

## Psychological well-being: analysis and interpretation applying Compositional Data Analysis methods

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### Summary

One of the applications of Psychology that makes use of Statistics is that which refers to the analysis of psychological well-being tests. However, this analysis was not systematic until Carol Riff proposed in 1989 a test to describe, analyze and interpret the psychological well-being of people. The model is based on six descriptive fields or dimensions of psychological well-being: self-acceptance, positive relations, autonomy, environmental mastery, purpose in life and personal growth. To measure these theoretical dimensions, an instrument known as “Scale of Physiological Well Being (SPWB)” was developed, with 120 items forming the original scale. Nowadays, there exist several different versions of the instrument in different languages and with different numbers of items. To interpret the results of the different dimensions, the scores of each of them must be added up and compared with the maximum and minimum possible score, since there are no existing ready-reckoners.

Application of the previous methodology generates certain problems when interpreting the results individually, since there is no normative or reference group with which to compare the results obtained. Applying methods of Compositional Data Analysis such as the centered log-ratio transformation (CLR), relative position ratios (RPR) of each individual can be obtained in relation to each of the indicators used. Positive values of the position ratio indicate that the subject is above the (geometric) mean of the normative group and negative values of the position ratio indicate that the subject is below the mean of the normative group. The average of the relative position ratios allows to obtain a global indicator or profile of subjective wellbeing for each individual, not only in relation to himself but also in the context of the group in which he has been analyzed.

**Key words:** Psychological well-being, psychometric, Compositional Data Analysis, Centered Logratio.

## 1. Introduction

Analysis and description of people well-being is a recurrent theme studied in the field of Psychology for a long time, and which is still under study today, due to its great interest. Well-being can be defined and interpreted from two perspectives: as a concept related to the

happiness of the individual and in this case we talk about the subjective well-being (SWB), or as a concept linked to the development of human potential and, in that case, it is called psychological well-being (PWB). This work focuses on this second approach, that is, the PWB is studied as a positive functioning of the individual in the different areas of his life.

In psychology, the measuring instruments are the psychological tests; the tests try to measure unobservable variables (psychological constructs) through observable indicators (items of the questionnaire) so that they can infer scores. In many occasions the psychological constructs are, as in this case, multidimensional.

Until 1989 studies on the welfare of people were carried out from a theoretical point of view and had not yet established a measurement instrument that had associated a measure of reliability or validity that would validate these studies. It is in this year when Carol Ryff proposes a multidimensional model for the study of psychological well-being; the model establishes six dimensions: positive relationships with other people (PRE), autonomy (AUT), control of the environment (CEN), personal growth (PGR), purpose in life (PLI) and self-acceptance (SAC).

The 1989 Carol Ryff PWB questionnaire was designed with 120 items, so that each dimension was formed by the sum of the scores on 20 items. Currently there are different versions of the questionnaire with a different number of items that have been validated in different countries. In the study that serves as an example for this work, the version of 14 items per dimension has been used, this is a total of 84 items, which is one of those recommended by the author. Therefore, to obtain the characterization and have a quantitative idea of the subjective psychological well-being of each person surveyed, the score of each of the 14 items of each dimension is carried out, resulting in a total of 6 scores.

The analysis and interpretation of the tests has traditionally been carried out through the analysis of the direct scores, the dimensions and the global construct. Thus, the higher the score in the "personal growth" dimension, it is inferred that the person puts more effort into developing their potential, in continuing to grow as a person and in maximizing their abilities. Depending on how high or low the individual's scores are in the sum of the different factors, a higher or lower level of psychological well-being can be established.

However, the way in which the dimensions are related to each other in a group is difficult to access if the data are not analysed as proportions in relation to the total instead of each value in isolation. Therefore, in this work we propose to take a step that complements the traditional study, making use of the concepts and methods of the Compositional Data Analysis. In this way, the focus of the study is not on the value of the degree of PBW of the individual, but on "how" or "in what proportions" that welfare is composed in such a way that the interrelations between the dimensions and the equilibria or imbalances between

them. On the other hand, the study can be expanded if subsets of individuals are studied in terms of sociodemographic variables.

## 2. Raw Data Analysis

The data set consists of information obtained through anonymous questionnaires online and in person from 623 people that answered the questionnaire completely. The confidentiality of the data has been explicitly guaranteed, since the answers to the test are not associated with personal data. On the other hand, three variable sociodemographic variables were made explicit that allow for partial studies that may be of interest; these variables are sex, educational level and age.

The statistical analysis of the data is done in a classical, when the questionnaires are not scaling (as it is our case) it can be done by calculating the statistics of each of the welfare dimensions. An illustration that is usual from this perspective is the one corresponding to values in Table 1 and represented in Figure 2.1, which shows the values of each dimension of two individuals selected at random (ID-387, ID-452), the maximum, the minimum and the average (arithmetic) of each dimension. The lines of values of both individuals would lead to affirm that they are quite similar in the first three and that in the last three ID-387 is slightly above ID-452; both individuals are located above the average in four of the dimensions (PRE, CEM, PLI, SAC).

Table 1. Statistical indicators for ID-387 and ID-452 (raw data).

	PRE	AUT	CEN	PGR	PLI	SAC
<b>ID-387</b>	77	59	71	73	71	78
<b>ID-452</b>	76	61	73	63	69	68
<b>MIN</b>	28	30	23	43	26	21
<b>MAX</b>	84	84	81	84	84	84
<b>MEAN</b>	65.02	61.37	57.12	66.68	63.46	61.50

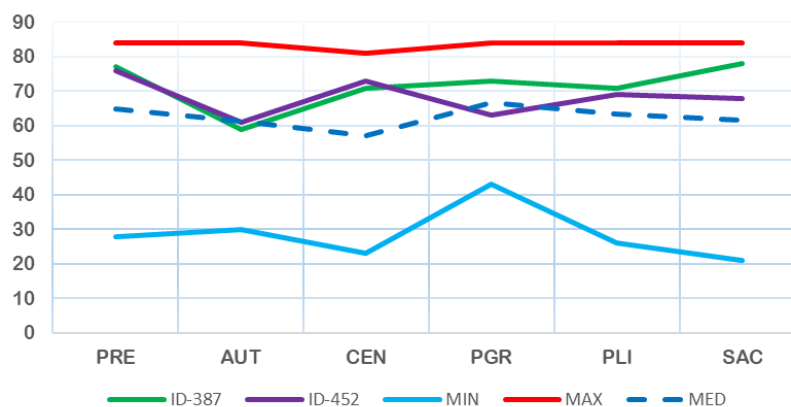


Figure 1. Graph of values of the direct data of two individuals (ID-387, ID-452) and the average, maximum and minimum statistical parameters.

### 3. The Compositional Data Approach

The compositional perspective starts calculating closed data from raw data values. Two “position ratios” can be obtained from corresponding proportions. First, applying the centered logratio transformation (CLR) for each individual, that is

$$(1)$$

The values obtained through Eq.(1) can be interpreted as a position ratio (PR1) of each of the dimensions of the individual in relation to their centrality estimator. This PR1 allows us to observe the balance or relative imbalance of each of the dimensions for each individual. For example, in Figure 2 the PR1 of two individuals are represented, together with three statistics: maximum, minimum and arithmetic mean of the values of the position ratios (values are in Table 2).

Table 2. Position ratios RP1 for ID-387 and ID-452, and statistical indicators.

	PRE	AUT	CEN	PGR	PLI	SAC
<b>ID-387</b>	-0.254	-0.463	-0.127	0.398	0.439	0.007
<b>ID-452</b>	0.077	-0.073	-0.166	0.064	0.114	-0.016
<b>MIN</b>	-0.508	-0.463	-0.601	-0.275	-0.441	-0.613
<b>MAX</b>	0.503	0.683	0.233	0.487	0.439	0.217
<b>MEAN</b>	0.041	-0.017	-0.091	0.072	0.017	-0.022

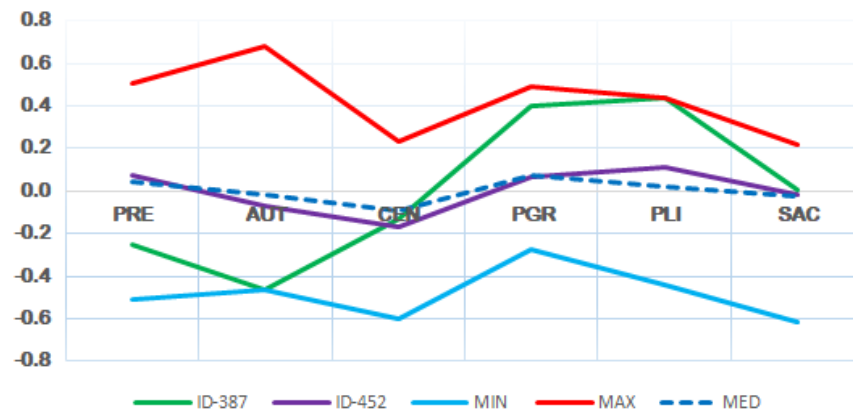


Figure 2. Graph of values of the position ratios PR1 of two individuals (ID-387, ID-452) and the lines corresponding to the values of the mean, the maximum and the minimum.

A second position ratio RP2 can be obtained to analyse the position of each dimension of the individual in relation to the average of each dimension. This position ratio is calculated by:

(2)

In Figure 3 the PR2 of the same two individuals are represented, together with three statistics: maximum, minimum and arithmetic mean of the values of the position ratios (values are in Table 3). Observe that, as it is obvious, arithmetic mean of that values is zero.

Note the remarkable difference between Fig.1 in relation to Fig.2 and Fig.3; both figures since, from this perspective, the individuals are noticeably different. In fact, ID-452 is a "balanced" individual in relative values with respect to the whole; on the other hand, the ID-387 is a more "unbalanced" individual and already has differences in its RP: the dimensions PRE, AUT are clearly below, the dimensions CEN, SAC are in the average and the dimensions PGR, PLI are clearly above of the set.

Table 3. Position ratios RP2 for ID-387 and ID-452, and statistical indicators.

	PRE	AUT	CEN	PGR	PLI	SAC
<b>ID-387</b>	-0.341	-0.493	-0.083	0.280	0.375	-0.018
<b>ID-452</b>	0.039	-0.054	-0.072	-0.005	0.099	0.009
<b>MIN</b>	-0.572	-0.493	-0.556	-0.358	-0.528	-0.652
<b>MAX</b>	0.422	0.630	0.271	0.353	0.375	0.232
<b>MEAN</b>	0.000	0.000	0.000	0.000	0.000	0.000

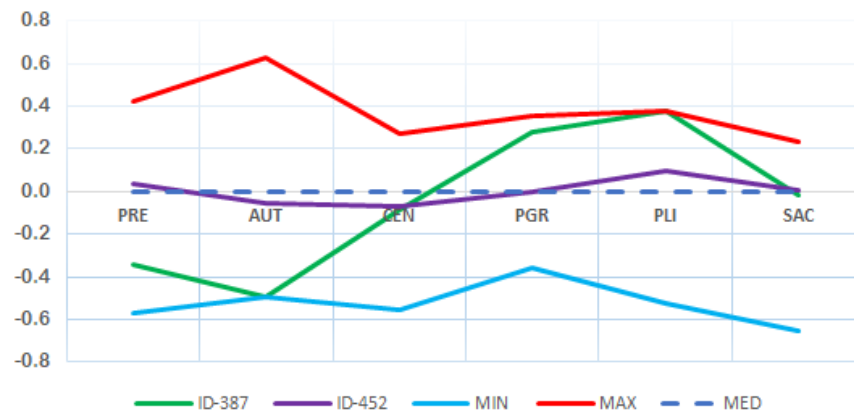


Figure 3. Graph of values of the position ratios PR2 of two individuals (ID-387, ID-452) and the lines corresponding to the values of the mean, the maximum and the minimum.

## Conclusions

- The analysis of psychological well-being tests from a traditional perspective can be completed by applying the concepts and methods of Compositional Data Analysis.
- This new perspective allows analysing not only the dimensions by themselves but

also interrelated, being able to reveal elements that cannot be detected through traditional analysis.

- The centered logratio transformation (CLR) allows establishing for each individual a position ratio as a measure of the relative situation of each of the dimensions in relation to the set represented by its geometric mean.
- Additionally, for each individual, a second position ratio of each dimension can be obtained, calculating the logratio of each proportion between the geometric mean of all the proportions. This ratio indicates the relative position of each proportion of the individual in relation to the average value of all of them in the tested population.

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