A SIMPLIFIED LPV MODEL FOR MOBILE ROBOTS NAVIGATION WITH AUDIO FEATURES

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Key words: LPV modeling, indoor robot localization, audio.

In this article a new simplified LPV (Linear Parameter Varying) theoretical model is presented which corresponds to the acoustic response of an industrial plant. The parameters of this model vary with the characteristics of the closed room. The aim of the model is to locate mobile robots in such environments. A general methodology is proposed in order to simplify the robots self-localization process in industrial plants using the acoustic signals generated by typical engines in those environments.

1. Introduction

Indoor robot localization is an important issue in the field of robotics. So far, usually for this purpose overall odometer, camera, infrared sensor, ultra sonic sensor, mechanical wave and laser are mainly used. Nowadays the role of acoustic perception in autonomous robots, intelligent buildings and industrial environments is increasingly important and in the literature there are different works [1, 2].

It is very interesting the use of audio sensors according on the application. In industrial environments this type of sensors offers its main advantages: they are cheaper than other type of sensors, they hold a reach greater than the ultrasound sensors and they can cover a large area of exploration (with low directivity), they are not sensitive in front of changing light conditions, like cameras. Although audio sensors present low resolution to detect obstacles this fact is not too much relevant in industrial environments.

In an industrial plant to establish the transmission characteristics of a sound between a stationary audio source and a microphone in closed environment there are different study models: 1) the beam theory applied to the propagation of the direct audio waves and reflected audio waves in the room [3]; 2) the development of a lumped parameters model similar to the model used to explain the propagation of the electromagnetic waves in the transmission lines [3] and the study of the solutions given by the wave equation [4]. Other authors propose a transfer function of a room, denoted RTF (Room Transfer Function) that carries out to industrial plant applied sound model [5-7]. In these works the complexity to achieve the RFTs is
evident as well as the need of a high number of parameters to model the complete acoustic response for a specific frequency range, even with almost ideal conditions.

In [8] authors presented a first approximation to a theoretical model set-up which modeled the RTF in an industrial plant in order to locate mobile robots in such environments. In the present work, this model will be studied in depth as well as the features in the proposed RTF that are more suitable to be used by a mobile robot to navigate in an industrial plant; we have simplified the methodology and our goal is to determine the x-y coordinates of the robot. In such a case, the obtained RFT will not present a complete acoustic response, but will be powerful enough to determine the robot’s position.

The acoustic response of an industrial plant depends mainly on its dimensions, but also on the different absorption coefficients of the materials that form the walls, the floor and the ceiling of the plant. Another added difficulty present to establish the model of the RTF of a real environment will be the presence of objects in the plant that will act as obstacles in the signal propagation, depending on their dimensions respect the signal wavelength. This fact will introduce a new losses’ factor that depends on the position and the kind of materials [3].

This work is focused in the audio waves that are generated by rotary engines present in industrial environments. Those engines generate typically a reverberant field due to the successive reflections of the waves with the environment bounds. In order to simplify the establishment of a theoretical model as well as the obtaining of the experimental model by identification, a filtering step is introduced. In this phase, the most important harmonic component in the emitted signal spectrum will be selected.

In authors’ previous works, [9-11] the navigation system was presented. That work investigated the feasibility of using sound features in the space domain for robot localization (in x-y plane) as well as robot’s orientation detection. A robust sound-based indoor robot’s pose (x, y, θ) detection system was proposed utilizing two microphones. For this reason, in this present work the navigation system will be skipped and the work will be focused in obtaining a more general model for rooms through audio features. This model is LPV (linear parameters varying) because the parameters of the model vary along the robot’s navigation. Besides, this is a simplified model with the objective to obtain a low computational time in the robot self-localization process. The calibration of these parameters is carried out by physical equations and experimentation altogether. This new model is validated through significant experiments in a room with a real robot. In the literature there are many works about LPV modeling [12-14].

2. Sound model in a closed room

2.1. Theoretical model

The acoustical response of a closed room (with rectangular shape), where the dependence with the pressure in a point respect to the defined (x,y,z) position is represented by the following wave equation:

\[ L_x \frac{\partial^2 p}{\partial x^2} + L_y \frac{\partial^2 p}{\partial y^2} + L_z \frac{\partial^2 p}{\partial z^2} + k^2 p = 0 \]

\[ L_x, L_y \text{ and } L_z \] denote the dimensions of the length, width and height of the room with ideally rigid walls where the waves are reflected without loss, (1) is rewritten as [15]:

\[ p(x, y, z) = p_1(x)p_2(y)p_3(z) \]

when the evolution of the pressure according to the time is not taken into account.
Then (2) is replaced in (1), and three differential equations can be derived and it is the same for the boundary condition. For example, $p_1$ must satisfy the equation:

$$\frac{d^2 p_1}{dx^2} + k_x^2 p_1 = 0$$

with boundary conditions in $x = 0$ and $x = L_x$:

$$\frac{dp_1}{dx} = 0$$

$k_x, k_y$ and $k_z$ constants are related by the following expression:

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

Equation (3) has as general solution:

$$p_1(x) = A_1 \cos(k_x x) + B_1 \sin(k_x x)$$

Through (3) and limiting this solution to the boundary conditions, constants in (5) take the following values:

$$k_x = \frac{n_x \pi}{L_x}; \quad k_y = \frac{n_y \pi}{L_y} \quad \text{and} \quad k_z = \frac{n_z \pi}{L_z}$$

being $n_x, n_y$ and $n_z$ positive integers. Replacing these values in (5) the wave equation eigenvalues are obtained:

$$k_{n_x, n_y, n_z} = \pi \left[ \left( \frac{n_x}{L_x} \right)^2 + \left( \frac{n_y}{L_y} \right)^2 + \left( \frac{n_z}{L_z} \right)^2 \right]^{1/2}$$

The eigenfunctions or normal modes associated with these eigenvalues are expressed by:

$$p_{n_x, n_y, n_z}(x, y, z) = C_1 \cos \left( \frac{n_x \pi x}{L_x} \right) \cos \left( \frac{n_y \pi y}{L_y} \right) \cos \left( \frac{n_z \pi z}{L_z} \right) e^{i \omega t}$$

$$e^{j \omega t} = \cos(\omega t) - j \sin(\omega t)$$

being $C_1$ an arbitrary constant and introducing the variation of pressure in function of the time by the factor $e^{i \omega t}$. This expression represents a three dimensional stationary waves space in the room. For the eigenfrequencies corresponding to (6), the eigenvalues can be expressed by:

$$f_{n_x, n_y, n_z} = \frac{c}{2\pi} k_{n_x, n_y, n_z}, \quad f_{n_x, n_y, n_z} = \sqrt{f_{n_x}^2 + f_{n_y}^2 + f_{n_z}^2}$$

and

$$f_{n_x, n_y, n_z} = \sqrt{\left( \frac{n_x c}{2L_x} \right)^2 + \left( \frac{n_y c}{2L_y} \right)^2 + \left( \frac{n_z c}{2L_z} \right)^2}$$

where $c$ is the sound speed. Therefore, the acoustic response of any close room presents resonance frequencies (eigenfrequencies) where the response of a sound source emitted in the room at these frequencies is maximum. The eigenfrequencies depend on the geometry of the room and also depend on the materials reflection coefficients, among other factors.

In our case, the transform function $f_1$ (that relates the distance between feature space coefficients of each signal vs. source signal (see [11]) with the distance between the points $(x, y)$ in the space domain) is represented by (7), considering that the pressure is the square of the amplitude of sound signals $(S_j)$ for an specific time. The solution of this equation is real because imaginary numbers are neglected, and represented as:

$$S_j(x, y) = C_2 \cos \left( \frac{n_{dz} \pi d}{L_{dz}} \right)$$
where $C_2$ is an arbitrary constant.

If temporal dependency pressure respect the time is not considered, (7) is:

\[ p_{n_x,n_y,n_z}(x, y) = C_1 \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right). \]

In our experiment, the used values are $L_x = 10.54m$, $L_y = 5.05m$, $L_z = 4m$ and $z = 0.7m$ considering a sound speed propagation of 345m/s, some of the corresponding resonant frequencies that are obtained are indicated in Table 1. Specifically, in this Table the resonant frequency corresponding to the propagation mode $(3, 0, 2)$ can be observed. This frequency is close to 100Hz that is selected from the signal spectrum when the climatic chamber is used as sound source.

<table>
<thead>
<tr>
<th>$n_x$</th>
<th>$n_y$</th>
<th>$n_z$</th>
<th>$f_{nx}$</th>
<th>$f_{ny}$</th>
<th>$f_{nz}$</th>
<th>$f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>49,1</td>
<td>68,3</td>
<td>43,1</td>
<td>94,54</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0</td>
<td>65,5</td>
<td>68,3</td>
<td>0,0</td>
<td>94,62</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>32,7</td>
<td>34,2</td>
<td>86,3</td>
<td>98,37</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2</td>
<td>49,1</td>
<td>0,0</td>
<td>86,3</td>
<td>99,25</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>102,5</td>
<td>0,0</td>
<td>102,48</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
<td>102,5</td>
<td>0,0</td>
<td>103,77</td>
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<td>1</td>
<td>65,5</td>
<td>68,3</td>
<td>43,1</td>
<td>103,98</td>
</tr>
</tbody>
</table>

When (10) is applied in the experiments rooms, for mode $(3, 0, 2)$, this equation indicates the acoustic pressure in the rooms depending on the x-y robot’s position, and this is:

\[ p_{n_x,n_y,n_z}(x, y) = C_1 \cos\left(\frac{3\pi x}{10.54}\right) \cos\left(\frac{1.4\pi}{4}\right) \]

With these ideal conditions, for an ideal value for constant $C_1 = 2$ and for an ideal room without audio losses, the theoretic acoustic response in the rooms for this absolute value of pressure, and for this propagation mode, can be seen in fig. 1 (left).

![Fig. 1. Room response for (left) propagation mode (3, 0, 2) and (right) propagation mode (1,1,2).](image)

The response shown in fig. 1 (left) can vary depending on the excited propagation mode, in fig. 1 (right) the response to $(1,1,2)$ propagation mode can be seen. The shape of both plots would be obtained for a sound source that excite only these propagation modes, but really the
acoustic response will be more complex as the propagation modes excited by the sound source are increased.

### 2.2. Transfer function

Since the dimensions of industrial plants will be comparable to the wavelength of audio signals present in the environment, distributed constant models can be used in order to model the audio waves propagation, in a similar way to those proposed for electromagnetic signals transmission lines.

Even in closed rooms, as we stated before for other authors working with RTF, the number of parameters is relatively high for the description of RTF in our environment. Since the objective of this work is to find the x-y coordinates through the processing of emitted audio signals for a fixed source, a new methodology is proposed in order to work without the need and constraint of a complete description of the plant’s acoustic response.

In [16] a model based in the sum of second order transfer functions is proposed; these functions have been build between a sound source located in a position $d_s$ emitting an audio signal with a specific acoustic pressure $P_s$ and a microphone located in $d_m$ which receives a signal of pressure $P_m$ with a time delay $t_{rm}$; each function represents the system response in front to a propagation mode.

The first contribution of this work is to introduce an initial variation to this model considering that the sound source has a fixed location, and then this model can be expressed as:

$$\sum_{n=1}^{M} \frac{K_m s e^{j \omega_m s}}{s^2 + 2 \xi_n \omega_n s + \omega_n^2}$$

Because our objective is not to obtain a complete model of the acoustic response of the industrial plant, it will not be necessary to consider all the propagation modes in the room and we will try to simplify the problem for this specific application without the need to work with models of higher order.

To implement this experiment the first step is to select the frequency of interest by a previous analysis of the audio signal frequency spectrum emitted by the considered sound source (an industrial machine). Those frequencial components with a significant acoustic power will be considered with the only requirement that they are close to one of the resonant frequencies of the environment. The way to select those frequencies will be through a band-pass digital filter centered in the frequency of interest. Right now, the term $M$ in the sum of our model will have the value $N$, being this new value the propagation modes resulting from the filtering process.

### 3. Proposed LPV model in a closed room

For a concrete propagation mode, the variation that a stationary audio signal receives at different robot’s position can be modeled, this signal can be smoothed by the variation of the absorption coefficient of the different materials that conform the objects in the room; those parameters are named $K_{nm}$ and $\xi_{nm}$, and (12) results:

$$H(s, d_m) = \frac{P_m(d_m, s)}{P_s(s)} = \sum_{n=1}^{N} \frac{K_{nm}(d_m) s e^{j \omega_n s}}{s^2 + 2 \xi_{nm}(d_m) \omega_n(d_m) s + \omega_n^2(d_m)}$$

where the subscript $n$ indicates the natural frequency associated to the considered propagation mode ($\omega_n$) and the subscript $m$ indicates the distance of the microphone to the sound source ($d_m$). This natural frequency ($\omega_n$) of the transfer function room depends on the room characteristics: $L_x$, $L_y$ and $L_z$ resulting in an LPV indoor model.
The distance \((d_m)\) from the robot to the sound source is given by the expression:

\[
d_m = \sqrt{(x_r - x_s)^2 + (y_r - y_s)^2 + (z_r - z_s)^2}
\]

where \((x_r,y_r,z_r)\) are the robot location coordinates and \((x_s,y_s,z_s)\) are the sound source coordinates.

Using (13) the module of the closed room in a specific transmission mode \(\omega_n\) is:

\[
|H(j\omega_{nm}, d_m)| = \frac{K_{nm}}{2\xi_{nm}\omega_n}
\]

Regarding (10) and assuming that the audio source only emits a frequency \(\omega_1\) with an amplitude \(P_s\) the room response in the propagation mode \(\omega_n\) for a specific coordinate \((x,y,z)\) of the room is:

\[
|H| = \frac{P_m}{P_s} = C_1 \cos \left( \frac{n_x \pi x}{L_x} \right) \cos \left( \frac{n_y \pi y}{L_y} \right) \cos \left( \frac{n_z \pi z}{L_z} \right)
\]

being \(z\) the height of the microphone from the floor, and with

\[f_n = \sqrt{f_x^2 + f_y^2 + f_z^2}\] and \(\omega_n = 2\pi f_n\).

For instance, if a one-dimensional environment is considered where the three modes are propagated, as it can be seen in fig. 2, and the observer is located at a distance \(d_m = 2\) m, the values of maximum pressure in this point are \(A_1\), \(A_2\) and \(A_3\). The total pressure received in this point is the sum of the partial pressures given by each mode.

![Fig. 2. Response in the one-dimensional case.](image)

For a specific mode and location, the system behaves as a pass band filter, presenting a maximum response at the resonant frequency for the considered mode and a lower response when the signal frequency moves away from the resonant frequency.

Taking into account these considerations, equaling (15) and (16), it results:

\[
\xi_{nm} = \frac{K_{nm}}{2\omega_n C_1 \cos \left( \frac{n_x \pi x}{L_x} \right) \cos \left( \frac{n_y \pi y}{L_y} \right) \cos \left( \frac{n_z \pi z}{L_z} \right)}
\]
If the filter is non-ideal then more than one transmission mode could be considered and therefore the following expression is obtained for a robot position $d_m$ and a resonant frequency $\omega_n$ given by the propagation mode $n_x, n_y, n_z$:

$$
\sum_{l=1}^{n} \frac{K_{nm}}{2\xi_{nm} \omega_n} = \sum_{l=1}^{n} C_l \left[ \cos \left( \frac{n_x \pi x}{L_x} \right) \cos \left( \frac{n_y \pi y}{L_y} \right) \cos \left( \frac{n_z \pi z}{L_z} \right) \right]
$$

If the emitted signal power by the sound sources is constant and the audio signal power acquired with the microphones varies along the robot’s position, then the pole positions in the $s$ plane, for the considered propagation mode, will vary in the different robot’s positions and their values will be:

$$
s_{1nm} = -\zeta_{nm} \omega_n + \omega_n \sqrt{1 - (\zeta_{nm})^2}
$$

$$
s_{2nm} = -\zeta_{nm} \omega_n - \omega_n \sqrt{1 - (\zeta_{nm})^2}
$$

It is worth noting that this model of reduced order gives good results in order to determine the robot’s position and, although it does not provide a complete physical description of the evolution of the different parameters in the acoustic response for the different robot’s positions, according to the physical model given by the wave equation in (10), the modules of the proposed transfer functions will vary following a sinusoidal pattern and the pole position in the $s$ plane will show those variation in the same fashion.

This is an LPV grey-box model where the damping factor and the natural frequency can be determined by theoretical laws and the parameter $K_{nm}$ is calibrated through experiment.

### 4. Experimental results

The methodology applied to determine the robot’s position is the following:

1) The robot acquires an audio signal in its current position and performs a LPV modeling process taking as input signal the filtered sound source signal and as output signal the acquired and filtered signal. The parameters corresponding to the obtained poles in this identification process will be the features components for further steps.

2) The Euclidean distances in the feature space are calculated between the current position and the different labeled samples.

3) The two first samples are chosen and the distance between them and the robot’s position are then calculated. Through a transformation function $f_T$ (see [11]) the distance in the feature domain is converted to a distance in the space domain. These two distances in the space domain give two possible positions by the crossing circles of distances.

4) To discriminate between both possible solutions, the angle between each one and the platform containing the microphone array (which contains a compass) are calculated, and the closest one to the platform angle will be chosen as discriminatory variable to select the current robot’s position.

5) Steps 3 and 4 are repeated with the remaining labeled samples, and the solution is chosen among the closest angle to the robot’s platform.

For an accurate explanation of the algorithm, the microphone array and the robot used in the experiments see [11].

The acoustic response of the environment is very directional, and this fact leads to consider some uncertainty in the determination of the transformation function which relates the distance in the feature space and the domain space.
The robot, in order to determine its location, will perform the identification process between the emitted sound signal by the sound sources and the acquired signal by the microphone. Furthermore, the robot incorporates a rotary platform allowing orientating the microphone to the audio source, determining this orientation angle accurately.

As it can be seen in fig. 3, the robot follows the trajectory indicated by the arrows. In the map the sound source is indicated (climate chamber). There are two kind of audio samples: R1, R2, R3, R4, R5, R6 and R7 which are used in the recognition step whereas M1, M2, M3, M4 and M5 are labeled samples used in the learning step.

The acquired signal in the climatic chamber will be used in the identification process. This signal is time-continuous and, initially, non-stationary; but because the signal is generated by revolving electrical machines it has some degree of stationarity when a high number of samples is used, in this case, 50,000 samples (1.13 seconds).

![Fig. 3. Robot environment: labeled audio signals and actual robot trajectory with unlabeled signals (R1, R2, R3, R4, R5, R6 and R7).](image)

![Fig. 4. Source signal (climate chamber) and its frequency spectrum.](image)

The fundamental frequency is located at 100Hz, see fig. 4, and there are also some significant harmonics above and below it. In the filtering step, in order to simplify the identification process only the fundamental frequency at 100Hz and its closer harmonics will be taken into account, specifically those corresponding to the frequencies 178, 147.6, 89 and 66.8 Hz, that get involved due to the non-ideality of the implemented filter.

The sampling frequency is 44,100Hz. Other lower frequencies could be used instead, avoiding working with a high number of samples, but this frequency has been chosen because...
in a near future a voice recognition system will be implemented aboard the robot and it will be shared with this audio localization system. To facilitate the plant identification process centering its response in the 100Hz component, the input and output signals will be filtered and, consequently, the input-output relationship in linear systems is an ARX model.

To do that, a band-pass filter is applied to the acquired sound signals by the robot, specifically a 6th-order digital Cauer filter. Fig. 5 shows the results of the filter for the input signal in, for instance, robot position \( R_4 \) in the climatic chamber.

![Fig. 5. R4 sound signal (left) and its filtered signal (right).](image)

The propagation modes closer to the frequency of 100Hz and its harmonics considered in the filtering process are indicated in Table 2.

**Table 2.** Considered propagation modes.

<table>
<thead>
<tr>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( n_z )</th>
<th>( f_{nx} )</th>
<th>( f_{ny} )</th>
<th>( f_{nz} )</th>
<th>( f ) (Hz)</th>
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<tr>
<td>4</td>
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<td>2</td>
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<td>0,0</td>
<td>86,3</td>
<td>99,25</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>16,4</td>
<td>68,3</td>
<td>129,4</td>
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<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>32,7</td>
<td>34,2</td>
<td>172,5</td>
<td>178,87</td>
</tr>
</tbody>
</table>

From the proposed theoretical model, the damping coefficient for these frequencies is function of gain \( K \) and the considered robot’s position \((x,y,z)\).

\[
\xi_{nm} = \frac{K_{nm}}{2\omega_n C_1 \left[ \cos \left( \frac{n_x \pi x}{L_x} \right) \cos \left( \frac{n_y \pi y}{L_y} \right) \cos \left( \frac{n_z \pi z}{L_z} \right) \right]}
\]  

(21)

In this model the acoustic energy dissipation has not been modeled and it is considered by gain \( K \) in the model and the constant \( C_1 \) that appears in the wave equation (with a value of 2, as it has been aforementioned) for a signal frequency considered in a specific robot’s position. The acoustic energy is degraded into thermal energy due to the losses in reflection that will depend on the materials mainly. It is important to consider that the presence of obstacles will introduce a new loss factor that will depend on the position and size of the obstacle and the material that form it [3]. The following expression can be used to model those effects:

\[
\frac{K_{nm}}{C_1} = K \xi e^{-mf}
\]

(22)

Then (21) can be written as follows
indicating that the higher the frequency the higher the damping ratio. From the experimental measurements in the plant, the parameters $K_2$ and $m$ can be determined in (22) which quantify the different losses in the acoustic signals considered in the environment. Good results are obtained for $K_2 = 50$ and $m = 0.015$, resulting

\begin{equation}
\frac{K_{nm}}{C_1} = 50 e^{-0.015 f}
\end{equation}

The LPV model is obtained from the theoretical laws and the experimental data acquired in labeled samples. The damping factor and the natural frequency are obtained by the expressions explained in the Section 3 and the previous equations (22)-(24). The amplitude constant $K_{nm}$ is calibrated by experimentation through the least square method [16]. This procedure is carried out for those 5 models. They have been validated with the error criteria of FPE (Function Prediction Error) and MSE (Mean Square Error), yielding values about 10e-10 and 3% respectively using 3000 for this validation. Besides, for the whole estimated models the residuals autocorrelation and cross-correlation between the inputs and residuals are uncorrelated, indicating the goodness of the models.

For instance, for labeled M5 sample the signal and its estimation can be seen in fig. 6 in the first experiment, validating the model.

**Fig. 6.** Original M5 signal and its estimation.

When observing the diagram of poles and zeros for the different transfer function models in the LPV modeling process for the labeled signals, the zeros represents the approximation of the delay for each of the 5 models for the propagation modes indicated in Table 2 using the labeled samples (see fig.7 (left)), and there is a significant variation in pole positions, due mainly to obstacles presence, reverberations among other effects (see fig. 7 (right), in this figure only the poles are shown for clarity). Therefore, we will focus in poles to determinate the points in the feature space.
Fig. 7. (Left) Poles and zeros positions with the LPV model; (right) poles positions only.
In fig. 9, the nominal transformation function and the limits for the uncertainty interval transformation functions can be seen.

**Fig. 8.** (Left) Output signal obtained by the model and (right) real experimental signal in each \((x, y)\) for M1, M2, M3, M4 and M5.

**Fig. 9.** Transfer function \(f_T\) with uncertainty interval.
Table 3. Results in experiments.

<table>
<thead>
<tr>
<th>Robot’s position</th>
<th>Real coordinates</th>
<th>Computed coordinates</th>
<th>Error X(%)</th>
<th>Error Y(%)</th>
</tr>
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<tbody>
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<td>(973,456)</td>
<td>-0,10</td>
<td>-1,08</td>
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<tr>
<td>R2(Pos2)</td>
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<td>(978,321)</td>
<td>0,41</td>
<td>-1,23</td>
</tr>
<tr>
<td>R3(Pos6)</td>
<td>(819,255)</td>
<td>(807,250)</td>
<td>-1,47</td>
<td>-1,96</td>
</tr>
<tr>
<td>R4(Pos10)</td>
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<td>(644,257)</td>
<td>-1,53</td>
<td>0,78</td>
</tr>
<tr>
<td>R5(Pos11)</td>
<td>(654,105)</td>
<td>(640,110)</td>
<td>-2,14</td>
<td>4,76</td>
</tr>
<tr>
<td>R6(Pos15)</td>
<td>(474,105)</td>
<td>(475,108)</td>
<td>0,21</td>
<td>2,86</td>
</tr>
<tr>
<td>R7(Pos19)</td>
<td>(294,105)</td>
<td>(282,104)</td>
<td>-4,08</td>
<td>-0,95</td>
</tr>
</tbody>
</table>

There exists another uncertainty of about \( \pm 7.5 \) degrees in the angle determination due to the rotary platform in the robot that contains the microphones. Finally, to determine the current robot’s position the solution that provides the closest angle to the robot’s platform will be chosen.

The results of our experiments are shown in Table 3. The average error in the X axis is -1.242% and in the Y axis is 0.454% in experiment 1 and 0.335% in the X axis and -0.18% in the Y axis, providing estimated x-y positions good enough and robust.

5. Conclusions

In this article a new simplified LPV theoretical model has been presented which corresponds to the acoustic response of an industrial plant. The aim of the model is to locate mobile robots in such environments. In this work authors presented a methodology to be applied in order to simplify the identification process in industrial plants using the acoustic signals generated by typical engines in those environments.

Because the model will be used by a mobile robot to navigate in an industrial plant, the methodology has been simplified and the goal is to determine the x-y coordinates of the robot. In such a case, the obtained RFT will not present a complete acoustic response, but it has been demonstrated that the results are very good.

With the methodology presented in this article some interesting results have been achieved encouraging the authors to keep on walking in this research field. The room feature extraction is carried out by identification of the RTFs in different robot’s positions, and the RTFs poles will defined the feature space. Besides to reinforce the localization, avoiding ambiguity and reducing uncertainty and incorporating robustness, a sensorial system is used aboard the robot to compute the angle between itself and the sound source. The obtained feature space is related with the space domain through a general approach with acoustical meaning. The validation of this novel approach is tested in a room with an industrial machine as sound source obtaining promising results. The results keep being very good when the uncertainty is incorporated in the transformation function.

References