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EVALUATION OF THE R&R AND THE COMPATIBILITY INDEX FOR NON-INDEPENDENT MEASUREMENTS

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Abstract: This paper describes the methodology of the compatibility criteria "E_n" and the methodology of R&R. With this paper we'll use the methodology R&R for the evaluation of the compatibility criteria between the staff of the laboratories, where independent measurements aren't insured.

Keywords: compatibility, uncertainty, R&R, Monte Carlo's Method.

1. INTRODUCTION

Currently in an increasingly competitive world, the differential values between companies or institutions are becoming more critical to the client, who has to choose one or the other. One important factor is quality. In this paper, a requirement found in the standard ISO 17025 [1] applicable for calibration or testing laboratories is presented. As it is stated in the ISO 17025, the centers must perform various tasks to ensure the quality of their measurements. Among other tests, there are two which are very important: the intercomparison exercises between laboratories and the repetitions between the staff of the same laboratory. These two tests are precisely the factor to be analyzed in this article.

The intercomparison exercises between laboratories, as stated in the ISO 17043 [2], describes different types of studies, being one of the most used the compatibility index, shown in (1)

$$E_n = \frac{|A-B|}{\sqrt{U_A^2+U_B^2}}, \quad (1)$$

where A and B are the corrections, real minus the averaged values, and U_A and U_B are the expanded uncertainty of A and B.

Laboratories compatibility is accepted if $E_n<1$.

This index is perfectly valid as long as we can say that the measurements have always been independent of each other, a claim that can be valid if the measurements are performed in different laboratories, and also if the laboratories do not know the values found on other laboratories. The problem arise when the repeatability study is performed between workers of the same laboratory, since the independence of the measures, although intended to be independent, can always be dependent because the measurements are done in the same technical facilities, and also because technicians can talk to each other about the tests. For this kind of cases we must change equation (1). Given the non-independence of measurements, we obtain a new index E'_n (2). This equation will be mathematically derived along the development of the article.

$$E'_n = \frac{|A-B|}{\sqrt{U_A^2+U_B^2-2\cdot U_{A,B}}}, \quad (2)$$

where $U_{A,B}$ is the contribution to the uncertainty of the correlation between A and B.

The aim of this paper is to show that the R&R methodology for the repeatability and reproducibility evaluation with technicians within the same laboratory is better than the use of compatibility index E_n. On Section 2 we show the compatibility index, E'_n, calculation with the contribution of the non-independence of measurements. On Section 3 we show the R&R method application for repeatability and reproducibility evaluation. On Section 4 we develop the uncertainty budget. On Section 5 we show the results for all cases and conclusions are presented on the last section.

2. COMPATIBILITY INDEX (E'_n) CALCULATION

To determine the correlation term, we will show the process from the beginning.

$$Y = \bar{X}_1 - \bar{X}_2 \quad (3)$$

Equation (3) will be the numerator of the equation (1). Once we have defined the function, we proceed by calculating the associated uncertainty, which in this case we do it following the criteria, set by the GUM guide [3] and develop the Taylor series up to the second order, with the purpose to observe the correlation term. The result is shown in (4).

$$u(Y) = \sqrt{\left(\frac{\partial Y}{\partial X_1} \cdot u_{X_1}\right)^2 + \left(\frac{\partial Y}{\partial X_2} \cdot u_{X_2}\right)^2 + 2 \cdot \frac{\partial Y}{\partial X_1} \cdot \frac{\partial Y}{\partial X_2} \cdot u_{X_1,X_2}} \quad (4)$$

Following the criteria of the GUM guide, we introduce the correlation coefficient, shown in (5)

$$u(Y) = \sqrt{\left(\frac{\partial Y}{\partial X_1} \cdot u_{X_1}\right)^2 + \left(\frac{\partial Y}{\partial X_2} \cdot u_{X_2}\right)^2 + 2 \cdot \frac{\partial Y}{\partial X_1} \cdot \frac{\partial Y}{\partial X_2} \cdot \rho(X_1, X_2) \cdot u_{X_1} \cdot u_{X_2}} \quad (5)$$

where the correlation index ρ is defined. It can take values between $-1 \leq \rho \leq +1$. The value ρ can be calculated by the standard deviations of the measurements as indicated in the GUM guide, show in (6)

$$\rho(X_1, X_2) = \frac{s(X_1, X_2)}{s_{X1} \cdot s_{X2}}, \quad (6)$$

where the value of s(X1, X2) is calculated by (7).

$$s(X_1, X_2) = \frac{1}{n \cdot (n-1)} \cdot \sum (X_1 - \overline{X_1}) \cdot (X_2 - \overline{X_2}) \quad (7)$$

Another option is to take the maximum value, +1, which is the worst case we can have because it implies a strong correlation. If ρ is equal to 0, it implies no correlation between the variables X1 and X2. Taking the hypothesis of the worst case, we get equation (8),

$$u(Y) = \sqrt{(1 \cdot u_{X_1})^2 + (-1 \cdot u_{X_2})^2 + 2 \cdot 1 \cdot (-1) \cdot 1 \cdot u_{X_1} \cdot u_{X_2}} \quad (8)$$

This lead to the equation (9), which corresponds to the denominator of equation (2).

$$u(Y) = \sqrt{{u_{X_1}}^2 + {u_{X_2}}^2 - 2 \cdot u_{X_1} u_{X_2}} \quad (9)$$

Through (9) we can calculate the combined uncertainty for cases where the measures are correlated. With (3) and (9) we can compare between the function and its uncertainty value, where we want the result of the quotient to be less than or equal to 1. We see that this result is given in (2).

3. APPLICATION OF THE R&R METHOD

For the applications of the R&R method, the procedure detailed in [4] has been followed.

First, a process where the R&R methodology is going to be used is defined: three technicians will perform 6 measurements and on every measurement 3 trials. From these we have 3 evaluations, 6 parts, and 3 trials. Measurements are generated automatically following expression (15) where Random is comprised from 0 to 1, with a rectangular distribution probability.

The values of the R&R study are represented through the GRR index, which are expressed in %. An example of R&R method application can be shown in Table 1, where arbitrary values are used to explain the method variables.

	Value	% of TV
EV	1,63	86
AV	0,00	0
GRR	1,63	86
PV	0,98	51
TV	1,90	0
ndc		1

Table 1. Results of the R&R test

The term EV evaluates the contribution of the repeatability and the term AV evaluates the reproducibility of the test.

It is considered that the appraisals are compatible as long as the GRR value is less than 30%, and the *ndc* ratio is greater than 5.

For the example in table 1, we can see that GRR=86% (>30%) and ndc=1 (<5) that means that appraisals are not compatible even can be reproducibility because of the value of AV.

4. UNCERTAINTY BUDGET

For every assessment a typical contribution considers the repeatability, the resolution, and the master expanded uncertainty.

The repeatability contribution is u_{sA} and the value is obtained with (10) and (11).

$$u_{SA} = \max(u_{1S;Pi}) = \max(u_{1S,P1}; u_{1S,P2}; u_{1S,P3}; u_{1S,P4}; u_{1S,P5}; u_{1S,P6}) \quad (10)$$

$$u_{1S,P1} = \sqrt{\frac{1}{3} \cdot \frac{\sum_{j=1}^{3} (x_{j,P1} - \overline{x_{P1}})^2}{3-1}} \quad (11)$$

Another contribution is the resolution of the device, which is shown on (12).

$$u_R = \sqrt{\frac{(Resolution)^2}{12}} \quad (12)$$

The uncertainty budget from [3] has been simplified and the result is shown on (13).

$$u = \sqrt{u_S^2 + u_R^2 + u_P^2} \quad (13)$$

And the expanded uncertainty of the measurement is calculated using (14), because the probability distribution is known, it is Gaussian.

$$U = 2.00 \cdot u \quad (14)$$

5. RESULTS

We analyze the results for various scenarios. On this work we propose to follow the structured order shown on (15)

$$AP1 = Random$$
$$AP2 = AP1 \cdot (1 + \frac{Random}{10}) \qquad (15)$$
$$AP3 = AP1 \cdot (1 + \frac{Random}{10})$$

We study three possible cases:

Case a:. $E_n < 1$, $E_n' > 1$, GRR > 30%

We find that the compatibility index E_n is less than 1. Nevertheless, we know the relationship between operators and we can calculate E_n' getting values greater than 1. Likewise, we can identify that the study of R&R has given a result that shows that they are not compatible, because GRR > 30%. The numerical values are shown in Table 2.

EXPERIMENTAL MEASUREMENTS

		P1	P2	P3	P4	P5	P6
AP 1	N1	9,846	19,806	29,697	39,123	49,088	59,37
	N2	9,891	19,969	29,429	39,268	49,273	59,593
	N3	9,175	19,513	29,147	39,061	49,932	59,111
	Correction	0,36266667	0,23733333	0,57566667	0,849333333	0,569	0,642
	Range	0,716	0,456	0,55	0,207	0,844	0,482
	std.deviation	0,40102411	0,23106781	0,2750297	0,106237156	0,44362935	0,24122396
AP2	N1	9,836	19,791	29,684	39,067	49,046	59,351
	N2	9,888	19,966	29,408	39,215	49,269	59,591
	N3	9,132	19,48	29,14	39,033	49,928	59,11
	Correction	0,38133333	0,25433333	0,58933333	0,895	0,58566667	0,64933333
	Range	0,756	0,486	0,544	0,182	0,882	0,481
	std.deviation	0,4222669	0,24615104	0,2720098	0,096767763	0,45860913	0,24050017
AP3	N1	9,84	19,798	29,678	39,054	49,047	59,326
	N2	9,885	19,968	29,374	39,222	49,272	59,582
	N3	9,174	19,469	29,092	39,039	49,931	59,066
	Correction	0,367	0,255	0,61866667	0,895	0,58333333	0,67533333
	Range	0,711	0,499	0,586	0,183	0,884	0,516
	std.deviation	0,39814193	0,25368681	0,29306882	0,101602165	0,45941303	0,25800258

RESULTS R&R

	Value	% of TV
EV	1,96	81
AV	0,00	0
GRR	1,96	81
PV	1,42	59
TV	2,42	0
ndc		1

RESULTS UNCERTAINTY BUDGET

	AP1	AP2	AP3
us	0,256	0,265	0,265
uE	0,000	0,000	0,000
uP	0,001	0,001	0,001
u	0,256	0,265	0,265
U	0,512	0,530	0,530

EVALUATION

	E_n	E_n'
AP1-AP2	0,03	1,15
AP2-AP3	0,01	7,06
AP1-AP3	0,04	1,45

Table 2. Results of the experimental test with 3 appraisals showing the R&R results, the uncertainty budget and the evaluation of the compatibility index.

Case b. $E_n < 1$, $E_n' < 1$, GRR < 30%

This case indicates that the measurements are completly independent, which is not possible due the considerations presented on (15).

Case c. $E_n < 1$, $E_n' < 1$, GRR > 30%

This case indicates that the R&R method induces an error, see Table 3. Making iterations we can find this case, as shown on Table 3.

The analysis of this particular case can be done from the conditions expressed on (16).

$$E_n = \frac{\Delta}{\sqrt{U_i^2 + U_j^2}}$$
$$E_n' = \frac{\Delta}{\sqrt{U_i^2 + U_j^2 - 2 \cdot U_i \cdot U_j}} \qquad (16)$$

From these, we obtain the result shown in (17), with the initial assumption that $E_n' > E_n$ and that the squared value of E_n/E_n' is negligible for mathematical operations of addition or subtraction.

$$\frac{U_j}{U_i} = 1 \pm \sqrt{2} \cdot \left(\frac{E_n}{E_n'}\right) \qquad (17)$$

EXPERIMENTAL MEASUREMENTS

		P1	P2	P3	P4	P5	P6
AP 1	N1	9,03	19,262	29,211	39,237	49,064	59,751
	N2	9,758	19,639	29,103	39,82	49,505	59,853
	N3	9,127	19,832	29,751	39,043	49,916	59,412
	Correction	0,695	0,42233333	0,645	0,633333333	0,505	0,328
	Range	0,728	0,57	0,648	0,777	0,852	0,441
	std.deviation	0,39529609	0,28990746	0,34717143	0,404403676	0,42608802	0,23087009
AP2	N1	9	19,259	29,135	39,206	48,978	59,751
	N2	9,741	19,631	29,013	39,814	49,489	59,85
	N3	9,077	19,831	29,739	38,983	49,908	59,358
	Correction	0,72733333	0,42633333	0,70433333	0,665666667	0,54166667	0,347
	Range	0,741	0,572	0,726	0,831	0,93	0,492
	std.deviation	0,40741175	0,29027803	0,38875356	0,430107351	0,46575781	0,26022875
AP3	N1	9,03	19,198	29,193	39,221	49,001	59,743
	N2	9,753	19,633	29,027	39,819	49,467	59,852
	N3	9,047	19,824	29,736	38,951	49,91	59,4
	Correction	0,72333333	0,44833333	0,68133333	0,669666667	0,54066667	0,335
	Range	0,723	0,626	0,709	0,868	0,909	0,452
	std.deviation	0,41260433	0,32082758	0,37082925	0,44420866	0,45454849	0,23587921

RESULTS R&R

	Value	% of TV
EV	2,48	95
AV	0,00	0
GRR	2,48	95
PV	0,85	32
TV	2,62	0
ndc		0

RESULTS UNCERTAINTY BUDGET

	AP1	AP2	AP3
us	0,246	0,269	0,262
uE	0,000	0,000	0,000
uP	0,001	0,001	0,001
u	0,246	0,269	0,262
U	0,492	0,538	0,525

EVALUATION

	E_n	E_n'
AP1-AP2	0,04	0,67
AP2-AP3	0,00	0,18
AP1-AP3	0,04	0,86

Table 3 Values found for case c

Considering only the statistical contribution of the uncertainty, which is the main contribution, we perform an empirical study of the measured samples, show non Table 4

	En < 1, En' < 1, GRR > 30%			En < 1, En' > 1, GRR > 30%		
nº	us1	us2	us2/us1	us1	us2	us2/us1
1	0,234009	0,248882	1,06	0,184	0,188	1,02
2	0,257	0,276	1,07	0,222	0,228	1,03
3	0,216	0,226	1,05	0,194	0,197	1,02
4	0,31	0,332	1,07	0,21	0,218	1,04
5	0,214	0,232	1,08	0,252	0,258	1,02
6	0,262	0,282	1,08	0,196	0,198	1,01
7	0,245	0,261	1,07	0,256	0,27	1,05
8	0,25	0,268	1,07	0,189	0,19	1,01
9	0,234	0,255	1,09	0,21	0,216	1,03
10	0,28	0,296	1,06	0,198	0,201	1,02
Average			1,07			1,02

Table 4 Empirical data for the case AP1, AP2.

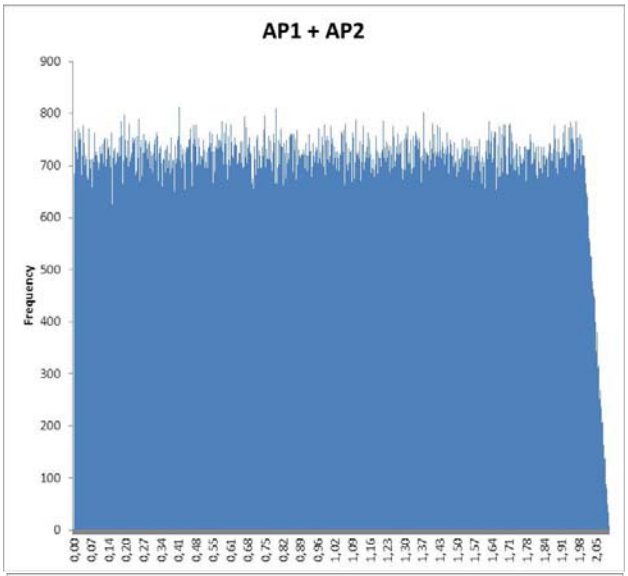
With the data form Table 4 and equation (17), we obtain:

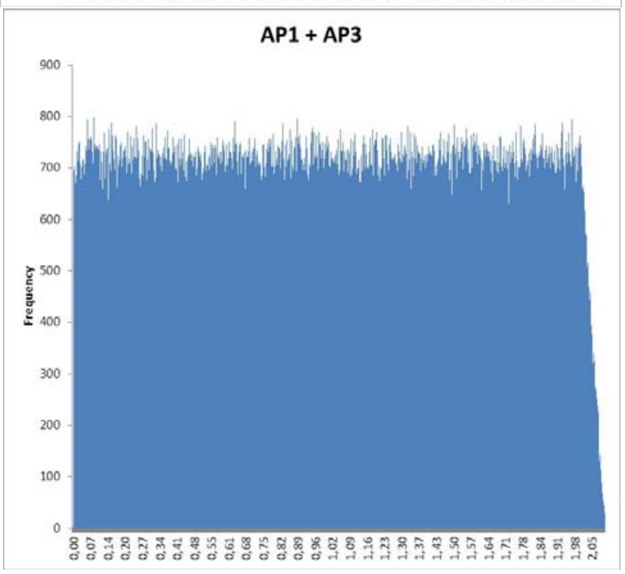
- $E_n < 1$, $E_n' < 1$, GRR > 30% the value of E_n/E_n' is 0.049.
- $E_n < 1$, $E_n' > 1$, GRR > 30% the value of E_n/E_n' is 0.014.

There is a limit value for the quotient us2/us1, of 1.05, which leads to a value of $E_n/E_n' = 0.035$.

From Table 4, we observe that for U_j/U_i lower than 1.05, the system detects that the variables have a certain level of correlation and this conditions that E_n' be greater than 1, but for U_j/U_i larger than 1.05, the variables show a small correlation and the calculus E_n' considers them as independent.

In order to better understand the reason of the duality between cases a and c, we have to think about the correlation between the three variables set on (15). Each one of these variables has a rectangular probability distribution, and the term that links all variables is the quotient, which in the case of (15) it has been set to 10. As this quotient increases the more delimited the probability distribution will be. Following the Monte Carlo method, $5{\cdot}10^5$ iterations are performed on the resulting function of the addition. The histogram is used to show the results for the representation of the probability distribution. On Figures 1, 2 and 3, is possible to observe the different frequency distributions obtained from the addition of AP2 with AP3, for a different denominator.





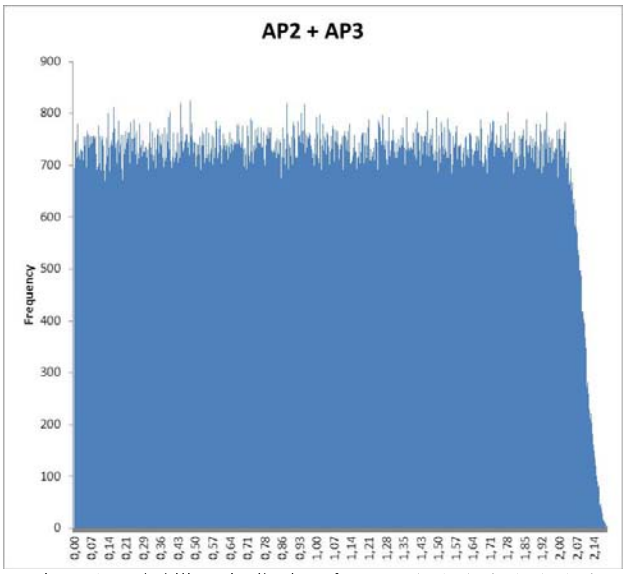
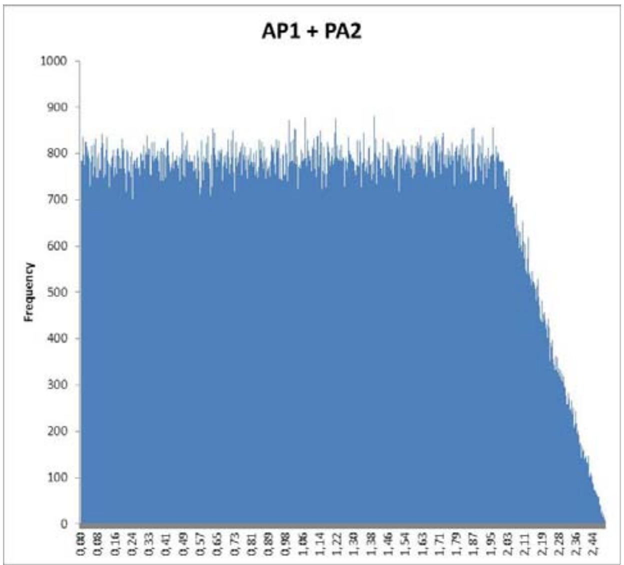
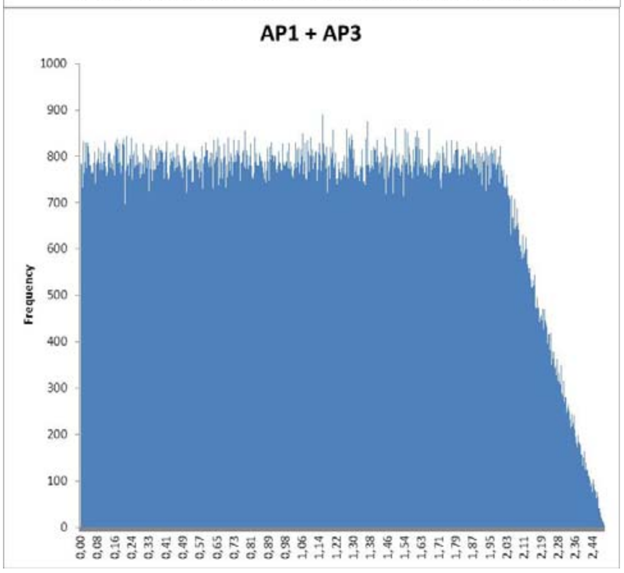


Figure 1. Probability Distributions for cases AP1+AP2, AP1+AP3 and AP2+AP3, all with a denominator of 10





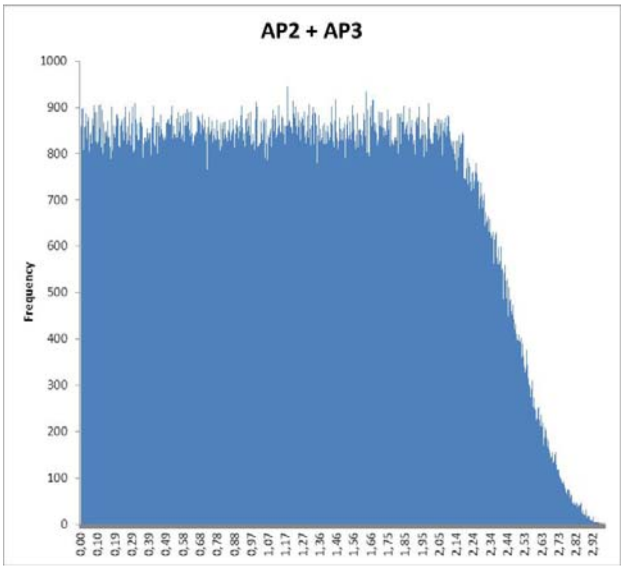


Figure 2. Probability Distributions for cases AP1+AP2, AP1+AP3 and AP2+AP3, all with a denominator of 2.

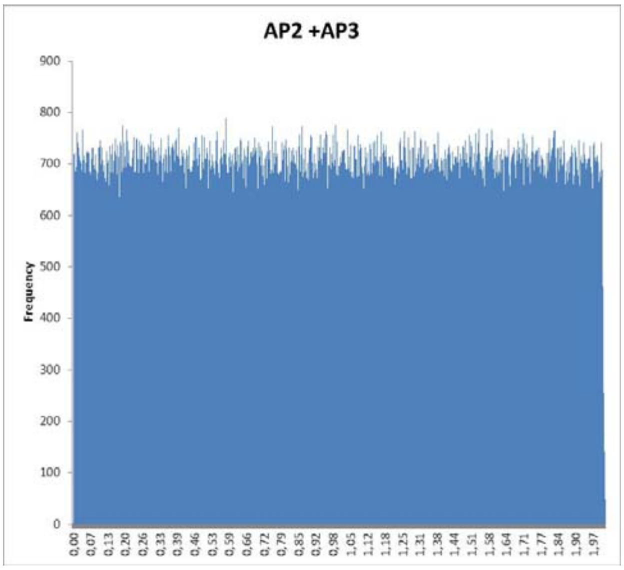
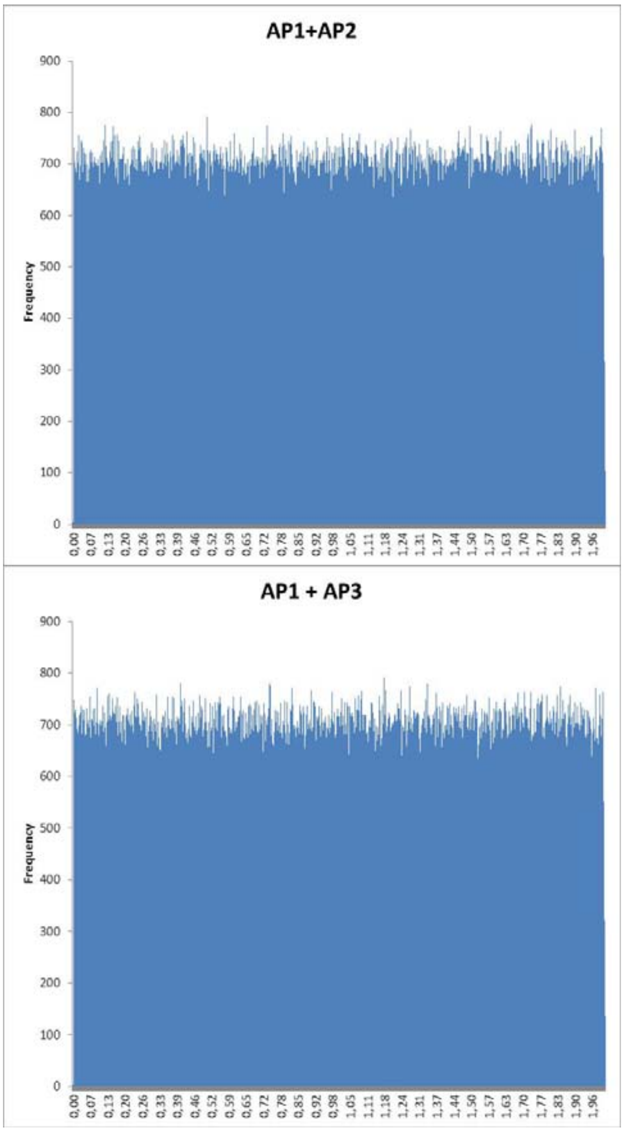


Figure 3. Probability Distributions for cases AP1+AP2, AP1+AP3 and AP2+AP3, all with a denominator of 100



It is possible to observe graphically on Figure 2, that the resulting probability distribution is not rectangular. Likewise, with a quotient of 2, the relation between variables is of 50%. On Figure 3 we observe that the probability distribution is rectangular, and this is because the relations between variables are close to the 100%, meaning that they are the same variables. In the case of the Figure 1, we can see an intermediate relation, which can lead to a possible case c.

6. CONCLUSION

On this paper we have shown a method to obtain R&R values and compare them with the compatibility criteria. The paper shows that we can use the R&R methodology because it presents better results than the compatibility with index E_n, since we have to evaluate the possible correlation between measurements. In the R&R method the correlation is automatically detected. A particular case has been studied and the input variables are tied-up lineally, where the contribution to the uncertainty of the statistical term is the most important contribution. This fact implies that this exercise is not valid for cases where the contribution of the statistical term is comparable to the uncertainty of the reference or to the contribution of the scale range of the measurement instrument.

A future way of work could be demonstrate that this methodology is valid for tied-up variables of other fields, for example tied-up to a triangular, Gaussian probability… etc.

7. ACKNOWLEDMENTS

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8. REFERENCES

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