Energy-Balance Control of PV Cascaded Multilevel Grid-Connected Inverters under Level-Shifted and Phase-Shifted PWM Modulations

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Abstract—This paper presents an energy-balance control strategy for a cascaded single-phase grid-connected H-bridge multilevel inverter linking $n$ independent PV arrays to the grid. The control scheme is based on an energy-sampled data model of the PV system and enables the design of a voltage loop linear discrete controller for each array ensuring the stability of the system for the whole range of PV arrays operating conditions. The control design is adapted to Phase-Shifted and Level-Shifted Carrier PWM to share the control action among the cascade-connected bridges in order to concurrently synthesize a multilevel waveform and to keep each of the PV arrays at its maximum power operating point. Experimental results carried out on a 7-level inverter are included to validate the proposed approach.

Index Terms—Cascaded-H bridge inverters, discrete-time control, grid-connected PV systems, multilevel modulations

I. INTRODUCTION

A low-carbon society is a current political trend in developed countries which promotes, among others, the connection of photovoltaic (PV) systems to the electrical grid. These systems can contribute to clean electricity production as they are harmless for the environment and reduce the dependence on polluting fossil fuels such as coal, oil, gas and nuclear. Additionally, the power scalability of PV generation facilitates its large-scale penetration and leads to grid-connected applications ranging from few kilowatts of small-residential PV systems primarily installed on roofs to several megawatts of large-scale PV power plants.

Nevertheless, since the cost per watt of the PV system is still high compared to other energy sources, current research focuses in both reducing manufacturing costs and increasing the energy production of the overall system. In particular, the power conditioning stage interfacing the PV modules to the grid has caught the attention of researchers since it must account for maximum power extraction from the energy source and an optimal energy transfer to the grid [1]. In this concern, string and multi-string power conditioning architectures have been proposed over the last two decades to improve the features of the central inverter based one [2-6]. Among the power converter topologies adopted in the aforementioned architectures, multilevel inverter ones are being investigated as an interesting option for grid-connected PV systems [7-19]. In particular, the cascaded H bridge multilevel inverter (CHB-MLI) topology as depicted in Fig.1 is especially attractive for grid-connected PV applications for several reasons, for instance [2], [8], [10]:

a) The output voltage level required for grid power injection can be achieved without the use of a transformer as the voltage boosting is shared between the DC series connection of PV modules and the cascade connection of H bridge outputs.

b) This topology allows the connection of independent strings of PV modules to the input DC links of the power stage. Since the DC link voltages can be independently controlled, the maximum power extraction of a reduced number of PV modules can be accomplished with the help of maximum power point tracking (MPPT) algorithms. This improves both PV system reliability and energy production when the PV modules operate under mismatching conditions such as in the case of partial shadowing.

c) Like other multilevel inverter topologies, the CHB-MLI allows the synthesis of staircase AC output waveforms with lower total harmonic distortion (THD) compared to those generated by two-level based inverters, thus releasing output filter requirements for the compliance of grid harmonic standards. Depending on the operation power level, this synthesis can be carried out either at the fundamental
frequency [20] or at higher switching frequencies using multilevel modulation [8], [10], [21], [22].

As shown in Fig.1, the control strategy of the CHB-MLI requires a set of voltage controllers to ensure independent voltage control of each PV array, a current controller driving the injected grid-current to assure overall power transfer at unity power factor and multilevel modulation to synthesize the staircase output voltage with low THD.

Except in the recent work of Cecati et al. [12] which suggests a fuzzy control approach to implement an extended version of this strategy allowing reactive power control, aforementioned controllers have been typically designed by continuous time linear control techniques. For instance, continuous time PI voltage controllers can be found in [15] and more recently in [16] which design follows the guidelines suggested in [23]. However, as pointed out in [24], these designs do not address system stability for the whole irradiance and temperature operating ranges since the nonlinear parametric dependence of the system resulting from the nonlinear current to voltage characteristics of the PV arrays is neglected.

Regarding output current control and multilevel modulation techniques, nonlinear current controllers based on sliding mode control and multiband hysteresis modulations operating at a variable switching frequency have recently been reported [18], [25]. However, linear PI or proportional resonant (PR) current controllers [26], [27] and fixed-frequency multilevel PWM modulations are preferred in low power applications to facilitate the design of the reactive components. In this regard and to the author’s knowledge, only PS-PWM (Phase Shifted PWM) has been applied to the CHB-MLI for grid-connected applications [15], [16], [19].

Collecting previous results [18-19], [24], the work here reported presents the design of a discrete-time linear voltage controllers for independent voltage control of each PV array, thus ensuring system stability for the whole irradiance operating range. Assuming a linear PR current controller, the paper also describes the design of the control signals driving each H-bridge for PS-PWM to further extend the results to LS-PWM (Level Shifted Carrier PWM).

The paper is organized as follows: the PV system modeling and the control strategy definition are presented in section II. Section III focuses on the energy-balance control approach and sets controller design criteria. Section IV addresses the generation of control signals driving the PV inverter switches for PS and LS-PWM modulations. Several different tests carried out on a 7-level inverter prototype linking three independent PV arrays to the grid are presented in section V for both modulations to experimentally verify the proposed approach. Finally, section VI draws the conclusions of this work.

II. SYSTEM MODELLING AND CONTROL STRATEGY DEFINITION

The grid-connected PV multilevel inverter under study is shown in Fig.1. It comprises PV generators and a power conditioning unit including full-bridge inverters whose outputs are series-connected to the grid. Each PV generator is connected at the input of one full-bridge. This section introduces the formal mathematical modeling of the system and the control strategy to be designed.

A. PV generators

All PV generators are formed by an array of series-connected PV panels with identical PV cells. The current-to-voltage relationship of the kth PV generator can be extrapolated from the PV cell model of Prince et al. [28] as follows:

\[ i_{kP} = I_{gk} \left( e^{\frac{-V_{k}}{\eta_{gk}}} - 1 \right) \quad k = 1, \ldots, n \]  

where \( i_{kP} \) and \( V_{k} \) are the output current and voltage respectively of the \( k \)th PV generator, \( I_{gk} \) represents the light-induced current, \( \eta_{gk} \) stands for the emission coefficient, \( I_{sat} \) is the reverse saturation current and \( V_{T} \) represents the thermal voltage of the semiconductor material. This model assumes that each generator can operate at different irradiance and temperature levels, i.e., each generator can exhibit a different Maximum Power Point (MPP) at any time.

Fig.1. Grid-connected PV multilevel inverter.

B. Power conditioning unit

As for the variables in Fig.1, the \( k \)th cascaded-bridge can be modeled as follows if no losses are considered:

\[ \begin{align*}
  v_{lk} &= u_{k}V_{Ck} ; \quad k = 1, 2, \ldots, n \\
  i_{lk} &= u_{k}I_{Ck} ; \quad k = 1, 2, \ldots, n 
\end{align*} \]  

where \( u_{k} \) stands for the control signal of each full-bridge which is assumed to operate under three-level modulation, thus restricting the values of the control signal to \( u_{k} \in \{-1,0,1\} \). The
lossless operation of each inverter cell can be derived in terms of instantaneous power since from (2) the following identity holds: 

\[ v_{c3} i_{th} = v_{th} i_L \]  

(3) 

The output voltage of the cascaded inverters, \( v_{th} \) is given by: 

\[ v_{th} = \sum_{k=1}^{n} v_{th} = \sum_{k=1}^{n} u_i v_{ck} \]  

(4) 

Finally, the system dynamics can be modeled by the following set of differential equations: 

\[ C_j \frac{dv_{ck}}{dt} = i_{pyk} - u_j i_j; \quad k = 1,...,n \]  

(5a) 

\[ L \frac{di_j}{dt} = \sum_{k=1}^{n} u_i v_{ck} - v_g(t) \]  

(5b) 

where \( v_g \) represents the grid voltage, which is assumed to be purely sinusoidal, i.e. 

\[ v_g(t) = A \sin(\omega_g t) \]  

(6) 

being \( \omega_g \) the grid angular frequency. It is worth noting that the dynamic description of the PV system is given by (1), (5) and (6), and involves the nonlinear \( i-v \) relationship of the PV arrays.

C. Control strategy

The control of the multilevel inverter must achieve the following goals: 

1. The operation of each PV generator at its own MPP independently of the ambient conditions, to assure the maximum power extraction of each array. 
2. The transfer of the overall DC power to the grid. This is performed by the output current \( i_L(t) \) which must be injected to the grid with low harmonic distortion at unity power factor. 
3. The synthesis of a multilevel step-like AC wave voltage, \( v_{th}(t) \), at the output of the cascaded converter.

The following paragraphs summarize the control strategy of the system according to the block-diagram in Fig.1.

The first goal requires the design of a voltage control loop per array which regulates the corresponding capacitor voltage \( v_{ck} \) to a reference value given by a MPPT algorithm (\( v_{c3} \) in Fig.1) at any time. This paper assumes that the above reference signals come from conventional Perturb & Observe MPPT algorithms [29].

With regard to the second goal, the injection of the DC power in phase with the grid entails the control of the output current \( i_L \) (i.e. the design of a current loop) to track with a fast transient and zero steady-state error a current reference \( i_L^* \) given by: 

\[ i_L^*(t) = K(t)v_g(t) = K(t)A \sin(\omega_g t) \]  

(7) 

where the current amplitude \( K(t)A \) must be time varying to deal with the time varying input DC power. When the output current reaches this reference, the transfer condition of the overall average power over a grid period can be formulated as:

\[ \frac{1}{T_g} \int_{(m-1)T_g}^{mT_g} \sum_{k=1}^{n} i_{pyk}(\tau)v_{c3}(\tau) d\tau = \frac{1}{T_g} \int_{(m-1)T_g}^{mT_g} K(\tau)[A \sin(\omega_g \tau)]^2 d\tau \]  

(8) 

However since the temperature and irradiance vary slowly within a grid period, so varies the input DC power. Therefore the average DC power can be approximated as:

\[ \frac{1}{T_g} \int_{(m-1)T_g}^{mT_g} \sum_{k=1}^{n} i_{pyk}(\tau)v_{c3}(\tau) d\tau \approx \sum_{k=1}^{n} i_{pyk}(mT_g)v_{c3}(mT_g) \]  

This assumption allows the value of \( K(t) \) to be updated only at the beginning of each grid period, i.e.: 

\[ K(t) = K_{(m-1)} \quad \text{for} \quad (m-1)T_g \leq t \leq mT_g, \quad m = 1,2,... \]  

Accordingly, the current reference given in (5) can be rewritten as:

\[ i_L^*(t) = K_{(m-1)}A \sin(\omega_g t) \]  

(9) 

hence the transfer condition of the average power given in (8) becomes:

\[ \sum_{k=1}^{n} i_{pyk}(mT_g)v_{c3}(mT_g) = K_{(m-1)}A^2/2 \]  

(10) 

In other words, if the current controller drives the output current to properly track a current reference given by (9-10), the overall average DC power is transferred to the grid. In particular, if this power is set to its maximum value by the voltage controllers, the maximum power transfer is achieved. It is worth noting that this approach leads to discrete-time relationships among the system variables sampled at the grid period, as proposed for the control design of high-power-factor preregulators [30-32].

Finally, the design of the “modulation” block of Fig.1 focuses on the local control goal. The block contains the same number of PWM modulators and cascaded inverter cells and delivers the control signals \( u_i \) driving each bridge. These signals are built based on both the information of the current loop which is related to the overall power transfer (d in Fig.1) and the information of the voltage loops (\( K_1, K_2, K_3 \) in Fig.1) which, in turn, is related to the power handled by each inverter. The multilevel step-like AC output voltage \( v_{th} \) is synthesized using PWM techniques based on Phase-Shifted (PS) and Level-Shifted (LS) triangular carriers. The details of the generation of these control signals for both modulations are addressed in section IV.

A direct attempt of control design from the system dynamics given in (1), (5) and (6) is cumbersome due, among others, to the current to voltage nonlinear relationship of the PV generators described in (1). The following approach undertakes this design to ensure at least the local stability of the system.

III. ENERGY-BALANCE CONTROL APPROACH

A. Energy-balance linear modeling

In terms of instantaneous power, the system can be characterized by the following power-balance equation:
\[ \sum_{k=1}^{n} \int_{(m-1)T_k}^{mT_k} \sum_{i=1}^{n} i_{P,ix} v_{C,i} \, dt = \sum_{i=1}^{n} v_{C,i} C_i \frac{dv_{C,i}}{dt} + L_i \frac{di_{L,i}}{dt} + i_{iy} v_y \tag{11} \]

Assuming that the output current \( i_y \) has reached the reference value \( i_{y'} \) given in (9-10) and integrating over a grid period yields:

\[ \int_{(m-1)T_k}^{mT_k} \sum_{i=1}^{n} i_{P,ix} v_{C,i} \, dt = \sum_{i=1}^{n} C_i \left[ v_{C,i}'^2 - v_{C,i(m-1)}'^2 \right] + \frac{K_{(m-1)} A^2 T_g}{2} \tag{12} \]

If \( E_{P,k,m} \) stands for the DC energy produced by the \( k \)th PV array during the \( m \) grid period and \( E_{C,m} \) is the energy stored in the capacitor, namely:

\[ E_{P,k,m} = \int_{(m-1)T_k}^{mT_k} \sum_{i=1}^{n} i_{P,ix} v_{C,i} \, dt \tag{13a} \]

\[ E_{C,m} = 0.5C_i v_{C,i}'^2 \tag{13b} \]

(12) can be rewritten as the following dynamic energy-balance equation:

\[ \sum_{k=1}^{n} \left[ E_{C,m} - E_{C,(m-1)} \right] = \sum_{k=1}^{n} E_{P,k,m} - \frac{K_{(m-1)} A^2 T_g}{2} \tag{14} \]

However, as pointed out in [24], the above dynamic description is still not complete since \( E_{P,k,m} \) and \( E_{C,m} \) are dependent one on another through the current to voltage nonlinear characteristic of the PV generator given in (1). From (1) and (13) this nonlinear dependence can be found as

\[ E_{P,k,m} = \int_{(m-1)T_k}^{mT_k} \frac{2}{C_i} E_{C,k} \left[ (l_{k1} + I_{sat1}) \exp \left( \frac{2E_{C,k}}{C_i \eta_i^k V_A} \right) \right] \, dt \tag{15} \]

This relationship can be linearized around a reference value \( E_{C,m}^* \), i.e.:

\[ E_{P,k,m} = E_{P,k,m}^* + \delta_k (E_{C,m} - E_{C,m}^*) \tag{16} \]

where:

\[ E_{P,k,m}^* = E_{P,k,m} (E_{C,m}^*) \quad ; \quad \delta_k = \frac{dE_{P,k,m}}{dE_{C,m}} \bigg|_{E_{C,m} = E_{C,m}^*} \tag{17a} \]

and, the slope \( \delta_k \) can be expressed as:

\[ \delta_k = \frac{2T_k (l_{k1} + I_{sat1})}{\sqrt{2C_i E_{C,m}^*}} - \frac{T_k I_{sat1}}{\sqrt{2C_i E_{C,m}^*}} \exp \left( \frac{2E_{C,m}^*}{C_i \eta_i^k V_A} \right) \left( 1 + \frac{2E_{C,m}^*}{C_i \eta_i^k V_A} \right) \tag{17b} \]

Replacing (16) into (14) leads to the following relationship which corresponds to a discrete-time linearized model of the grid-connected PV system:

\[ \sum_{k=1}^{n} \left[ E_{C,m} - E_{C,(m-1)} \right] = \sum_{k=1}^{n} \left[ E_{P,k,m}^* + \delta_k (E_{C,m} - E_{C,m}^*) \right] - \frac{K_{(m-1)} A^2 T_g}{2} \tag{18} \]

This model demonstrates that if the energy of each capacitor \( E_{C,m} \) is regulated to the reference value \( E_{C,m}^* \) corresponding to the \( k \)th PV array, namely

\[ E_{C,m} = E_{C,(m-1)} = E_{C,m}^* \quad ; \quad k = 1, 2, ..., n \tag{19} \]

then, according to (13a), (18) collapses to (10), i.e.:

\[ \sum_{k=1}^{n} E_{P,k,m} = \sum_{k=1}^{n} E_{P,k,m}^* + \sum_{k=1}^{n} \delta_k (E_{C,m} - E_{C,m}^*) \tag{20} \]

The desired power transfer is thus achieved. It must be pointed out that since the reference current and voltage values of each PV array are set by the irradiance and temperature operating conditions, the only way to force (20) is by controlling the variable \( K_{(m-1)}. \) To ensure control of all capacitors, a set of \( n \) auxiliary variables \( K_k \) with \( k = 1, ..., n \) is defined so that:

\[ \sum_{k=1}^{n} K_k (m-1) = K_{(m-1)} \tag{21} \]

and in this case (18) becomes:

\[ \sum_{k=1}^{n} \left[ E_{C,m} - E_{C,(m-1)} \right] = \sum_{k=1}^{n} E_{P,k,m}^* + \sum_{k=1}^{n} \delta_k (E_{C,m} - E_{C,m}^*) \tag{22} \]

Therefore if the dynamics of each bridge is modeled as:

\[ E_{C,m} - E_{C,(m-1)} = E_{P,k,m}^* + \delta_k (E_{C,m} - E_{C,m}^*) \tag{23} \]

the auxiliary variable \( K_k (m-1) \) can be controlled to set the capacitor energy to its reference value, resulting in

\[ \frac{K_k (m-1) A^2 T_g}{2} = E_{P,k,m}^* ; \quad k = 1, 2, ..., n \tag{24} \]

It is worth emphasizing that proper control of each auxiliary variable leads to the desired steady-state of the overall system given in (20) since from (21) and (23) it can be written:

\[ \sum_{k=1}^{n} K_k (m-1) A^2 T_g = \sum_{k=1}^{n} E_{P,k,m} \tag{25} \]

Finally note that if the reference \( E_{C,m}^* \) is set to the MPP of the PV arrays, the maximum power is transferred to the grid.

One of the main benefits of the previous approach is that a linear discrete-time model of the dynamics of each bridge is obtained. Applying the \( Z \) transform to (23) yields:

\[ E_{C,k}(z) = \left[ E_{P,k}^*(z) + \delta_k (E_{C,k}(z) - E_{C,k}^*) \right] \frac{z}{z-1} \tag{26} \]

\[ - \frac{K_k(z) A^2 T_g}{2(z-1)} ; \quad k = 1, 2, ..., n \tag{27} \]

This model allows the design of a linear discrete-time controller \( G_{C,k}(z) \) to control the auxiliary variable \( K_k(z) \) according to the block diagram derived from (26) and shown in Fig. 2:
This controller is hereafter referred to as the “energy-balance controller” and is designed with the help of the powerful linear control tools in the Z domain to ensure the local stability of the corresponding control loop.

B. Control design guidelines for local stability

This section presents the main design guidelines for the controllers of the cascaded inverter. A complete design of a laboratory prototype built to experimentally verify the proposed approach is presented in section V.

B.1. Energy-balance controllers

The energy-balance controller design follows the same approach in [24], which is summarized below for the sake of completeness.

From the block-diagram of Fig. 2, the closed-loop transfer function can be written as

\[ E_{cl}^* (z) = \frac{zE_{pVk}^* (z) - E_{cl}(z)}{z - 1 - \left(z\delta_k + 0.5A^2T_s G_{cl}(z)\right)} \]

A PI digital controller is chosen to ensure the control of \( E_{cl} \) with zero steady-state error and to fix a desired transient response. Accordingly, \( G_{cl}(z) \) can be written as:

\[ G_{cl}(z) = \gamma_k \frac{z - \alpha_k}{z - 1} \]

Hence, the closed-loop characteristic equation, noted as \( P_i(z) \), results in

\[ P_i(z) = (1 - \delta_k)z^3 + \left(\delta_k - 2 - \frac{\gamma_k A^2T_s}{2}\right)z + 1 + \frac{\gamma_k \alpha_k A^2T_s}{2} \]  

(27)

The set of controller parameters \( \gamma_k \) and \( \alpha_k \) ensuring the system stability is derived by applying the Jury test to (27), yielding the following design restrictions:

\[ \begin{align*}
\delta_k &< 1; \quad \alpha_k < 1; \quad \gamma_k < 0 \\
\frac{4(\delta_k - 2)}{A^2T_s(1 + \alpha_k)} &< 2\delta_k \\
\frac{\gamma_k}{\alpha_k A^2T_s} &< 0
\end{align*} \]  

(28)

Accordingly, the following design guidelines are adopted:

a) The zero of the controller \( (\alpha_k) \) will be located close to the unit circle to compensate the destabilizing effect of the integral action.

b) The controller gain \( (\gamma_k) \) will be adjusted to ensure the stability conditions given in (28).

These restrictions involve the operating conditions of the PV arrays through the parameter \( \delta_k \) defined in (17b).

C.2. Current controller

The energy-balance control approach assumes that the output current \( i_L \) has reached the reference \( i_L^* \). This assumption requires the design of a current controller to ensure that \( i_L \) tracks the sinusoidal current reference \( i_L^* \) given in (9) with a fast transient response and zero steady-state error. The closed-loop dynamics of the inverter can be represented by the following block-diagram:

As extensively reported in [26-27], a linear Proportional + Resonant (PR) controller is especially suitable to track a sinusoidal current reference. The transfer function of this controller, \( G_{PR}(s) \), is given by:

\[ G_{PR}(s) = K_p s + \frac{K_s}{s^2 + \omega^2_s} \]  

By following the design guidelines in [26-27], the controller concurrently ensures a fast dynamics, zero-steady state error at the tracking frequency and local stability.

IV. CONTROL SIGNALS FOR PS-PWM AND LS-PWM MODULATIONS

The proper operation of the current loop leads to a duty cycle \( d \) ensuring the transfer of the overall average DC power to the grid. The last step of the control design is the generation of the control signal of each modulator \( u_k \) from the duty cycle \( d \) to drive the power handled by each bridge according to the energy balance control. This generation depends on the modulation technique used to build the multilevel output voltage \( v_H \). To show this dependence, Fig.4 illustrates the voltages of a 3 full-bridge cascaded inverter for the particular case of the same input normalized DC voltages when a sinusoidal signal of normalized amplitude is applied to PS-PWM (Fig.4a) and LS-PWM (Fig.4b) modulators operating at the same carrier frequency:
As previously reported in [8], PS-PWM leads to an even power distribution among the inverters but to an uneven distribution if LS-PWM is applied, as it can be seen from the output voltage plot of each bridge \((vH_1, vH_2, vH_3)\) of Fig.4a and b. However the work of Angulo et al. [33] modifies the LS-PWM strategy by introducing the concept of “rotating carrier” which allows the power balance by modifying the carrier assigned to each inverter over time. This concept periodically assigns to each inverter (with a period \(T_{rot}\)) the carriers of different shifted levels according to the sequence in Fig. 5, during the same time interval \(T_a\). It is worth noting that achieving power balance requires both the sequence in Fig. 5 and the same time interval assignment.

In contrast, the case under study requires an unbalanced power distribution among the inverters to deal with different operating conditions of the PV arrays: the previous strategies must therefore be modified to make sure that each inverter handles the power of its corresponding PV array.

To handle this, a modifications of the PS-PWM strategy is proposed as follows:

### A) PS-PWM:

The duty-cycle \(d_i\) of inverter \(k\) is computed as:

\[
d_i = \frac{K_i}{K} \frac{d}{\sum_{k=1}^{K} K_i}
\]

The PS-PWM and the corresponding control signals are shown in Fig.6 for the case of 3 inverters:

![PS-PWM carriers](image)

### B) LS-PWM

For Level-Shifted modulation the control design takes advantage of the rotating carrier concept and applies the same rotating carrier assignment to each inverter as in Fig.5, but the time during which this assignment prevails is modified as:

\[
T_k = \frac{K_i}{\sum_{k=1}^{K} K_i} \frac{T_{rot}}{K_i}
\]

The resulting assignment is shown in Fig.7 for the case of 3 inverters:

![LS-PWM strategy](image)
three inverters:

Fig. 7. Modified LS-PWM modulation strategy

V. EXPERIMENTAL VALIDATION

A laboratory prototype of a grid-connected PV multilevel inverter including three cascaded inverter cells was built to experimentally verify the proposed approach. The energy-balance and current controllers described in Section III were implemented in a field programmable gate array (FPGA, Xilinx Spartan 3). The FPGA was used to generate the PS-PWM carriers and the rotating carriers of the LS-PWM addressed in section IV. It must be pointed out that the voltage PWM carriers and the rotating carriers of the LS-PWM Xilinx Spartan 3). The FPGA was used to generate the PS-PWM components of the power stage were set to MOSFETs controlled by IR21084 drivers. The reactive filter inductance. Three Solar Array Simulators (SAS) addresses in section IV. It must be pointed out that the voltage PWM carriers and the rotating carriers of the LS-PWM addressed in section IV. It must be pointed out that the voltage PWM carriers and the rotating carriers of the LS-PWM Xilinx Spartan 3). The FPGA was used to generate the PS-PWM components of the power stage were set to MOSFETs controlled by IR21084 drivers. The reactive filter inductance. Three Solar Array Simulators (SAS) (Agilent E4350B #J02) were also used to emulate the electrical behavior of the PV arrays. The power vs voltage curves programmed in the SAS Voltage (V)

Since the maximum SAS open circuit voltage is of 86V, the multilevel inverter prototype was connected to the single phase grid through a step-up power transformer of 20:3 turns ratio. This resulted in a grid voltage amplitude of 33V and a frequency of 50Hz at the transformer primary side. Therefore, the values of A and \( T_g \) were set to A=33V and \( T_g=20\text{mS} \). The same energy-balance controller, referred to \( G_c(z) \), was designed for the three arrays. The following design procedure assumes the worst operating case to ensure system stability over a wide range of operation.

As pointed out in section III.B.1, parameter \( \alpha_k \) must be set to a value close to 1 to mitigate the instability effect of the integral component of \( G_c \). Therefore, to focus on the selection of the parameter \( \gamma_k \), \( \alpha_k \) is fixed to 0.875. The design of the controller gains (\( \gamma_k \)) is constrained by the stability conditions of (28) which depend on parameter \( \delta_k \). Moreover, the expression of \( \delta_k \) given in (17b) must fulfill the stability restriction \( \delta_k <1 \) for the whole range of irradiances under which the PV array operates. The worst case corresponds to the maximum irradiance value. Note from (17.b) that this restriction leads to the operating range of the array voltage (\( v_{c2} \)) (i.e. of the capacitor stored energy \( E_{c2} \)) ensuring the system stability under this irradiance. This range can be evaluated by a numerical simulation of \( \delta_k \) vs \( v_{c2} \) as shown in Fig. 9 for an irradiance of 1000W/m² and the parameters previously defined. As can be seen, the lower voltage limit of 22.35V is reached when \( \delta_k=1 \) while the upper one of 30V corresponds to the open circuit voltage of the PV array.

In addition, (28) constrains the values of \( \gamma_k \) in terms of the parameter \( \delta_k \). Setting \( \delta_k=0.9 \) to ensure the condition \( \delta_k<1 \), the values of parameter \( \gamma_k \) ensuring system stability are constrained to: -0.053 <\( \gamma_k <0.047 \). It can be proved that a value of \( \gamma_k=-0.05 \) assures system stability for a sufficiently wide operating range.
27) for a fixed controller gain of $K_p=140$ and $K_c=50000$ to obtain a fast transient response and zero tracking error at the grid frequency.

To ensure stability, the capacitor voltage and voltage reference values must be within the stability limits. This condition requires that both capacitors are turned-on.

Fig. 10 shows the root locus of the system in terms of $\delta_k$ (see 27) for a fixed controller gain of $\gamma=0.05$ and an irradiance of 1000W/m². The plot confirms the stability prediction and the system response is stable for $\delta_k<0.953$ (or, equivalently, for a voltage $v_C<22.59V$). This stability condition requires that both capacitor voltage and voltage reference values be within the limits set by the MPPT algorithm remain within these limits at any time. Otherwise, the capacitor is pre-charged near the open circuit voltage of the PV array and regulated to an arbitrary voltage value falling within the stability limits. A partially or totally shaded panel will operate at the voltage arbitrarily fixed by the shading intensity of the array.

Otherwise, the capacitor is pre-charged near the open circuit voltage of the PV array and regulated to an arbitrary voltage value falling within the stability limits. A partially or totally shaded panel will operate at the voltage arbitrarily fixed by the shading intensity of the array.

B.2 Current controller

Following the design procedure given in [26-27], the PR-controlled controller parameters were fixed to $K_p=140$ and $K_c=50000$ to obtain a fast transient response and zero tracking error at the grid frequency.

B.3 Modulation parameters

The paper assumed the same value of the carrier frequencies in order to further make a coherent comparison in terms of harmonic distortion and overall efficiency for both modulations. The choice of the carrier frequency must take into account that the energy-balance control design is valid as long as the capacitor voltage remains within the validity range of the PV array linearized model. Note that, since the number of switching events per grid period is higher in PS-PWM (see Fig.4), the capacitor voltage dynamics is better controlled under this modulation. Accordingly, the carrier frequency must be designed for the LS-PWM case to ensure a number of switching events high enough to control the capacitor voltage within the validity range of the PV array linearized model. In this concern, the following criteria have been adopted:

1) A proper energy balance between the different inverter cells and the power delivered to the grid requires a minimum number of carrier rotations per grid period. The case study has considered 9 rotations per grid period, being this number a multiple of the number of inverter cells. Hence, the rotating carrier period is $T_{rot}=2.2ms$, which has been finally adjusted to $T_{rot}=2.15ms$.

2) The number of carrier cycles during a rotation period must be high enough to preserve an acceptable resolution of the control action (i.e. the number of switching events) for any operating conditions of the PV arrays. In particular, as evidenced in Figs. 7 and 16, the resolution is compromised when, according to eq.(31), the lowest number of carrier cycles is assigned to the cell handling the lowest power. The case study has adopted 42 carrier cycles per rotating period, to ensure an acceptable control action resolution for irradiances ranging from 500W/m² to 1000W/m². Moreover, since this value is a multiple of the number of inverter cells, when all the PV arrays operate under the same irradiance, the same number of cycles, namely 42/3=14 cycles, are assigned to each inverter cell. On the other hand, each of the 6 carriers required in LS-PWM for the present design has been implemented into the FPGA by means of 512 levels, this finally leading to the following carrier frequency value:

$$f_{carrier} = \frac{1}{2 \cdot A_{carrier} \cdot \text{div}_{clk} \cdot T_{clk200MHz}} = \frac{1}{2 \cdot 512 \cdot 9 \cdot 10} = 19531Hz$$

where:

- $A_{carrier}$ = Number of levels/carrier = 512;
- $\text{div}_{clk}$ = scaling factor of the FPGA clock signal = 10.
- $T_{clk200MHz}$ = FPGA clock period = 5ns.

C. Experimental Results

A series of experimental tests were carried out to validate the proposed control approach. The design parameters of the experimental set-up are resumed in Table I:

<table>
<thead>
<tr>
<th>TABLE I. DESIGN PARAMETERS OF THE PV INVERTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power stage</td>
</tr>
<tr>
<td>C1 = C2 = C3</td>
</tr>
<tr>
<td>L</td>
</tr>
<tr>
<td>Grid Voltage</td>
</tr>
<tr>
<td>Panels</td>
</tr>
<tr>
<td>Energy-balance controllers(FPGA)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
</tr>
<tr>
<td>Current controller (FPGA)</td>
</tr>
<tr>
<td>$K_c$</td>
</tr>
<tr>
<td>Carrier frequency</td>
</tr>
<tr>
<td>Rotating period(LS-PWM)</td>
</tr>
</tbody>
</table>
Test 1—Start-up and steady-state behavior under uniform irradiance

The same power vs voltage curve corresponding to an irradiance of 1000W/m² (see Fig.8) was programmed in the three SAS and the reference voltages were set to \( v_{C1} = v_{C2} = v_{C3} = 25V \). Fig. 11 shows the start-up behavior of the inductor current \((i_L)\) and the capacitor voltages \((v_{C1}, v_{C2}\) and \(v_{C3}\)) for PS-PWM (left) and LS-PWM (right). As can be seen, the voltages evolve from the PV array open circuit voltages (30V) to the reference ones, and the transient response of the injected current is smooth. Fig. 12 presents the primary transformer voltage and the injected current \((i_L)\) in steady state. Note that the output current is always in phase with the grid voltage. Fig. 13 shows the output voltage of the multilevel converter \((v_{o})\) and confirms that both modulations operate with seven levels, as expected. Fig. 13 also illustrates the injected current \((i_L)\) and evidences a greater current amplitude ripple for LS-PWM. This is due to the different switching patterns of both modulations:

<table>
<thead>
<tr>
<th>TABLE II: QUALITY INDICES FOR PS AND LS PWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDEX</td>
</tr>
<tr>
<td>THD (%)</td>
</tr>
<tr>
<td>Efficiency (η%)</td>
</tr>
<tr>
<td>Cos(ϕ)</td>
</tr>
</tbody>
</table>

These results suggest that, under the same carrier frequency operation, both modulations exhibit an excellent displacement factor, thus confirming the proper operation of the P+R current controller. In addition, LS-PWM leads to better efficiency but worse THD than PS-PWM, this being attributable to the different switching patterns of both modulations. On the other hand, a FPGA-based design operating at a lower switching frequency to reduce switching losses would be envisaged for the PS-PWM case, but would result more difficult if LS-PWM is adopted since the lowest switching frequency is limited by both the ratio rotating frequency/grid frequency and the control action resolution supported by the FPGA platform, as detailed in section V B.3.

Test 2—Steady-state behavior under non-uniform irradiance

In this test the SAS were programmed to emulate system operation under three different irradiances, namely 1000W/m², 800W/m², and 500W/m² and the reference voltages were set to their MPP values, i.e. \( v_{C1} = 25.2V, v_{C2} = 24.7V v_{C3} = 24V \) (see Fig. 8). Fig.15 shows a zoom of the steady-state capacitor voltages and confirms that they have reached their reference values. Since each inverter handles a different power, the control values delivered by the energy-balance controllers \((K_k)\) were different, this leading to different duty cycles in PS-PWM (see 30) or different time interval assignation in LS-PWM (see 31). Fig. 16 shows a scaled version of the control signals internally generated by the FPGA for both modulations. As it can be seen in Fig.16 (left), inverters handling higher input power are driven by control signals of higher amplitude, in accordance with (30). Similarly, in (Fig.16 right), higher time intervals are assigned to inverters handling higher power, in accordance with (31).

Test 3—Robustness to irradiance changes:

As can be deduced from Fig.2, the input \( E_{PV_k} \) (i.e the energy of the PV array \( k \)) can be considered as a perturbation of \( E_{C_k} \) (i.e the voltage of capacitor \( C_{k} \)). To check the robustness of the energy-balance control, the SAS were programmed to emulate an abrupt irradiance change according to the pattern in Table III and the reference voltage values were held at \( v_{C1} = v_{C2} = v_{C3} = 25V \). Fig. 17 shows the evolution of the capacitor voltages \((v_{C1}, v_{C2}\) and \(v_{C3}\)) and the injected current \((i_L)\). Note that, after a small transient response the voltage across the capacitors maintains its reference value, thus confirming a proper voltage regulation in the presence of irradiance changes.

<table>
<thead>
<tr>
<th>Irradiance (W/m²)</th>
<th>Time interval (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq t &lt; 1.6 )</td>
<td>1.6 \leq t &lt; 5.2</td>
</tr>
<tr>
<td>( 5.2 \leq t &lt; 7.2 )</td>
<td>7.2 \leq t &lt; 10</td>
</tr>
<tr>
<td>( \text{Irrad}_1 )</td>
<td>1000 \hspace{1cm} 800 \hspace{1cm} 800 \hspace{1cm} 800</td>
</tr>
<tr>
<td>( \text{Irrad}_2 )</td>
<td>1000 \hspace{1cm} 1000 \hspace{1cm} 800 \hspace{1cm} 800</td>
</tr>
<tr>
<td>( \text{Irrad}_3 )</td>
<td>1000 \hspace{1cm} 1000 \hspace{1cm} 1000 \hspace{1cm} 800</td>
</tr>
</tbody>
</table>

Test 4—MPPT emulation:

The following two tests aimed to emulate an MPPT algorithm by varying the reference capacitor voltages. In the first test, all reference voltages were fixed at the same value which changed according to the pattern in Table IV. In the second one, the reference voltages simultaneously varied to different reference values following the pattern in Table V.

<p>| Table IV |</p>
<table>
<thead>
<tr>
<th>Reference Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0s \leq t &lt; 1.8s )</td>
</tr>
<tr>
<td>( v_{C1} )</td>
</tr>
<tr>
<td>( v_{C2} )</td>
</tr>
<tr>
<td>( v_{C3} )</td>
</tr>
</tbody>
</table>

<p>| Table V |</p>
<table>
<thead>
<tr>
<th>Reference Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0s \leq t &lt; 3s )</td>
</tr>
<tr>
<td>( v_{C1} )</td>
</tr>
<tr>
<td>( v_{C2} )</td>
</tr>
<tr>
<td>( v_{C3} )</td>
</tr>
</tbody>
</table>

Figs. 18 and 19 show the output current and the capacitor voltages for both tests and both modulations. As it can be seen, the capacitor voltages follow their respective voltage references after a short transient, validating the proposed control approach.
VI. CONCLUSIONS

This paper addresses the control design of a CHB-MLI grid-connected PV system which can operate under PS or LS Pulse Width Modulations. The energy-balance model of the system and the linearization of the PV array electrical characteristics allow the design of discrete-time PI linear voltage controllers ensuring independent control of PV arrays operation. This design which can be easily carried out by conventional discrete-time linear control techniques, takes into account the stability restrictions given by the Jury test which depend on the operating points of the PV arrays. In contrast to other works, the obtained criteria to choose the parameters of the controllers is one of the features of the proposed approach, since they ensure system stability for the whole operating range of the PV arrays in terms of irradiance and temperature. Furthermore, the definition of a set of auxiliary control variables allows the synthesis of the control signals driving each bridge not only for Phase Shifted PWM but also for Level Shifted PWM by modifying the rotating carrier concept.

A set of laboratory tests carried out on a 7-level CHB-MLI grid-connected PV system has experimentally validated the proper operation of the energy-balance control for both modulations under uniform and non uniform irradiance as well as under abrupt irradiance and MPPT algorithm changes. Moreover these results have also shown that LS-PWM leads to worse THD but better efficiency than PS-PWM. In this regard, a FPGA-based design operating at a lower switching frequency to reduce switching losses would be envisaged for the PS-PWM case, but would result more difficult if LS-PWM is adopted since the lowest switching frequency is limited by both the ratio rotating frequency/grid frequency and the control action resolution supported by the FPGA platform.

REFERENCES

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Carlos Meza


Figure 11. Start up behavior of inductor current ($i_L$) and capacitor voltages ($v_{C1}$, $v_{C2}$ and $v_{C3}$) for both PS-PWM (left) and LS-PWM (right).

Figure 12. Steady state primary transformer voltage and injected current ($i_L$) for both PS-PWM (left) and LS-PWM (right).

Figure 13. Steady state output voltage of the multilevel converter ($v_H$) and injected current ($i_L$) for both PS-PWM (left) and LS-PWM (right).
Figure 14. Spectrum of the inductor current of the multilevel converter: PS-PWM (left) and LS-PWM (right).

Figure 15. Zoom of the capacitor voltages: PS-PWM (left); LS-PWM (right).

Figure 16. Steady state PS-PWM control signals (left). Zoom of steady state LS-PWM sequence assignment (right).
Figure 17. Irradiance change. Inductor current ($i_L$) and capacitor voltages ($v_{C1}$, $v_{C2}$ and $v_{C3}$) for both PS-PWM (left) and LS-PWM (right).

Figure 18. Capacitor voltage regulation. Inductor current ($i_L$) and capacitor voltages ($v_{C1}$, $v_{C2}$ and $v_{C3}$) for both PS-PWM (left) and LS-PWM (right).

Figure 19. Capacitor voltage regulation. Inductor current ($i_L$) and capacitor voltages ($v_{C1}$, $v_{C2}$ and $v_{C3}$) for both PS-PWM (left) and LS-PWM (right).