BASIC METRIC GEOMETRY

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ETSEIB

Contents

Introduction

Legend

In this document the following colour code has been used to differentiate the data (what is known), the operations (procedures that must be executed to obtain the result) and the solutions (what is sought):

Black or White	DATA
Blue	OPERATIONS
Red	SOLUTIONS

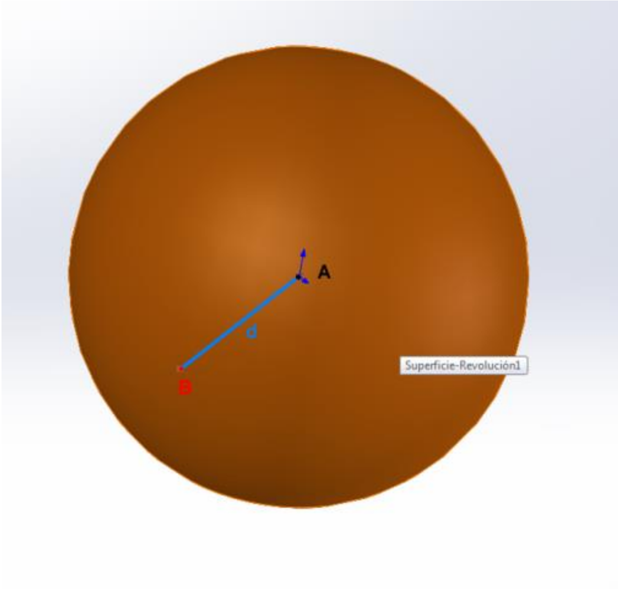
SDW	ACCESS TO SOLIDWORKS SOLUTION FILE
STEP	ACCESS TO SOLUTION FILE IN STEP FORMAT
VIDEO	ACCESS TO THE VIDEO RECORDING OF THE PROCEDURE

Instructions for reading the graphics:

BLACK COLOR = Fixed elements (they do not move or transform).
RED COLOR = Variable elements (those that modify their position in the space after the data is entered based on the fixed elements).
BLUE COLOR = Elements that contain the construction data.

THE POINT

1 Distance between two points



VIDEO SDW

Case 1: Wire mesh

Description:

Distance **d** between the point **B** and the point **A**.

What I should I before:

-

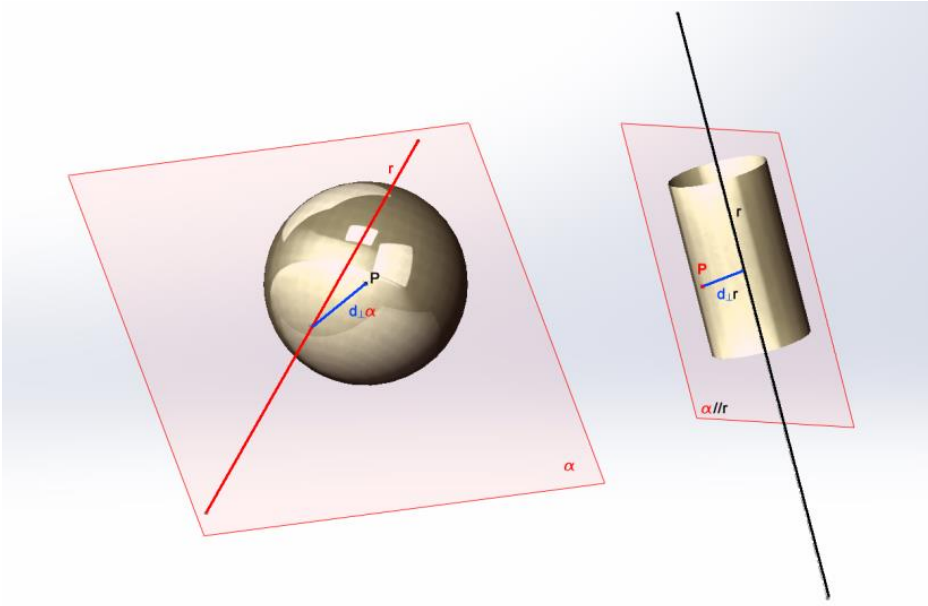
Construction:

The distance between two points is measured as the length of the segment connecting the two points.

Solutions:

If one of the points (***A***) is fixed and the (***B***) is not, there are infinite positions for point ***B*** that meet to be at a distance ***d*** from ***A***. All these positions are on the surface of a sphere of centre ***A*** and radius the distance (***d***) between the two points.

2 Distance between a point and a line



VIDEO SDW

Case 1: Wire mesh

Description:

Distance ***d*** from the point ***P*** to the line ***r***

What I should know before:

[Distance between two points]

Construction:

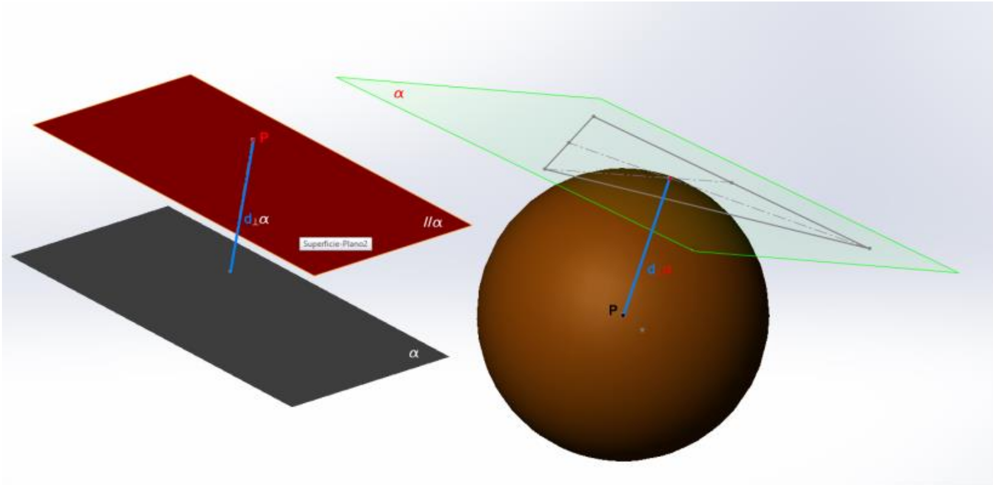
The distance ***d*** from point ***P*** to line ***r*** is measured as the length of the segment ***d*** between the point and the line. The segment should be the smallest possible of those that fall between the point and the line, so it must be perpendicular to the line.

Solutions:

If point ***P*** is fixed (left construction) and the line is not, there are infinite solutions for the line. All of them are tangent to sphere of centre point ***P*** and being the radius the distance between the point and the line.

If the line ***r*** is fixed (right construction) and the point is not, there are infinite solutions for the point. All of them are located on the surface of a cylinder of axis line ***r*** and radius the distance ***d***.

3 Distance between a point and a plane



VIDEO SDW

Case 1: Wire mesh

Description:
Distance ***d*** between the point ***P*** and the plane ***α***

What I should know before:

1. Distance between two points. The distance will be measured between the point ***P*** and one point of the plane *α*.
2. Distance between a point and a line. The segment ***d*** that measures the distance between the point ***P*** and the plane *α* is perpendicular to all the lines of the plane *α*, but it only maintains the same distance ***d*** with those that are coincident with the intersection point between ***d*** and *α*.

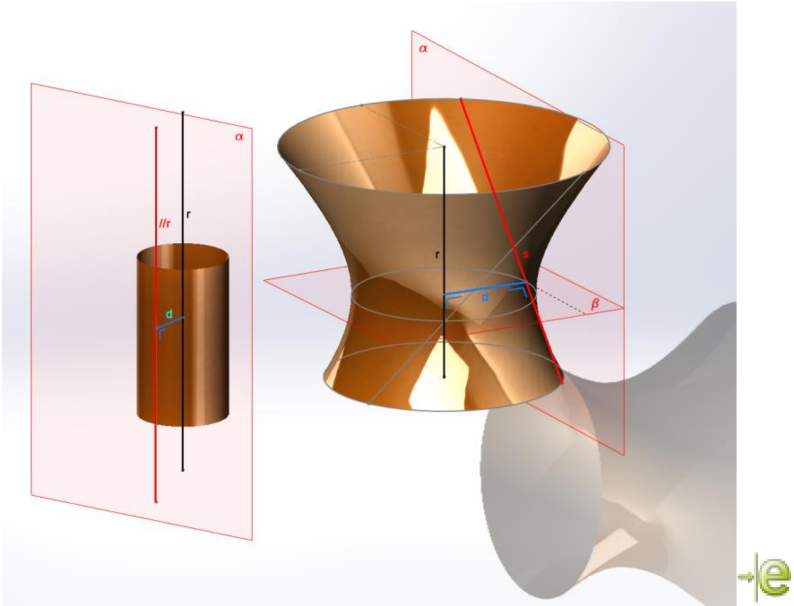
Construction:
The distance between point ***P*** and plane *α* is measured by a straight segment between the point and the plane. The distance should be the minimum possible between the point and the plane, thus, the segment ***d*** must be perpendicular to the plane *α*.

Solutions:

1. If *α* is a fixed plane (left construction) and the point ***P*** is not, there are infinite solutions for ***P***, all of them located on a plane //*α* placed at a distance ***d*** from *α*.
2. If ***P*** is a fixed point (right construction) and the plane *α* is not, there are infinite solutions for *α*. All of them are tangent to a sphere of centre point***P*** and radius ***d***.

THE LINE (straight line)

4 Distance between two lines



VIDEO SDW

Case 1: Wire mesh

Description:
Distance **d** between the line ***r*** and the line ***s***

What I should know before:

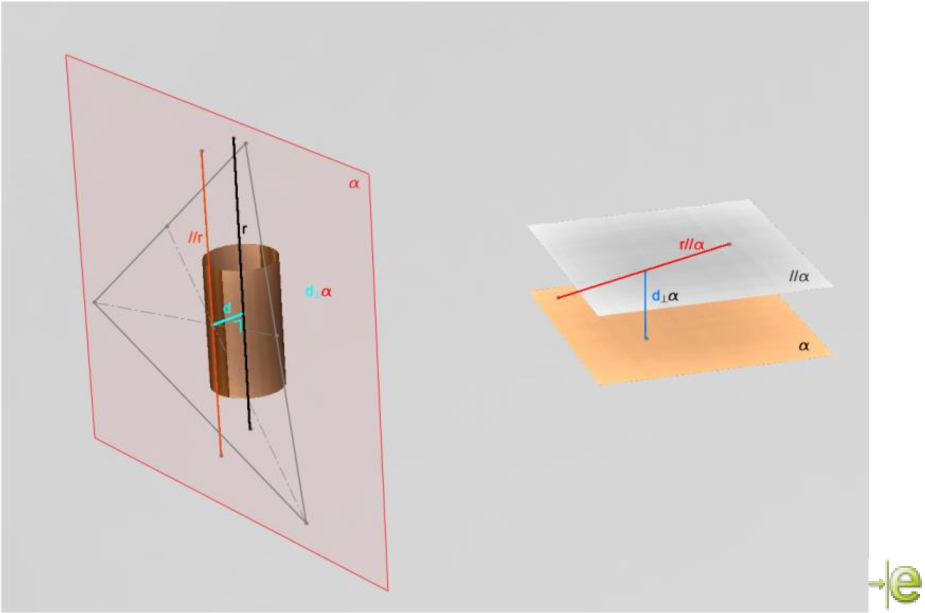
1. Distance between a point and a line
2. For the distance between two lines to be different from 0 both lines must be contained in parallel planes.

Construction:
The distance between two lines, ***r*** and ***s***, is measured with a straight segment perpendicular to ***r*** and perpendicular to ***s***. The segment that meets this condition is perpendicular to the plane direction defined by the two lines ($//\alpha$).

Solutions:

1. If the line ***r*** is fixed (left construction) and the line ***s*** (//r) line is not, there are infinite solutions for ***s***. All of them are generators of a cylinder of axis the linr ***r*** and its radius is **d**.
2. If the line ***r*** is fixed and the line ***s*** is neither fixed nor parallel to ***r*** (right construction), there are infinite solutions for ***s***. All of them are generators of a hyperboloid of revolution of axis the line ***r*** and has by generatrix line ***s***. The hyperboloid would be fully defined if the angle formed by the lines ***r*** and ***s*** is also known (see angle between two lines)

5 Distance between line and plane



VIDEO SDW

Case 1: Wire mesh

Description:

The distance between a line and a plane is greater than 0 when the line is contained in a non coincident parallel plane.

What I should know before:

1. distance between point and plane
2. distance between two lines

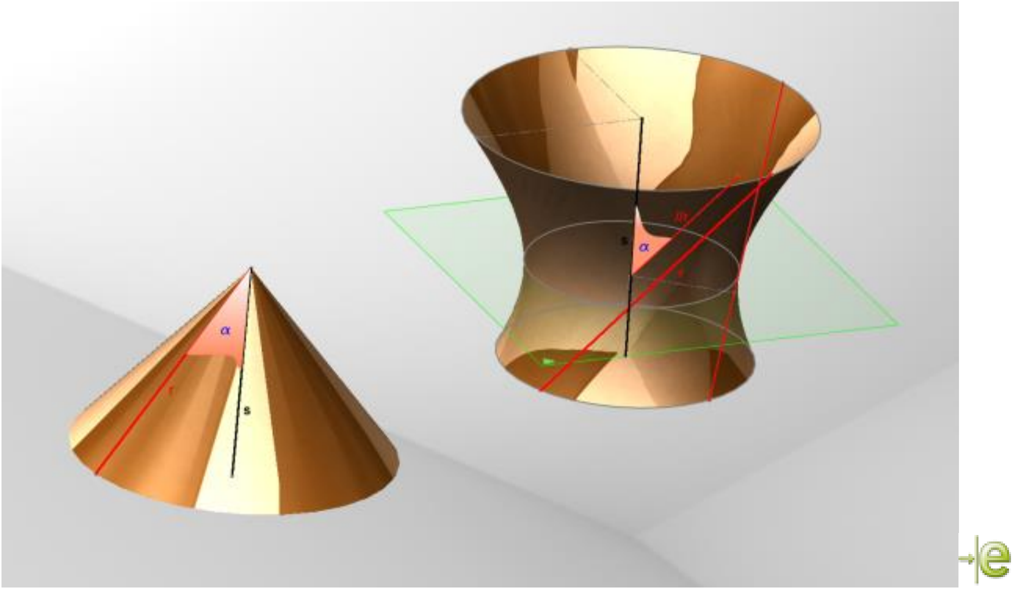
Construction:

The distance between the line ***r*** and the plane α is measured as the length of the straight segment (**d**) between the ***r*** line and the plane α. The segment **d** must be perpendicular to plane α, and therefore it will also be perpendicular to line ***r*** (because it is contained in a plane parallel to the plane α)

Solutions:

1. If the line ***r*** is fixed (left construction) there are infinite solutions for the plane α. All of them will be tangent to a cylinder of axis line **r** and radius segment **d**.
2. If the plane α is fixed (right construction) there are infinite solutions for line***r***. All of them are placed on a plane parallel to α at a distance **d** from α. In this case there are two solutions: two parallel planes to α, one oneach side of it.

6 Angle between lines



VIDEO — Case 1: Wire mesh | VIDEO — Case 2: Solids | SDW

Description:
The angle between two lines is measured on the plane they define. The intersection of the two lines divides the plane in four zones that are symmetrical to each other from the point of intersection of the lines. Two additional angles (complementary angles) can be measured.

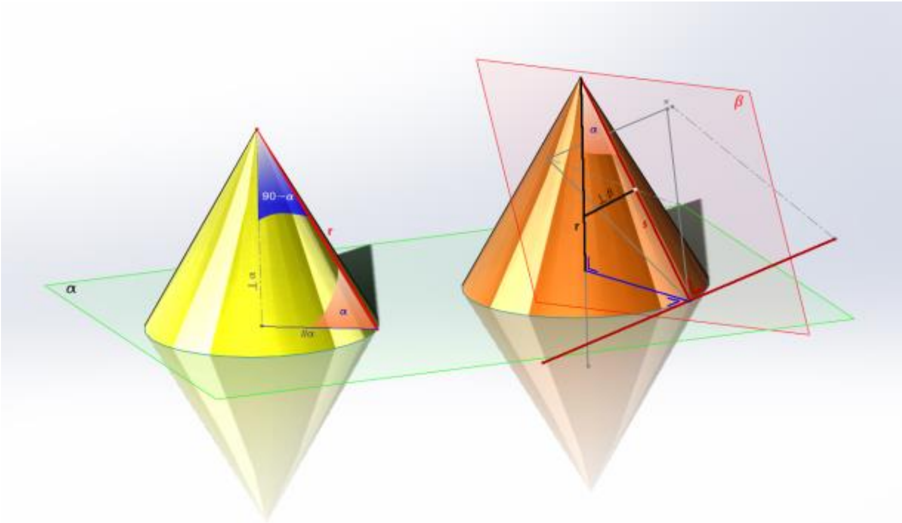
What I should know before:
The intersecting lines define a beam of parallel planes. One of these planes can be obtained with two lines which intersect and parallel to the given ones. Each of the intersecting lines is contained in one of the beam planes.

Construction:
The angle between two lines is measured directly between them in case the lines intersect. If the lines cross, it is measured between two lines that intersect and are parallel to the given ones.

Solutions:

1. In the case that the lines intersect (construction on the left) and one of them remains fixed (**s**); the other line (***r***) rotates around the first one occupying the surface of a cone. The vertex of the cone is the point of intersection of the lines and its semi-aperture angle is the angle defined between the two lines.
2. In the event that the lines intersect (construction on the right) and one of them is fixed (**s**), the other (***r***) rotates around the first one generating the surface of a hyperboloid of revolution.

7 Angle between a line and plane



VIDEO	VIDEO	VIDEO	SDW
Case 1: Wire mesh	Case 2: Solids left	Case 3: Solids right	

Description:

The angle between a line and a plane is measured between the line and its orthogonal projection on the plane.

What I should know before:

A line and its orthogonal projection on a plane form a right triangle in which one of the legs is perpendicular to the plane, the other is parallel to it and the hypotenuse is the line which maintains a particular angle with the plane. The angle we are looking for is measured between the line (i.e. the hypotenuse) and its projection, and the complementary angle will be measured between the line and the line normal to the plane.

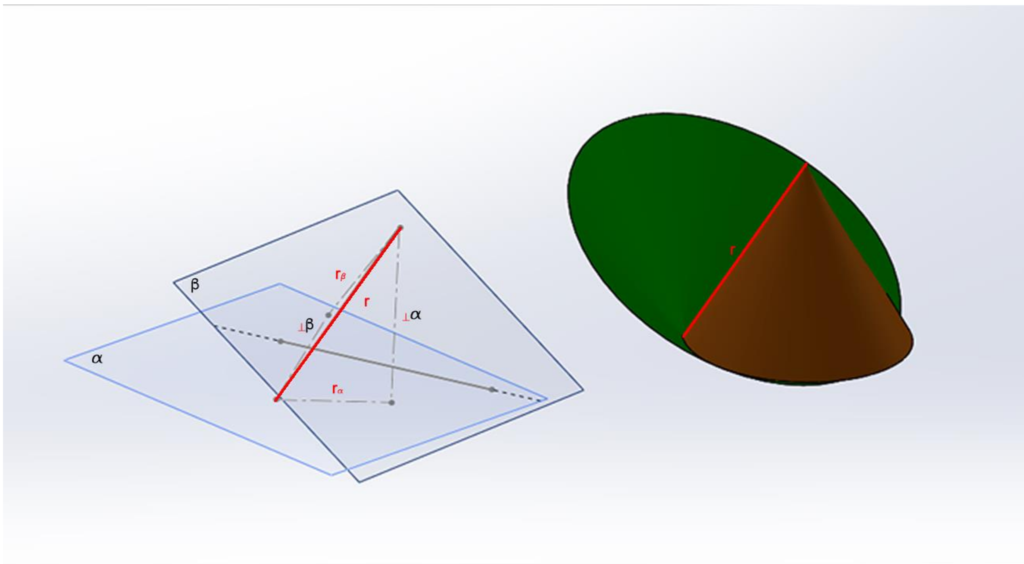
Construction:

1. Direct: A line parallel to the plane is drawn from any point of the line, and from another point a second line normal to the plane is drawn. Both lines need to intersect to be contained in the same plane. The angle we are looking for is measured between the line parallel to the plane and the other line..
2. Indirect: the line normal to the plane is drawn from any point contained in the line. The complementary angle between the two lines can be measured.

Solutions:
If the plane is fixed (left construction), there are infinite solutions for the line, all of them will be contained on the surface of the cone of axis the line normal to the plane ($\perp \alpha$) and of generatrix the line ***r***.

1. If the line is fixed (right construction), there are infinite solutions for the plane (β). All of them will be tangent to the surface of a cone of axis the line ***r*** and of generatrix the line **s**, which is contained in a plane perpendicular to **β** that also contains line ***r***.

8 Angle between a line and two planes



VIDEO SDW

Description:
The angle between the line ***r*** and two other planes α and β, is measured between the line and the orthogonal projection of line ***r*** on each one of the planes (**$r\alpha$** and **$r\beta$**, respectively).

What I should know before:

[Angle between line and plane](#).

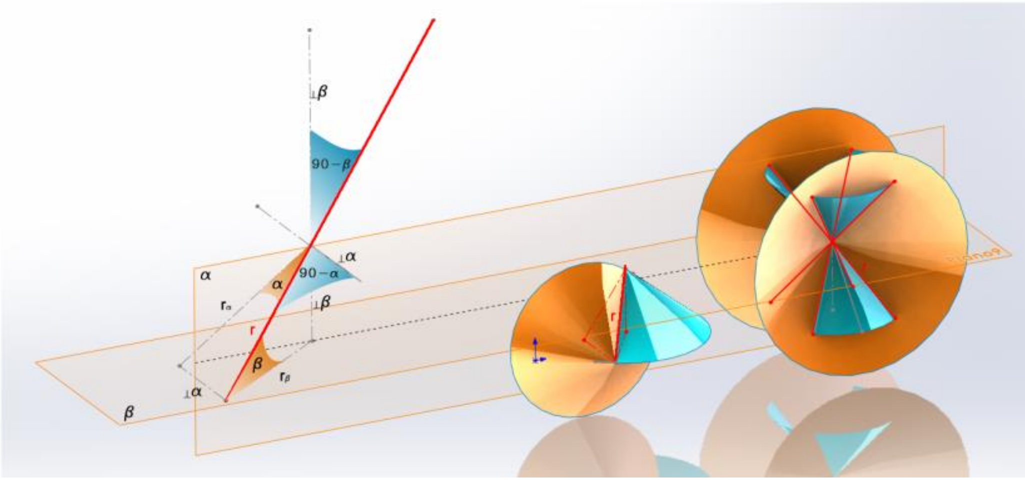
Construction:

1. Direct: a segment connecting the two planes is drawn and two right triangles are formed by ***r***, $\perp\alpha$, **$r\alpha$** on one side and ***r***, $\perp\beta$, **$r\beta$** ,on the other. The angles we are looking for will be measured between ***r*** and **$r\alpha$**, and ***r*** and **$r\beta$**.
2. Indirect: the line ***r*** y and the two lines normal to the planes $\perp\alpha$ y $\perp\beta$ are drawn. Between ***r*** and each one of the normal lines the complementary angle is measured.

Solutions:

There are four solutions that are obtained by symmetry with the two reference planes (one for each one of the quadrants defined by both planes).

9 Angle between a line and two orthogonal planes



VIDEO
Case 1: Solids

VIDEO
Case 2: Wire mesh

SDW

Description:

The line **r** has an angle α with one plane and an angle β with another plane orthogonal to α.

What I should know before:

[Angle between a line and a plane]
[Angle between a line and two non-orthogonal planes]

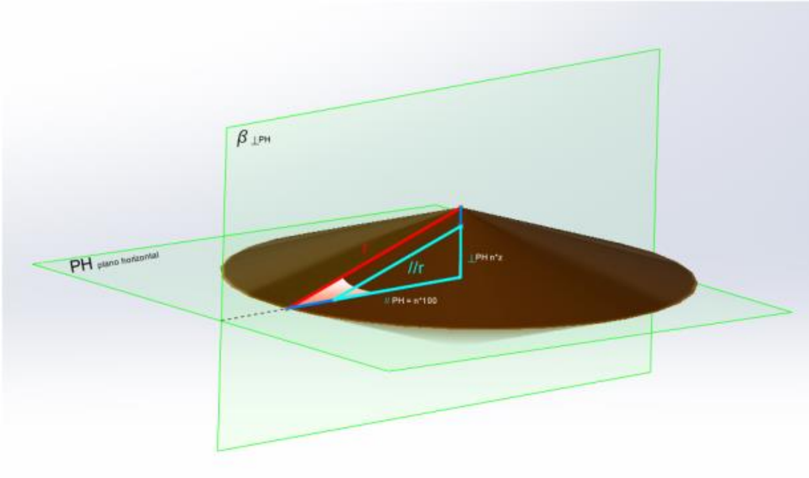
Construction:

1. Direct: the segment connecting the two planes is drawn and two right triangles are formed by $r \perp \alpha$, $r\alpha$ on one side, and $r \perp \beta$, $r\beta$ on the other. The angles we are looking for are measured between r and $r\alpha$, and r and $r\beta$.
2. Indirect: the line r and the two lines normal to the planes are projected on them ($\perp\alpha$ y $\perp\beta$). The complementary angle between the line and each plane is measured between r and each one of the normal lines.

Solutions:

There are 4 solutions that are obtained by symmetry with the planes.

10 Slope of a line



GRAPHIC LEGEND

PLANO HORIZONTAL = HORITZONTAL PLANE

VIDEO
Case 1: Solids

VIDEO
Case 2: Wire mesh

SDW

Description:

The slope of a line is the angle which the line forms with the horizontal plane expressed in the percentage (is the relation between its height increase and its horizontal displacement).

What I should know before:

Angle between line and plane

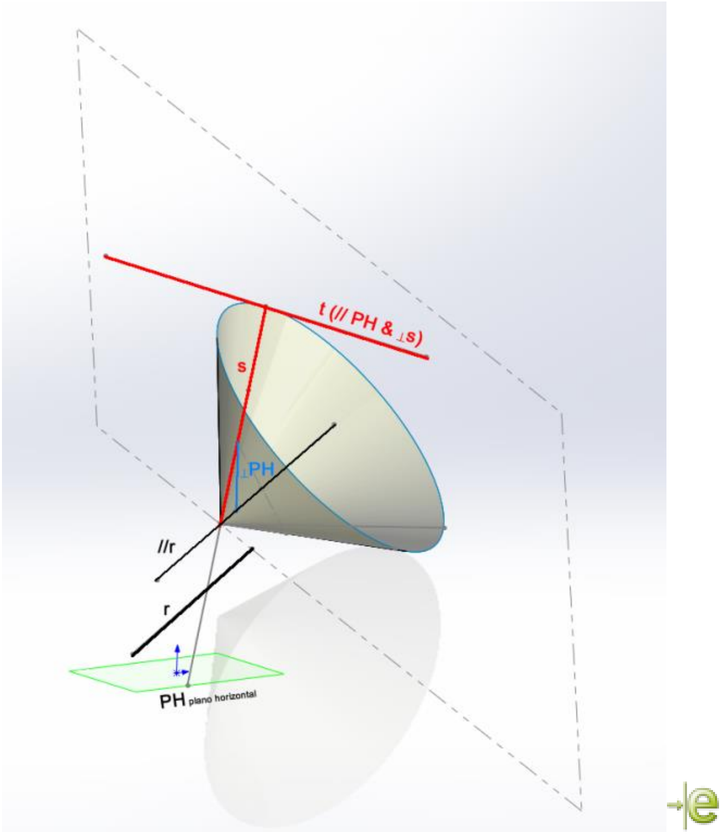
Construction:

1. Direct: a right triangle is constructed with a leg perpendicular to the horizontal plane and other parallel to it. The hypothenuse needsto be parallel to the line ***r***. The ratio ***z/100*** is imposed between the legs of the right rectangle, being **z** the length of the leg perpendicular to the horizontal plane and ***100*** the length of the leg parallel to the horizontal plane.
2. Indirect: the angle (arc tangent of (**z/*100***)) is measured between the leg perpendicular to the horizontal plane and line ***r***.

Solutions:

There are infinite solutions for line ***r***. All of them are generatrix of a cone of axis the leg perpendicular to the horizontal plane.

11 Maximum slope of a line



GRAPHIC LEGEND
PLANO HORIZONTAL = HORITZONTAL PLANE

VIDEO SDW

Case 2: Wire mesh

Description:
From the different possible positions of a line ***s*** contained on the surface of the cone, identify the one which has the greater angle with the horizontal plane. This case occurs when the line ***s*** rotates around another line (***r***).

What I should know before:
The maximum possible slope of a line is 90º and the minimum is 0º; when the line occupies the vertical and horizontal position, respectively.

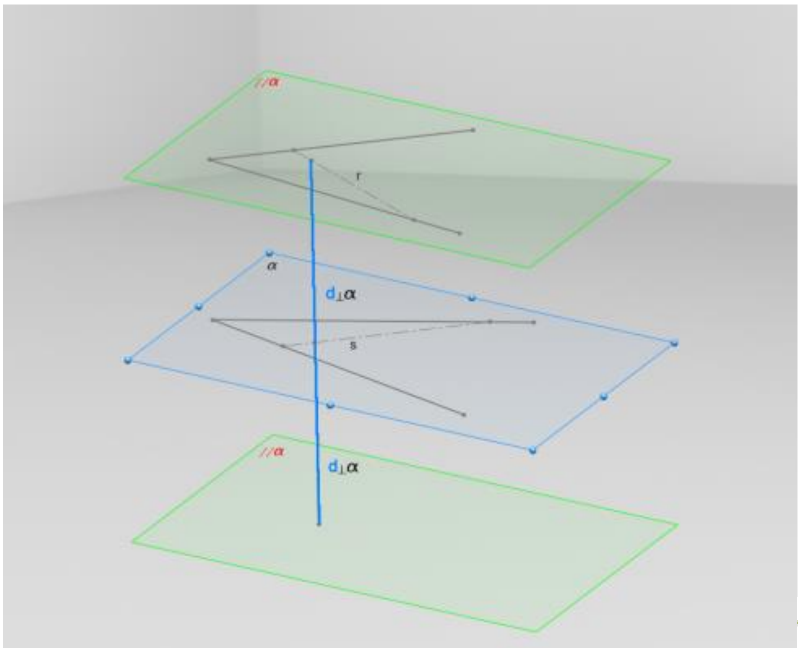
Construction:

Option 1: The line ***t***, coplanar with the base of the cone, tangent to it and parallel to the horizontal plane is drawn at the end of the line ***s***.
Option 2: A parallel line to line ***r*** that intersects with line ***s*** is drawn. A segment perpendicular to the horizontal plane (⊥ ***HP***) from a point contained in line ***r*** to a point in the straight ***s*** is drawn.

Solutions:
There is only 1 solution for the maximum slope and only another one for the minimum.

THE PLANE

12 Distance between planes



VIDEO SDW

Case 1: Wire mesh

Description:

The distance from one plane to another will be different from zero when the planes are parallel and not coincident.

What I should know before:

- distance between a line and a plane
- distance between a point and a plane
- distance between two lines

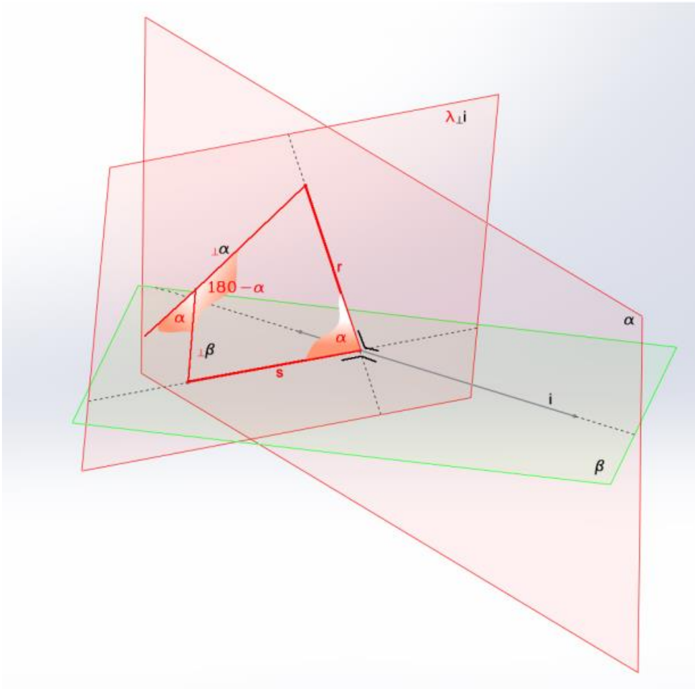
Construction:

The distance between the plane α and the plane $//\alpha$ is measured with a segment (**d**) connecting the two planes. The segment **d** must be perpendicular to plane α.

Solutions:

For a given plane α there are two solutions that are at a distance **d**. Each of the solutions is in one of the two semi-volumes in which plane α divides the space

13 Angle between two planes



VIDEO | VIDEO | SDW

Case 1: Wire mesh | Case 2: Solids

Description:

The angle between two planes α and β is measured between the lines ***r*** and ***s***, resulting from the intersection of the two planes with another plane λ perpendicular to them.

What I should know before:

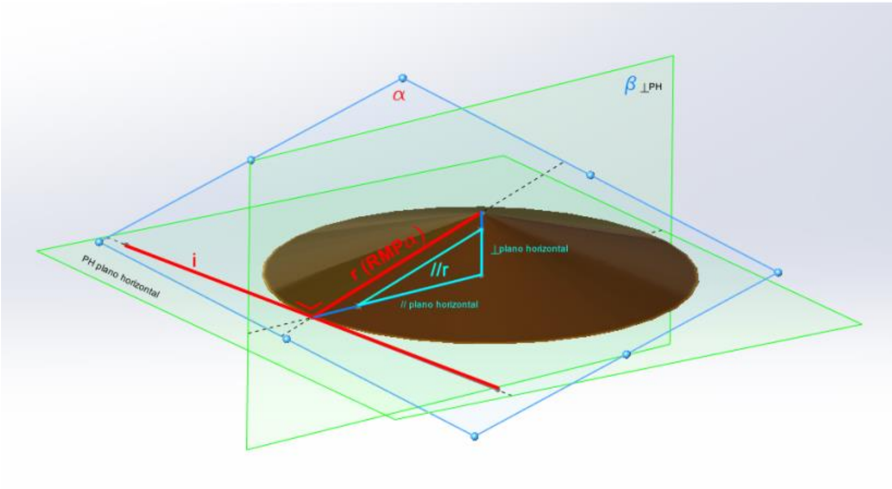
1. If one plane (λ) is perpendicular to two other planes (α y β) it will be perpendicular to the line ***i*** intersection of both.
2. All the lines contained on plane (λ) are perpendicular to line***i***.
3. The angle between the normal line $\perp\alpha$ and the normal line $\perp\beta$ with the two planes (α y β) is the same as the angle between the two planes.

Construction:

1. Direct: draw the lines ***r*** and ***s*** perpendicular to line ***i*** intersection between the planes. The angle is measured between the two lines.
2. Indirect: draw the normal line to the planes α and β so these lines intersect at one point. Measure the angle between the normal lines.

Solutions: There are 2 Solutions, each of which corresponds to one of the regions in which each of the planes divides the space. The four regions of space resultant give symmetrical solutions.

14 Slope of a plane



GRAPHIC LEGEND

PLANO HORIZONTAL = HORITZONTAL PLANE

VIDEO	VIDEO	SDW
Case 1: Wire mesh	Case 2: Solids	

Description:

The slope of a plane is the angle that it forms with the horizontal plane expressed in the percentage of the relationship between height and horizontal displacement.

What I should know before:

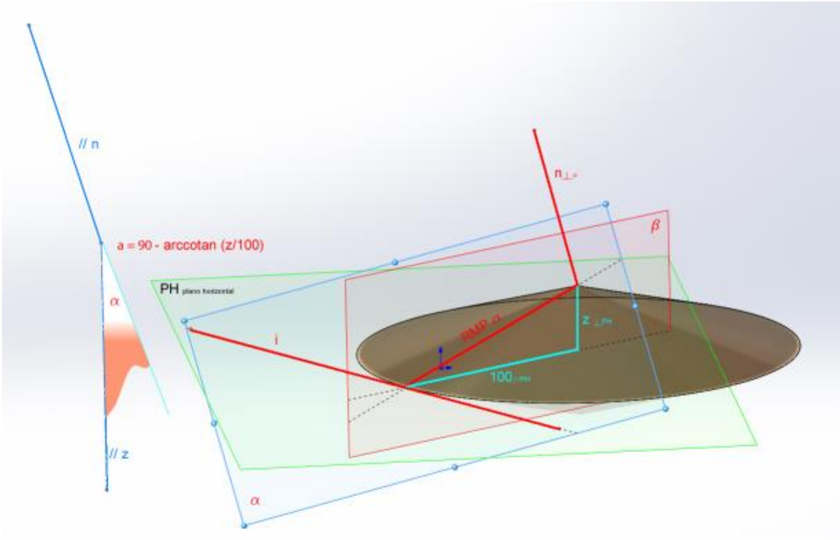
[slope of a straight](#)
[angle between two planes](#)

Construction:

1. Direct: draw the line ***r*,** contained on the plane, which has the same slope with the horizontal plane (slope of a line). Line ***r*** will be the line of maximum slope of plane α (***RMP***α) if it is perpendicular to a horizontal line of the plane (***i*** is the intersection between the plane α and the HP).
2. Indirect: [See slope of a plane measured by the normal](#)

Solutions: To obtain plane α there are infinite solutions. All of them are tangent to a cone whose axis is perpendicular to the ***HP*** and has as a generatrix line ***r*** (***RMPα***)

14.1 Slope of a plane measured by the normal



GRAPHIC LEGEND
PLANO HORIZONTAL = HORITZONTAL PLANE

VIDEO SDW

Case 1: Wire mesh

Description:

The slope of a plane α is measured between the line perpendicular to the horizontal plane and the line perpendicular to the plane α.

What I should know before:

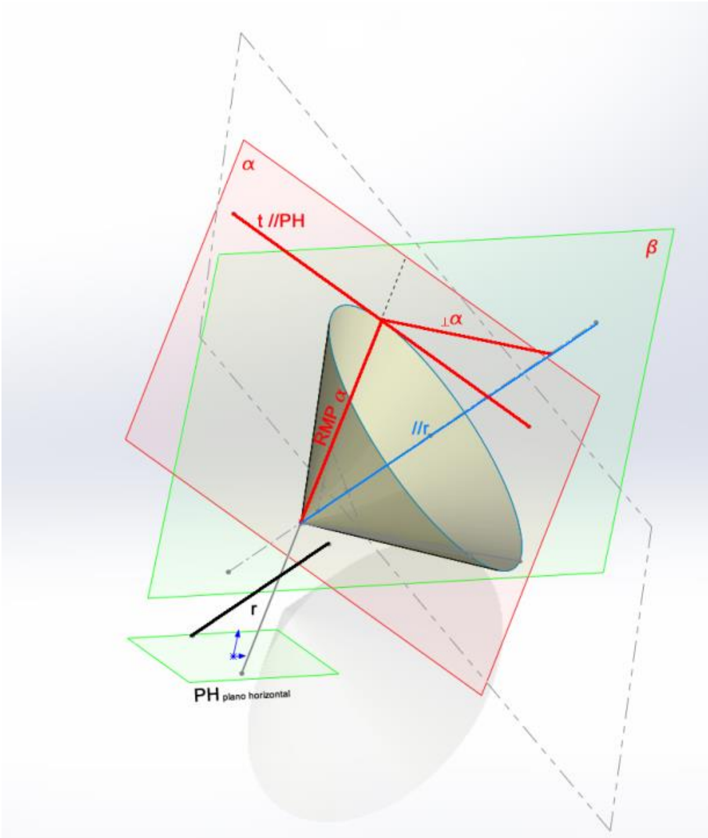
- Angle between a line and a plane
- Angle between two planes
- Slope of a line
- Slope of a plane

Construction:

The line perpendicular to the horizontal plane **//z** and the line perpendicular to the plane α (//**n**) are drawn so that both intersect. The angle (90-arc tangent (**z** /100) is imposed between the two, with line **z** being the corresponding height to a 100 units of horizontal displacement). The plane perpendicular to α //**n** is drawn.

Solutions: There are infinite solutions. All of them are perpendicular to any generatrix of a cone of axis //z and of generatrix //n.

15 Maximum slope of a plane



GRAPHIC LEGEND

PLANO HORIZONTAL = HORITZONTAL PLANE

VIDEO SDW

Case 1: Wire mesh

Description:

The plane with the greater angle with the horizontal plane is identified from the different positions of a plane α. The case arises when the plane rotates around a line (***r***).

What I should know before:

Maximum slope of a line

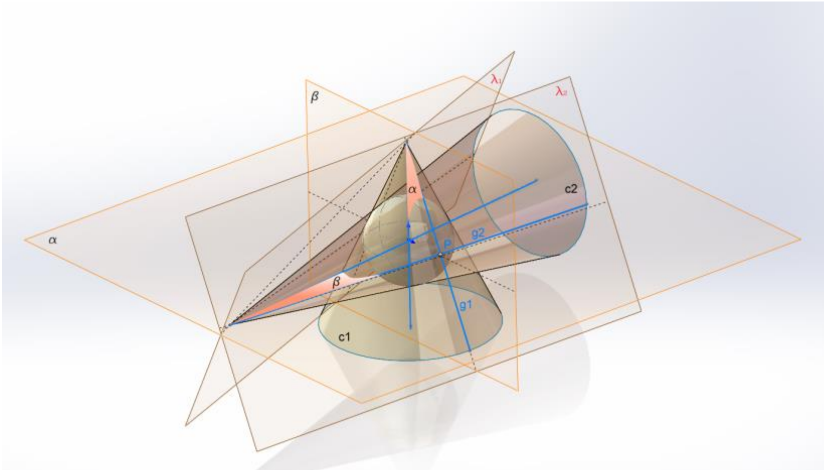
Construction:

It is similar to the maximum slope of a line. In this case the line behaves as the line of maximum slope of the plane α. The plane α will be defined with the **RMP**α and the line tangent to the base of the cone and parallel to the HP (***t//HP***).

Solutions:

There is only 1 solution for the line of maximum slope and another one for the minimum.

16 Angles of a plane with two planes



VIDEO
Case 1: Wire mesh

VIDEO
Case 2: Solids

SDW

Description:

The plane λ forms an angle α with the plane α and angle β with the plane β.

What I should know before:

Angle between two planes
Distance from point to line

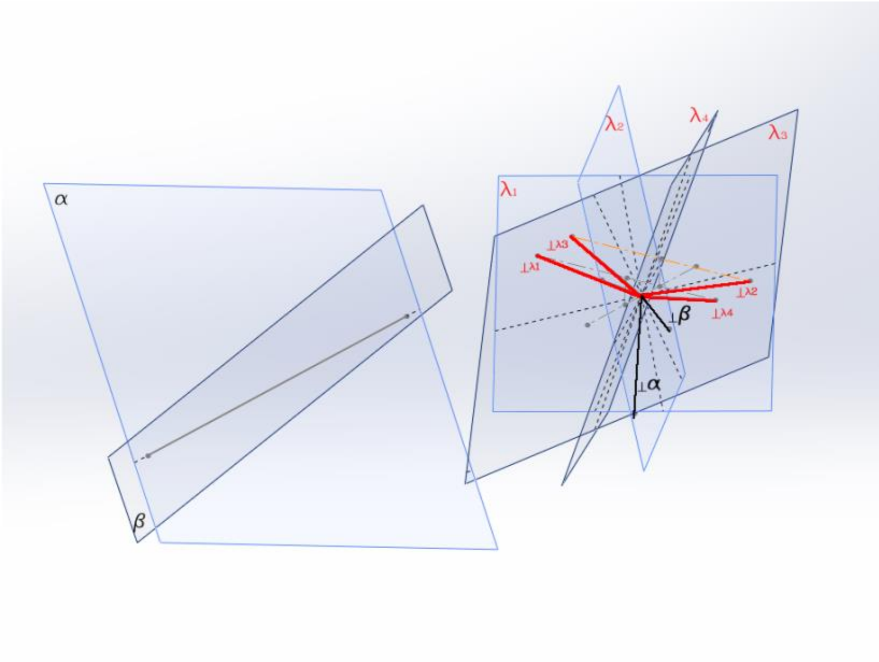
Construction:

The plane λ is tangent to two cones that share the same inscribed sphere. The two cones are constructed so that their axes intersect and the distance from the intersection point to the generatrix of each cone is the same. The cones are defined with their axis perpendicular to the plane on which the angle is measured and their generatrix (***g1*** for one of the cones and ***g2*** for the other) forms the complementary angle with the perpendicular planes to their corresponding axes (**90-**α for one and **90-**β for the other). The resultant plane is obtained by the two generatrix (***g1*** and ***g2***) that intersect at point ***P***, being ***P*** a point of the inscribed sphere.

Solutions:

There are 4 symmetrical solutions to the reference planes.

16.1 Angles of a plane with two planes measured by the normal



VIDEO SDW

Case 1: Wire mesh

Description:

The plane λ forms an angle α with the plane α and an angle β with the plane β.

What I should know before:

[Angle between two planes](#)
[Distance from a point to a line](#)

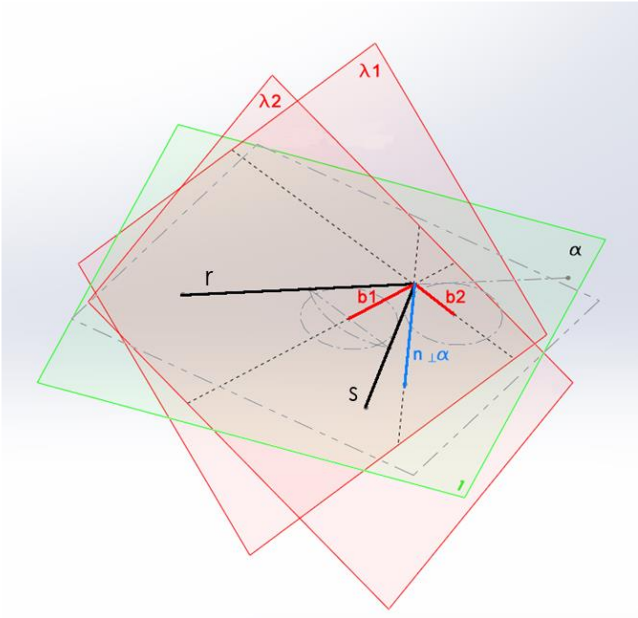
Construction:

The normal lines of the three planes (α, β and λ)are drawn so that all of them intersect at the same point. The angles given between each two of the normal lines are measured (α between $\perp\alpha$ and $\perp\lambda$; β between $\perp\beta$ and $\perp\lambda$. A plane perpendicular to the normal line obtained in each case is drawn.

Solutions:

There are 4 symmetrical solutions to the reference planes.

17 Bisector plane of two lines that intersect



VIDEO SDW

Case 1: Wire mesh

Description:

The bisector plane λ of two lines ***r*** and ***s*** is the plane that forms the same angle with the two lines and which contains the direction of the perpendicular line to the plane α defined by the lines. **Any line belonging to the bisector plane will form identical angles with the given lines**. The bisector plane contains the intersection point of the two lines, however, any other parallel plane to it will present the same properties.

What I should know before:

Angle between a line and a plane

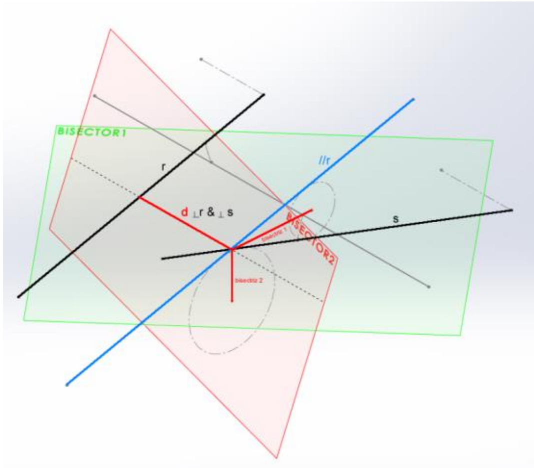
Construction:

The bisector lines ***b1*** and ***b2***, and contained in plane α, are drawn between the lines ***r*** and ***s***. The line ***n*** perpendicular to the plane α is defined. The bisector planes ***λ1*** (***b1*** and ***n***) and ***λ2*** (***b2*** and ***n***) are built.

Solutions:

There are 2 solutions perpendicular to each other.

18 Bisector plane of crossing lines



VIDEO
Case 1: Wire mesh

VIDEO
Case 2: Reference

SDW

Description:

The bisecting plane λ of two intersecting lines ***r*** and ***s*** is the plane that forms the same angle with both lines and contains the line perpendicular to the plane direction defined by the lines. The lines belonging to the bisector plane form identical angles with the given lines. The bisector plane contains the segment that measures the distance between the lines, however, any other plane parallel to this one will present the same properties.

What I should know before:

[Distance between two lines](#)
[Angle between line a and a plane](#)

Construction:

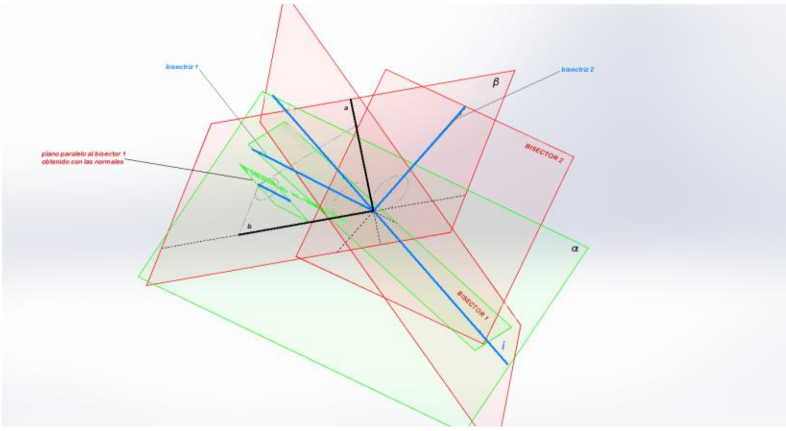
This case is simplified to the case of two lines that intersect.

The segment ***d*** perpendicular to line ***r*** and to line ***s*** is drawn. A parallel line to one of the lines (***//r***) that intersects the other line (***s***) at the point of intersection between ***d*** and ***s*** is drawn. Bisector lines ***b1*** and ***b2*** are drawn between the lines ***//r*** and ***s***. The bisector planes ***λ1*** (***b1*** and ***d***) and ***λ2*** (***b2*** and ***d***) are built.

Solutions:

There are 2 solutions perpendicular to each other.

19 Bisector plane of two planes



GRAPHIC LEGEND

Plano paralelo al bisector 1 obtenido con las normales = Bisector plane obtained by normal lines

Bisectriz = Bisector line

VIDEO

Case 1: Wire mesh

SDW

Description:

The bisector plane of two planes α and β is the one containing all the points at the same distance from the two planes. The lines contained in this plane form equal angles with the two reference planes.

What I should know before:

Angle between two planes
Bisector plane of two lines that intersect

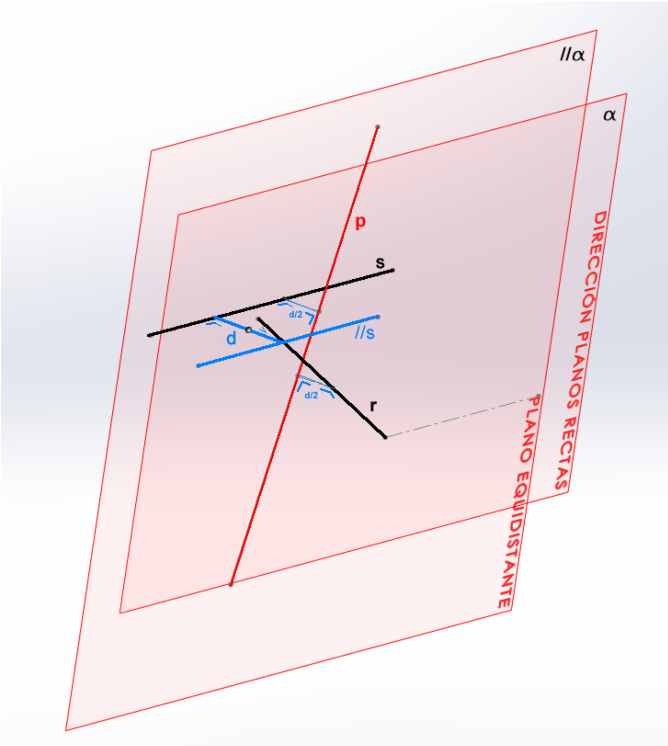
Construction:

1. **When the two planes are known**, the reference geometry tool is applied to obtain the equidistant plane of the two planes.
2. **If the planes are not known** the problem needs to be solved following the wire-mesh method to obtain the bisector plane of two lines that intersect. In this case the lines are the lines ***a*** and ***b***, which are the lines used to measure the angle between the two planes (***a*** and ***b*** are perpendicular to the line ***I***, intersection of the planes). The bisector plane is defined with the bisectrix of lines ***a*** and ***b*** and the line ***i*** intersection of the two planes.
3. If solved with the normal lines of the planes, parallel planes to the bisector ones are obtained (the bisectrix line of the normal lines will be parallel to the bisectrix line of lines ***a*** and ***b***).

Solutions: There are two solutions.

There are 2 solutions perpendicular to each other.

20 Equidistant plane from crossing lines



GRAPHIC LEGEND
PLANO EQUIDISTANTE = EQUIDISTANTE PLANE
DIRECCIÓN PLANOS RECTAS = PLANES AND LINES DIRECTION

VIDEO SDW
Case 1: Wire mesh

Description:
The equidistant plane from two crossing lines ***r*** and ***s*** is a plane parallel to the plane direction defined by the two lines and located at the same distance from both of them. Any line ***p*** contained in the equidistant plane will be also equidistant from the two given lines.

What I should know before:
[Distance between two planes]
[Distance between line and plane]
[Distance between two lines]

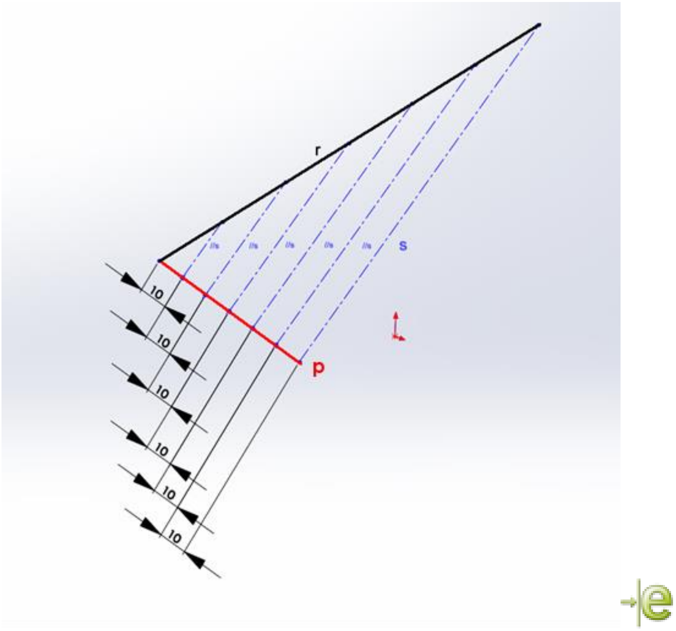
Construction:
A parallel line to one of the lines (**//s**) that intersects the other (**r**) is drawn. The plane α is defined with the **//s** and r. The segment of minimum distance **d** between ***r*** and ***s*** is drawn. A plane parallel to α that contains the middle point of **d** is defined.

Solutions:

There is one solution.

PROPORTIONS

21.1 Division of a segment by Thales



VIDEO SDW

Case 1: Wire mesh

Description:

To divide a segment into proportional or equal parts is done by projecting known divisions from another segment intersecting the first one. A beam of parallel lines is used to relate both segments.

What I should know before:

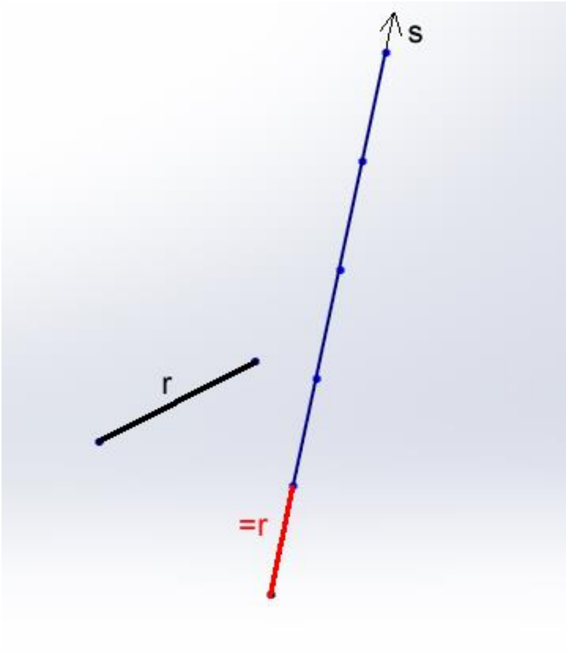
Construction:

Any straight segment ***s*** intersecting a given segment ***r*** is drwan. The new segment ***s*** is divided equally using known measures. The ends of the two segments are joined (***p***) defining the direction of the lines that will be used to relate both segment, thus obtaining the division into equal segments of the given line.

Solutions:

There is one solution.

21.2 Proportional segment to another



VIDEO SDW

Case 1: Wire mesh

Description:

A proportional relation between two segments can be established by aligning different segments equal to the smallest one.

What I should know before:

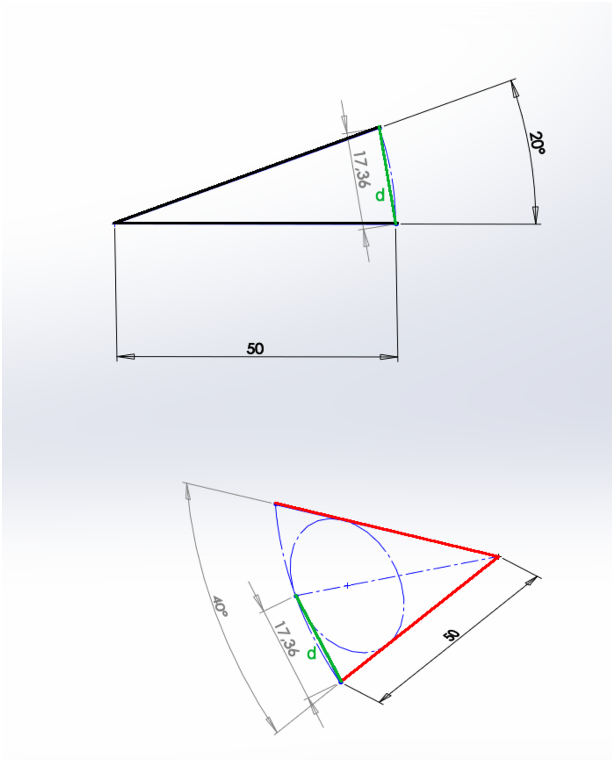
Construction:

A segment **=r** of the same length is plotted as ***r*** in the direction of ***s***. This segment is aligned with as many copies of itself as necessary to be a defined time longer than ***r***.

Solutions:

There is one solution.

Proportion between angles



VIDEO
22 Proportional angles between lines

VIDEO
23 Equal angles between line and three known planes

VIDEO
24 Proportional angles between line-line and line-plane

VIDEO
25 Proportional angles between planes

VIDEO
26 Proportional slopes

SDW

Description:

The angle between two lines is proportional to the angle between two other lines.

What I should know before:

Two triangles that are equal or proportional have equal angles between their sides.

Construction:

An arc of known radius is drawn between the reference lines, with its centre at the intersection point of the lines. The triangle formed by the arc string and the two segments between the intersection point of the lines and the arc string is defined. An arc of equal radius is drawn between the lines whose angle must be a fixed number of times greater than that of the reference. The corresponding number of arc strings equal to the one of the reference are drawn (in the figure the angle will be twice the one of the reference). Each of the arc strings shall be of the same length as the arc string obtained at the reference angle.

Solutions:

There is one solution

References

YouTube channels and lists:

- Geometría Métrica
- Metric Geometry
- Diseño y Tecnología

Knowing authors and teams

- Joaquin Fernandez
- Alba Ramos Cabal
- LAM
- ETSEIB
- UPC

Contents of the same programme

1. Basic Metric Geometry. UPC 2019.
2. Solids of Revolution. Procedures. UPC 2019.
3. Exercises, Problems and Practices. UPC 2019.

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Joaquín Fernández and Alba Ramos Cabal, Barcelona, August 2019