

Film Melting Heat Transfer from a Horizontal Open Surface

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In this internal report, consideration is given to the velocity of melting from a horizontal open surface and with particular reference to polar caps and large glaciers. In current models, the melting velocity is critically modulated by surface albedo as well as the rate of the advancing melting front, which makes that the melting velocity decrease inasmuch that melting take place. However, as will be discussed, when the surface is horizontal an open the rate of melting is not any longer controlled by the advancing melting front but rather by the formation of a micrometric liquid film which driven by capillarity. This mechanism of heat transfer which can be properly called as film-melting heat transfer is important for two reasons, namely. On one hand it predict a constant velocity (independent of the position of the front of melting), and on the other hand, the velocity of melting is at least one order of magnitude higher. The above mechanism could have important repercussion in application of the melting of polar caps and large glaciers if it is considered that during the melting process even initial uneven or sloping surfaces sooner or later become horizontally flat under the action of gravity if they are homogeneously heated and then an abrupt increase in the rate of melting can be expected. Understanding the processes and feedbacks involve during the entire process of melting is a prerequisite for accurately forecasting mass loss and the contribution to global sea level rise.

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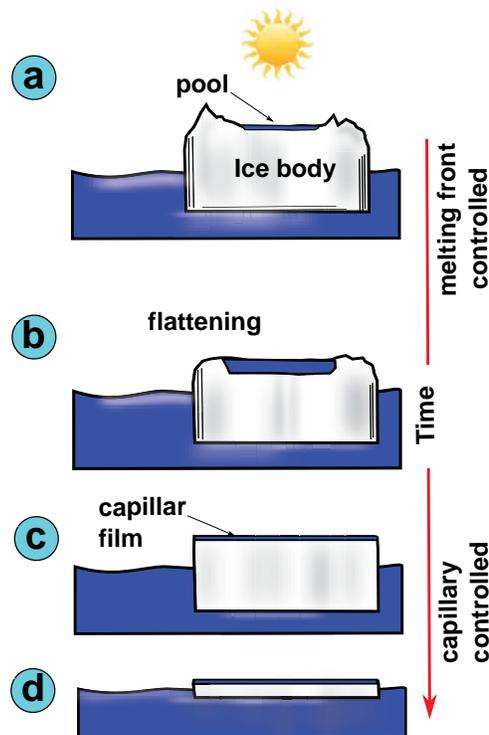


FIG. 1. Melting process form a horizontal uneven ice sheet. (a) An initially uneven ice body is heated from the top and the process is controlled by the thickens of the melted layer. (b) Inasmuch that melting progress, flattening is happening. (c) capillarity limits the thickness of the film layer. (d) melting is highly enhanced.

1. Introduction

The polar regions are important drivers of the worlds climate. When the "everlasting ice" melts at an increasing rate fresh water flows into the sea, changing ocean currents and the living conditions for marine organisms, hence, understanding the processes and feedbacks involve during the entire process of melting is a prerequisite for accurately forecasting mass loss and then forecasting the contribution to global sea level rise, (Planche C, et al, 2014), (Zhu H et al, 2017), (Hammond, M.D, et al 2016).

Recent studies across the world show that polar caps and large glaciers -including greenland, are losing ice much more rapidly than in past years. For example, accelerated ice melt has been reported from southwest Greenland, a part of the island that hadn't been known to be losing ice that rapidly. Previously, the scientific focus was on Greenland's southeast and northwest regions, where large glaciers stream iceberg-sized chunks of ice into the Atlantic ocean. In current models, melt is critically modulated by surface albedo as well as the rate of the advancing melting front, however, as it will be discussed, because the gravitational flattening of the surface during the melting process, capillarity starts to control the melting rate by the formation of a micrometric film which can enhance the rate of melting at least one order of magnitude. The study have important implications for the melting of polar caps in the future if it is considered that during the melting process, sooner or later an uneven or sloping ice surface and homogeneously heated becomes flat.

Melting rate from a horizontal open ice flat sheet

Consider a 1-dimensional semi-infinite and initially uneven surface of an ice -body upon melting. The body is heated from the top surface and extends to the bottom, as pictorially depicted in Fig. 1.

The most simple model (Stefan 1891) considers some simplifying assumptions which are reasonably acceptable in application of melting of large ice-bodies. In the Stefan problem, heat is stored as latent heat, sensible heat stored is negligible, heat transfer is only by conduction, there is no convection or, at least, it is negligible in comparison with conduction. The temperature at the surface is T_o and is kept constant at that level for any later time. On the other hand, the temperature at the melting front is the melting temperature T_m , and therefore the difference of temperature between the surface and the front of melting or superheat $\Delta T_s = T_o - T_m$ is kept constant inasmuch that the front of melting moves.

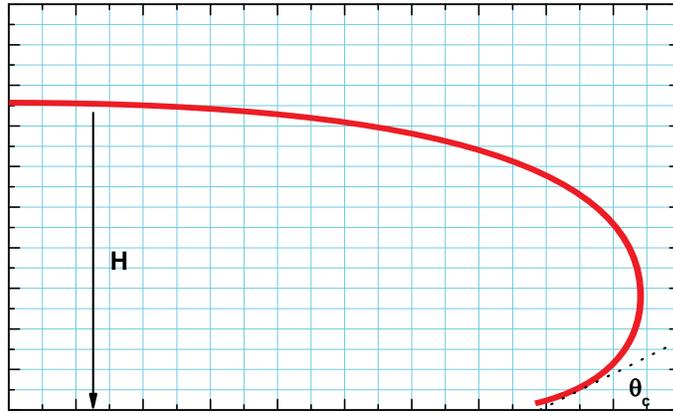


FIG. 2. Profile curve of the edge of a puddle where the contact angle is 180° , (Gennes PG, et al, 2004).

The heat released when the melting front is moving is equal to the heat that leaves at the surface, and then

$$\frac{dq(t)}{dt} = \Delta h \frac{ds}{dt} \quad (1)$$

where q is the heat flux per area; Δh the melting enthalpy; s is the distance from the surface to the front of melting, and t is time. On the other hand, because heat is only transferred by conduction, the temperature change from the location of the melting front at distance s from the surface follows the Fourier's law and therefore is linear. Then, the heat flux density at the surface as a function of the distance of the melting front s is given by

$$\frac{dq(t)}{dt} = \kappa_l \frac{\Delta T_s}{s} \quad (2)$$

where κ_l is the thermal conductivity of the liquid; $\Delta T_s = T_o - T_m$ is the superheat temperature.

Substituting Eq.(2) into Eq.(1) we obtain

$$\frac{ds}{dt} = \frac{\kappa_l \Delta T_s}{\Delta h s} \quad (3)$$

Nevertheless the preceding analysis is valid for confined surfaces in which it is assumed that the body is bounded and then the thickness of the liquid layer grows as the melting progress (See Fig. 1- (a) and (b)). This assumption is, of course, not allowable when the surface is open and flat in which case the maximum thickness of the liquid layer is limited by the formation of a liquid film. This film will have a thickness which is fixed by the action of water's strong surface tension. This *film-melting heat transfer* mechanism could be particularly important in application of the melting of polar caps and large glaciers if it is considered that during the melting process even initial uneven or sloping surfaces sooner or later become horizontally flat under the action of gravity and homogeneous heating (See Fig. 1- (c) and (d)) and then an abrupt increase in the rate of melting could be expected.

The specific profile curve of the film depends on the specific contact angle with the surface which for the water-ice system is $\approx 12 \pm 1^\circ$. For the sake of illustration, Fig. 2 shows the profile curve of the interface when the contact angle is 180° . The thickness of the film -far away from the edges, H , is given (Gennes PG, et al, 2004).

$$H = \sqrt{\frac{2\gamma(1 - \cos(\theta))}{\rho g}} \quad (4)$$

where H is the thickness of the liquid film layer, γ is the surface tension of the liquid; θ is the contact angle; g is the acceleration due to gravity ; and ρ is the density of the liquid. It is easy to see that, with typical values of the

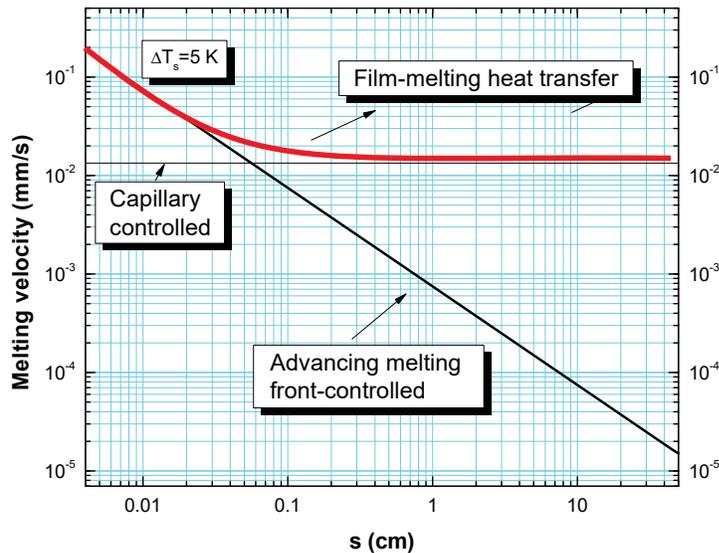


FIG. 3. Deviation of the velocity of melting because the formation of the liquid film from a horizontal open ice flat sheet.

parameter for water-ice system: $\gamma = 7 \times 10^{-2} \text{ N/m}$; $\rho = 10^3 \text{ kg/m}^3$; and $g = 9.8 \text{ m/s}^2$; $\approx 12 \pm 1^\circ$, the thickness of the liquid film controlling the heat transfer is around $\approx 600 \mu\text{m}$. Proceeding as before, the heat flux density is now given by

$$\frac{dq(t)}{dt} = \kappa_l \frac{\Delta T_s}{H} \quad (5)$$

which inserted into Eq.(1) gives the melting velocity

$$\frac{ds}{dt} = \frac{\kappa_l \Delta T_s}{\Delta h H} \quad (6)$$

or inserting Eq.(4) yields

$$\frac{ds}{dt} = \frac{\kappa_l \Delta T_s}{\Delta h} \sqrt{\frac{\rho g}{2\gamma(1 - \cos(\theta))}} \quad (7)$$

• Discussion

In order to obtain some comparative idea of the velocity of melting predicted by Eq.(3) and Eq.(7), we assume some typical values of the parameters: $\kappa_l = 0.5 \text{ W/(mK)}$; $\Delta h = 3.34 \times 10^8 \text{ J/m}^3$; $\gamma = 7 \times 10^{-2} \text{ N/m}$; $\theta \approx 12^\circ$; $\rho = 10^3 \text{ kg/m}^3$; $g = 9.8 \text{ m/s}^2$. The resulting curves are shown in Fig. 3 for a superheat $\Delta T_s = 5 \text{ K}$, where it was considered an initially flat sheet.

Referring to Fig. 3 it is seen, that once the surface is becoming flat, the formation of the liquid film $\approx 600 \mu\text{m}$ is controlling the velocity of melting, and approach a constant value independent of the time or amount of melting.

Summary of results and conclusions

In this note, consideration was given to the velocity of melting from a horizontal open ice flat sheet and with particular reference to polar caps and glaciers. It was shown that once the ice sheet becomes flat, the melting velocity is largely enhanced up to one order of magnitude higher by the formation of a thin micrometric liquid film controlled by capillarity. In addition, the velocity of melting becomes constant. The above result is important to be considered because predicts that the rate of melting of polar caps al large glaciers could be substantially accelerated in the future if it is considered that even initial uneven or sloppy ice surfaces sooner or later become flat under the action of gravity if they are homogeneously heated.

4. Nomenclature

Δh = enthalpy of melting
 ΔT_s = superheat temperature
 g = gravity
 H = liquid film thickness
 q = heat flux
 s = distance from the melting front to surface
 t = time

Greek symbols

κ_l = thermal conductivity of the liquid
 θ = contact angle
 γ = surface tension

Subscripts symbols

l = liquid
 m = melting
 s = superheat

5. Acknowledgements

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6. References

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