Maximal Entanglement in Quantum Computation

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I. Extended Abstract

Entanglement is a quantum phenomenon that occurs when two or more quantum systems cannot be described independently from the others. Several experiments have highlighted the fact that this is a *genuine* quantum property that goes beyond any classical description [1]. One may expect that such a distinctive trait of quantum mechanics have several physical applications. Indeed, entanglement can also be understood as the resource that enables genuine quantum protocols such as cryptography based on Bell inequalities [2] and teleportation [3]. In addition, large entanglement is expected to be present in quantum registers when a quantum algorithm produces a relevant advantage in performance over a classical computer such as Shor's algorithm [4].

In recent years the quantum computing has dived fully into the experimental realm. Control of quantum systems has improved so much that quantum computing devices have become a near term reality. Private companies have also joined the field. Since 2016, IBM offers cloud based quantum computation platform [5]. It is not the only company that has launched this kind of service: Rigetti Computing also allows the use of its 19-qubits device on the cloud [6]. Although both companies are betting for superconducting qubits, their respective device characterization is not the same. As more quantum devices are appearing, it is important to find some methods to test their quality when running sophisticated quantum algorithms. In addition, there is a need to set up a thorough benchmarking strategy for quantum computers that prove the usefulness of these devices to perform tasks that classical computation can not achieve.

The main goal of this work is to present quantum algorithms to test current quantum computers. First, we introduce the exact circuit that simulates exactly the XY model Hamiltonian. This model can be solved analytically. Then, the results extracted after the experiment can be compared with the theoretical values and the possible discrepancies can be used to test the quality of the quantum device. Next, we present a hard but necessary test for a quantum computer: the simulation of absolutely maximally entangled (AME) states. Since entanglement is behind quantum advantage, quantum devices must be able to generate and hold highly entangled states.

A. Exact simulation of XY Hamiltonian on a Quantum Computer

Let's consider the existence of a quantum circuit that *disentangles* a given Hamiltonian and transforms its entangled eigenstates into product states. This circuit will be represented by an unitary transformation U_{dis} such that [7]

$$\widetilde{\mathcal{H}} = U_{dis}^{\dagger} \mathcal{H} U_{dis}, \tag{1}$$

where $\mathcal{H}$ is the model Hamiltonian and $\widetilde{\mathcal{H}}$ is a noninteracting Hamiltonian that can be written as $\widetilde{\mathcal{H}} = \sum_i \epsilon_i \sigma_i^z$. This diagonal Hamiltonian contains the energy spectrum ϵ_i of the original one and its eigenstates correspond to the computational basis states. Then, we will have access to the whole spectrum of the model by just preparing a product state and applying U_{dis}.

In general, to find this operation will be hard. However, for the case of XY Hamiltonian, the steps to obtain the U_{dis} quantum gate can be based on the analytical solution of the model: *i)* Implement the Jordan-Wigner transformation to map the spins into fermionic modes. *ii)* Perform the Fourier transform to get fermions to momentum space. *iii)* Perform a Bogoliubov transformation to decouple the modes with opposite momentum. Thus, the construction of the disentangling gate can be done by pieces:

$$U_{dis} = U_{JW} U_{FT} U_{Bog}. \tag{2}$$

Figure 1 shows the result of the exact simulation of an $n = 4$ spin chain with an Ising model interaction in three quantum computers. The Ising model is a particular case of the XY model. The quantum circuit used for this experiment is detailed in Ref. [10]. Left figure plots the expected value of the ground state transverse magnetization as a function of the external field. The right part of Fig. 1 shows the simulation of time evolution of the $|\uparrow\uparrow\uparrow\uparrow\rangle$ state. The results show a clear disagreement between the theoretical value and the experimental values. However, the error sources are systematic, as indicates the simulation of the time evolution.

B. Absolute Maximal Entanglement on a Quantum Computer

AME states are n qudit quantum states with local dimension d such that every reduction to $\lfloor n/2 \rfloor$ parties is maximally mixed. Such states are maximally entangled when considering the entropy of reductions as a measure of multipartite entanglement, that is, the average entropy of these states is $S = \lfloor n/2 \rfloor$ when taking the logarithm in d basis.

The existence of AME states for n qudit systems, denoted as AME(n, d), is a hard open problem. Only for the case of

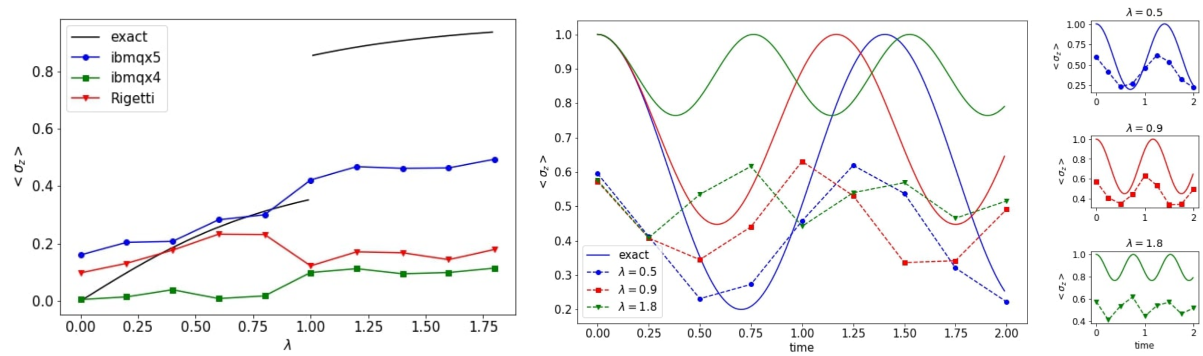
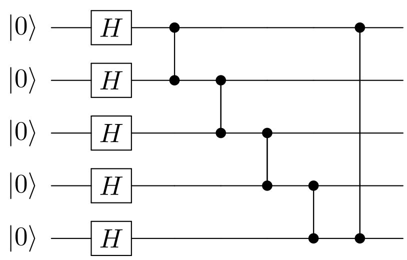


Fig. 1. *Left*. Expected value of $\langle\sigma_z\rangle$ of the ground state of a $n=4$ Ising spin chain as a function of transverse field strength λ. Solid line represents the exact result in comparison with the experimental simulations represented by scatter points. The best simulation comes from ibmqx5 device, which is an expected result since the number of gates used is lesser than with the other devices because of qubits connectivity. *Right*. Time evolution simulation of transverse magnetization, $\langle\sigma_z\rangle$, for the state $|\uparrow\uparrow\uparrow\uparrow\rangle$ of a $n=4$ Ising spin chain. Left plot compares the exact result with the experimental run in the ibmqx5 chip for different values of λ. Right plots detailed the results for each λ to compare them with the theoretical values. Although the magnetization is lesser than expected, the oscillations follow the same theoretical pattern.

qubit systems, i.e. $d=2$, the problem is fully solved for any n: an AME$(n,2)$ exists only for $n=2,3,5,6$. [11], [12]. For instance, Bell states and GHZ states are AME for bipartite and three partite systems of any d, respectively.

We propose to construct AME states quantum circuits using graph states. As an example, the AME state of five qubits can be implemented in a quantum computer using the following circuit:



where H gates are Hadamard gates and vertical lines correspond with Controlled-Z (CZ) gates. In addition, some graph states work for any local dimension d. Then, the above circuit can be used to obtain AME states of five qudits of any dimension d by replacing Hadamard gates by Fourier gates and CZ gates by its generalized version.

II. Conclusions

We have presented two methods that can be used to test and benchmark quantum computers. The experimental results show that the current devices have still many error sources. However, they are capable to perform sophisticated algorithms that have interest, for instance, in condense matter physics and error correcting codes.

III. Acknowledgment

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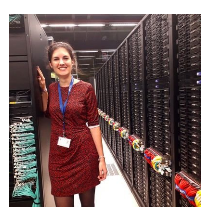

Alba Cervera-Lierta recieved her BSc degree in Physics from Universitat de Barcelona (UB), Spain in 2014. The following year, she completed her MSc degree in Astrophysics, Particle Physics and Cosmology from UB. She started her PhD in 2015 in Institut de Ciències del Cosmos and since 2017 she is also a technical researcher in Quantic group in BSC.