**Título / Title**

# ADC QUANTIZATION REQUIREMENTS.

# (ISSUE 2)


**Abstract**: In this document, we analyze the impact of the quantization at the input of the demultiplexer. The required number of bits of the Analog-to-Digital-Converter (ADC) is obtained, along with the corresponding quantization losses.

With respect to issue 1 (document adcqr/26/06/00/1), issue 2 analyzes the impact of the following aspects:

1) Increasing the dynamic per carrier from 12dB to 15dB.
2) Use of Reed Solomon codes along with turbo codes (with the same specified dynamics).
3) Use of 8-PSK (the Es/No losses of 3.5dB of 8-PSK with respect to QPSK were not considered in issue 1).



**Authors**: J. Riba, M. Lamarca, F. Rey, J. Sala, G. Vàzquez, X. Villares

## LIST OF ABBREVIATIONS, ACRONYMS AND PARAMETERS

| | |
|---|---|
| $E_s$ | Energy per channel symbol |
| $N_o$ | Noise spectral density |
| $At$ | Maximum attenuation per carrier |
| $B$ | Number ADC quantization bits |
| $X_{max}$ | Clipping level at the ADC input |
| $N$ | Sampling-frequency to symbol-rate ratio |
| $Q$ | Maximum number of carriers |
| $E_{s\,max} / N_o$ | Maximum $E_s / N_o$ per carrier |
| $\sigma_{qg}^2$ | Granular quantization noise variance |
| $\sigma_{qc}^2$ | Clipping quantization noise variance |
| $erfc(x)$ | Complementary error function |
| $\gamma$ | Clipping-to-granular quantization noise variance |
| $f_s$ | Sampling frequency |
| $T_s$ | Mean time of channel symbol |

| | |
|---|---|
| ADC | Analog-to-Digital-Converter |
| LSB | Least significant bit |
| MSB | Most significant bit |

## SPECIFICATIONS

| | |
|---|---|
| Modulation | QPSK and 8-PSK |
| Symbol rates | $NR_s$, with n = 1,2,4,8 |
| Number of carriers | Q=36 |
| Tree option | Tree 5222 (document [psotdd/1]) |
| Sampling frequency | N Rs (N=120) (document [psotdd/1]) |
| Minimum Eb/No Turbo Code 6/7 | 7 dB |
| Minimum Es/No Turbo Code 6/7 | **9.3 dB** |
| Minimum Eb/No Turbo Code 4/5 | 6 dB |
| Minimum Es/No Turbo Code 4/5 | **8 dB** |
| Minimum Eb/No Reed Solomon | 9.4 dB |
| Minimum Es/No Reed Solomon | **12 dB** |
| Input dynamic per carrier | **15 dB** |

## MAIN CONCLUSSION OF THE STUDY

1) Scenario A. Maximum Es/No per carrier=9.3+15 = **24.3 dB**

| **Codes** | **Modulation** |
|---|---|
| Turbo | QPSK |

| ADC quantization bits (B) | Normalized total input power to the ADC (dB) | Quantization losses (dB) |
|---|---|---|
| **8** | -12.5 | <0.035 |
| **9** | -13 | <0.01 |
| **10** | -13.5 | <0.003 |

2) Scenario B. Maximum Es/No per carrier =12+15 = **27 dB**

| **Codes** | **Modulation** |
|---|---|
| Turbo | QPSK |
| Reed Solomon | |

| ADC quantization bits (B) | Normalized total input power to the ADC (dB) | Quantization losses (dB) |
|---|---|---|
| **8** | -12.5 | <0.06 |

| | | |
|---|---|---|
| **9** | -13 | <0.02 |
| **10** | -13.5 | <0.005 |

3) Scenario C. Maximum Es/No per carrier =(12+15)+3.5[#1] +1.8[#2] = 32.3 dB

[#1] 3.5dB accounts for the Eb/No losses of the 8-PSK with respect QPSK.
[#2] 1.8dB (=10*log(3/2)) accounts for the Es/No increment of the 8-PSK with respect QPSK.

| **Codes** | **Modulation** |
|---|---|
| Turbo | QPSK |
| Reed Solomon | 8-PSK |

| ADC quantization bits (B) | Normalized total input power to the ADC (dB) | Quantization losses (dB) |
|---|---|---|
| **8** | -12.5 | **<0.2** |
| **9** | -13 | <0.06 |
| **10** | -13.5 | <0.02 |

## QUANTIZATION LOSSES

### Granular quantization noise variance

The normalized granular noise variance is given by:

$$\overline{\sigma}_{qg}^{2} = \sigma_{qg}^{2} / X_{max}^{2} = \frac{2^{-2B}}{3}$$

### Clipping quantization noise variance

The normalized clipping noise variance depends on the ADC input histogram f(x) as follows:

$$\overline{\sigma}_{qc}^{2} = \sigma_{qc}^{2} / X_{max}^{2} = \frac{2}{X_{max}^{2}} \int_{X_{max}}^{\infty} \left(x - X_{max}\right)^{2} f(x) dx$$

The worst case scenario corresponds to the case of Q active carriers at the minimum symbol rate. In this situation we have the longest histogram tails. As an upperbound we take the limiting case of a Gaussian histogram, motivated by the central limit theorem. Then, the normalized clipping noise variance is:

$$\overline{\sigma}_{qc}^{2} = \left(\overline{P} + 1\right) erfc\left(\frac{1}{\sqrt{2\overline{P}}}\right) - \sqrt{\frac{2\overline{P}}{\pi}} e^{-\frac{1}{2\overline{P}}}$$

where:

$$\overline{P} = P / X_{max}^{2}$$

$$erfc(x) = \int_x^{\infty} e^{-t^2/2} dt$$

is the normalized total input power.

**Total quantization noise variance**

The total normalized quantization noise variance is then a function of $\overline{P}$ and $B$.

$$\overline{\sigma}_q^2 = \overline{\sigma}_{qg}^2 + \overline{\sigma}_{qc}^2 = \frac{2^{-2B}}{3} + \left(\overline{P}+1\right) erfc\left(\frac{1}{\sqrt{2\overline{P}}}\right) - \sqrt{\frac{2\overline{P}}{\pi}} e^{-\frac{1}{2\overline{P}}}$$

Figure 1 shows the normalized quantization noise variance as a function of $\overline{P}$ for different number of bits, $B$.

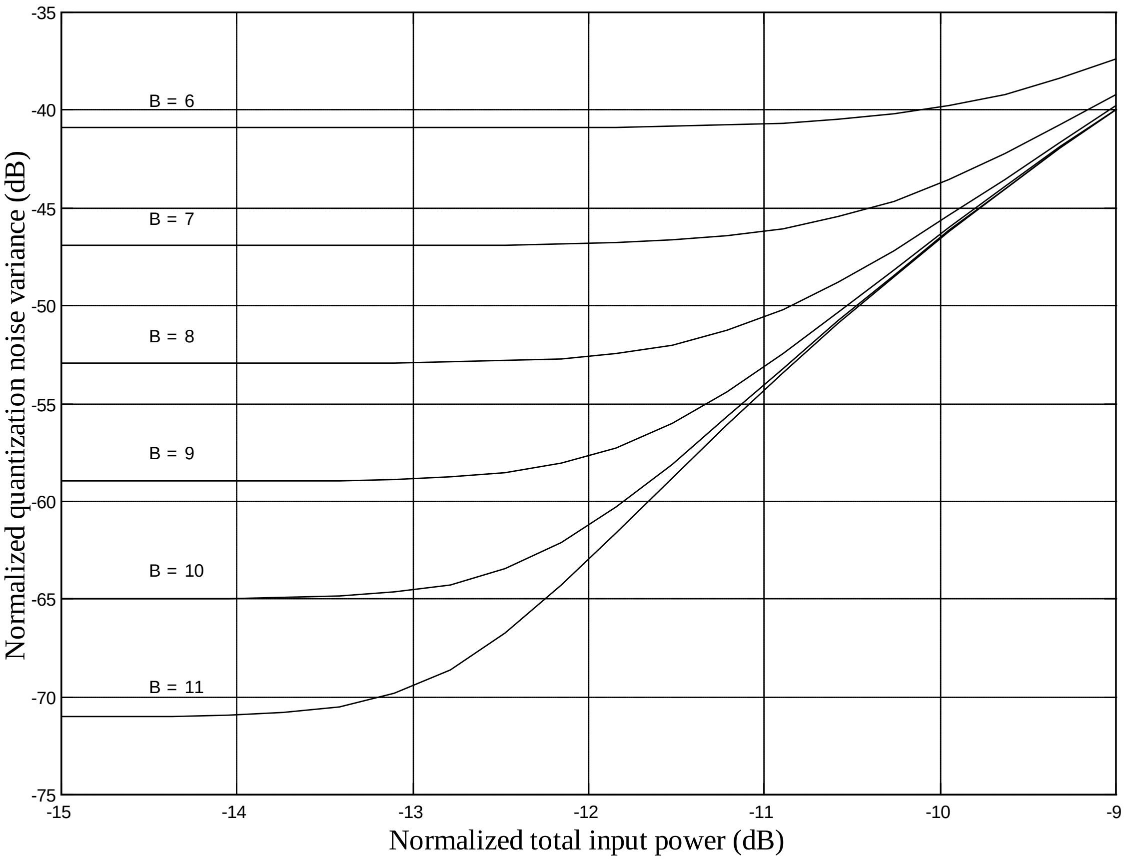


Figure 1. Normalized (to $X_{max}^2$) quantization noise variance as a function of the normalized (to $X_{max}^2$) total (signal plus noise) input ADC power.

**Additive noise variance**

Assuming an ideal pass-band Nyquist filter of bandwidth fs, the additive noise variance is:

$$\sigma_n^2 = N_o f_s = N_o \frac{N}{T_s} = N_o \frac{N}{\frac{QE_{s\,max}}{P-\sigma_n^2}} = \frac{P-\sigma_n^2}{\frac{Q}{N} E_{s\,max} / N_o}$$

Isolating $\sigma_n^2$ yields:

$$\overline{\sigma}_n^2 = \sigma_n^2 / X_{max}^2 = \frac{\overline{P}}{\frac{Q}{N} E_{s\,max} / N_o + 1}$$

**Quantization losses**

Assuming that the granular and clipping quantization noises are white, the quantization losses can be expressed as follows:

$$L_q = 10 \log\left(1 + \frac{\overline{\sigma}_q^2}{\overline{\sigma}_n^2}\right)$$

We have considered the following three scenarios:

| Scenario indentifier | A | B | C |
|---|---|---|---|
| Description | Only turbo codes (only QPSK) | Turbo codes and Reed Solomon (only QPSK) | Turbo codes and Reed Solomon (QPSK and 8-PSK) |
| Maximum Es/No | 9.3+15 = 24.3 dB | 12+15 = 27 dB | 27 + 3.5[#1] + 1.8[#2] = 32.3 dB |

[#1] 3.5dB accounts for the Eb/No losses of the 8-PSK with respect QPSK.
[#2] 1.8dB (=10*log(3/2)) accounts for the Es/No increment of the 8-PSK with respect QPSK.

Figures 2a, 2b and 2c show the quantization losses as a function of the total normalized input power for different number of ADC quantization bits (B=8,9 and 10).

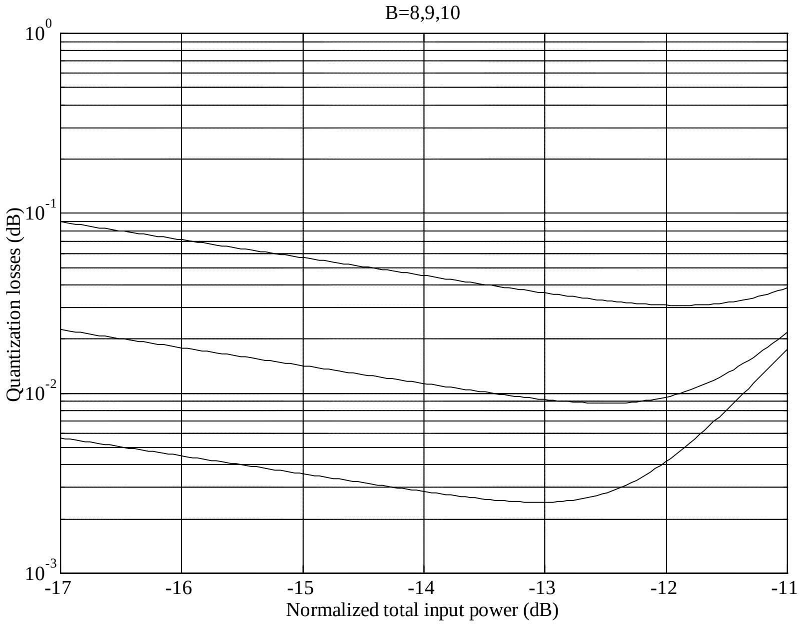


Figure 2a. Scenario A. Quantization losses as a function of the normalized (to $X^2_{max}$) total (signal plus noise) input ADC power. Number of quantization bits: B=8,9,10.

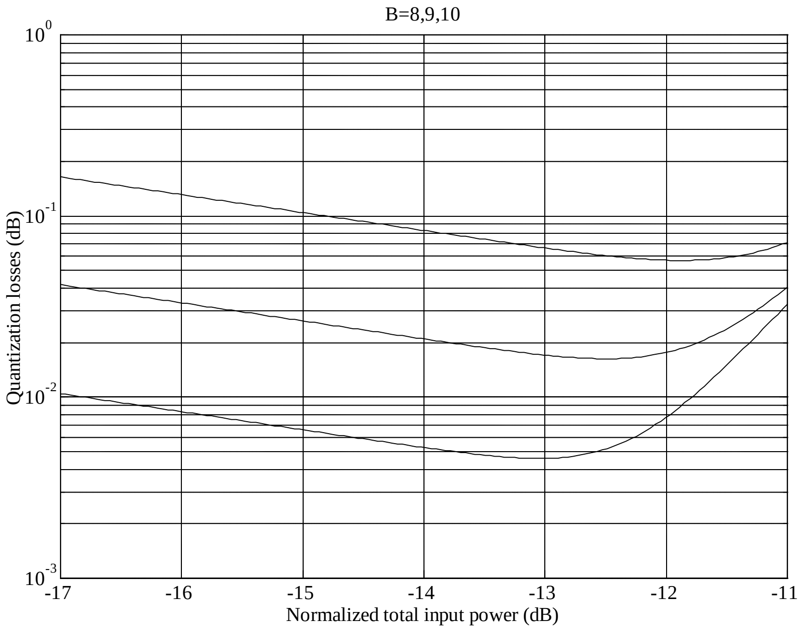


Figure 2b. Scenario B. Quantization losses as a function of the normalized (to $X^2_{max}$) total (signal plus noise) input ADC power. Number of quantization bits: B=8,9,10.

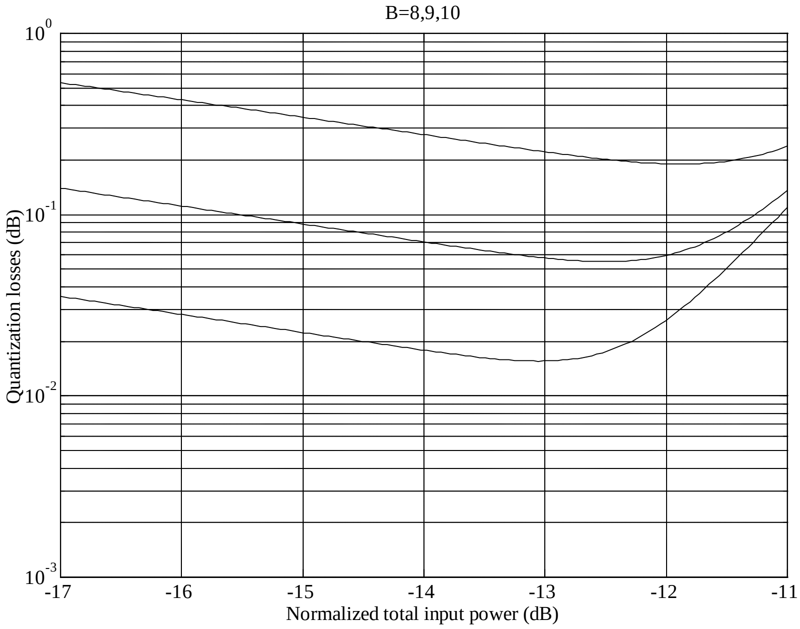


Figure 2c. Scenario C. Quantization losses as a function of the normalized (to $X^2_{max}$ ) total (signal plus noise) input ADC power. Number of quantization bits: B=8,9,10.

The total input power should be fixed to yield minimum quantization losses. However, as the curves are more flat at the left of the minimum, it is recommended to fix a input power slightly lower than the optimal point.

Finally, figure 3 shows the quantization losses for B=8,9,10, as a function of the maximum Es/No per carrier. These losses correspond to the minimum losses obtained at the optimal normalized total ADC input power.

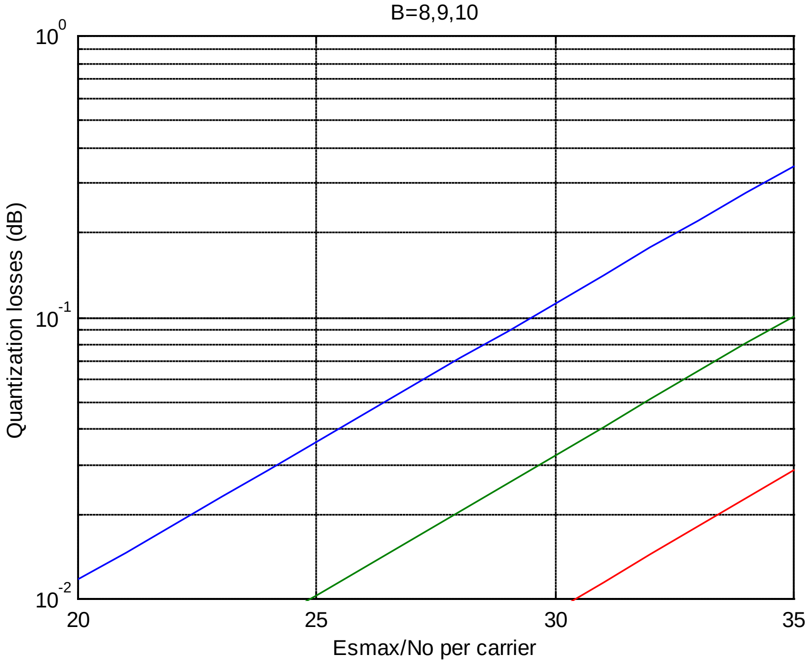


Figure 3. Minimum quantization losses as a function of the maximum Es/No per carrier. Number of quantization bits: B=8,9,10.

## REFERENCES

[psotdd/1] J. Sala, M. Lamarca, F. Rey, J. Riba, G. Vàzquez, X. Villares, "PRELIMINAR STUDY OF THE DVB-F DEMULTIPLEXER (DRAFT)", UPC/TSC/GPS/psotdd/1, 23/05/00, 11 pages.

[adcqr/26/06/00/1] J. Riba, M. Lamarca, F. Rey, J. Sala, G. Vàzquez, X. Villares "ADC QUANTIZATION REQUIREMENTS", UPC/TSC/GPS/adcqr/26/06/00/1, 26/06/00, 8 pages.