Hydrodynamic model, simulation and linear control for Cormoran-AUV

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Abstract—this work shows the mathematic calculation for obtention of a Cormoran-AUV hydrodynamic model, it also shows a linear control design for a path tracking. The model has been simplified to three degrees of freedom of movement and the whole system has been simulated using Matlab Simulink Software. The system has been linearizated for different velocities to design a linear control for each one of them. However, all resulting systems can be controlled by a unique linear control due characteristics of the vehicle. The designed control is a PD controller, which avoids the position error since the pole of the vehicle model is at the origin. Different paths have been simulated using this control and their results have been comparated in both rising time as establish time.

Keywords—Hydrodynamic model, linear control, autonomous underwater vehicle.

I. INTRODUCTION

Cormoran (see figure 1) is a low cost oceanic observation vehicle, hybrid between AUV (Autonomous Underwater Vehicles) and ASV (Autonomous Surface Vehicles), which has been built in Mediterranean Institute of Advanced Studies (IMEDEA) of Mallorca (Spain) by the oceanographic group, in colaboration with the University of the Balearic Islands.

The principle of movement is based on the navigation over the marine surface where the vehicle follows a predetermined path in the mission. The path is defined by a series of waypoints, in which the vehicle stops and dives vertically to obtain a profile of a water column. Subsequently, the vehicle rises to the surface and transmits the most relevant data (temperature, salinity, depth and global position using GPS) through GSM messages. After sending this data, the vehicle continues to the next waypoint defined by the mission [1].

In order to create a mission path it is necessary to have two things: A reliable mathematical model and a control for the maneuvers of the vehicle, which follows the fixed path under a certain tolerance.

Figure 1 shows a picture of the Cormoran – AUV.

This paper has been organized as follows: In section II is presented a mathematical model of the vehicle’s dynamics over the surface simplified to three degrees of freedom and shows its implementation in simulink. Section III shows a linearization of the sytem. Section IV shows a control design for the path traking using a PD controller. Section V shows the results. And finally, section VI shows the conclusions and future work.

II. 3 DOF HYDRODINAMIC MODEL OF THE VEHICLE

Due to the movement described before, heave, roll and pitch are not taken into consideration. Therefore the characterization of the vehicle can be achieved through a three degrees of freedom that include the advance (x), the lateral displacement (y) and yaw angle (ψ).

Vectors of position, velocity and force can be expressed as shown in (1), where \( \eta \) is the position, \( \nu \) is the velocity and \( \tau \) is the force.

\[
\begin{align*}
\eta &= [x, y, \psi]^T \\
\nu &= [u, v, \tau]^T \\
\tau &= [X, Y, N]^T
\end{align*}
\]

A. Vehicle dynamics

The generalized non linear dinamic equation of any ship can be expressed as shown in (2), which involves accelerations, velocities and forces [2].

\[
M_{RB} \ddot{\nu} + C_{RB}(\nu)\nu = \tau_{RB}
\]

Where

- \( M_{RB} \) is the matrix of masses and inertias.
- \( C_{RB} \) is the centripetal and coriolis matrix
- \( \tau_{RB} \) is the generalized vector of hydrodinamic forces and moments (generated by hydrostatic forces, hydrodinamic forces, added masses, lift forces, propulsion of motors and environmental disturbances.
- \( \nu \) is the velocity vector.

All of these matrices have different coefficients depending of the characteristics of the ship. The principle of movement of AUV Cormoran is based in two things: A thruster that gives force in x axis for the movement, and a rudder to
provide direction to the vehicle. The general model of marine vehicles was simplified to three degrees of freedom using these considerations. The resulting equations are presented in (3), (4) and (5), which are functions of speed, mass, vehicles’ propulsion and a set of hydrodynamic coefficients.

\[
m u - m v r = X_u \dot{u} - Y_v r - Y_r r^2 + X_{\text{prop}} \dot{u} + X_{\text{prop}} \tag{3}
\]

\[
m \dot{v} + m u r = Y_i \dot{v} + Y_r \dot{r} + X_i u r + Y_i \dot{r} \tag{4}
\]

\[
I \dot{r} = N_i \dot{v} + N_r \dot{r} + Y_i u r - (X_u - Y_i) u v + v + N_{\text{prop}} \dot{v} + N_{\text{prop}} \dot{r} + N_{\text{prop}} u^2 \delta_r \tag{5}
\]

Figure 2 shows the block diagram of this dynamic, where propulsion of the thruster and rudder angle are the inputs of the system.

III. LINEARIZATION OF THE SYSTEM

First of all, it is necessary to obtain a system linearization in order to design a linear control system [3]. The model is linearized around the speed, assuming that the forward speed \( u \) is constant and \( v \) and \( r \) speeds are smaller compared to the forward speed, several velocities were used to linearize the system. Similar approaches are made in works on the Remus vehicle [4], as well as the AUV-Infante [5]. Consequently, the point of work is \((u, v, r) = (u_0, 0, 0)\).

Applying Taylor series approximations [6], linear vehicle model is achieved, expressed in matrix form (6). In this model, the propulsion engine \( X_{\text{prop}} \) and rudder angle \( \delta_r \) are considered inputs to the system.

\[
\begin{bmatrix}
-2X_{\text{prop}} & 0 & 0 \\
0 & -Y_{\text{wif}} & m - X_u - Y_{\text{wif}} \\
0 & X_u - Y_v - N_{\text{wif}} & -Y_r - N_{\text{wif}}
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta v \\
\Delta r
\end{bmatrix} =
\begin{bmatrix}
-2X_{\text{prop}} u_0 \\
0 \\
0
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 \\
0 & m - Y_v & -Y_r \\
0 & -N_v & I - N_r
\end{bmatrix}
\begin{bmatrix}
\Delta \dot{u} \\
\Delta \dot{v} \\
\Delta \dot{r}
\end{bmatrix} +
\begin{bmatrix}
X_{\text{prop}} \\
Y_{\text{wif}} u_0^2 \delta_r \\
N_{\text{wif}} u_0^2 \delta_r
\end{bmatrix}
\tag{6}
\]

A. Analysis of poles and zeros

In order to do an analysis of poles and zeros the transfer function \( G_p \) has been calculated, which are in function of yaw angle respect the action of the rudder for different forward speeds, from \( u_0 = 0.3 m/s \) to \( u_0 = 3.3 m/s \). This array of transfer functions had a pole and a zero very close; therefore they were canceled in order to simplify the transfer function. The type of transfer function for different speeds is presented in (7).

\[
G_p(s) = \frac{-b}{s(s + a)}, \text{ with } a, b \in \mathbb{R}^+ \tag{7}
\]

This transfer function has a negative value in numerator, a pole in origin and other pole in the left-half of the s-plane. This indicates that it is a critically stable system.

As a first step a proportional controller has been used to

![Fig. 2. Block diagram of vehicle dynamics](image)

![Fig. 3. Simulink implementation](image)
guarantee the stability of the system. In this case it is necessary that roots of the characteristic polynomial of the closed loop system are negative. This condition is achieved iff \( k < 0 \).

\[
p(s) = s^3 + as - kb
\]  
(8)

Figure 4 shows the closed loop poles for different linearization in different forward speeds. The gain chosen for these systems is -1 in order to make the system stable. It should be noted that while the speed increases the system is more stable with a fast response of the system and the same time maintaining the overshoot.

IV. CONTROL DESIGN

A controller must be developed to track a predefined path. The controller must lead the nonlinear system to a desired dynamic. This dynamic is defined in (9), which specifies the maximum overshoot and settling time desired.

\[
\xi \leq 0.707
\]

\[
t_{ss} \leq 0.8seg
\]

In other words

\[
\phi \leq 45^\circ
\]

\[
\sigma \leq -5
\]

(9) (10)

Figure 4 shows this desired dynamic zone with a green line, in where several linearizations at slow velocities are outside of this zone and it becomes necessary develop a control.

A. PD controller

Since the linearized model with yaw as output has a pole in the origin, PD controller is enough to eliminate the position error and achieve the desired dynamic area. For example, the equation (11) shows the design of the PD controller for constant velocity of 0.3 m/s.

\[
G_c(s) = 22.25(s + 5.44)
\]

(11)

Eventhough PD controllers were calculated for different velocities, only one is needed to move the poles of all systems to the target zone due the system at high forward speed has poles more stables. Specfically, the control designed for the smallest velocity (0.3 m/s) is enough to move all poles to the target zone.

V. RESULTS

A unique linear controller has been applied to the vehicle as it is described above. In this case it is used the controller designed for the smallest velocity (11).

A. PD controller in linearized systems

Figure 5 shows the closed loop poles using PD controller (11) in the different linearizations from 0.3m/s up to 3.3m/s. It shows that when increasing speed, the poles become more stables. Figure 5b shows the same representation at low speeds. Poles at smallest velocity are the poles on the boundary of the target zone (10).

Figures 6 and 7 show the step response of linearized system at two velocities: 0.3m/s and 3.3m/s. Linearized system perform time response and overshoot conditions (9) in both cases.

A. PD controller in linearized systems

\[
0.707 \leq \xi \leq 0.8seg
\]

\[
\phi \leq 45^\circ
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\sigma \leq -5
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IV. CONTROL DESIGN

A controller must be developed to track a predefined path. The controller must lead the nonlinear system to a desired dynamic. This dynamic is defined in (9), which specifies the maximum overshoot and settling time desired.

\[
G_c(s) = 22.25(s + 5.44)
\]  
(11)
B. PD controller in nonlinear system

This PD controller has been applied to nonlinear system. Figure 8 shows step response when the vehicle navigates at two different velocities: 0.3 m/s and 3.3 m/s. This shows that PD controller don’t work correctly at low velocities. It indicates that nonlinear terms have more influence in the system at low velocities, and system has performance very different to linearized system.

Fig 8. Step responses with PD controller in nonlinear system

Above indicates that designed PD controller is valid for a certain range of speeds. In order to determine this range different forward speed have been simulated. The results indicates that PD controller works at least at a speed of 0.97 m/s. Figure 9 shows step response for this velocity.

C. Tracking of a variable yaw

In order to follow a preestablished path we define a variable yaw in time as reference of the system. Figure 10 shows a simulations of trajectory using different forward speeds. It shows the misfit of the vehicle in the XY plane because the control is over yaw but not over XY plane.

VI. CONCLUSIONS AND FUTURE WORK

This paper presented a nonlinear model of the Cormoran vehicle using three degrees of freedom. The model has been linearized at different velocities to analyse their behavior. In order to develop a path tracking, a PD controller has been designed and it has been determined the range of velocities that validates the linear approximation of model. This controller performs a specific desired dynamics in vehicle; It eliminates the position error for a certain range of forward speeds. The paper also shows that this controller is not satisfactory at low velocities. As future work we should study a control strategy that solves the nonlinearities at low speeds, in order to give the same dynamic for all forward velocities of the vehicle.

ACKNOWLEDGMENT

This work has been funded by the Spanish Ministry of Science and the European Union (FEDER), project n°: CTM2010-16274 and project n°: CTM2009-08867.

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