Modification of the Perrine-Baum Diagram to Improve the Calculation of High Voltage Electric Lines

Ricard Horta-Bernús, and Martí Rosas-Casals

***Abstract*— Through the history of Electrical Engineering education, vectorial and phasorial diagrams have been used as a fundamental learning tool. At present, computational power has replaced them by long data lists, the result of solving equation systems by means of numerical methods. In this sense, diagrams have been shifted to an academic background and although theoretically explained, they are not used in a practical way within specific examples. This fact may be against the understanding of the complex behavior of the electrical power systems by students. This article proposes a modification of the classical Perrine-Baum diagram construction to allowing both a more practical representation and a better understanding of the behavior of a high-voltage electric line under different levels of load. This modification allows, at the same time, the forecast of the obsolescence of this behavior and line's loading capacity. Complementary, we evaluate the impact of this tool in the learning process showing comparative undergraduate results during three academic years.**

***Index Terms*—High-voltage, electric line, learning tool, forecasting, vectorial diagram and phasorial diagram.**

I. Introduction

NOWADAYS the usefulness of vectorial diagrams to study or to project electrical lines has been limited by computational power and numerical methods. Nonetheless, vectorial diagrams are an essential tool in order to achieve a complete conceptual understanding of electrical lines, both from a physical and dynamical point of view. This does not occur when using the available commercial software since the results and calculations are provided in numerical tables that require a careful study to reach conclusions.

This article presents a method to partially modify the general theory of the Perrine and Baum electrical diagram [1], [2], [3] (referenced sometimes also as Blondel and Thielemans diagram [4]). This method allows its practical drawing when applied to real cases in order to evaluate different loading rates and to detect the moment in time when the demand for electric power exceeds the capacity of the active line [5]. The graphical information given by the modified diagram is equivalent to the analytical results obtained by numerical computation, but without the effort involved in analyzing results in tabular form. From a graphical perspective, excellent CAD computing tools can actually facilitate and enhance the drawing of the diagrams.

This paper is organized as follows. In Section II we present the basic concepts related to the Perrine-Baum diagram construction. The modified version of the diagram is explained in Section III. Section IV provides an example of application aimed to forecast obsolescence in electric lines. Section V details the educational experience and evaluates the modified diagram usage by students. Finally, Section VI draws conclusions.

II. Perrine-Baum Diagram of an Electric Line

The aim of Perrine-Baum diagram (PBd) is to graphically represent the functioning equations of an electric line. These are the following:

$$\begin{Bmatrix} \overline{V}_1 = \overline{AV_2} + \overline{BI_2} \\ \overline{I}_1 = \overline{CV_2} + \overline{DI_2} \end{Bmatrix} \; ; \; \begin{Bmatrix} \overline{V}_2 = \overline{AV_1} - \overline{BI_1} \\ \overline{I}_2 = -\overline{CV_1} + \overline{DI_1} \end{Bmatrix} \quad (1)$$

Where V and I are respectively the voltage and the intensity of line in origin (subscript 1) or end (subscript 2). Parameters A, B, C and D define a line as a quadripole. Considering the interests of this article, only equations V_1 and V_2 will be represented graphically.

One main problem of the graphical representations of this phasorial diagram is the relative dimensions of the vectors (i.e., vectors AV_2 and BI_2 can differ in one order of magnitude) [1]. This fact prevents the drawing of the whole diagram in useful paper formats (i.e., either A4 or A2) not allowing useful readings. Fig. 1 shows the relative position of vectors AV_2 and BI_2 considering as references that (a) the incoming tension vector V_2 is at zero degrees and (b) a unit power factor. Published as it is in some canonical references [1], [4], this diagram is theoretical and presents problems of representation when trying to use it with real cases. The relative dimensions of the vectors involved in the diagram are not real since some of them are exceedingly greater than others. When using relative real dimensions, vector BI_2 is much smaller (10 to 15

Manuscript received October 26, 2011.
R. Horta-Bernús is with the Electrical Engineering Department, Universitat Politècnica de Catalunya, Campus Terrassa, 08222 Barcelona, Spain (e-mail: rhorta@ee.upc.edu).
M. Rosas-Casals is with the Thermal Machinery Department, Universitat Politècnica de Catalunya, Campus Terrassa, 08222 Barcelona, Spain (e-mail: rosas@mmt.upc.edu).

times) than vector AV_2. When drawn on standard format papers (i.e., A4, A3 or even A2) a lack of precision will prevent to achieve a diagram of practical interest. It should be taken into account that the operating point of the line is inside the area defined by the triangle whose hypotenuse is BI_2.

III. Modified Drawing of the Perrine-Baum Diagram

Fig. 2 incorporates the power factor (f) and shows how vector *FH* (BI_2 in Fig.1) moves or changes its magnitude when the former varies. Triangle *OEF* is invariant, since both tension V_2 and coefficients a' and a", which depend solely on the physical characteristics of the electric line, are fixed. However, triangle *FGH* is not fixed since the main purpose of the diagram is to have the possibility of reading the starting line voltages V_1 under different charging states.

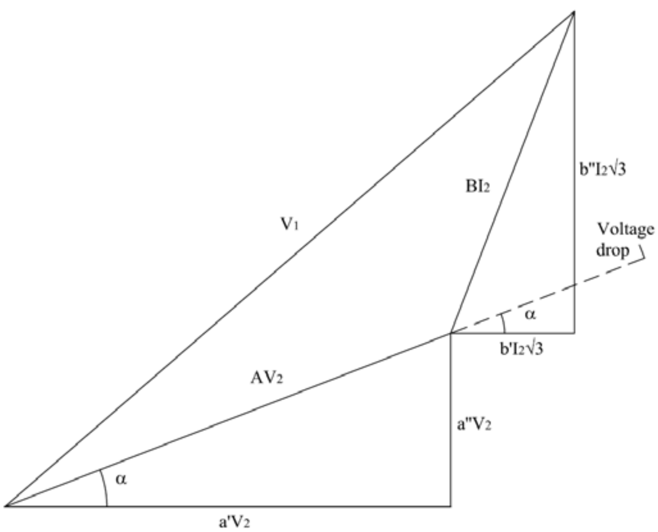

Fig. 1. Graphical representation of functioning equations (1).

Considering that triangle *OEF* is invariant, the easiest solution will be to draw only triangle *FGH*. The extension of the straight line *OF* will be drawn inside triangle *FGH*, from this moment on referred as voltage-drop-straight-line (dashed line in Fig. 1). Indeed, this straight-line will be used to compensate the absence of triangle *OEF* and to allow the reading of tension V_1 with no need of drawing point O. This straight-line will be determined by (a) its angle with segment $b'I_{2N}3^{1/2}$ and (b) the 0% voltage drop point. The aforementioned angle is the angle α of vector *A*, which is the result of calculating the coefficients corresponding to the starting equations system (1). The location of the zero voltage drop point will not be necessarily coincident with the origin of the coordinates of an electric diagram. For example, for lines under 60-80 km long, *A* equals 1 with precision enough and in this case, zero voltage drop meets the coordinates origin. In case *A* module is not 1 (lines over 80 km long), then zero voltage drop will be located above the voltage drop axis, and to a certain distance of the coordinates origin. This distance will be calculated as follows:

$$D = \frac{V_2 - AV_2}{\text{voltage scale}} \quad (2)$$

Starting at *D*, the voltage drop axis will be divided according to the voltage drop percentage (voltage drop over nominal voltage). The voltage drop percentage lines will be drawn perpendicular to the latter. These straight-lines should actually be semi-circles drawn with its center located at the origin of vector V_1. If there is no possibility of drawing vector V_1 due to its longitude, then obviously there will be no possibility of drawing semi-circles either. Percentage voltage drop lines will replace them (Fig. 5). The error committed is usually very small and can be easily accepted within the required accuracy parameters. This is due to the fact that, as stated before, V_1 is 10 to 15 times greater than BI_2 and the arc measured inside triangle *FGH* (Fig. 2) can be assimilated by the proposed straight-lines.

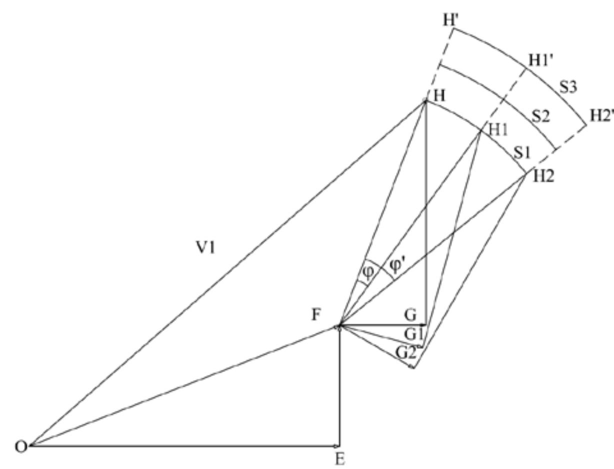

Fig. 2. Modification of the *FGH* triangle for different f and I_2 values.

We can, nonetheless, evaluate the importance of such error by means of trigonometric relations. Fig. 2 and 3 reflect the following relations:

$$\alpha = \sin^{-1}\left(\frac{C}{A}\right) \; ; \; B = A\cos\alpha \; ; \; A - B = A - A\cos\alpha \quad (3)$$

For the sake of clarity, for a line with 380,000 V and 455 A, meaning a transport power of 300 MVA, we have:

$$\alpha = 6.67^{\circ} \; ; \; B = 152.47\ \text{cm} \; ; \; A - B = 1.04\ \text{cm}$$

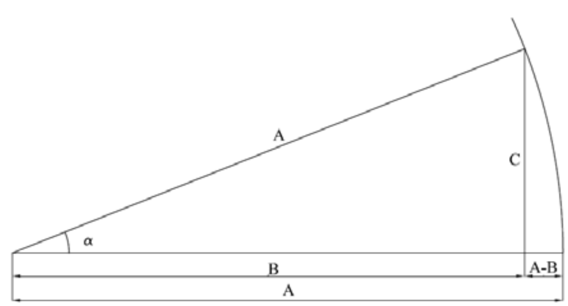

Fig. 3. Graphical representation of committed error.

TABLE I
TECHNICAL DATA OF 300 MVA AND 380 KV ELECTRIC LINE

Length	200 km
Nominal voltage	380,000 V
Frequency	50 Hz
Power	300 MVA
Height over sea level	350 m
Average temperature	25ºC
Weather	90 days of rain per year
Composition	Al-Ac (26+7)
Sections	241.68 mm2 (Al); 39.42 mm2 (Ac); Total = 281.10 mm^2
Copper section	152.01 mm^2
Power-Conductor diameter	21.79 mm^2
Radius of Power-Conductor	10.89 mm^2
Electric Resistance at 20ºC	0.119 Ω/km
Configuration of the conductors	Fig. 4

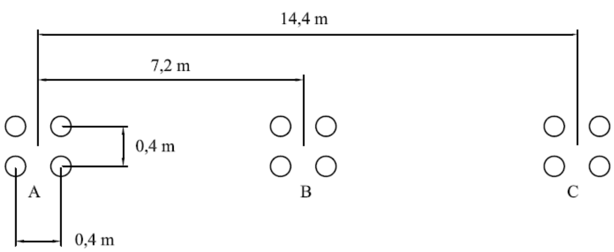

Fig. 4. Spatial distribution for the electric line described in Table I.

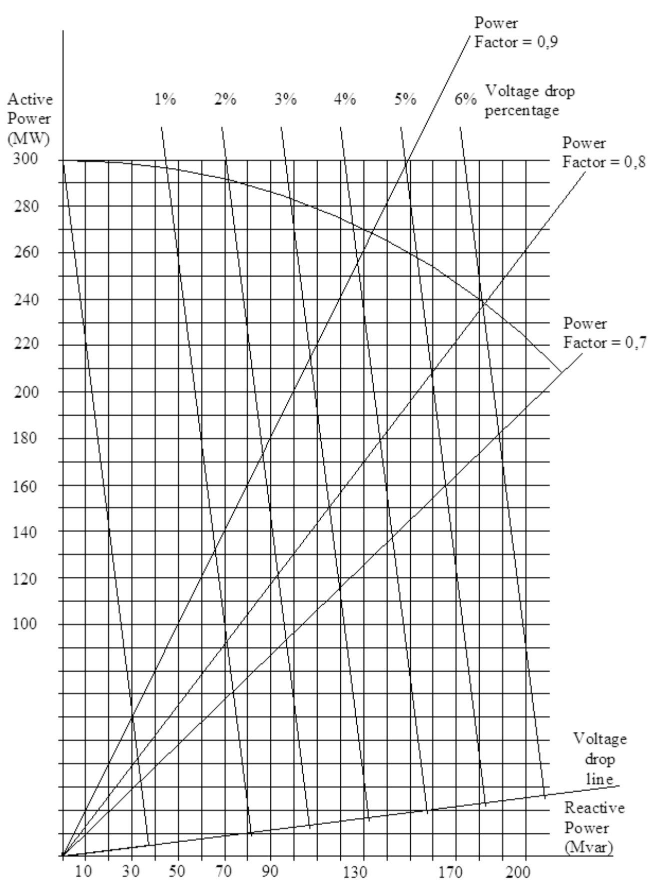

Fig. 5. Perrine-Baum diagram for Table I and Figure 4 data.

This means an absolute error of 2,605 V over 380,000 V, which is 0.685%.

Table I and Fig. 4 give the usual data needed to (a) obtain parameters *A*, *B*, *C* and *D* in (1) and (b) finally construct the PBd, which is showed in Fig. 5.

IV. FORECASTING OBSOLESCENCE WITH THE PERRINE-BAUM DIAGRAM

The diagram in Fig. 5 shows all theoretical possibilities of line operation between unit and 0.7 inductive power factors. In real conditions the line functioning point is located within a specific area of the diagram. This area is limited by two geometric spaces. The first is related to the working power factor (PF). In practice, a high voltage line is designed to work with a PF lower than 0.9 to reduce, among other minor reasons, excessive power loss. The second is related to the maximum apparent power of transport. In Fig. 5, the inner area of the arc starting at 300 MW nominal active power limits the effective working area of the line. The performance of the line over this arc would mean to increase the density of allowable current established by the technical regulations [6]. Fig. 6 shows now the real working area for an active power higher than 210 MW (at 70% full load) with data extracted from Ref. [7]. This modified diagram is now useful in order to read the capacity of active power transport with different power factors. As seen in Fig. 6, the line offers a capacity of transport of 268 MW when PF = 0.9 and 285 MW when PF=0.95. If one or more parallel axis, graduated in annual dates, are incorporated within the active power axis, the expected dates when some active power is reached can be easily read. This feature allows the introduction of the forecasting issue in the student’s curricula, not always an easy assignment to present with limited time and scope.

For the graduation of the axis, many criteria can be considered. During the last decades several models and mathematical algorithms have been used with the purpose of predicting electric energy demand, the most common being co-integration techniques with uni- and multivariate modeling, autoregressive moving average models, neuronal network models, econometric models or genetic algorithms [8], [9], [10], [11]. The variables used in these models may come from different sources like energy costs, energy consumption historical data, GDP, income per capita, annual average temperature, degree days, etc. [12], [13], [14], [15]. In order for these methodologies to reliably predict future energy demands, some additional and fundamental new issues related to energy production and consumption tendencies (like decentralized planning, energy efficiency, technological evolution, recycling, integrated planning and renewable energies [13], [16]) must be taken into account. Up until now, the forecast of the energetic demand and planning has not paid enough attention to the size of the city as a variable that affects the efficiency of the energy consumption per capita. In particular, allometric scaling laws which correlate electric energy consumption in urban areas with population, can be used as another tool to correct the aforementioned consumption forecasting algorithms [17], [18], [19].

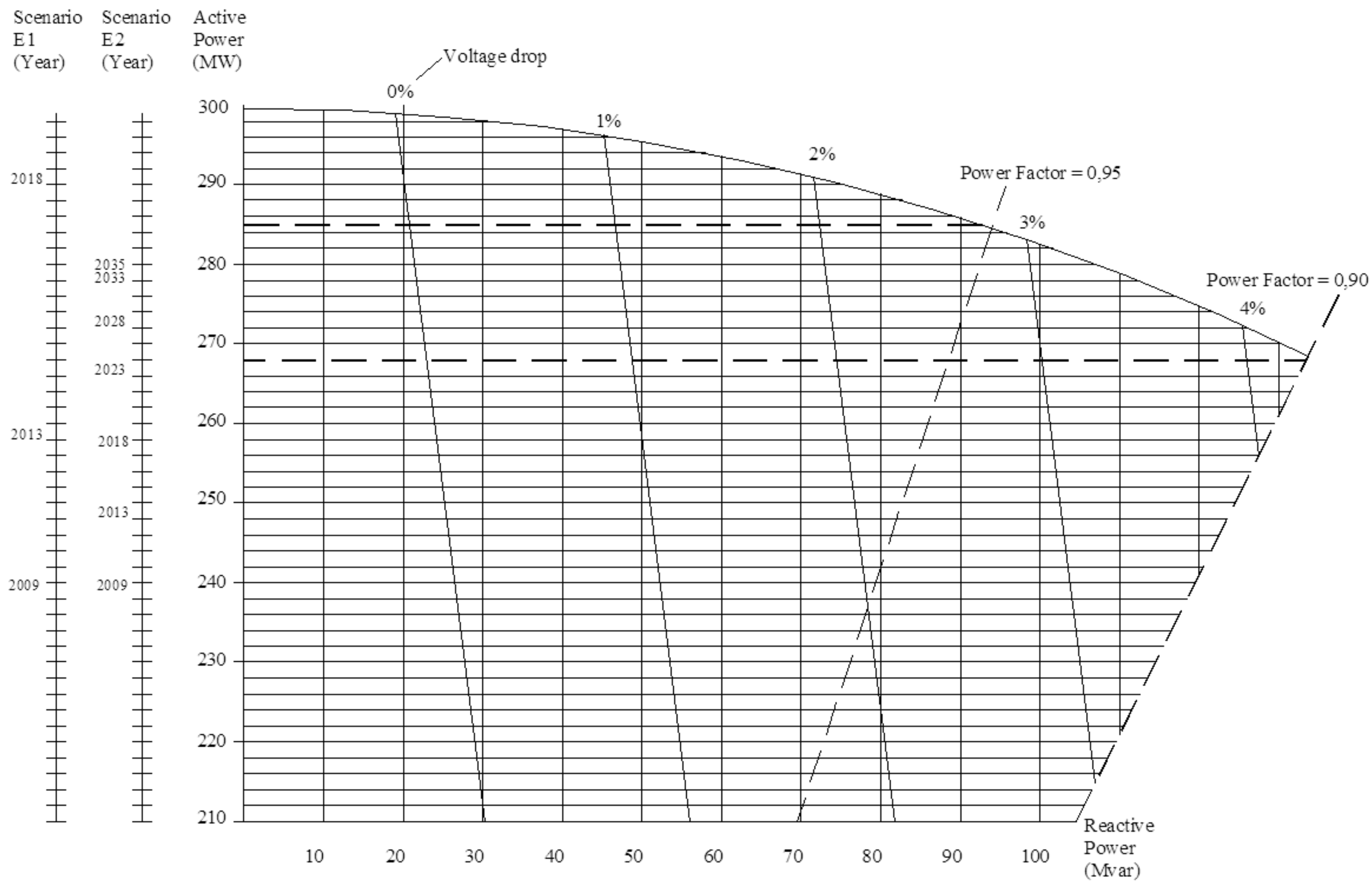

Fig. 6. Real working area for the modified Perrine-Baum diagram, with two examples of graduation of the temporary axis: escenarios E1 and E2.

Some recent references use these techniques to explain the differences in electric consumption as a function of the cities' geographical location [20] or depending on the household, services and administration partial consumptions [21].

Fig. 6 shows two examples of graduation of the temporary axis based on two different energy scenarios for Málaga, a city located within the autonomous region of Andalusia (Spain), with a population of 568,305 and global electric energy consumption of 3,013,733 MWh in 2009. The first scenario (E1) considers the electric energy demand growth under allometric scaling criteria. The second scenario (E2) is based on a forecast study of the energy consumption in Spain until 2035 [16]. Thus we can obtain the limits of voltage drop or power factor for a specific electric power demand. For instance, according to scenario E1 (Fig. 6), from 2018 onwards it is not possible to exceed a voltage drop of 2 % and according to scenario E2, if we want to exceed 268 MW (consumption for year 2024) we need to achieve a higher power factor.

V. Learning with the Modified Perrine-Baum Diagram

One objective for the Grade in Electrical Engineering taught in the School of Engineering of Terrassa (Universitat Politècnica de Catalunya, BarcelonaTech) is to prepare the students to become competent in designing and projecting a high- voltage line. The Grade syllabus, established in 2009, progressively replaced the previous syllabus corresponding to the Technical Engineering of Electricity that started back in 2004. In the new syllabus this objective will be achieved through some specific subjects, one of them is "Calculation and Design of High-Voltage Electric Lines" included in 2011-2012 academic year. This subject replaces "Analysis and Design of Lines and Networks" from the old syllabus. This change of syllabus has been used to implement the modified PBd as means of providing a new calculation and design tool and to improve students' learning process. To assure its utility some training has taken place during 2010-2011 academic year (the last academic year teaching 2004 syllabus) and the results have been compared with the two previous academic years.

At present, professors and students have at their disposal computational tools that make the numerous complex calculations necessary to design a High-Voltage electric line much easier. Additionally, the laborious vectorial diagrams have been superseded by these new tools since they have been either eliminated from the programs or they are only theoretically explained, without real calculation examples. Although in the past few years graduated students have gained calculation speed thanks to these tools, they have lost capacity to analyze and interpret the results given by the computer programs. The capacity of analysis and interpretation is aimed to detect incorrect or absurd computer results or those results out of range from the technological possibilities. The PBd allows to detect at a glance line voltage drops for different loading states and power factors. It even allows achieving in a rapid manner the reactive power of a temporary capacitor battery in case reactive energy is needed. Obviously, these results can be more accurate by using the existing calculation computer programs but these programs do not offer an overview of the line where to see at a glance the existing possibilities with no technical limits. For example, if somebody needs to know the closest villages to a specific village by using a coordinates list (i.e., latitude and longitude) of these villages, she will be able to calculate with accuracy the respective distances. But, in case the person is offered a map, then the knowledge of the relative location will be much more improved. The student may consider this diagram like a map of the load state of an electric line. The authors of this article believe that the PBd is the best one achieving this purpose. However, to achieve the best performance of this tool the diagram will require the modifications introduced in this article.

The students that take “Analysis and Design of Lines and Networks” learn first how to calculate electric and physical constants of a high-voltage electric line. The values of these constants will help them to pose and solve functioning equations of an electric line. At this point the students have to analyze the results of the equations system (1) and decide whether the section and geometrical distribution may satisfy the needs of electric energy transport for which it was projected. Afterwards, some more theoretical diagrams are explained as the zero load diagram and short circuit diagram, and some others more practical like the circular power or lost active energy diagrams. Finally, the modified PBd is incorporated as an item in order to better display the results of (1) in a graphical form and use it like a practical design tool.

The outcome of the implementation of the diagram is shown in the Table II where students results are compared for academic years 2008-2009 and 2009-2010 and also the academic year that this was implemented, 2010-2011. Table II shows the results of the final tests of this subject as well as the percent accuracy upon 3 questions (Q1, Q2, Q3) of similar difficulty corresponding to three different academic years that have the purpose of evaluating the analytic/interpretative capacity of the results obtained through the calculation of the computer programs. It is important to notice that the maximum value is 10 and only students that finally passed the subject have been considered. The students that quitted or failed the subject have been excluded since most of them do not take the exams in which this new tool is implemented. According to the results indicated in the Table II, the improvement is remarkable during 2010/2011 academic year, when this new tool was implemented.

TABLE II
RESULTS OF IMPLEMENTING PBD DURING LAST THREE ACADEMIC YEARS

Academic Year	08/09	09/10	10/11
Number of students	25	28	33
Final Average Mark	6.7	6.3	7.5
Average Mark Q1	5.2	5.5	7.8
Average Mark Q2	5.4	5.0	6.8
Average Mark Q3	5.1	5.8	7.2
Maximum Standard Deviation	1.52	2.12	1.91
Maximum Standard Error	0.43	0.38	0.41

VI. CONCLUSION

The modified PBd provides great amount of information and offers quick responses to technical questions related to the functioning of a line with no need of solving vectorial equations systems. Here we have presented two modifications introduced in the original diagram. On one hand, a practical drawing system that allows using suitable graphical dimensions to represent results. On the other, the incorporation of one tool capable to evaluate the moment in time when line demand exceeds the nominal power of transport. Obsolescence and load forecasting can be in this way easily introduced in the learning program, without the many subtleties involved in a thorough treatment of the planning and load trending estimation techniques and methods. Moreover, the influence of scaling laws, hardly considered in the concluding predictions of regional energy consumption models, can be also incorporated in this tool. However, the diagram allows the incorporation of as many axes as different settings the designer intends to evaluate. As the results indicate, the implementation of this tool in the learning program has been positive and the inclusion of the diagram in the subject “Calculation and Design of Electric Lines” of the new syllabus corresponding to the Grade of Electrical Engineering taught in the Escola d’Enginyeria de Terrassa-Universitat Politècnica de Catalunya (BarcelonaTech) we believe will improve the students learning capacity and habilities.

REFERENCES

[1] L. M. Checa, *Lineas De Transporte De Energía.* Barcelona: Marcombo, 1988.

[2] C. H. Holladay, "A Graphic Method for the Exact Solution of Transmission Lines," *American Institute of Electrical Engineers, Transactions of the,* vol. XLI, pp. 785-793, 1922.

[3] F. A. C. Perrine and F. G. Baum, "The Use of Aluminium Line Wire and Some Constants for Transmission Lines," *American Institute of Electrical Engineers, Transactions of the,* vol. XVII, pp. 345-384, 1900.

[4] E. Ras, *Teoría De Líneas Eléctricas.* Barcelona: Marcombo, S.A., 1973.

[5] R. Horta and J. I. Candela, *Teoria, Càlcul i Deisseny De Línies Elèctriques.* Barcelona: Edicions UPC, 2001.

[6] J. Moreno, F. Garnacho, P. Simón and J. Rodríguez, *Reglamento De Lineas Eléctricas De Alta Tensión y Sus Fundamentos Técnicos.* Madrid: Paraninfo, 2009.

[7] REE. Demanda de energía eléctrica en el sistema peninsular. boletín mensual. enero 2011. 2011.

[8] Z. W. Geem and W. E. Roper, "Energy demand estimation of South Korea using artificial neural network," *Energy Policy,* vol. 37, pp. 4049-4054, 10, 2009.

[9] M. Kankal, A. Akpınar, M. İ. Kömürcü and T. Ş. Özşahin, "Modeling and forecasting of Turkey’s energy consumption using socio-economic

and demographic variables," *Appl. Energy,* vol. 88, pp. 1927-1939, 5, 2011.

[10] G. E. Nasr, E. A. Badr and M. R. Younes, "Neural Networks in forecasting electrical energy consumption," *FLAIRS-01 Proceedings,* pp. 489, 2001.

[11] H. K. Ozturk, O. E. Canyurt, A. Hepbasli and Z. Utlu, "Residential-commercial energy input estimation based on genetic algorithm (GA) approaches: an application of Turkey," *Energy Build.,* vol. 36, pp. 175-183, 2, 2004.

[12] J. M. Beaujean and J. P. Charpentier, "A review of energy models," International Institute for Systems Analisis, Luxenburg, Austria, 1978.

[13] S. Jebaraj and S. Iniyan, "A review of energy models," *Renewable and Sustainable Energy Reviews,* vol. 10, pp. 281-311, 8, 2006.

[14] F. Egelioglu, A. A. Mohamad and H. Guven, "Economic variables and electricity consumption in Northern Cyprus," *Energy,* vol. 26, pp. 355-362, 4, 2001.

[15] I. Dincer and S. Dost, "Energy intensities for Canada," *Appl. Energy,* vol. 53, pp. 283-298, 1996.

[16] M. Valero, "La demanda de electricidad en España. Previsión para el 2035," *Boletín Económico De ICE,* pp. 23, 2010.

[17] M. Kleiber, "Body size and metabolism," *Hilgardia,* vol. 6, pp. 315-351, 1932.

[18] C. Kühnert, D. Helbing and G. B. West, "Scaling laws in urban supply networks," *Physica A.,* vol. 363, pp. 96-103, 4/15, 2006.

[19] L. M. A. Bettencourt, J. Lobo, D. Helbing, C. Kühnert and G. B. West. Growth, innovation, scaling, and the pace of life in cities. *P. Natl. Acad. Sci. USA 104(17),* pp. 7301-7306. 2007.

[20] Xiaowu Zhu, Aimin Xiong, Liangsheng Li, Maoxin Liu and X. S. Chen. Scaling law between urban electrical consumption and population in china. *Computer Science 4(Complex Science),* pp. 853-864. 2009.

[21] R. Horta, M. Rosas and S. Valverde. Discerning electricity consumption patterns from urban allometric escaling. *Complexity in Engineering Conference* 2010.

Ricard Horta-Bernús received the M. Eng. degree in electrical engineering in 1995 and the M. Sc. degree in industrial organization in 2000 and investigating sufficiency in 2010 all of Universitat Politècnica de Catalunya (BarcelonaTech), Terrassa, Barcelona, Spain. Since 1996, he has been Full Professor with the Department of Electrical Engineering Universitat Politècnica de Catalunya (BarcelonaTech), Barcelona, Spain. His research interest includes allometric scaling and complexity applied to electrical networks, mainly to study its robustness and fragility.

Marti Rosas-Casals received his PhD degree in 2009 from the Universitat Politècnica de Catalunya (BarcelonaTech). He is currently Full Professor with the Department of Thermal Machinery, at the Engineering Faculty in Terrassa Campus. His major fields are energy, infrastructure networks and complex systems.