{R} = \mathbf{C}_y^\dagger + \mathbf{Q}\mathbf{L}, \tag{11}$$

where $\mathbf{C}_y^\dagger = \mathbf{C}_y^T(\mathbf{C}_y\mathbf{C}_y^T)^{-1}$ is the *Moore-Penrose pseudoinverse* of $\mathbf{C}_y$, and $\mathbf{L} \in \mathbb{R}^{(n_x - n_y)\times n_y}$ is a given matrix. Using the new LMI variable matrices $\mathbf{X}_Q$, $\mathbf{X}_R$ and $\mathbf{Y}_R$, a suboptimal H_∞ SOF controller $\mathbf{u}(t) = \widehat{\mathbf{K}}\mathbf{y}(t)$

can be obtained by solving the following LMI optimization problem:

$$\mathcal{P}'_o : \begin{cases} \text{minimize } \gamma_o \\ \text{subject to } \mathbf{X}_Q > 0, \mathbf{X}_R > 0, \gamma_o > 0, \\ \quad \begin{bmatrix} \Xi_1 & * & * \\ \mathbf{E}^T & -\gamma_o \mathbf{I}_{n_w} & * \\ \Xi_2 & [\mathbf{0}]_{n_z \times n_w} & -\gamma_o \mathbf{I}_{n_z} \end{bmatrix} < 0, \end{cases} \tag{12}$$

where $\Xi_1 = \text{sym}\big(\mathbf{AQX}_Q\mathbf{Q}^T + \mathbf{ARX}_R\mathbf{R}^T + \mathbf{BY}_R\mathbf{R}^T\big)$ and $\Xi_2 = \mathbf{C}_z\mathbf{QX}_Q\mathbf{Q}^T + \mathbf{C}_z\mathbf{RX}_R\mathbf{R}^T + \mathbf{D}_z\mathbf{Y}_R\mathbf{R}^T$. If the LMI optimization problem $\mathcal{P}'_o$ can be successfully solved, attaining an optimal value $\widehat{\gamma}_o$ for the matrices $\widehat{\mathbf{X}}_Q$, $\widehat{\mathbf{X}}_R$ and $\widehat{\mathbf{Y}}_R$, then the output-feedback control matrix $\widehat{\mathbf{K}} = \widehat{\mathbf{Y}}_R(\widehat{\mathbf{X}}_R)^{-1}$ defines a SOF controller with an asymptotically stable closed-loop matrix $\mathbf{A}_{\widehat{K}}$, and the corresponding H_∞-norm satisfies $\gamma(\widehat{\mathbf{K}}) \leq \widehat{\gamma}_o$.

3 ILMI DESIGN PROCEDURE

In MDOF mechanical systems subjected to broadband excitations, the secondary resonant modes can make a significant contribution to the vibrational response. Broadly speaking, in the H_∞ controller approach, the contribution of the different resonant modes is described by the magnitude of the FRF peaks, and the design procedure is focused on minimizing the magnitude of the largest FRF peak. However, this strategy does not necessarily imply a good attenuation level in the secondary FRF peaks. In fact, as it will be shown by the example provided in the next section, an improved overall behavior can often be attained by suboptimal H_∞ controllers due to their better performance on the secondary FRF peaks.

In this section, we present an ILMI procedure that pursues obtaining suboptimal H_∞ SOF controllers with high-performance characteristics. In a preliminary step, we set $\mathbf{K} = \mathbf{K}^{(0)} = [\mathbf{0}]_{n_u \times n_y}$ in (9) and solve the restricted optimization problem $\mathcal{P}_o$, which is now an LMI optimization problem with the variable matrix $\mathbf{X}$. Assuming that the state-matrix $\mathbf{A}$ is asymptotically stable, $\mathcal{P}_o$ is feasible and will produce an optimal matrix $\widehat{\mathbf{X}}^{(0)}$ and the optimal γ-value $\gamma\big(\mathbf{K}^{(0)}\big)$, which is the H_∞-norm of the uncontrolled system. Next, following the discussion in (Palacios-Quiñonero et al. 2014), we select the L-matrix

$$\mathbf{L}^{(0)} = \mathbf{Q}^\dagger \widehat{\mathbf{X}}^{(0)} \mathbf{C}_y^T \left(\mathbf{C}_y \widehat{\mathbf{X}}^{(0)} \mathbf{C}_y^T \right)^{-1} \tag{13}$$

with $\mathbf{Q}^\dagger = \big(\mathbf{Q}^T\mathbf{Q}\big)^{-1}\mathbf{Q}^T$ and consider the LMI optimization problem $\mathcal{P}'_o$ in (12) with the R-matrix

$$\mathbf{R}^{(0)} = \mathbf{C}_y^\dagger + \mathbf{Q}\mathbf{L}^{(0)}. \tag{14}$$

By solving $\mathcal{P}'_o$, we obtain a new output-feedback gain matrix $\mathbf{K}^{(1)}$, which can be substituted in (9) to define a feasible LMI optimization problem $\mathcal{P}_o$. After completing the preliminary step, the iterative procedure can continue as follows:

- *Step i.a*: Substitute the matrix $\mathbf{K}^{(i)}$ in (9) and solve the LMI optimization problem $\mathcal{P}_o$ to obtain the optimal matrix $\widehat{\mathbf{X}}^{(i)}$ and the value $\gamma\big(\mathbf{K}^{(i)}\big)$
- *Step i.b*: Set $\mathbf{L}^{(i)} = \mathbf{Q}^\dagger \widehat{\mathbf{X}}^{(i)} \mathbf{C}_y^T \left(\mathbf{C}_y \widehat{\mathbf{X}}^{(i)} \mathbf{C}_y^T \right)^{-1}$ and compute $\mathbf{K}^{(i+1)}$ by solving the LMI optimization problem $\mathcal{P}'_o$ in (12) corresponding to the R-matrix $\mathbf{R}^{(i)} = \mathbf{C}_y^\dagger + \mathbf{Q}\mathbf{L}^{(i)}$

Remark. Using the results in (Palacios-Quiñonero et al. 2014), it can be proved that the LMI optimization problems $\mathcal{P}_o$ and $\mathcal{P}'_o$ in the i-th step are feasible, and the values $\gamma\big(\mathbf{K}^{(i)}\big)$ are a non-increasing sequence.

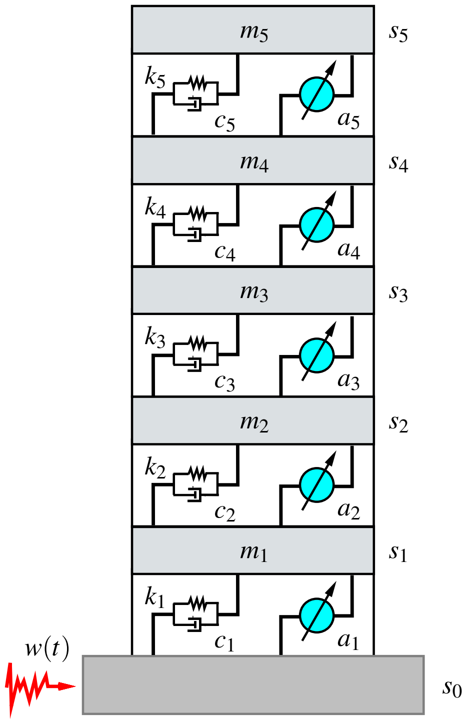

Figure 1: Five-story building structure equipped with a complete system of interstory force-actuation devices.

4 NUMERICAL RESULTS

4.1 *Building Model*

Let us consider a 5-story building model as the one schematically depicted in Figure 1, where s_0 represents the ground level; s_i, $i = 1, \ldots, 5$, denotes the i-th story; m_i, k_i and c_i are the mass, stiffness and damping coefficients of s_i, respectively; a_i is an interstory actuation device implemented between the stories s_{i-1} and s_i, which produces opposite forces of magnitude $|u_i(t)|$ on the associated stories; and $w(t)$ is the seismic ground-acceleration disturbance. The lateral displacement of the structure can be described by the state-space model

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}w(t), \tag{15}$$

where

$$\mathbf{x}(t) = \left[\begin{array}{c} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{array} \right] \tag{16}$$

is the state vector, $\mathbf{q}(t) = [q_1(t), \ldots, q_5(t)]^T$ is the vector of story displacements with respect to the ground, and $\mathbf{u}(t) = [u_1(t), \ldots, u_5(t)]^T$ is the vector of control actions. The matrices in (15) have the following form:

$$\mathbf{A} = \left[\begin{array}{cc} [\mathbf{0}]_{5\times 5} & \mathbf{I}_5 \\ -\mathbf{M}^{-1}\mathbf{K}_s & -\mathbf{M}^{-1}\mathbf{C}_d \end{array} \right], \tag{17}$$

$$\mathbf{B} = \left[\begin{array}{c} [\mathbf{0}]_{5\times 5} \\ \mathbf{M}^{-1}\mathbf{T}_u \end{array} \right], \quad \mathbf{E} = \left[\begin{array}{c} [\mathbf{0}]_{5\times 1} \\ -[\mathbf{1}]_{5\times 1} \end{array} \right], \tag{18}$$

where $[\mathbf{1}]_{5\times 1}$ is a column vector of dimension 5 with all its entries equal to 1, $\mathbf{M} = \mathrm{diag}(m_1, \ldots, m_5)$ is the mass matrix and

$$\mathbf{K}_s = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \tag{19}$$

is the stiffness matrix. $\mathbf{C}_d$ is the damping matrix, which can be computed in a similar way to the stiffness matrix when the damping coefficients c_i are known; otherwise, an approximate damping matrix can be obtained by setting a proper damping ratio on the building modes (Chopra 2017). Finally, $\mathbf{T}_u$ is the control-input matrix, which, for the proposed actuation scheme, has the following form:

$$\mathbf{T}_u = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{20}$$

The interstory drift $r_i(t)$, $i = 1, \ldots, 5$, is the relative displacement between the stories s_i and s_{i-1}. Using the matrix $\mathbf{C}_r = \{\mathbf{T}_u\}^T$, the vector of interstory drifts $\mathbf{r}(t) = [r_1(t), \ldots, r_5(t)]^T$ can be computed in the form

$$\mathbf{r}(t) = \mathbf{C}_r\mathbf{q}(t). \tag{21}$$

To define a performance index for the controller designs discussed in the next subsection, we consider the controlled-output vector

$$\mathbf{z}(t) = \mathbf{C}_z\mathbf{x}(t) + \mathbf{D}_z\mathbf{u}(t), \tag{22}$$

defined by the matrices

$$\mathbf{C}_z = \begin{bmatrix} \mathbf{C}_r & [\mathbf{0}]_{5\times 5} \\ [\mathbf{0}]_{5\times 5} & [\mathbf{0}]_{5\times 5} \end{bmatrix}, \quad \mathbf{D}_z = \alpha \begin{bmatrix} [\mathbf{0}]_{5\times 5} \\ \mathbf{I}_5 \end{bmatrix}, \tag{23}$$

where α is a scaling factor that can be used to adjust the intensity of the control action. With this choice, we have

$$\mathbf{z}^T(t)\,\mathbf{z}(t) = \sum_{i=1}^{5} r_i^2(t) + \alpha^2 \sum_{i=1}^{5} u_i^2(t). \tag{24}$$

Table 1: Mass and stiffness coefficients for the five-story building model.

story	1	2	3	4	5
mass ($\times 10^5$ Kg)	2.15	2.09	2.07	2.05	2.66
stiffness ($\times 10^8$ N/m)	1.47	1.13	0.99	0.89	0.84

Table 2: γ-values obtained in the ILMI procedure.

step i	0	1	2
$\gamma(\mathbf{K}^{(i)})$	0.3619	0.0725	0.0694

In the design of the SOF controllers, we consider the vector of interstory velocities $\dot{\mathbf{r}}(t)$ as feedback information. In this case, the vector of measured outputs $\mathbf{y}(t) = \dot{\mathbf{r}}(t)$ can be written in the form $\mathbf{y}(t) = \mathbf{C}_y\mathbf{x}(t)$ with the measured-output matrix

$$\mathbf{C}_y = \left[[\mathbf{0}]_{5\times 5}\ \mathbf{C}_r \right]. \tag{25}$$

4.2 *Controller designs*

In this section, we consider the state-space building-model corresponding to the mass and stiffness values collected in Table 1 (Kurata et al. 1999) and the damping matrix (in Ns/m)

$$\mathbf{C}_d = 10^5 \times \begin{bmatrix} 2.895 & -0.736 & -0.114 & -0.040 & -0.028 \\ -0.736 & 2.448 & -0.740 & -0.134 & -0.077 \\ -0.114 & -0.740 & 2.263 & -0.719 & -0.196 \\ -0.040 & -0.134 & -0.719 & 2.137 & -0.862 \\ -0.028 & -0.077 & -0.196 & -0.862 & 1.598 \end{bmatrix}, \tag{26}$$

which has been computed by setting a 2% of relative damping in the building modes. For this particular building configuration, we design a suboptimal H_∞ SOF controller $\mathbf{u}(t) = \widetilde{\mathbf{K}}\mathbf{y}(t)$ using the ILMI procedure presented in Section 3. The proposed controller only uses the interstory velocities as feedback information and it is selected based on both the associated γ-value and its behavior on the secondary resonant peaks. Additionally, two optimal H_∞ controllers are designed and are taken as a reference in the performance assessment: (i) an ideal H_∞ state-feedback controller $\mathbf{u}(t) = \widehat{\mathbf{G}}\mathbf{x}(t)$ that uses the full state as feedback information, and (ii) an H_∞ SOF controller $\mathbf{u}(t) = \widehat{\mathbf{K}}_h\mathbf{y}(t)$ that uses the interstory velocities as feedback information and is computed with the Matlab function `hinfstruct`. All the controllers are obtained using the controlled-output in (22) defined by the scaling factor $\alpha = 10^{-7.35}$. Specifically, by solving the LMI optimization problem $\mathcal{P}_s'$ in Subsection 2.1, we obtain the state-feedback control gain-matrix $\widehat{\mathbf{G}}$ presented in Figure 2, which produces an optimal H_∞-norm $\gamma(\widehat{\mathbf{G}}) = 0.0691$. Next, by using the function `hinfstruct` with the measured-output matrix $\mathbf{C}_y$ given in (25), we obtain the following output-feedback control gain-matrix

$$\widehat{\mathbf{K}}_h = 10^6 \times \begin{bmatrix} -4.339 & -4.471 & -3.849 & -2.879 & -1.681 \\ -5.074 & -5.342 & -4.704 & -3.598 & -2.134 \\ -4.824 & -5.168 & -4.611 & -3.574 & -2.140 \\ -3.940 & -4.289 & -3.875 & -3.040 & -1.835 \\ -2.439 & -2.680 & -2.440 & -1.928 & -1.170 \end{bmatrix}, \tag{27}$$

$$\widehat{\mathbf{G}} = 10^6 \times \begin{bmatrix} -2.271 & 0.134 & 0.290 & -0.240 & -0.778 & -1.212 & -0.845 & -0.916 & -1.037 & -1.468 \\ 4.979 & -2.441 & -0.160 & -0.043 & -0.679 & 0.343 & -1.525 & -1.228 & -1.317 & -1.841 \\ 0.314 & 5.778 & -3.173 & -0.399 & -0.304 & -0.083 & 0.203 & -1.825 & -1.451 & -1.928 \\ 0.835 & 0.261 & 5.945 & -4.123 & -0.322 & -0.137 & -0.238 & 0.123 & -1.954 & -1.916 \\ 0.502 & 0.641 & -0.017 & 5.627 & -4.824 & -0.098 & -0.197 & -0.227 & 0.257 & -2.281 \end{bmatrix}.$$

Figure 2: Control gain-matrix corresponding to the optimal H_∞ state-feedback controller $\mathbf{u}(t) = \widehat{\mathbf{G}}\mathbf{x}(t)$.

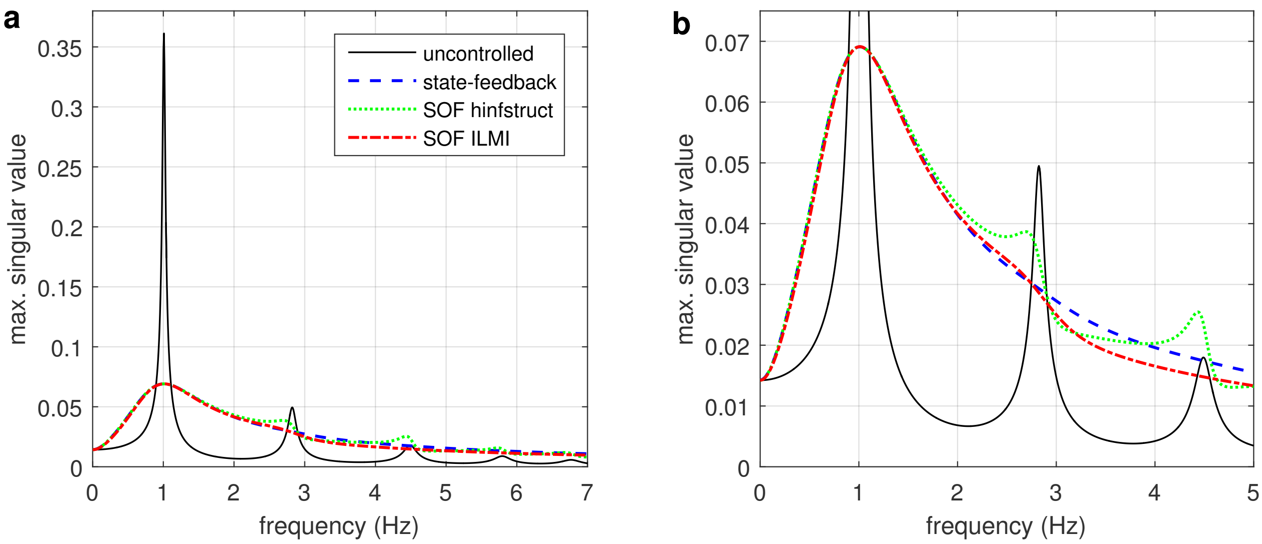

Figure 3: Maximum singular values of the Frequency Response Functions corresponding to the uncontrolled building (black solid line), the optimal H_∞ state-feedback controller $\mathbf{u}(t) = \widehat{\mathbf{G}}\mathbf{x}(t)$ (blue dashed line), the SOF controller $\mathbf{u}(t) = \widehat{\mathbf{K}}_h\mathbf{y}(t)$ computed with the Matlab function `hinfstruct` (green dotted line), and the SOF controller $\mathbf{u}(t) = \widetilde{\mathbf{K}}\mathbf{y}(t)$ computed with the proposed ILMI procedure (red dash-dotted line). (a) Overall view. (b) Close view.

with an associated H_∞-norm $\gamma(\widehat{\mathbf{K}}_h) = 0.0691$. It should be observed that, despite the limited feedback information, the SOF controller produced by the `hinfstruct` function attains the same γ-value as the optimal H_∞ state-feedback controller. In the case of the proposed ILMI procedure, after completing two steps as described in Section 3, we obtain the following output-feedback control gain-matrix $\widetilde{\mathbf{K}} = \mathbf{K}^{(2)}$:

$$\widetilde{\mathbf{K}} = 10^6 \times \begin{bmatrix} -5.181 & -3.062 & -2.992 & -2.830 & -2.049 \\ -4.263 & -6.247 & -4.664 & -2.625 & -1.668 \\ -4.284 & -4.451 & -5.579 & -3.187 & -1.815 \\ -4.188 & -3.061 & -3.069 & -4.236 & -1.970 \\ -2.960 & -1.676 & -1.588 & -2.117 & -2.710 \end{bmatrix}. \quad (28)$$

The γ-values corresponding to the different steps of the ILMI procedure are collected in Table 2, where $\gamma\big(\mathbf{K}^{(0)}\big) = 0.3619$ is the H_∞-norm of the uncontrolled configuration, and $\gamma\big(\mathbf{K}^{(2)}\big) = \gamma\big(\widetilde{\mathbf{K}}\big) = 0.0694$ is the H_∞-norm of the selected SOF controller. The frequency-response plots of the uncontrolled building and the considered controlled configurations are displayed in Figure 3, where the magnitude of the main peaks indicate the corresponding H_∞-norm. The plots of the overall view presented in Figure 3(a) show that all the proposed controllers produce a good attenuation level of the main resonant-peak. Additionally, the plots of the close view in Figure 3(b) evidence the superior performance in the secondary resonant-peaks of the suboptimal SOF controller obtained with the proposed ILMI procedure when compared with the optimal *SOF hinfstruct controller*.

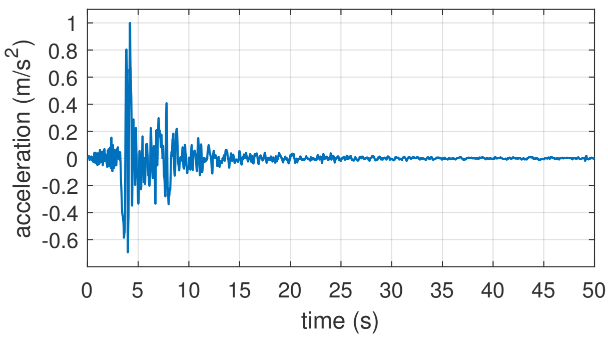

Figure 4: North-South Northridge 1994 ground acceleration seismic record scaled to an acceleration-peak of 1m/s.

4.3 *Seismic response*

To complement the information provided by the frequency-response plots, we have carried out a set of time-response numerical simulations using the scaled North-South Northridge 1994 seismic record as ground-acceleration input (see Figure 4). The absolute peak-values of the interstory-drifts, absolute story-accelerations and control-efforts are displayed in Figure 5. The plots in Figure 5(c) show that all the proposed controllers require a similar level of control effort. Looking at the plots in Figure 5(a) and 5(b), it can also be appreciated that significant and similar levels of reduction in the interstory-drift and absolute story-acceleration peak-values are attained by the optimal H_∞ state-feedback controller (blue dashed line with circles) and the *SOF ILMI controller* (red dash-

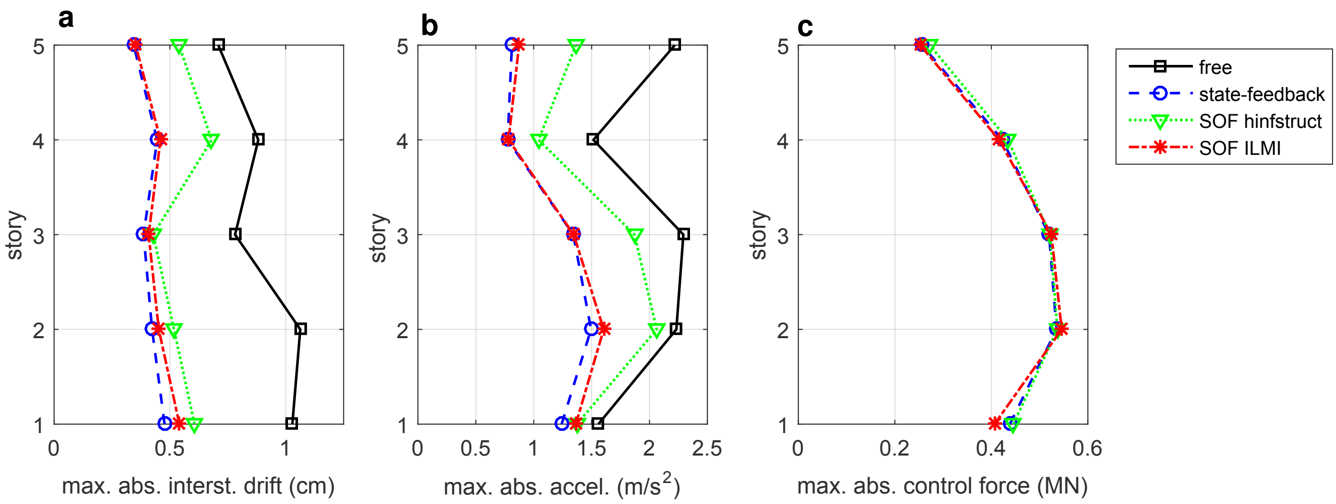

Figure 5: Time-response peak-values corresponding to the uncontrolled building (black solid line with squares), the optimal H_∞ state-feedback controller $\mathbf{u}(t) = \widehat{\mathbf{G}}\mathbf{x}(t)$ (blue dashed line with circles), the SOF controller $\mathbf{u}(t) = \widehat{\mathbf{K}}_h\mathbf{y}(t)$ computed with the Matlab function `hinfstruct` (green dotted line with triangles), and the SOF controller $\mathbf{u}(t) = \widetilde{\mathbf{K}}\mathbf{y}(t)$ computed with the proposed ILMI procedure (red dash-dotted line with asterisks) (a) Maxim absolute interstory-drifts (cm). (b) Maximum absolute story-accelerations (m/s^2). (c) Maximum absolute control-efforts (10^6N).

dotted line with asterisks). In contrast, the plots corresponding to the SOF hinfstruct controller (green dotted line with triangles) in Figure 5(a) and 5(b) show a noticeable increase of the interstory-drift peak-values in the upper building-levels, and a remarkable loss of performance in the story-acceleration response in all the building levels. The obtained results are consistent with the reduced effectiveness of the SOF hinfstruct controller in the secondary resonant-peaks indicated by the frequency-response plots in Figure 3(b) and the broad-band characteristics of the considered near-fault impulsive-type seismic excitation (Ohtori et al. 2004).

5 CONCLUSIONS

In this paper, we have presented a novel Iterative Linear Matrix Inequality (ILMI) procedure for designing static output-feedback (SOF) controllers with high-performance characteristics. The proposed procedure is based on a transformation of the LMI variables and the produced SOF controllers can be effective in reducing the vibrational response of MDOF systems subjected to broad-band excitations. To demonstrate the effectiveness of the proposed methodology, a suboptimal H_∞ SOF controller has been designed for the seismic protection of a five-story building equipped with a distributed set of interstory actuators. The corresponding frequency and time responses have been studied and compared with the responses produced by an optimal H_∞ state-feedback controller with full-state information, and an H_∞ SOF controller computed with the Matlab function `hinfstruct`. The obtained results indicate that, despite the reduced feedback information, the SOF ILMI controller attains a level of performance similar to that achieved by the full-state controller, and it has an improved behavior when compared with the SOF hinfstruct controller.

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