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LMI Control Design of Structural Systems with Experimental Study

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ABSTRACT

A nonlinear robust control is developed for active mass damper system subject to external perturbation. This nonlinear controller is composed by the sum of a linear term plus a chattering component. The linear term is designed using linear matrix inequality (LMI) theory. Then, the chattering term is added to improve controller performance. Lyapunov theory is used to validate our control design. According with experiments, where a flexible two levels building with active mass damper and external perturbation is employed, they show that this chattering term improves controller performance. However, when a fault occurs, this chattering term is complaining.

Keywords: *robust control, structural systems, LMI techniques.*

1 INTRODUCTION

In this paper, a nonlinear robust control is developed for active mass damper systems subject to external perturbation. This is conceptually similar to active mass damper being studied in earthquake mitigation research facilities to reduce damage from earthquakes on high rise buildings [1, 2, 3]. For the purpose of maintaining the seismic response of structures within safety levels, service and comfort limits, the combination of passive base isolators and feedback controllers has been proposed in recent years [1]. In these systems, the presence of structural or dynamic faults is relevant.

The purpose is to design a control system to counteract the effects of the external disturbance on the structure. Here, a solution is presented using the robust H_∞ control theory [4] by the method of linear matrix inequality (LMI) [5, 6, 7] that can be applied to structural control [2]. Then, to improve controller performance, a chattering term is added, in terms of the sign of the local velocity, where Lyapunov theory is used to prove stability [8]. An experimental shaking table is used to simulate earthquakes exciting the flexible modes of a tall structure [3]. It is composed by a flexible two levels building with active mass damper at the top of the building (see Fig. 1). Only acceleration measurements at level one and two are available, and the position measurement of the cart (the active mass damper) situated at the building top. However, a velocity observer is introduced to define this chattering controller in terms of the cart position. Experiments are done to study the effectiveness of the LMI-chattering controller when sinusoidal chirp external perturbation and system faults are present.

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Figure 1: Quanser Shaking Table II.

2 STRUCTURAL MODEL

The model is derived using Lagragian formulation, where the dynamic equations are obtained and then a linear model is derived by linearizing about the quiescent (latent) point [3]. The states are defined as:

$$x = \left[x_c \, x_{f1} \, x_{f2} \, \dot{x}_c \, \dot{x}_{f1} \, \dot{x}_{f2} \right]^T \tag{1}$$

where x_{fi} is the position of the floor i=1,2 (floor deflection), and x_c is the cart position (see Fig. 1). The linear model about the quiescent point is defined by [3]:

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{2}$$

where $u(t)$ is the control input and the matrices are defined as [3]:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 278.43 & -18.69 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & -766.49 & 5.98 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 3.01 & 0 & -0.96 \end{bmatrix}' \tag{3}$$

The available measurements are x_c, $\ddot{x}_{f1}$ and $\ddot{x}_{f2}$. That means we can introduce as the measured output the variable $y(t) = \left[x_c(t) \quad \ddot{x}_{f1}(t) \quad \ddot{x}_{f2}(t) \right]^T$ and from system (2)-(3) the next definition is derived:

$$y(t) = C\,x(t) + D\,u(t) \qquad (4)$$

with

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & 766.49 & 5.98 & 0 & 0 \end{bmatrix} \quad , \quad D = \begin{bmatrix} 0 & 0 & -0.96 \end{bmatrix}^T .$$

3 CONTROL DESIGN

In the structure, the active damper is located at the top of the building. For this reason, we want to study the cart position and floor two acceleration, where the cart is located, when an external disturbance $w(t) \in L_2$ occurs. Therefore, a performance variable (virtual output) z(t) is defined as $z(t) = \begin{bmatrix} x_c(t) & \ddot{x}_{f2}(t) & u(t) \end{bmatrix}^T$. The state-space representation of system (2)-(4) yields

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_1 w(t) \\ z(t) &= C_1 x(t) + D_{12} u(t) \\ y(t) &= Cx(t) + Du(t) + D_{21} w(t) \end{aligned} \qquad (5)$$

where

$$B_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T, C_1 = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 & 0 & 0.1 \end{bmatrix}^T, D_{21} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

Matrix B_1 is defined to take into account that the external perturbation $w(t)$ is produced on the ground. Matrices C_1 and D_{12} are defined to increase the weight of the cart position (where the controller is located) in front of the external perturbation. The above dynamic model satisfies the f standard H_∞ assumptions [5]. An H_∞ controller $u(t)$ is designed as a dynamic control strictly proper [6]:

$$K : \begin{cases} \eta(t) = A_k\, \eta(t) + B_k\, y(t) \\ u(t) = C_k\, \eta(t) \end{cases} \qquad (6)$$

To improve the performance of the dynamic controller (6), a chattering term is added to $u(t)$:

$$\begin{aligned} & u(t) = C_k \eta(t) + u_{chat}(t) \\ & \text{with} \\ & u_{chat}(t) = -\delta\, sign(\dot{x}_c(t)) \end{aligned} \qquad (7)$$

where δ is a positive constant design parameter. Locally, the cart moves as a single degree-of-freedom system with mass m:

$$m\ddot{x}_c(t) = u_{chat}(t) + w_c(t) \quad (8)$$

where $w_c \in L_2$ is the local perturbation on the cart. To prove the local stability of system (5) with controller (6) – (7), we use Lyapunov theory. First, for system (8), consider Lyapunov function $V_2 = \frac{1}{2} m\dot{x}_c^2$. Then, for the whole system (5), consider the Lyapunov function $V = V_1 + V_2$, where V_1 is the H_∞ Lyapunov function, verifying the H_∞ inequality. Asymptotic stability of the unperturbed closed-loop system (5) with (7) is concluded.

4 EXPERIMENTAL VALIDATION

To test the obtained robust controller against disturbance and faults, the controller effectiveness is studied experimentally. To compare the performance of the H_∞ control (6) versus the nonlinear control including chattering (7), three scenarios are implemented: (a) response in front of external perturbation withput faults; (b) response from perturbation with system fault (adding a mass to the platform); and (c) response from perturbation when a sensor mails.

Shake Table II is an instrumental shake table developed by Quanser Inc. [3]. The system is comprised of a shake table, a universal power module, a data acquisition card (DAC) along with its external terminal board, and a PC running control software. The PC sends and receives signals through the DAC using WinCon. Designed to simulate earthquakes and evaluate the performance of active mass dampers, the shake table consists of a 1 Hp brushless servo motor driving a lead screw. The lead screw drives a circulating ball nut which is coupled to the 18” °— 18” table (see Fig. 1). The table itself slides on low friction linear ball bearings on 2 ground-hardened shafts. It can drive a 15 Kg. mass at 2.5 g. Maximum travel is ±7cm. In this paper, the external perturbation is a sine chirp wave (Quanser Chirp block in Fig. 2) with increasing frequency from 0.1Hz to 0.7Hz and target time 20s (5s for the first experiment), the total time of the experiments.

Using Matlab’s Robust Control Toolbox [9] to compute (6), the performance index $\gamma = 40.3$ is obtained and the control matrices are:

$$A_k = \begin{bmatrix} -57.99 & -4.89 & -27.66 & 303.85 & -212.17 & 2.12 \\ 0.18 & -3.24 & -1.32 & 52.52 & 4.54 & -0.29 \\ -41.11 & -16.74 & -28.22 & 392.42 & -162.13 & -8.47 \\ 247.39 & 26.78 & 137.73 & -1655.66 & 967.55 & 4.66 \\ 245.41 & 49.72 & 129.82 & -1935.56 & 786.63 & 4.01 \\ 800.71 & 463.07 & 537.60 & -1120.36 & 2231.03 & -99.36 \end{bmatrix}$$

We do not have velocities measurements, so we use [15] to aproximate $\dot{x}_c$ in (7). We consider δ=0.5. Fig. 2 pictures the acceleration of floors 2 for cases under study. The experiment takes 5 seconds. From Fig. 2 it is observed that the robust response is effectively improved by adding the chattering term.

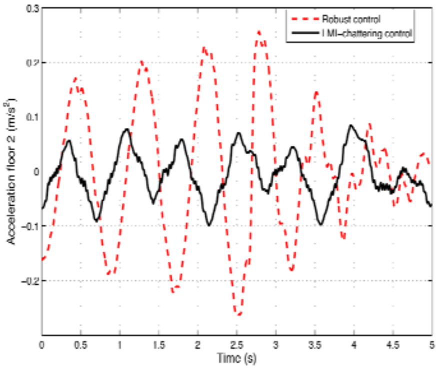

Figure 2: Acceleration of Floor 2 using robust control versus chatering-LMI control.

4.1 Experiments with structural fault

We add a mass to the structure (see Fig. 3), introducing important changes at the system parameters (3). Fig. 4 pictures the acceleration of floors 1 and 2 for cases under study. The experiment takes 20 seconds. When the mass is added to the structure, values in matrices (3) are strongly modified.

Figure 3. Shake Table II with a hardware-simulated structure fault.

From Fig. 4, it is observed that the robust response is not considerable improved by adding the chattering term, due the residual noise that effects the chattering control term performance (see Fig. 5).

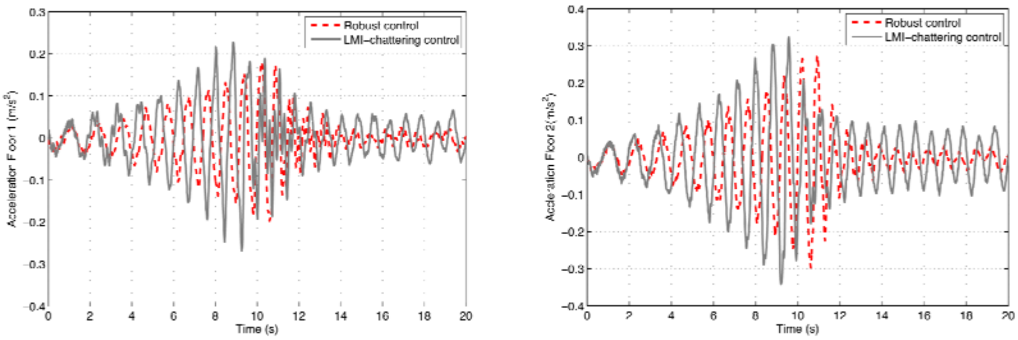

Figure 4. Accelerations of Floor 1 and 2 using robust control (6) versus LMI-chattering control (7), when structural fault appears.

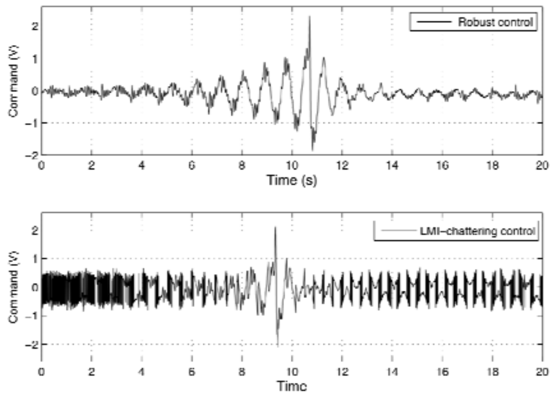

Figure 5. Robust control (6) versus LMI-chattering control (7), when structural fault appears.

4.2 Experiments with sensor faults

We disconnect the sensor from floor 1 (only sensor in floor 2 actuates). Fig. 6 pictures the acceleration of floor 2 for the two control cases under study. Again, the experiment takes 20 seconds. The control performance showed in Fig. 7 demonstrates the bad effect of residual noise on the chattering term. When a sensor fault occurs, the H_∞ control (6) is more robust without the chattering term. But when no fault occurs, the chattering term adds robustness to the controller, as showed in the first experiment. We can infer that control in (6) has better tolerance when information is missing or when the system parameters are changed.

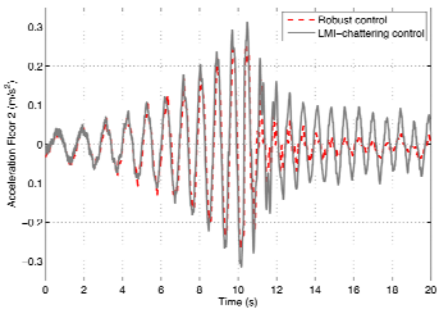

Figure 6. Floor 2 acceleration using (6) and (7) controls, when sensor fault appears.

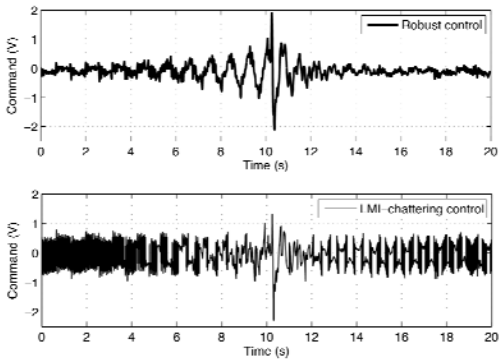

Figure 7. Signal from robust control (6) and LMI-chattering control (7), when sensor fault appears.

5 CONCLUSION

A nonlinear H_∞ control was developed for active mass damper systems subject to sinusoidal perturbation. This robust controller was composed by two terms: a linear term (robust dynamic control) and a chattering term (sign function of velocity). The linear term was designed using linear matrix inequality (LMI) theory. Then, the chattering term was added to improve controller performance and Lyapunov theory validated the control design. According with experiments, where a flexible two levels building with active mass damper and seismically exited was employed, the chattering term improved controller performance when an external disturbance appears. However, when a fault occurs, the chattering term does not improve the system performance. Also, from experiments, it was appreciated that LMI control without chattering has better fault tolerance when information is missing or when the system parameters are modified. The robust dynamic control can be considered as a passive fault-tolerant control.

6 ACKNOWLEDGEMENTS

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