Mimetic Loop Quantum Cosmology

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Considering as usual that the underlying geometry of our universe is well described by the spatially flat Friedmann-Lemaître-Robertson-Walker line element, in this work we establish that the background of a holonomy corrected Loop Quantum Cosmology (LQC) could be equivalent with a simple modified version of the mimetic gravity. We also analyze the scalar and tensor perturbations of this modified mimetic model. We find that at the level of scalar perturbations, the modified mimetic model is exactly equivalent to the LQC while at the level of tensor perturbations, the modified mimetic gravity is indistinguishable from the General Relativity.

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1. INTRODUCTION

The understanding of our universe in its extremely early phase ($\sim$ Planck scale $\sim$ high energy regime) is not absolutely complete in the framework of Einstein's General theory of Relativity. Within this framework, as long as the matter sector does now allow any exotic nature, the singularity is inevitable [1–4]. A singularity means $-$ the break down of the space-time structure $-$ the failure of the physical laws $-$ the fundamental flaw in the description of our universe. Aside from the singularity issue, the explanations for the flatness problem, horizon problem, baryon asymmetry, magnetic monopole problem, and some more were also in great demand. To explain such puzzles, it was found that the universe should undergo through a phase of rapid accelerated expansion, something like an exponential type, known as the inflationary paradigm [5, 6] and consequently, the theory of inflation became quite successful in explaining most of the serious issues. Nonetheless, the singularity could not be framed within such formalism and stayed as a shortcoming of the General theory Relativity. We note that we are considering the classical General Relativity to understand the behavior of the universe in the Planck's scale where quantum effects should be extremely important. Thus, the quantum description of gravity came into existence. The investigations in the last several years, remind two promising theories of quantum gravity, namely the *String theory* [7] and the *Loop Quantum Gravity* (LQG) [8]. In the current work we shall consider the cosmological description for the later one, that means the *Loop Quantm Cosmology* (LQC), see [9] for a review on the developments of LQC.

The *Loop Quantum Cosmology* is a very promising alternative to the inflationary paradigm. It has been found that LQC provides a matter-ekpyrotic bouncing scenario [10, 11], which contains a phase transition in the contraction regime and as a consequence allowing our universe to be reheated via the particle production mechanism, whose theoretical values of the power spectrum, spectral index and its running agree with their observational estimations extracted from the joint analysis of BICEP2/Keck Array and Planck [14]. We also refer to some earlier works where the authors show that inflation [12] and bounce [13] are generically obtained in this context.

A suitable connection between LQC and the modified gravity theories was established in a series of articles [15–18] showing that, in the framework of the usual spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe, the holonomy corrected LQC model can be equivalent to modified gravity if one works with an invariant scalar that depends only on the square of the Hubble parameter [15–18]. One may recall that such a scalar quantity in already known in some viable modified gravity models, for instance, the scalar could be the torsion that appears in the teleparallel gravity models where the spacetime is equipped with an unusual Weitzenböck connection [19], and a preferred orthonormal basis in the tangent bundle of the spacetime manifold must be selected. As a second thought, one may recall the extrinsic curvature scalar in the context of the Arnowitt-Deser-Misner (ADM) formalism of GR [20]. One may find that, in the above two frameworks, if the background universe is perfectly described by the spatially flat FLRW spacetime, then for the synchronous co-moving coordinates [21], the scalar becomes '$-6H^2$'.

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Although both the modified gravity models mentioned above are believed to be viable for the expansion history of the universe, but one may recall that − both of them share a common trouble − the preferred coordinate system. Locally, the teleparallel gravity is not Lorentz invariant [22] and concerning the extrinsic curvature scalar in the ADM formalism of GR, it requires to fix the slicing [21]. Thus, essentially, both the modified gravity theories have some shortcomings. It might be important to refer to some recent investigations which argue that if the spin connection is also considered along with the pure-tetrad formalism for the teleparallel gravity, the local invariance problem might be resolved [23, 24]. However, the consequences of such proposals where in particular the spin connection is important for the teleparallel models need some more time for a decisive conclusion. On the other hand, for the ADM formalism of GR, we are not concerned with any kind of proposal, at least at the time of writing this paper. Thus, we conclude that the scalar we find in both these modified gravity theories, is not gauge invariant. The construction of such gauge invariant scalars is not so easy, while they are not impossible though. Such scalars can be built with the Carminati-McLenaghan invariants [26] (see [25] for the definition of these invariants) or the d'Alembertian of a mimetic field [31, 32].

Once such gauge invariant scalars are found, the realization of the cosmic bounce becomes simple working in the phase space (H, ρ), where H denotes de Hubble parameter and ρ the energy density of the universe. In particular, the simplest route to build such bouncing backgrounds is to consider the closed curves in the (H, ρ) phase space crossing the $H = 0$ axis, at least twice. Now, for a given f-theory, the modified Friedmann equation becomes a first order differential on f, that relates f and ρ, both as a function of the invariant scalar. The solution of the modified Friedmann equation returns the corresponding f theory that leads to the background depicted by the corresponding curve. We remark that the simplest closed curve in the (H, ρ) phase space is an ellipse which depicts the holonomy corrected Friedmann equation in LQC (see for details [26]).

In the present work we use the scalar provided by the modified mimetic gravity, (the D'Alembertian of the mimetic field with a negative sign), and show that there is a f-theory which reproduces exactly the same background as the holonomy corrected LQC. We extend our analysis considering both the scalar and tensor perturbations. The results show that at the level of scalar perturbations, the modified mimetic model coincides with the LQC. However, for the tensor perturbations, as has recently pointed out in [28], the mimetic field does not have any influence, and thus, one obtains the well-known tensor perturbed equations for General Relativity.

The paper has been organized in the following way. In section 2 we introduce the modified mimetic model and its evolution in the background level. The perturbation equations have been presented in section 3 where in particular, the scalar and tensor perturbations are shown in subsections 3.1 and 3.2, respectively. Finally, we close the entire work in section 4 with the main findings of the work. The units used throughout the paper are $\hbar = c = 1 = M_{pl} = 1$, where M_{pl} is the reduced Planck's mass (the other notations have their usual meanings), with the convention that a temporal vector v_μ satisfies $v_\mu v^\mu < 0$. The meanings of some frequently used quantities in this work are, (a) $\phi_{,\mu} \equiv \partial_\mu \phi = \nabla_\mu \phi$, (b) $\bar{\phi}$ as the unperturbed part of ϕ ; (c) f_χ is the derivative of f with respect to χ and V_ϕ is the derivative of V with respect to ϕ .

2. MODIFIED MIMETIC GRAVITY

In recent time, the cosmology with mimetic gravity is getting an impressive attention [40–46] for explaining the recent observational evidences (also see a recent review in this direction [29]). Here we allow an extension of the original mimetic gravity in terms of the following action

$$S = \int \sqrt{-g} \left(\frac{1}{2} R + \lambda(\varphi_{,\mu}\varphi^{,\mu} + 1) + f(\chi) + \mathcal{L}_{matt} \right) d^4x, \tag{1}$$

where R is the scalar curvature, φ is the mimetic field satisfying $\varphi^{,\mu}\varphi_{,\mu} \equiv \nabla^\mu \varphi \nabla_\mu \varphi = -1$; $\chi \equiv -\Box\varphi = -\nabla^\mu \varphi_{,\mu}$; λ is a Lagrange multiplier and we have assumed that the matter sector of the universe is filled with a scalar field ϕ with potential $V(\phi)$, which is minimally coupled to gravity, and whose Lagrangian is given by

$$\mathcal{L}_{matt} = -\frac{\phi_{,\mu}\phi^{,\mu}}{2} - V(\phi). \tag{2}$$

Although, one can extend the action (1) for $F(R)$ gravity [30] with the replacement $R \to F(R)$, however, in this work we only consider the simplest case described in the above action (1). The dynamical equations making the variation of (1) with respect to $g_{\mu\nu}$, are [32]

$$G_{\mu\nu} = T_{\mu\nu} - \tilde{T}_{\mu\nu} - 2\lambda\varphi_{,\mu}\varphi_{,\nu}, \tag{3}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, is the Einstein tensor, $T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi)\right)g_{\mu\nu}$, is the stress tensor, and

$$\tilde{T}_{\mu\nu} \equiv g_{\mu\nu}\left(\chi f_\chi - f - \varphi_{,\alpha}\chi^{,\alpha} f_{\chi\chi}\right) + f_{\chi\chi}\left(\varphi_{,\nu}\chi_{,\mu} + \varphi_{,\mu}\chi_{,\nu}\right). \tag{4}$$

The dynamical equation for the mimetic field is obtained after performing the variation of the action with respect to φ as follows

$$\partial_\mu\left[\sqrt{-g}\left(f_{\chi\chi}\chi^{,\mu} + 2\lambda\varphi^{,\mu}\right)\right] = 0, \tag{5}$$

and the variation of the scalar field ϕ leads to the well-known conservation equation

$$-\Box\phi + V_\phi = 0. \tag{6}$$

Remark 2.1 *The dynamical equation for the scalar field (6) could also be deduced from the conservation equation* $\nabla_\mu T^\mu_\nu = 0$*, and the dynamical equation for the mimetic field must also be obtained taking the divergence of (3). From the Bianchi identity* $\nabla_\mu G^\mu_\nu = 0$*, and the conservation equation* $\nabla_\mu T^\mu_\nu = 0$*, it is verified that* $\nabla_\mu(\tilde{T}^\mu_\nu + 2\lambda\varphi^{,\mu}\varphi_{,\nu}) = 0$*. Further, following some algebraic calculations, one can see that the equation* $\varphi^\nu\nabla_\mu(\tilde{T}^\mu_\nu + 2\lambda\varphi^{,\mu}\varphi_{,\nu}) = 0$ *is equivalent to equation (5).*

2.1. Background equations

In this section we explicitly describe the background equations for the action (1) in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry. We work with the synchronous observers, that means, the line element takes the form $ds^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j$, where $a(t)$ is the expansion scale factor of the universe. The simplest solution of $\bar{\varphi}_{,\mu}\bar{\varphi}^{,\mu} = -1$, is $\bar{\varphi}(t) = t$, leading to $\bar{\chi} = 3H$. Thus, equation (5) becomes

$$\partial_t\left[a^3(3\dot{H}\bar{f}_{\chi\chi} + 2\bar{\lambda})\right] = 0, \tag{7}$$

whose general solution is $\bar{\lambda} = \frac{C}{2}a^{-3} - \frac{3}{2}\dot{H}\bar{f}_{\bar{\chi}\bar{\chi}}$. For simplicity, we take $C = 0$, thus, the $0-0$ component of the field equations (3) becomes

$$\rho = 3H^2 + \bar{f} - 3H\bar{f}_\chi, \tag{8}$$

which depicts a curve in the phase space of (H, ρ). The $i-0$ component identically vanishes whereas the $i-i$ component yields the Raychaudhuri equation

$$\left(1 - \frac{3}{2}\bar{f}_{\chi\chi}\right)\dot{H} = -\frac{1}{2}(\rho + P), \tag{9}$$

which is equivalent to the conservation equation $\dot{\rho} = -3H(\rho + P)$, where P is the pressure associated with the energy density ρ.

Therefore, for given a curve $\rho = \bar{g}(3H) = \bar{g}(\bar{\chi})$, in order to obtain the corresponding $\bar{f}(\bar{\chi})$ one has to solve the first order differential equation (8) written as

$$\bar{\chi}\bar{f}_\chi - \bar{f} - \frac{1}{3}\bar{\chi}^2 + \bar{g}(\bar{\chi}) = 0, \tag{10}$$

whose solution is,

$$\bar{f}(\bar{\chi}) = +\frac{1}{3}\bar{\chi}^2 - \bar{\chi}\int\frac{\bar{g}(\bar{\chi})}{\bar{\chi}^2}d\bar{\chi}. \tag{11}$$

As an example, we consider the holonomy corrected Friedmann equation in LQC, i.e., we take the following ellipse

$$\rho = \bar{g}(\bar{\chi}) = \frac{\rho_c}{2}\left(1 \pm \sqrt{1 - \frac{4\bar{\chi}^2}{3\rho_c}}\right) \Longleftrightarrow H^2 = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right), \tag{12}$$

which leads to the mimetic f-theory

$$\bar{f}(\bar{\chi}) = \frac{1}{3}\bar{\chi}^2 + \frac{\rho_c}{2}\left(1 - \sqrt{1-s^2} - s\arcsin(s)\right), \tag{13}$$

where $s \equiv \frac{2}{\sqrt{3\rho_c}}\bar{\chi}$, and the functions $\sqrt{1-s^2}$, $\arcsin(s)$ are the bi-valued [26]. In order to be well defined we have to take the following prescription. The ellipse has two branches where the upper part corresponds to $\rho = \frac{\rho_c}{2}\left(1+\sqrt{1-\frac{4\bar{\chi}^2}{3\rho_c}}\right)$ and the lower one is depicted by $\rho = \frac{\rho_c}{2}\left(1-\sqrt{1-\frac{4\bar{\chi}^2}{3\rho_c}}\right)$. We choose the sign of the square root as positive (respectively, negative) in the lower (respectively upper) branch and $\arcsin(s) \equiv \int_0^s \frac{1}{\sqrt{1-\bar{s}^2}}d\bar{s}$, in the lower branch while $\arcsin(s) \equiv \int_0^s \frac{1}{\sqrt{1-\bar{s}^2}}d\bar{s} + \pi$, in the upper one, having the same criteria for the sign of the square root, obtaining that the function $\bar{f}$ is continuous throughout the ellipse.

Finally, a simple calculation shows that, for this particular theory, the $0-0$ and $i-i$ equations respectively become

$$H^2 = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_c}\right), \tag{14}$$

and

$$\dot{H} = -\frac{\rho+P}{2}\left(1 - \frac{2\rho}{\rho_c}\right), \tag{15}$$

which are nothing but the modified field equations in the holonomy corrected LQC [9].

A final remark is in order: The simplest route to obtain the background equations goes as follows [27]. Let us consider the metric $ds^2 = -N^2dt^2 + a^2\delta_{ij}dx^idx^j$, where we introduce the lapse function N. In this case the constraint $\varphi^{,\mu}\varphi_{,\mu} = -1$, reads $\dot{\bar{\varphi}} = N$, and thus, $\bar{\chi} = \frac{3H}{N}$. Then, introducing these results in (1) we obtain the reduced action

$$S_{red} = \int a^3N\left(-\frac{3H^2}{N^2} + \bar{f}\left(\frac{3H}{N}\right) + \mathcal{L}_{matt}\right)dt, \tag{16}$$

where the matter Lagrangian is

$$\mathcal{L}_{matt} = \frac{\dot{\bar{\phi}}^2}{2N^2} - V(\bar{\phi}). \tag{17}$$

We note that performing the variation of the reduced action (16) with respect to the lapse function N and at the end making $N = 1$, without any loss of generality, one obtains the Friedmann equation (8). Moreover, performing the temporal derivative of (16) and using the conservation equation $\dot{\rho} = -3H(P+\rho)$, one may find the Raychaudhuri equation (9).

3. PERTURBATIONS

The behavior of any cosmological theory in the large scale of the universe is the most important subject for investigation. Thus, following the evolutions of the mimetic modified gravity model at the background level in section 2.1, in this section, we calculate the scalar and tensor perturbations for the present model using the longitudinal gauge.

3.1. Scalar Perturbations

In the Newtonian gauge the perturbed line element can be written as

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)a^2\delta_{ij}dx^idx^j, \tag{18}$$

because in mimetic gravity $\delta(T_i^j - \tilde{T}_i^j - 2\lambda\varphi_i\varphi^j)$ vanishes for $i \neq j$ [33]. One may identify Φ and Ψ as the perturbation quantities. We perturb the scalar field as, $\varphi = t + \delta\varphi$, then the equation $\varphi_{,\mu}\varphi^{,\mu} = -1$ leads to $\Phi = \delta\dot{\varphi}$

At the linear order, a simple calculation leads to

$$\chi = 3H - 3\delta\ddot{\varphi} - 3H\delta\dot{\varphi} - \frac{1}{a^2}\Delta\delta\varphi. \tag{19}$$

And perturbing the equation (3), the $i-0$, $i-i$ and $0-0$ equations respectively take the following forms

$$\dot{\Phi}+H\Phi=\frac{1}{2}\dot{\bar{\phi}}\delta\phi-\frac{1}{2}\bar{f}_{\chi\chi}\delta\chi, \tag{20}$$

$$2\left(\ddot{\Phi}+4H\dot{\Phi}+(3H^2+2\dot{H})\Phi\right)=-\dot{\bar{\phi}}(\Phi\dot{\bar{\phi}}-\delta\dot{\phi})-\bar{V}_\phi\delta\phi-\delta(\chi f_\chi-f-\varphi^{,\mu}\chi_{,\mu}f_{\chi\chi}), \tag{21}$$

$$2\left(3H^2\Phi+3H\dot{\Phi}-\frac{1}{a^2}\Delta\Phi\right)=\dot{\bar{\phi}}(\Phi\dot{\bar{\phi}}-\delta\dot{\phi})-\bar{V}_\phi\delta\phi-\delta(\chi f_\chi-f+\varphi^{,\mu}\chi_{,\mu}f_{\chi\chi})+2\delta\lambda\ . \tag{22}$$

On the other hand, the general solution of the equation (5) is

$$f_{\chi\chi}\chi^{,\mu}+2\lambda\varphi^{,\mu}=\xi^\mu, \tag{23}$$

where ξ^μ is a vector field satisfying $\nabla_\mu\xi^\mu=0$. Then, one has

$$2\lambda=-\xi^\mu\varphi_{,\mu}+f_{\chi\chi}\chi^{,\mu}\varphi_{,\mu}, \tag{24}$$

and by perturbing it, one obtains

$$2\delta\lambda=\delta\xi_0-\partial_t(\bar{f}_{\chi\chi}\delta\chi)+\dot{\bar{f}}_\chi\delta\dot{\varphi}, \tag{25}$$

which leads to

$$\delta\lambda=\frac{1}{2}\delta\xi_0+\frac{\bar{f}_{\chi\chi}}{2}(3\ddot{\Phi}+\frac{1}{a^2}\Delta\Phi)+\frac{3}{2}(H\bar{f}_{\chi\chi}+\dot{\bar{f}}_{\chi\chi})\dot{\Phi}+\frac{3}{2}(2\dot{H}\bar{f}_{\chi\chi}+H\dot{\bar{f}}_{\chi\chi})\Phi+(\frac{1}{2}\dot{\bar{f}}_{\chi\chi}-H\bar{f}_{\chi\chi})\frac{1}{a^2}\Delta\delta\varphi. \tag{26}$$

Now, adding the equations $0-0$ and $i-i$ and using $i-0$, one gets

$$\ddot{\Phi}-\frac{1}{a^2}\Delta\Phi+H\dot{\Phi}+2\dot{H}\Phi=\ddot{\bar{\phi}}\delta\phi+\delta\lambda. \tag{27}$$

Then, writing $i-0$ as follows

$$\ddot{\bar{\phi}}\delta\phi=2\frac{\ddot{\bar{\phi}}}{\dot{\bar{\phi}}}(\dot{\Phi}+H\Phi)(1-\frac{3}{2}\bar{f}_{\chi\chi})-\frac{\ddot{\bar{\phi}}}{\dot{\bar{\phi}}}\bar{f}_{\chi\chi}\frac{1}{a^2}\Delta\delta\varphi, \tag{28}$$

collecting terms and introducing the notation $\Omega\equiv\frac{1}{1-\frac{3}{2}\bar{f}_{\chi\chi}}$, one gets

$$\ddot{\Phi}-\frac{\Omega}{a^2}\Delta\Phi+\left(H-2\frac{\ddot{\bar{\phi}}}{\dot{\bar{\phi}}}-\frac{\dot{\Omega}}{\Omega}\right)\dot{\Phi}+\left(2\left(\dot{H}-H\frac{\ddot{\bar{\phi}}}{\dot{\bar{\phi}}}\right)-H\frac{\dot{\Omega}}{\Omega}\right)\Phi=\frac{1}{2}\left[\Omega\delta\xi_0+\frac{2}{3}\dot{H}\partial_t\left(\frac{(\Omega-1)\Delta\delta\varphi}{a^2\dot{H}}\right)\right]. \tag{29}$$

Dealing with the Newtonian gauge, in order to obtain the same equations as in holonomy corrected LQC, we have to choose $\delta\xi_0$ so that the right hand side of (29) vanishes.

Introducing the notation $\epsilon\equiv\frac{\dot{\Omega}}{2\Omega}$ and using conformal time, it could be written as

$$\Phi''-\Omega\Delta\Phi+2\left(\mathcal{H}-\left(\frac{\bar{\phi}''}{\bar{\phi}'}+\epsilon\right)\right)\Phi'+2\left(\mathcal{H}'-\mathcal{H}\left(\frac{\bar{\phi}''}{\bar{\phi}'}+\epsilon\right)\right)\Phi=0, \tag{30}$$

which for the f-theory given in (12) coincides with the corresponding equation in LQC (see equation (108) in [34]) because in that case $\Omega=1-\frac{2\rho}{\rho_c}$.

Remark 3.1 *The expression of $\delta\xi_0$ is composed by the terms which cancel the ones in (26) and (28) containing the Laplacian, i.e., $\delta\xi_0=-\dot{\bar{\phi}}^2\partial_t\left(\frac{\bar{f}_{\chi\chi}\Delta\delta\varphi}{a^2\dot{\bar{\phi}}^2}\right)$, and its choice, which is equivalent to those of $\delta\lambda$, is not arbitrary, but we do not have a full justification of it. In fact, when f is linear on χ, we obtain GR background, and for our choice of $\delta\xi_0$, we also obtain that the equation for the Newtonian potential is the same as in GR, because in this case $\delta\xi_0$ vanishes. Moreover, when f is quadratic, i.e., $f=\frac{k}{2}\chi^2$, the dynamical equations are $H^2=\frac{\rho}{3(1-3k)}$ and $\dot{H}=-\frac{\rho+P}{2(1-3k)}$, which* *are equivalent, after performing the transformation $t = \tilde{t}\sqrt{1-3k}$, to the equations of GR. Therefore, using the time $\tilde{t}$ the equation for scalar perturbations has to be*

$$\Phi_{\tilde{t}\tilde{t}} - \frac{1}{a^2}\Delta\Phi + \left(\tilde{H} - 2\frac{\bar{\phi}_{\tilde{t}\tilde{t}}}{\bar{\phi}_{\tilde{t}}}\right)\dot{\Phi} + 2\left(\tilde{H}_{\tilde{t}} - \tilde{H}\frac{\bar{\phi}_{\tilde{t}\tilde{t}}}{\bar{\phi}_{\tilde{t}}}\right)\Phi = 0, \tag{31}$$

where $\tilde{H} = a_{\tilde{t}}/a$, and $h_{\tilde{t}}$ denotes the derivative of h with respect to $\tilde{t}$. And using the cosmic time t, the equation (31) becomes

$$\ddot{\Phi} - \frac{\Omega}{a^2}\Delta\Phi + \left(H - 2\frac{\ddot{\bar{\phi}}}{\dot{\bar{\phi}}}\right)\dot{\Phi} + 2\left(\dot{H} - H\frac{\ddot{\bar{\phi}}}{\dot{\bar{\phi}}}\right)\Phi = 0, \tag{32}$$

which coincides with (29), when its right hand side term vanishes. From our viewpoint, this argument enforces our choice, although for the moment this is a point that remains open and deserves future investigation.

To end this section we will calculate the Mukhanov-Sasaki (M-S) equation for scalar perturbations. First of all, note that the equation $i-0$ could be written as

$$\frac{d}{dt}\left(\frac{a\Phi}{H}\right) = \frac{a\Omega\dot{\bar{\phi}}^2}{2H^2}\left[\frac{H\delta\phi}{\dot{\bar{\phi}}} + \Phi + \frac{H}{a^2\dot{\bar{\phi}}^2}\bar{f}_{\chi\chi}\Delta\delta\varphi\right]. \tag{33}$$

On the other hand, the equation $0-0$ has the form

$$2\left(3H^2\Phi + 3H\dot{\Phi} - \frac{1}{a^2}\Delta\Phi\right) = \dot{\bar{\phi}}(\Phi\dot{\bar{\phi}} - \delta\dot{\phi}) - \bar{V}_{\phi}\delta\phi - 3H\bar{f}_{\chi\chi}\delta\chi + \delta\xi_0, \tag{34}$$

and after a cumbersome calculation, if one choose $\delta\xi_0$ such that the right hand side of (29) vanishes, one can see that it is equivalent to the following one

$$\frac{1}{a^2}\Delta\Phi = \frac{\dot{\bar{\phi}}^2}{2H}\frac{d}{dt}\left[\frac{H\delta\phi}{\dot{\bar{\phi}}} + \Phi + \frac{H}{a^2\dot{\bar{\phi}}^2}\bar{f}_{\chi\chi}\Delta\delta\varphi\right]. \tag{35}$$

Now, introducing the variables

$$v = a\left(\delta\phi + \frac{\dot{\bar{\phi}}}{H}\Phi + \frac{1}{a^2\dot{\bar{\phi}}}\bar{f}_{\chi\chi}\Delta\delta\varphi\right), \quad z = a\frac{\dot{\bar{\phi}}}{H}, \quad u = \frac{2\Phi}{\dot{\bar{\phi}}\sqrt{\Omega}}, \quad \text{and} \quad \theta = \frac{1}{z\sqrt{\Omega}}, \tag{36}$$

and using the conformal time, one obtains the M-S equations

$$\sqrt{\Omega}\Delta u = z\left(\frac{v}{z}\right)', \qquad \theta\left(\frac{u}{\theta}\right)' = \sqrt{\Omega}v. \tag{37}$$

Finally, performing the Laplacian in the second equation and using the first one, we obtain

$$v'' - \Omega\Delta v - v\frac{z''}{z} = 0, \tag{38}$$

and inserting the second equation in the first one, we get

$$u'' - \Omega\Delta u - u\frac{\theta''}{\theta} = 0, \tag{39}$$

which coincides with the M-S in LQC for scalar perturbations [35].

As we can see from the previous equation, the square of the velocity of sound, namely c_s^2, is equal to $\Omega = \left(1 - \frac{3}{2}\bar{f}_{\chi\chi}\right)^{-1}$, which could exhibit the well-known gradient instability in the mimetic gravity [28, 36]. In the particular case of the f-theory that leads to the same background as holonomy corrected LQC, i.e., for the f given in equation (13), one has $c_s^2 = 1 - \frac{2\rho}{\rho_c}$ meaning that the gradient inestability appears in the upper branch of the ellipse. Fortunately, in the matter-ekpyrotic bouncing scenario the pivot scale mode leaves, in the contracting phase, the

Hubble radius in the lower branch [11]. So, this instability has no effect in the pivot scale mode because outside the Hubble mode the long-wavelength approximation holds.

Finally, we can see that the *curvature fluctuation in co-moving coordinates* [37], namely

$$\mathcal{R} \equiv \frac{v}{z} = \frac{H\delta\phi}{\dot{\phi}} + \Phi + \frac{H}{a^2\dot{\phi}^2}\bar{f}_{\chi\chi}\Delta\delta\varphi, \tag{40}$$

which is invariant for the change of slicing $t \to \tilde{t} = t+\zeta(t)$ because the Newtonian potential transforms as $\tilde{\Phi} = \Phi+H\zeta(t)$ and the scalar fields as $\delta\tilde{\phi} = \delta\phi - \dot{\phi}\zeta(t)$, and $\delta\tilde{\varphi} = \delta\varphi - \zeta(t)$, has the usual form as in GR or $f(T)$ gravity [38]

$$\mathcal{R} = \Phi - \frac{H}{\dot{H}}\left(\dot{\Phi} + H\Phi\right). \tag{41}$$

3.2. Tensor Perturbations

Concerning the tensor perturbations for the modified mimetic model, the perturbed metric is given by

$$ds^2 = -dt^2 + a^2(\delta_{ij} - h_{ij})dx^i dx^j, \tag{42}$$

where h_{ij} is a symmetric, traceless and transverse tensor, that means, $h^i_i = \partial_i h^{ij} = 0$. The constraint in this case now becomes $\varphi^{,\mu}\varphi_{,\mu} = -1$ which leads to $\delta\dot{\phi} = 0$, and thus, at linear order one arrives at $\chi = 3H - \frac{1}{a^2}\Delta\delta\varphi$.

From this result, we can see that for the tensor perturbations, the $i-0$ equation becomes $\Delta\delta\varphi = 0$, which leads to $\delta\chi = 0$. As a consequence, the right hand side of the $i-j$ equation vanishes leading to $\delta G^i_i = 0$, which is equivalent to the well-known equation of tensor perturbations in GR

$$\ddot{h}^j_i + 3H\dot{h}^j_i - \frac{1}{a^2}\Delta h^j_i = 0. \tag{43}$$

This is a feature of this theory which is indeed different from the Teleparallel LQC [35, 38], Extrinsic curvature LQC [21] or even from the holonomy corrected LQC [39], where in all the mentioned theories, the equation for tensor perturbations differs from that of GR.

4. CONCLUDING REMARKS

General Relativity is a successful theory of gravity that describes the evolution of the universe almost in a satisfactory way. The theory of inflation was found to be an essential addition in this context. Nonetheless, the initial singularity issue, which had been found to be inevitable, inspired to search for other alternatives for inflation where the singularity does not appear. The *Loop Quantum Cosmology* is an effect of that, which is considered to be a viable alternative to the inflationary paradigm. The model also provides with a bounce of the universe in its early phase and hence the singularity problem is naturally avoided. In the present work, considering that the underlying geometry of the universe is best described by the usual spatially flat Friedmann-Lemaître-Robertson-Walker line-element, we have shown that a modified version of the mimetic gravity (see [40, 41] for the introduction of mimetic gravity theory) could be equivalent to LQC and thus, this gravity theory enjoys the same properties as in LQC.

The introduction of mimetic gravity in the literature of modern cosmology is very new [40, 41], at least in compared to other cosmological theories and within a few years it has gained a considerable attention in the scientific community [31, 32, 42–46]. We have shown that a modification of the mimetic gravity through the introduction of a functional $f(\chi)$ into the Lagrangian, where χ is the d'Alembertian of the mimetic field, could exhibit some interesting properties based on the choice of $f(\chi)$. At the background level, the modified version of the mimetic gravity may lead to an equivalent structure to that of the LQC for the appropriate f-function. While on the other hand, concerning the study of the cosmological perturbations, the model returns different characteristics respectively for the scalar and tensor perturbations. For the scalar perturbations, the modified mimetic gravity returns same equations as in LQC whereas for the tensor perturbations, we show that mimetic field does not exhibit any extra feature in compared to the General Relativity, that means, at the level of tensor perturbations, mimetic gravity coincides with the General Relativity.

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