Input observability analysis of fixed speed wind turbine

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Abstract: This paper deals with the concept of input observability in a fixed speed wind turbine. A linear system has been calculated from the nonlinear equations of the squirrel cage induction generator, supposing it connected directly to the grid and assuming a steady state operating point. The observability of the input from the output of the system could be an interesting way to know if its possible to develop some new controls without introduce special sensors in the system. Furthermore, it is interesting to analyse which is the parameter variation margin of the wind turbine from input-observable state to non-input observable, in order to obtain some restrictions to design future controllers, or limit the operating points.

Key–Words: Input Observability, Input Observability Margin, Linearisation, Squirrel Cage Induction Generator.

1 Introduction

In the theory of continuous linear time-invariant dynamical control systems, the most popular and the most frequently used mathematical model is given by the following differential state equation and algebraic output equations

$$\left.\begin{array}{ll}\dot{x}(t) & = Ax(t) + Bu(t)\\ y(t) & = Cx(t) + Du(t)\end{array}\right\} \qquad (1)$$

where x is the state vector, y is the output vector, u is the input (or control) vector, $A \in M_n(\mathbb{R})$ is the state matrix, $B \in M_{n\times m}(\mathbb{R})$ is the input matrix, $C \in M_{p\times n}(\mathbb{C})$ is the output matrix, and $D \in M_{p\times m}(\mathbb{C})$ is the feedthrough (or feedforward) matrix.

The first equation is known as the state equation and the second one as the output equation.

Applying the Laplace transform to the equation 1 an input-output relation

$$\hat{y}(s) = H(s)\hat{u}(s) \qquad (2)$$

where $H(s) = C(sI - A)^{-1}B + D$, is obtained. The matrix $H(s)$ is a proper rational matrix called transfer matrix, which converts the dynamical relation between input and output into a simple matrix operations.

From now on the system 1, will be denoted as a quadruple of matrices in the form (A, B, C, D).

In control theory, the observability concept is a measurement of how well internal states of a system can be inferred by knowledge of its external outputs. If a dynamical equation is observable all the modes of the equation can be observed at the output. The concept of observability was introduced by Rudolf E. Kalman for linear dynamic systems (see [4, 5]). Roughly speaking, the concept of observability denotes the possibility of reconstructing the full trajectory from the observed data. The exact definition varies slightly within the framework or the type of models applied. Moreover, there are different concepts regarding observability which depend on the type of dynamical system, for instance input observability.

Input observability is the related notion for the input variable of the system, allowing to distinguish the observable states which are not-controllable from the states that are both controllable and observable. It must be pointed out that an observable system is not necessarily input observable, and an input observable system is not necessarily observable.

The recent increasing of wind power in the electrical network is well known. Therefore, study and ensure the input-observability of Fixed-Speed Wind Turbines (FSWT) could be interesting, in order to obtain different candidates, such as parameters or state variables, to be regulated.

In this paper, the state observability and input observability of Fixed-Speed Wind Turbines (FSWT) are studied, and the input observability margin (distance to the bounds) are analysed under different conditions.

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2 Observability and input observability

Formally, a system is said to be state observable at t_0 if there exists a finite $t_1 > t_0$ such that for any state x_0 at time t_0 the knowledge of the input $u_{[t_0,t_1]}$ and the output $y_{[t_0,t_1]}$ over the interval $[t_0, t_1]$ suffices to determine the state x_0. Otherwise the system is said to be unobservable at t_0.

For time-invariant linear systems in the state space representation, a convenient test exists in order to evaluate if a system is observable.

In order to formulate an easily computable algebraic observability criteria for time-invariant linear systems in the state space representation, the so-called observability matrix O is introduced, which is known as observability matrix and defined as follows,

$$O = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix} \tag{3}$$

and the following theorem is verified.

Theorem 1 *Dynamical system 1 is state observable if and only if* rank $O = n$.

The rationale for this test is that if n rows are linearly independent, then each of the n states can be obtained through linear combinations of the output variables $y(k)$.

The conditions of state observability can be also calculated in terms of the transfer matrix of the system.

The necessary and sufficient condition for a full state observability is that a cancellation does not occur in the transfer matrix. If a cancellation occurs, none off-mode can be seen in the output.

An observer permitting the selection of the system characteristic polynomial is possible to construct for a given observable system. A significant result is the following well knowing theorem.

Theorem 2 *The observability character is invariant under output injection.*

In fact, a state feedback controller and linear observer can be combined to form an stabilizing controller for a controllable and observable linear system by using the estimate state in the feedback control law.

Similar to the state observability of dynamical control system, it is possible to define the so-called input observability for the input vector $u(t) \in \mathbb{R}^p$ of dynamical system.

Definition 3 *The dynamical system (A, B, C, D) is said to be input observable if the state sequence $\{x_0, x_1, \dots, x_{n-1}\}$ is uniquely determined by the knowledge of the output sequence $\{y_0, y_1, \dots, y_{n-1}\}$ for a finite number of steps $n - 1$.*

For a linear continuous-time system, for example 1, described by matrices A, B, C, and D, the input observability matrix is defined as follows,

$$iO \in M_{pn\times(n+(n-1)m)}(\mathbb{C}) \tag{4}$$

with

$$iO(A,B,C,D) = \begin{pmatrix} C & D & 0 & & \dots & 0 \\ CA & CB & D & \ddots & & \vdots \\ CA^2 & CAB & CB & \ddots & & \\ \vdots & \vdots & & \ddots & & \\ CA^{n-1} & CA^{n-2}B & CA^{n-3}B & \dots & CB & D \end{pmatrix}$$

for simplify and if confusion is not possible it is renamed as iO.

The following result is obtained.

Theorem 4 *Dynamical system 1 is input observable if and only if iO has full rank.*

It should be pointed out, that the state observability depends only on the state variables and output equation, whereas the input observability depends also on the control vector. Therefore, these two concepts are not necessarily related as is shown in the following examples.

Example 1.

a) Let (A, B, C, D) be a system with

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ D = 0.$$

$$\operatorname{rank}\begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \operatorname{rank}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 3$$

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then, the system is state observable, but

$$\text{rank}\begin{pmatrix} C & & \\ CA & CB & \\ CA^2 & CAB & CB \end{pmatrix} = \text{rank}\begin{pmatrix} 1&0&0&0&0\\0&0&1&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&0 \end{pmatrix} = 4 < 5$$

then, the system is not input observable.

b) Let (A, B, C, D) be now, a system where

$$A = \begin{pmatrix} 0&1&0\\0&0&0\\0&0&0 \end{pmatrix},\ B = \begin{pmatrix} 0&0\\1&0\\0&1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1&0&0 \end{pmatrix},\ D = 0.$$

$$\text{rank}\begin{pmatrix} C\\CA\\CA^2 \end{pmatrix} = \text{rank}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&0 \end{pmatrix} = 2 < 3$$

then, the system is not state observable, but

$$\text{rank}\begin{pmatrix} C & & \\ CA & CB & \\ CA^2 & CAB & CB \end{pmatrix} = \text{rank}\begin{pmatrix} 1&0&0&0&0\\0&1&0&0&0\\0&0&0&1&0 \end{pmatrix} = 3$$

then, the system is input observable.

It is well know that a manner to understand the properties of a dynamical system is treating it by purely algebraic techniques. The main aspect of this approach is defining an equivalence relation preserving the required properties. There are many interesting and useful equivalence relations between linear systems, in this paper the feedback and output injection transformations are considered.

Theorem 5 *The input observability character is invariant under feedback and output injection.*

Proof:

A more general result is going to be proven. In fact, input observability is invariant under feedback and output injection and basis change in the state input and output spaces.

Considering the following matrix

$$M(A,B,C,D,E) = \begin{pmatrix} A & B & E & 0 & 0 & & & & \\ C & D & 0 & 0 & 0 & & & & \\ 0 & 0 & A & B & E & & & & \\ 0 & 0 & C & D & 0 & & & & \\ & & & & & \ddots & & & \\ & & & & & & A & B & E & 0 \\ & & & & & & C & D & 0 & 0 \\ & & & & & & 0 & 0 & C & D \end{pmatrix} \in M_{x\times y}(\mathbb{C}),$$

$x = (n-1)n + np$, $y = n^2 + nm$, for $E = I$, renaming $M(A,B,C,D,E)$ as $M(A,B,C,D)$ in order to avoid any misunderstanding.

An equivalent system (A_1, B_1, C_1, D_1) is considered. It is defined by using the equivalence relationship deduced from considering the correspondent transformations to make both feedback and output injection and to change the basis into state input and output spaces.

$$\begin{aligned} A_1 &= PAP^{-1} + F^C CP^{-1} + PBF^B + F^C DF^B \\ B_1 &= PBR + F^C DR \\ C_1 &= SCP^{-1} + SDF^B \\ D_1 &= SDR. \end{aligned}$$

Taking the following matrices

$$\mathbf{P} = \begin{pmatrix} P & F^C & 0 & 0 & & & & \\ 0 & S & 0 & 0 & & & & \\ 0 & 0 & P & F^C & & & & \\ 0 & 0 & 0 & S & & & & \\ & & & & \ddots & & & \\ & & & & & P & F^C & 0 \\ & & & & & 0 & S & 0 \\ & & & & & 0 & 0 & S \end{pmatrix}$$

and

$$\mathbf{Q} = \begin{pmatrix} P^{-1} & 0 & 0 & 0 & & & & & \\ F^B & R & 0 & 0 & & & & & \\ 0 & 0 & P^{-1} & 0 & & & & & \\ 0 & 0 & F^B & R & & & & & \\ & & & & \ddots & & & & \\ & & & & & P & 0 & 0 & 0 \\ & & & & & F^B & R & 0 & 0 \\ & & & & & 0 & 0 & P^{-1} & 0 \\ & & & & & 0 & 0 & 0 & P^{-1} \end{pmatrix}$$

the following equality is obtained

$$\mathbf{P}M(A,B,C,D)\mathbf{Q} = M(A_1, B_1, C_1, D_1).$$

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So

$$\text{rank}\, M(A,B,C,D) = \text{rank}\, M(A_1,B_1,C_1,D_1).$$

On the other hand, making both block row and columns elementary transformations to the matrix $M(A,B,C,D)$, it can be obtained that

$$\text{rank}\, M(A,B,C,D) = n(n-1) + \text{rank}\begin{pmatrix} C_1 & D_1 & 0 & \dots & & 0 \\ C_1A_1 & C_1B_1 & D_1 & \dots & & 0 \\ C_1A_1^2 & C_1A_1B_1 & C_1B_1 & \dots & 0 & \\ \vdots & \vdots & & \ddots & & \vdots \\ C_1A_1^{n-1} & C_1A_1^{n-2}B_1 & \dots & \dots & C_1B_1 & D_1 \end{pmatrix}$$

and the proof is concluded. □

3 Bounding the distance from a input observable system to a non input observable one

Observability and input observability are generic conditions, it means that if a system is observable and/or input observable it is the same for all perturbed systems contained in a small neighborhood of the system.

For that reason, it is important to obtain a bound for the value of the radius of a ball which is neighborhood of a input observable system, containing only systems which are also input observable.

The aimed distance can be deduced from the Frobenius norm. For a given matrix $A = (a_{ij}) \in M_{n\times m}(\mathbb{R})$, its Frobenius norm is defined as $\|A\| = \sqrt{\sum_{ij} a_{ij}^2}$.

This norm leads to the natural definition of the norm of quadruples of matrices and the corresponding definition of the distance between two quadruples of matrices.

Definition 6 *Given a quadruple (A,B,C,D), its norm can be defined as*

$$\|(A,B,C,D)\| = \sqrt{\|A\|^2+\|B\|^2+\|C\|^2+\|D\|^2}$$

and the distance between the quadruples $X = (A_1,B_1,C_1,D_1)$, $Y = (A_2,B_2,C_2,D_2)$ is

$$d(X,Y) = \|(A_1-A_2, B_1-B_2, C_1-C_2, D_1-D_2)\|.$$

Finally, the distance between a input observable system and the nearest non-input observable one is defined as

$$\inf \|(\delta A, \delta B, \delta C, \delta D)\|$$

where $(\delta A, \delta B, \delta C, \delta D)$ is a quadruple such that $(A+\delta A, B+\delta B, C+\delta C, D+\delta D)$ does not satisfies the given property.

The starting point to find a bound is the relationship between the associated matrix iO norm of the system (A,B,C,D) and the norm of the same quadruple.

Proposition 7 *For all system (A,B,C,D),*

$$\|M(A,B,C,D)\| \leq \sqrt{n}\|(A,B,C,D)\|.$$

Proof:

It suffices to compute.

$$\|M(A,B,C,D)\|^2 = (n-1)(\|A\|^2+\|B\|^2+\|I\|^2) + n(\|C\|^2+\|D\|^2) \leq n(\|A\|^2+\|B\|^2+\|I\|^2+\|C\|^2+\|D\|^2) = n\|(A,B,C,D)\|^2.$$

□

Theorem 8 *Given an input observable system (A,B,C,D) a lower bound for the distance to the nearest non-input observable system is given by*

$$\|(\delta A, \delta B, \delta C, \delta D)\| \geq \frac{1}{\sqrt{n}} \sigma_{\text{inf}} M(A,B,C,D)$$

where $\sigma_{\text{inf}} M(A,B,C,D)$ denotes the smallest non-zero singular value of $M(A,B,C,D)$.

Proof:

It is known that

$$\text{rank}\, M(A,B,C,D) = \min((n-1)n + np, n^2+nm) = r$$

and $(A+\delta A, B+\delta B, C+\delta C, D+\delta D)$ is not input observable,

$$\text{rank}\, M(A+\delta A, B+\delta B, C+\delta C, D+\delta D) < r.$$

It is also well known that the smallest perturbation in the Frobenius norm which reduces the rank of a matrix A with $\text{rank}\, A = r$ from r to $r-1$ is $\sigma_r(A)$ which is the smallest non-zero singular value of A. Therefore, the norm of the perturbation of the matrix $M(\delta A, \delta B, \delta C, \delta D)$ must be at least $\sigma_{\text{inf}} M(A,B,C,D)$. The only fact which needs to be noted is that

$$\|M(A+\delta A, B+\delta B, C+\delta C, D+\delta C, I)\| \leq \|M(A,B,C,D,I)\| + \|M(\delta A, \delta B, \delta C, \delta C, 0)\|$$

and taking into account Proposition 7, it is obtained

$$\sqrt{n}\|(\delta A, \delta B, \delta C, \delta D)\| \geq \|M(\delta A, \delta B, \delta B, \delta C, \delta D, 0)\| \geq \sigma_{\text{inf}} M(A,B,C,D).$$

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Hence, a bound for the distance from (A, B, C, D) to the nearest non input observable system can be determined as

$$\|(\delta A, \delta B, \delta C, \delta D)\| \geq \frac{1}{\sqrt{n}} \sigma_{\text{inf}} M(A, B, C, D).$$

□

4 Modelling of FSWT

The global analysed system is a wind power generator connected directly to the grid. The system under study is drawn in Figure 1.

The linear system is defined by means of the squirrel cage induction generator differential equations. The differential equations of the generator are time dependant. Its inputs are the voltage of the grid. The outputs are the active and reactive power delivered by the wind power generator.

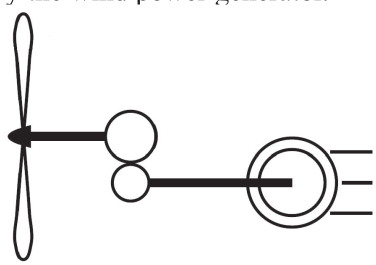

Fig. 1 Scheme of a Fixed Speed Wind Turbine

Supposing the system to be in steady state. This hypothesis implies constant slip. Therefore, the system can be described as:

$$\dot{X} = \frac{d}{dt} \begin{pmatrix} \Delta i_{sq} \\ \Delta i_{sd} \\ \Delta i_{rq} \\ \Delta i_{rd} \end{pmatrix} =$$

$$-\frac{1}{L_s L_r - M^2} \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ -\alpha_2 & \alpha_1 & -\alpha_4 & \alpha_3 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 \\ -\alpha_6 & \alpha_5 & -\alpha_8 & \alpha_7 \end{pmatrix} \begin{pmatrix} \Delta i_{sq} \\ \Delta i_{sd} \\ \Delta i_{rq} \\ \Delta i_{rd} \end{pmatrix}$$

$$+\frac{1}{L_s L_r - M^2} \begin{pmatrix} L_r & 0 \\ 0 & L_r \\ -M & 0 \\ 0 & -M \end{pmatrix} \begin{pmatrix} \Delta v_{sq} \\ \Delta v_{sd} \end{pmatrix} =$$

$$AX + B \tag{5}$$

where

$$\begin{aligned} \alpha_1 &= L_r r_s \\ \alpha_2 &= M^2 \dot{\theta}_r + (L_s L_r - M^2)\dot{\theta} \\ \alpha_3 &= -M r_r \\ \alpha_4 &= M L_r \dot{\theta}_r \\ \alpha_5 &= -M r_s \\ \alpha_6 &= -M L_s \dot{\theta}_r \\ \alpha_7 &= L_s r_r \\ \alpha_8 &= (L_s L_r - M^2)\dot{\theta} - L_s L r \dot{\theta}_r \end{aligned}$$

The α_i parameters have constant value. They are dependant of the machine parameters such as stator and rotor impedance. Moreover Δ indicates a little variation of the selected operating point.

4.1 Linearising the System

The desired output signals, both active and reactive power are nonlinear functions, described as:

$$\begin{aligned} Q_s &= \tfrac{3}{2}(v_{sd} i_{sq} - v_{sq} i_{sd}) \\ P_s &= \tfrac{3}{2}(v_{sd} i_{sd} + v_{sq} i_{sq}) \end{aligned} \tag{6}$$

Then, it is necessary linearise these equations to obtain the linear system of the outputs.
Hence, applying Taylor's approximation around the steady state operating point to these equations.

$$Q_{ss} = \tfrac{3}{2}((v_{sd0} i_{sq0} - v_{sq0} i_{sd0}) \\ +(v_{sd0}\Delta i_{sq} - v_{sq0}\Delta i_{sd} + i_{sq0}\Delta v_{sd} - i_{sd0}\Delta v_{sq}))$$

$$P_{ss} = \tfrac{3}{2}((v_{sd0} i_{sd0} + v_{sq0} i_{sq0}) \\ +(v_{sd0}\Delta i_{sd} + v_{sq0}\Delta i_{sq} + i_{sd0}\Delta v_{sd} + i_{sq0}\Delta v_{sq})) \tag{7}$$

where

$$\begin{aligned} Q_{ss0} &= (v_{sd0} i_{sq0} - v_{sq0} i_{sd0}) \\ P_{ss0} & \ (v_{sd0} i_{sd0} + v_{sq0} i_{sq0}) \end{aligned}$$

where the values with the 0-subscript are the constant values corresponding to the steady state operating point.
To simplify the calculations, it is used to linearize the system a small variation in the power values.

$$\begin{aligned} \Delta Q_{ss} &= Q_{ss} - Q_{ss0} \\ \Delta P_{ss} &= P_{ss} - P_{ss0} \end{aligned} \tag{8}$$

Then, the output system described as Y = CX + DU can be written as follows:

$$\begin{pmatrix} \Delta Q_{ss} \\ \Delta P_{ss} \end{pmatrix} = \begin{pmatrix} v_{sd0} & -v_{sq0} & 0 & 0 \\ v_{sq0} & v_{sd0} & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta i_{sq} \\ \Delta i_{sd} \\ \Delta i_{rq} \\ \Delta i_{rd} \end{pmatrix} + \begin{pmatrix} -i_{sd0} & i_{sq0} \\ i_{sq0} & i_{sd0} \end{pmatrix} \begin{pmatrix} \Delta v_{sq} \\ \Delta v_{sd} \end{pmatrix} \tag{9}$$

Obtaining the lineal dynamical system

$$\left.\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX + DU \end{aligned}\right\}$$

with matrices A and B as in (4) and C and D as in (8).

4.2 Observability and input observability of FSWT

State observability and input observability of FSWT are analysed in the following subsection.

It is necessary to compute rank O and rank iO, in order to apply theorems (1) and (3) into this particular case.

To be able to ensure the state observability of Fixed Speed Wind Turbine (FSWT), the following inequalities must be accomplished.

- $M^2 L_r^2 + \dot{\theta}_r^2 + M^2 r_r^2 \neq 0$
- $v_{sd0}^2 + v_{sq0}^2 \neq 0$

The first inequality is always true, since all involved parameters are real numbers ($\mathbb{R}$). The second inequality is true meanwhile the wind turbine is connected to a power system or it is not affected by a ”black out”.

Therefore,

$$\text{rank} \begin{pmatrix} C \\ CA \end{pmatrix} = 4.$$

Consequently,

$$\text{rank} \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{pmatrix} = 4$$

where it can be conclude that the system is observable.

Considering the input observability, another inequality must be guaranteed.

- $-i_{sd0}^2 - i_{sq0}^2 \neq 0$

It is true when the wind turbine is under any operation such as generator or consumption, except when it is stopped.

So, in this case the system is always input observable.

4.3 Bound estimation for input observability of FSWT

Some parameters are fixed, in order to estimate the inpunt observability margin of the system under different operating points.

The used electrical parameters corresponding to FSWT model are presented in Table 1, which are obtained from [8].

Parameter	S.I. Units
r_s	$1.1 \cdot 10^{-3}$
r_r	$1.3 \cdot 10^{-3}$
L_s	$3.0636 \cdot 10^{-3}$
L_r	$3.0686 \cdot 10^{-3}$
M	$2.9936 \cdot 10^{-3}$

Table 1: Electrical generator parameters

Aiming to define different operating points of the FSWT, different values for v_{sd0}, v_{sq0}, i_{sd0}, i_{sq0}, $\dot{\theta}_r$ and $\dot{\theta}$ have been considered. Since, this can imply different system input observability bounds.

For example, taking the rotor speed as $\dot{\theta}_r = (1 - 0.02) \cdot 2\pi \cdot 50$, the grid frequency as $\dot{\theta} = 2\pi \cdot 50$, the quadrature component of the grid voltage as $v_{sq0} = 0$ and both direct and quadrature component of the current flowing among the generator and the grid as $i_{sd0} = 150$ and $i_{sq0} = -100$, respectively. Keeping variable the direct component of the grid voltage, the boundaries are shown in Table 2. The values have been computed using Matlab.

v_{sd0} in V	Bound
10	0.0806
50	0.6923
100	1.3749
200	2.0334
300	2.3565
500	2.6535
690	2.7762

Table 2: Input observability margin for different operating points of the FSWT

From Table 2 can be observed that v_{sd0} values close to one of the inequalities previously defined present margins smaller than the other operating points, as it is expected.

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5 Conclusion

This paper has presented the concepts of observability and input observability. Moreover, by means of two different examples have been demonstrated the non equivalence of both concepts. Also, a linear system has been calculated from the nonlinear equations of the squirrel cage induction generator. Observability and input observability of the FSWT have been demonstrated using the A, B,C, B matrices. Moreover, the demonstration is made with a generic system. Therefore, it can be ensured not only for an example, meanwhile the system does not match any of the inequalities proposed. Moreover, bound estimation for input observability of FSWT has been done for different operating points. From these results, it can be concluded that the voltage of the system, in this case, can be observed and estimated from active and reactive power produced by the generator. One step further could be include the mechanics of the wind turbine in the system, since if this conditions are also ensured the wind speed could be estimated from the system outputs without requirement of any special sensor.

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