

Analysis of PMSM using FE analysis: a didactical approach for EMF determination and cogging torque reduction.

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Abstract—FE analysis is a useful technique, but the interpretation of the results are sometimes confuse and dark. In this paper we shown some methods to determine the back emf in PMSM and how determine and reduce the cogging torque. Our students use these methods and comparing the obtained results, they learn about the post-processing magnitudes in FE analysis and its interpretation; probably the most difficult task in this calculation method. Additionally these results are compared to a prototype and the results are agreed with the prediction.

I. BACK EMF DETERMINATION

Between the machine parameters, back emf is in general the first step for the determination of the overall performances of a design, and in the following it will be shown how it is possible to deduce accurately back emf at the machine terminals starting from FEM analysis.

The logical sequence of this technique starts from the knowledge of the magnetic vector potential: back emf is computed by means of a small number of integral data closely related to the flux through a surface. A FEM solution in terms of vector potential is considered below. The flux linked with a coil is:

$$\phi_S(B) = \int_S B \cdot ndS \quad (1)$$

Where S is the area of the cross section of the coil and B(x,y) is the flux density (to obtain the total linked flux these value must be multiplied by the number of turns)

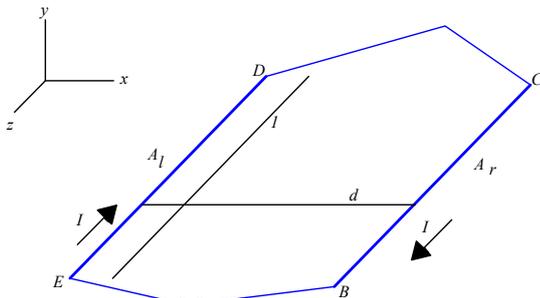


Figure 1. Elemental coil

By substituting in Eq. 1 the expression for B as a function of A:

$$\phi_S(B) = \int_S \nabla \times A \cdot ndS \quad (2)$$

Application of Stokes' theorem transforms Eq. (2) in the following:

$$\phi_S(B) = \int A \cdot dl \quad (3)$$

On the basis of the 2D assumptions previously made, the flux of B can be rewritten as:

$$\phi_S(B) = (A_r - A_l) \cdot l \quad (4)$$

In a practical case multiple conductors occupies a slot and has nonzero dimensions (figure 2), they aren't a point.

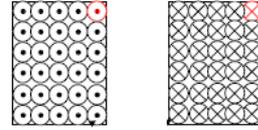


Figure 2. Multiple conductor

In this case to determine the total linked flux it's possible to evaluate the average flux in a slot using the following expression (valid for unity of depth):

$$\begin{aligned} \phi &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \phi_k = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N (A_{kl} - A_{kr}) = \\ &= \lim_{N \rightarrow \infty} \frac{S}{S} \frac{1}{N} \sum_{k=1}^N (A_{kl} - A_{kr}) = \frac{1}{S} \lim_{N \rightarrow \infty} \sum_{k=1}^N (A_{kl} - A_{kr}) \Delta S \\ &= \frac{1}{S} \left(\int_S A \, dS \Big|_l - \int_S A \, dS \Big|_r \right) \end{aligned} \quad (5)$$

And then calculate the total linked flux by multiplication with the number of turns and depth:

$$\phi = \frac{N}{S} \left(\int_S A \, dS \right) \cdot l \quad (6)$$

This is the value of the linked flux with a coil at time t_0 ; values for different times can be obtained with new relative positions between stator and rotor. However, the linked

flux of the coil at time t_1 is the same linked flux at t_0 with the coil which is at an angle $(t_1 - t_0)\omega_r$ from the current coil, where ω_r is the rotating speed of the rotor. This allows limiting the number of FEM calculations to one, if the magnetic structure is isotropic. When the geometry under consideration presents a small number of coils per pole and per phase, the relevant magnetic structures present not constant characteristics through space. Therefore, the number of FEM calculations to perform is at least two: the first one when the axis of the magnets is superimposed to the axis of a slot, the second one when the axis of the magnets is superimposed to the axis of a tooth. When the flux linked through a coil (or a phase) is known, the computation of the relevant back emf is in fact a simple task, by applying the Faraday law:

$$e(t) = -\frac{d\phi(t)}{dt} = -\frac{d\phi(\theta)}{d\theta} \frac{d\theta}{dt} = -\frac{d\phi(\theta)}{d\theta} \omega_r \quad (7)$$

Where θ is the angular position, in a reference frame strongly connected to the rotating field, of the coil axis and ω_r is the angular velocity of the rotating field. From the above equation it is evident that numerical derivation of the linked flux is the basis for the determination of the back emf. There are some possibilities to calculate the back emf:

A. Direct flux derivation. Accuracy reachable in this way is poor, since the linked flux is known in a small number of points of the interval. If the linked flux varies suddenly near the point under consideration, the numerical values of higher derivatives are not negligible, and this results in great numerical errors. Linked flux can be approximated by means of analytical functions, such as Fourier expansions. If an analytical approximation for the linked flux is determined, the back emf can be found by means of analytical derivation that can be obtained without numerical errors and with no significant computational efforts. Back emf is calculated by means of analytical derivation as in the following Eq:

$$e(\theta) = -\frac{d\phi(\theta)}{d\theta} = -\sum_{k=1}^N k \phi_k \sin(k \theta) \quad (8)$$

B. Calculate directly the back emf using the expression (In fact this expression cannot be applied to a coil in a slot, but it's a rough approximation):

$$e_{conductor} = B \cdot L \cdot v \quad (9)$$

II. COGGING TORQUE

Determination of cogging torque requires the calculation of torque for a set of positions including at least one tooth pitch. In this case it is interesting to compute at least two or more tooth pitch to show the cyclic variation

of the cogging torque. To compute cogging torque Maxwell's tensor method is used:

$$T = v_0 \cdot \int B_n B_t r \cdot dl \quad (10)$$

The number N_p of periods of the cogging torque variation during a rotation of a slot pitch is [1, 3]:

$$N_p = \frac{2p}{GCD(Q, 2p)} \quad (11)$$

GCD is the Great Common Divider, Q is the number of slots and p is the pole pairs. The resulting cogging torque can be described using a Fourier series expansion as:

$$T_{cog} = \sum_{k=1}^{\infty} T_k \cdot \sin(k \cdot N_p \cdot Q \cdot \theta_m + \varphi_k) \quad (12)$$

One of the techniques to reduce the cogging torque is shift the PM. The optimal displacement is:

$$\delta_{opt} = \frac{360^\circ}{N_s \cdot N_p} \quad (13)$$

N_s is the number of stacks.

III. SIMULATION AND EXPERIMENTAL RESULTS

First is analyzed a PMSM with these main characteristics (Machine 1):

- 6 poles
- 9 slots
- Stator diameter: 50 mm
- Airgap: 1 mm
- Machine length: 65 mm
- Conductors/slot: 45

Figure 3 shown 2D radial section of the machine. Ne35 grade permanent magnet is used for all analyzed machines.

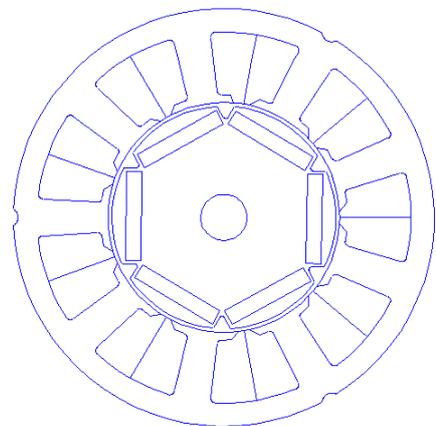


Figure 3. 2D section of tested machine 1.

The following pictures show the obtained results of tested machine. Figure 4 show the flux density through the airgap, figure 5 show linked flux in every phase as a function of rotor position, figure 6 shown back emf calculated using expression (7) and linked flux by phase, figure 7 show back emf calculated using expression (9) and figure 8 show calculated back emf using expression (8) and Fourier development of linked flux (four terms)

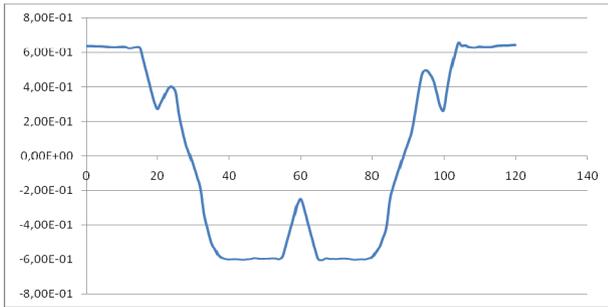


Figure 4. Airgap density Flux (T) Machine 1.

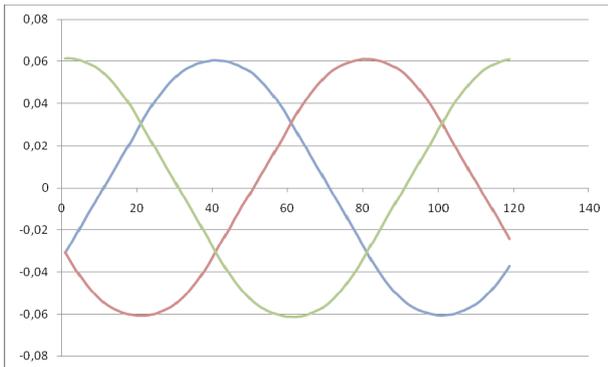


Figure 5. Linked flux by phase (Wb) Machine 1.

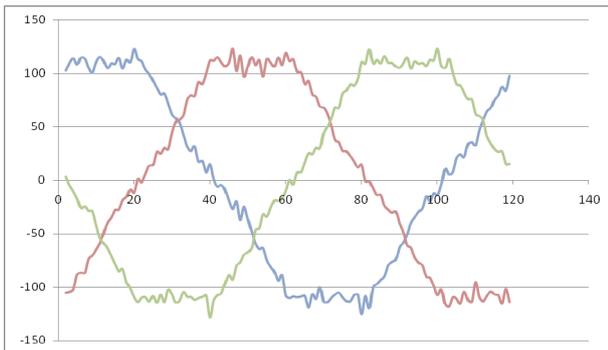


Figure 6. Back emf using direct derivation. Machine 1.

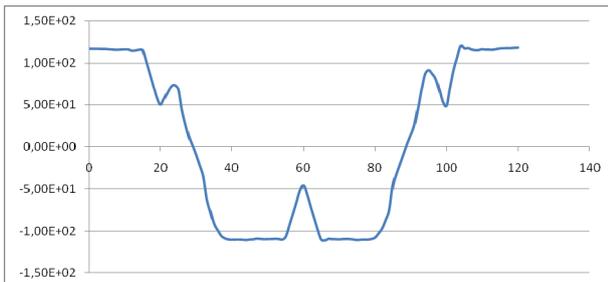


Figure 7. Back emf using BLv. Machine 1.

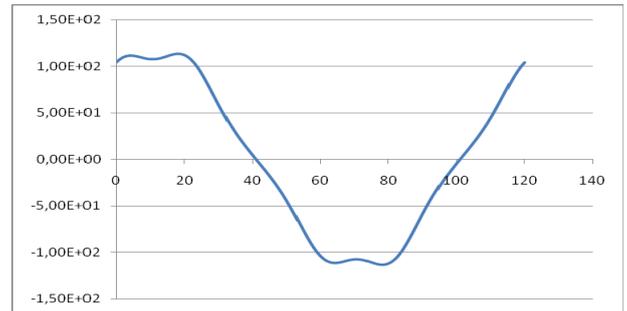


Figure 8. Back emf using Fourier approximation of flux. Machine 1.

Figure 9 show cogging torque for this machine. Where it's possible to see the periodicity and verify theoretical expression: $N_p \cdot Q = 18$.

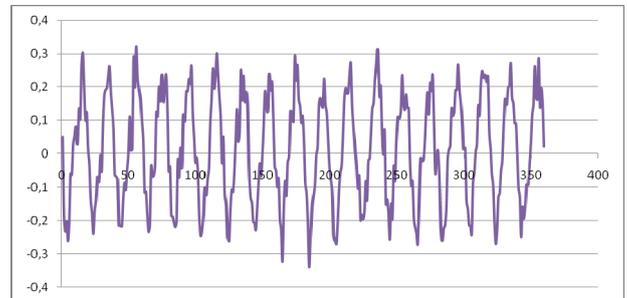


Figure 9. Cogging torque. Machine 1.

As a second example we analyze a PMSM with these main characteristics (Machine 2):

- 6 poles
- 36 slots
- Stator diameter: 55 mm
- Airgap: 1mm
- Machine length: 40 mm
- Conductors/slot: 32

Figure 10 shown 2D radial section of the machine. This is a Interior Permanent Magnet Synchronous Machine (IPMSM) The permanent magnet has Ne35 grade.

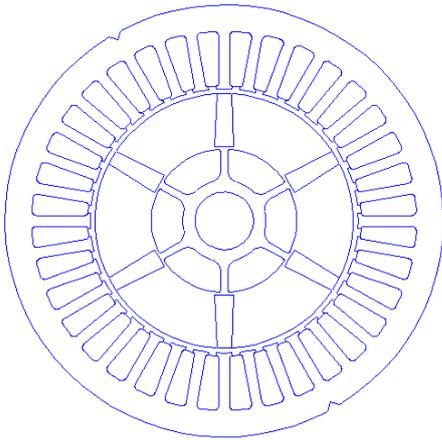


Figure 10. 2D section of tested machine 2.

The following pictures show the obtained results for the analyzed machine. In figure 11 it can be seen the calculated back EMF calculating direct derivation of flux using expression (7)

In figure 12 BLv approximation is used (expression (9)) Figure 13 show back emf calculated using Fourier approximation of flux (four terms)

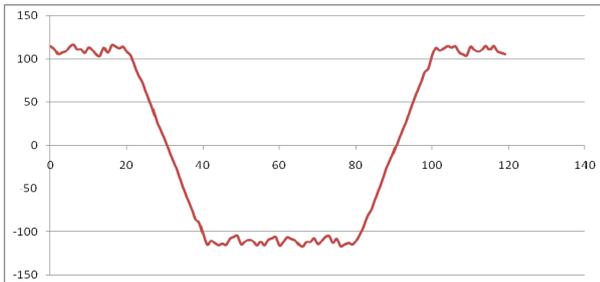


Figure 11. back EMF using direct derivation. Machine 2.

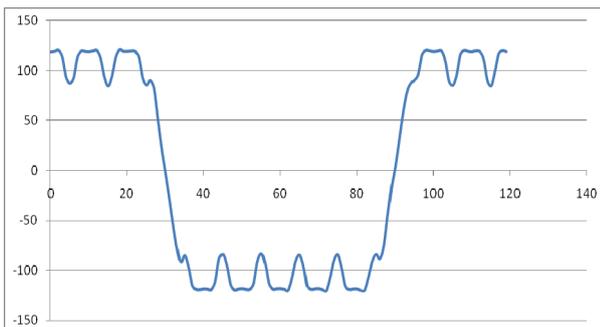


Figure 12. Back emf using BLv. Machine 2.

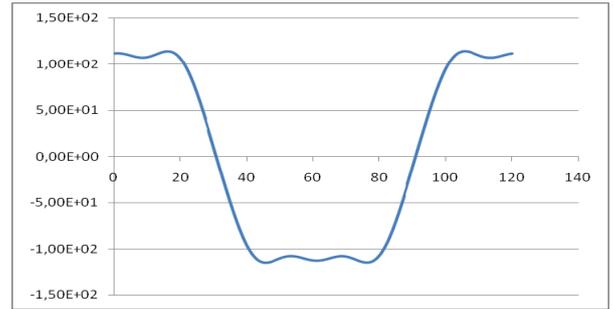


Figure 13. Back emf using Fourier approximation of flux. Machine 2.

The figure 14 show the cogging torque; it can be seen the periodicity of this and verify that is the predicted by theory analysis: $N_p \cdot Q = 36$.

Also figure 14 show the cogging torque obtained shifting the PM. For this case it's shown the obtained results using 2 sections; In this case the optimal angle is determined using expression (13); and the shifted angle is: $\delta = 5^\circ$.

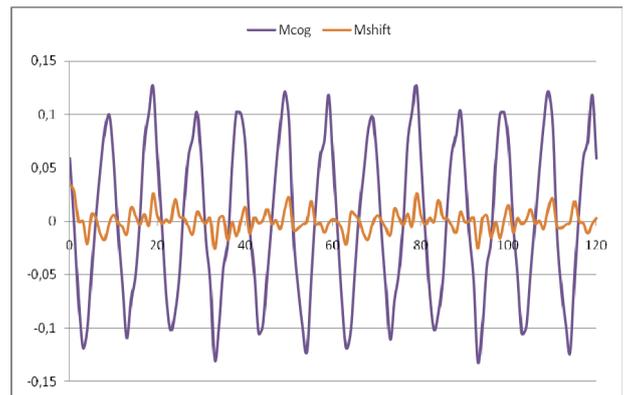


Figure 14. Cogging torque using optimal shift PM method. Machine 2.

As a final example it's shown the results for a Permanent Magnet assisted Synchronous Reluctance Motor (PMASRM) with these main characteristics (Machine 3):

- 4 poles
- 24 slots
- Stator diameter: 50 mm
- Airgap: 1mm
- Machine length: 45 mm
- Conductors/slot: 38

Figure 15 show a photo of the tested prototype. The permanent magnet aren't placed.



Figure 15. Prototype. Machine 3.

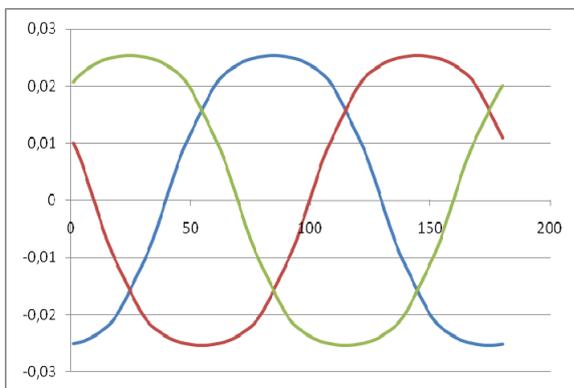


Figure 16. Linked flux by phase (Wb) Machine 3.

Figures 17 show calculated back emf. Figure 18 show measured back emf, figure 19 show back emf calculated using BLv approximation and figure 20 show back emf calculated using Fourier approximation. Figure 21 show calculated cogging torque.

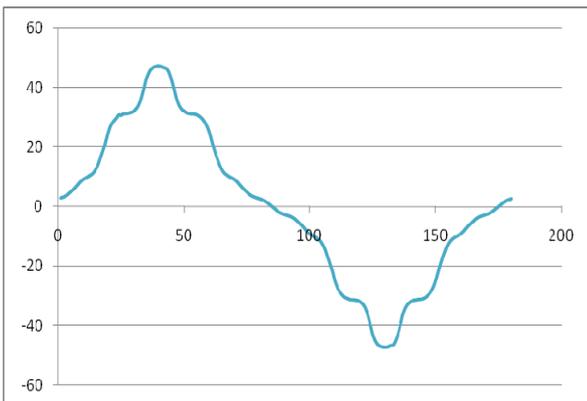


Figure 17 . Calculated Back emf using direct derivation. Machine 3.

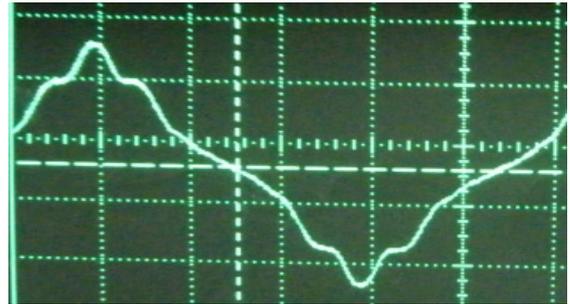


Figure 18 . Measured Back emf. Machine 3.

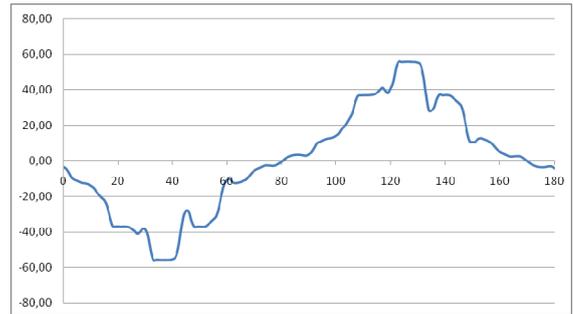


Figure 19. Back emf using BLv. Machine 3.

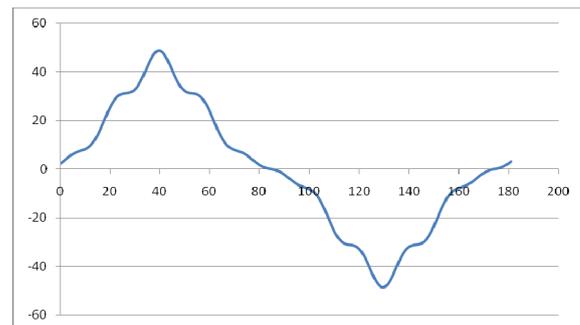


Figure 20. Back emf using Fourier approximation of flux. Machine 3.

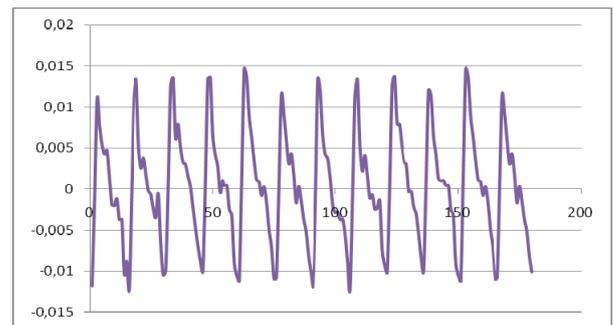


Figure 21. Cogging torque. Machine 3.

Table I summarizes these results for the three analyzed machines.

Table I.
Calculated results for machines 1, 2 and 3.

	Machine 1	Machine 2	Machine 3
Back EMF derivative (V)	123	116	47
Back EMF BLv (V)	120	120	55.7
Back EMF Fourier (V)	113	114	48.5
T_{cogging} (Nm)	0.31	0.12/0.03 (shift)	0.015
$N_p \cdot Q$	18	36	24

Maximum measured value is 46 V. Differences between measured and calculated values spans from 22% to 2.2%. See table 2 for details.

Table II.
Calculated and Measured values for machine 3.

	Machine 3	$\mathcal{E}(\%)$
Back EMF derivative (V)	47	2.17
Back EMF BLv (V)	55.7	21.1
Back EMF Fourier (V)	48.5	5.4
Measured value of back emf (V)	46	----

IV. CONCLUSIONS

- Some methods are explained to determine the back emf and cogging torque in an educational environment.
- BLv method is faster (only 1 or 2 simulation points) than other analyzed methods (they need more simulation points, at least 1 period)
- BLv approximation provides a lower precision in front of the precision obtained using the other analyzed methods
- How minimize the cogging torque using shift PM method is shown.
- In practical sessions several stator slot, rotor geometries and quality of permanent magnets are proposed. Students must investigate what combination produces better results attending to the cogging torque, back emf, etc.
- Students learn about complex concepts using FE analysis avoiding complicate handmade calculations.

V. REFERENCES

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