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## **GEOPLEX: Back-to-back converter for an electrical machine application**

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# A back-to-back converter for an electrical machine application.

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## Abstract

The architecture of a back-to-back converter, needed to feed the rotor windings of a 3-phase doubly-fed induction machine and such that power can flow both ways, is described, from both the modelling and experimental viewpoints. In particular, a PCHS model using selected GSSA variables is described in detail. Some new and improved results concerning the detailed PCHS structure of a GSSA expansion of a PCHS are also presented.

## 1 Introduction

A back-to-back converter is needed for a control of a doubly-fed induction machine (an induction machine fed from both the rotor and the stator) because in some operation ranges the rotor energy may come back to the converter [1]. A back-to-back converter has the feature that the power can flow to any direction. Figure 1 shows a back-to-back converter made of a full bridge AC/DC monophasic boost-like rectifier and a 3-phase DC/AC inverter.

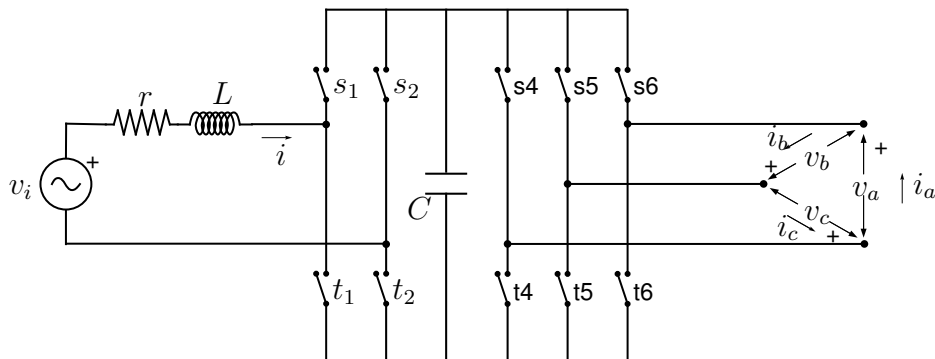


Figure 1: Back-to-back converter.

The whole converter has an AC monophasic voltage input and its output are 3-phase PWM voltages which feed the rotor windings of an electrical machine. This system can be split into two parts: a dynamical subsystem (the full bridge rectifier, containing the storage elements), and an static subsystem (the inverter, acting like a transformer from the energy point of view).

## 2 Port-controlled Hamiltonian model

Port-controlled Hamiltonian Systems (PCHS) describe, from an energetic point of view, a large kind of systems [3]. An explicit PCHS takes the form

$$\begin{aligned}\dot{x} &= (\mathcal{J} - \mathcal{R})\nabla H + gu \\ y &= g^T \nabla H\end{aligned}\quad (1)$$

where  $x$  is the state (in hamiltonian variables),  $H(x)$  is the hamiltonian function (representing its energy)<sup>1</sup>,  $\mathcal{J}(x) = -\mathcal{J}^T(x)$  is the interconnection or structure matrix,  $\mathcal{R}(x) = \mathcal{R}^T(x) \geq 0$  is the dissipation matrix and  $u$  and  $y$  are the port variables connected with the dynamical system through the  $g(x)$  matrix. It can be seen that PCHS are passive, with port variables  $(u, y)$  and storage function  $H$ .

### 2.1 PCHS model of a full bridge AC/DC monophasic boost rectifier

Figure 2 shows a full bridge AC/DC monophasic boost rectifier, where  $v_i = v_i(t) = E \sin(\omega_s t)$  is a monophasic AC voltage source,  $L$  is the inductance (including the effect of any transformer in the source),  $C$  is the capacitor of the DC part,  $r$  takes into account all the resistance losses (inductor, source and switches), and  $v_l = v_l(t)$  is the DC voltage of the load/output port.  $s_1, s_2, t_1$  and  $t_2$  are the switch states (with  $t_1 = \bar{s}_1, t_2 = \bar{s}_2, s_2 = \bar{s}_1$ ) defined by  $S = S(t) \in \{1, -1\}$ .

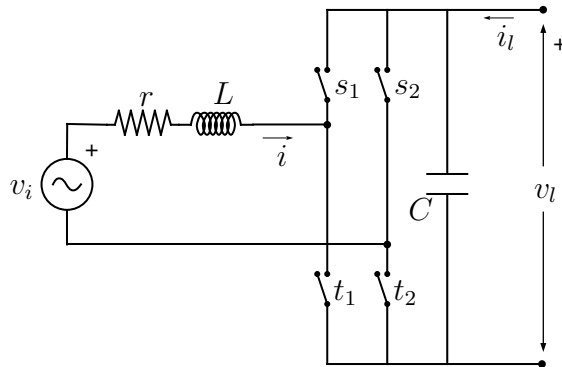


Figure 2: A full-bridge AC/DC monophasic boost rectifier.

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<sup>1</sup> $\nabla$  operator represents the gradient of the function. For simplify the notation, these gradients are taken as a column vectors throughout this paper.

We consider two ports, the first one in the AC part and the second one in the DC part, both positive in the incoming power direction. The system equations are

$$\begin{aligned}\dot{\lambda} &= -\frac{S}{C}q - \frac{r}{L}\lambda + v_i \\ \dot{q} &= \frac{S}{L}\lambda + i_i,\end{aligned}\tag{2}$$

where  $\lambda = \lambda(t) = Li$  is the inductor linking flux,  $q = q(t) = Cv_i$  is the charge in the capacitor and  $i_i$  is the current required by the output port.

The control objectives of this subsystem are

- the DC value of  $v_i$  voltage should be equal to a desired constant  $v_i^*$ , and
- the power factor of the converter should be equal to one. This means that the inductor current  $i = Li_d \sin(\omega_s t)$ , where  $i_d$  is an appropriate value to achieve the first objective via energy balance.

Using the equation (2), the system can be expressed as a port controlled hamiltonian system (1), with the inductor flux ( $\lambda$ ) and the capacitor charge ( $q$ ) as hamiltonian variables:

$$x^T = (\lambda, q),$$

with Hamiltonian function

$$H(\lambda, q) = \frac{1}{2L}\lambda^2 + \frac{1}{2C}q^2,$$

and with structure and dissipation matrices

$$\mathcal{J} = \begin{pmatrix} 0 & -S \\ S & 0 \end{pmatrix} \quad \mathcal{R} = \begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix},$$

and the port connections

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad u = \begin{pmatrix} v_i \\ i_i \end{pmatrix}.$$

## 2.2 Dirac structure of a DC/AC 3-phase inverter

A DC/AC 3-phase inverter is a static system which, from a DC voltage source (typically a capacitor with high capacitance) extracts a 3-phase PWM voltage. Figure 3 shows an inverter scheme, where  $v_{DC} \in \mathbb{R}$  is the DC voltage,  $i_{DC} \in \mathbb{R}$  is the DC current,  $i_a, i_b$  and  $i_c$  are the 3-phase currents and  $v_a, v_b$  and  $v_c$  are the 3-phase voltages (phase to phase).  $s_4, s_5, s_6 \in \{0, 1\}$ ,  $t_4, t_5$  and  $t_6$  are switches where  $t_n = \bar{s}_n$ . This system can be seen as a Dirac Structure [3], which transforms from the DC variables to the  $abc$  variables. The equations of this system are

$$\begin{aligned}v_{abc} &= f v_{DC} \\ i_{DC} &= -f^T i_{abc}\end{aligned}\tag{3}$$

where  $v_{abc}^T = (v_a, v_b, v_c) \in \mathbb{R}^3$ ,  $i_{abc}^T = (i_a, i_b, i_c) \in \mathbb{R}^3$  and

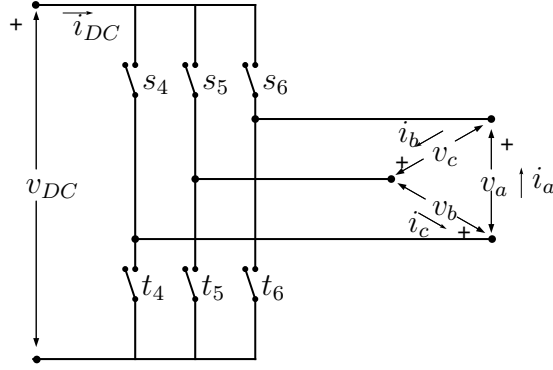


Figure 3: DC/AC 3-phase inverter.

$$f = \begin{pmatrix} S_6 - S_4 \\ S_5 - S_6 \\ S_4 - S_5 \end{pmatrix}.$$

Notice that both ports are taken with the incoming positive power convention. It is easy to show that this system is power preservative. Indeed, using the power definition ( $P = vi^T$ ) and (3)

$$v_{DC}i_{DC} = -v_{DC}i_{abc}^T f = -f v_{DC} i_{abc}^T = -v_{abc} i_{abc}^T.$$

### 2.3 PCHS of the whole system

Using the interconnection rules a PCHS of the whole system can be obtained. The following equations describes the connection between the two parts

$$\begin{aligned} v_l &= v_{DC} \\ i_l &= -i_{DC}. \end{aligned} \tag{4}$$

Figures 4 show the port representation of the boost converter (a), the inverter (b) and the connection (c).

Connecting the two systems according to (4), the whole system yields again an explicit PCHS with the Hamiltonian variables

$$x^T = ( \lambda \quad q ),$$

the Hamiltonian function

$$H = \frac{1}{2L}\lambda^2 + \frac{1}{2C}q^2,$$

the structure and dissipation matrices

$$\mathcal{J} = \begin{pmatrix} 0 & -S \\ S & 0 \end{pmatrix} \quad \mathcal{R} = \begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix},$$

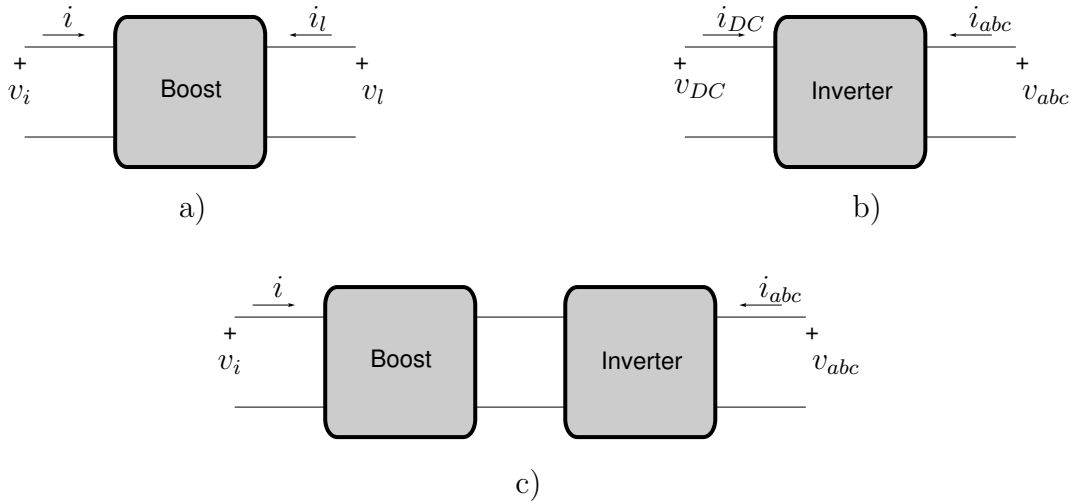


Figure 4: Interconnection block scheme.

which are the same that those of the full-bridge rectifier (it is the only dynamical part) and

$$g = \begin{pmatrix} 1 & O_{1 \times 3} \\ 0 & f^T \end{pmatrix} \in \mathbb{R}^{2 \times 4}, \quad u = \begin{pmatrix} v_i \\ i_{abc} \end{pmatrix} \in \mathbb{R}^4,$$

which describes the effect of the inverter in the system.

### 3 Generalized state space averaging model

This section presents the GSSA model of the system. The GSSA formalism allows reduce a tracking problem to a regulation one. In this method, the state and control variables are expanded in a Fourier-like series with time-dependent coefficients (for periodic behavior, the coefficients will evolve to constants). GSSA, with a fixed period, can be applied to the rectifier subsystem using the grid frequency, but can't be used for the inverter subsystem which works under a large rang of frequencies (depending on the desired speed of the electrical machine and PWM frequency).

#### 3.1 Generalized averaging for port controlled hamiltonian systems

This subsection presents results which combine the PCHS and GSSA formalisms. Detailed presentations can be found in [2], [5], [8] and [9] for GSSA. In this subsection we present some elementary results relating both.

Assume a Variable Structure System (VSS) such that the change in the state variables is small over the time length of an structure change, or such that one is not interested about the fine details of the variation. Then one may try to formulate a dynamical system for the time average of the state variables

$$\langle x \rangle(t) = \frac{1}{T} \int_{t-T}^t x(\tau) d\tau, \quad (5)$$

where  $T$  is the period, assumed constant, of a cycle of structure variations.

Let our VSS system be described in explicit port hamiltonian form

$$\dot{x} = [\mathcal{J}(S, x) - \mathcal{R}(S, x)] \nabla H(x) + g(S, x)u, \quad (6)$$

where  $S$  is a (multi)-index, with values on a finite, discrete set, enumerating the different structure topologies. For notational simplicity, we will assume in this Section that we have a single index (corresponding to a single switch, or a set of switches with a single degree of freedom) and that  $S \in \{0, 1\}$ . Hence, we have two possible dynamics, which we denote as

$$\begin{aligned} S = 0 &\Rightarrow \dot{x} = (\mathcal{J}_0(x) - \mathcal{R}_0(x)) \nabla H(x) + g_0(x)u, \\ S = 1 &\Rightarrow \dot{x} = (\mathcal{J}_1(x) - \mathcal{R}_1(x)) \nabla H(x) + g_1(x)u. \end{aligned} \quad (7)$$

Note that controlling the system means choosing the value of  $S$  as a function of the state variables, and that  $u$  is, in most cases, just a constant external input.

From (5) we have

$$\frac{d}{dt} \langle x \rangle(t) = \frac{x(t) - x(t - T)}{T}. \quad (8)$$

Now the central assumption of the SSA approximation method is that for a given structure we can substitute  $x(t)$  by  $\langle x \rangle(t)$  in the right-hand side of the dynamical equations, so that (7) become

$$\begin{aligned} S = 0 &\Rightarrow \dot{x} \approx (\mathcal{J}_0(\langle x \rangle) - \mathcal{R}_0(\langle x \rangle)) \nabla H(\langle x \rangle) + g_0(\langle x \rangle)u, \\ S = 1 &\Rightarrow \dot{x} \approx (\mathcal{J}_1(\langle x \rangle) - \mathcal{R}_1(\langle x \rangle)) \nabla H(\langle x \rangle) + g_1(\langle x \rangle)u. \end{aligned} \quad (9)$$

The rationale behind this approximation is that  $\langle x \rangle$  does not have time to change too much during a cycle of structure changes. We assume also that the length of time in a given cycle when the system is in a given topology is determined by a function of the state variables or, in our approximation, a function of the averages,  $t_0(\langle x \rangle)$ ,  $t_1(\langle x \rangle)$ , with  $t_0 + t_1 = T$ . Since we are considering the right-hand sides in (9) constant over the time scale of  $T$ , we can integrate the equations to get<sup>2</sup>

$$\begin{aligned} x(t) &= x(t - T) + t_0(\langle x \rangle)[(\mathcal{J}_0(\langle x \rangle) - \mathcal{R}_0(\langle x \rangle)) \nabla H(\langle x \rangle) + g_0(\langle x \rangle)u] \\ &\quad + t_1(\langle x \rangle)[(\mathcal{J}_1(\langle x \rangle) - \mathcal{R}_1(\langle x \rangle)) \nabla H(\langle x \rangle) + g_1(\langle x \rangle)u]. \end{aligned}$$

Using (8) we get the SSA equations for the variable  $\langle x \rangle$ :

$$\begin{aligned} \frac{d}{dt} \langle x \rangle &= d_0(\langle x \rangle)[(\mathcal{J}_0(\langle x \rangle) - \mathcal{R}_0(\langle x \rangle)) \nabla H(\langle x \rangle) + g_0(\langle x \rangle)u] \\ &\quad + d_1(\langle x \rangle)[(\mathcal{J}_1(\langle x \rangle) - \mathcal{R}_1(\langle x \rangle)) \nabla H(\langle x \rangle) + g_1(\langle x \rangle)u], \end{aligned} \quad (10)$$

where

$$d_{0,1}(\langle x \rangle) = \frac{t_{0,1}(\langle x \rangle)}{T},$$

with  $d_0 + d_1 = 1$ . In the power converter literature  $d_1$  (or  $d_0$ , depending on the switch configuration) is referred to as the duty cycle.

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<sup>2</sup>We also assume that  $u$  varies slowly over this time scale; in fact  $u$  is constant in many applications.

One can expect the SSA approximation to give poor results, as compared with the exact VSS model, for cases where  $T$  is not small with respect to the time scale of the changes of the state variables that we want to take into account. The GSSA approximation tries to solve this, and capture the fine detail of the state evolution, by considering a full Fourier series, and eventually truncating it, instead of just the “dc” term which appears in (5). Thus, one defines

$$\langle x \rangle_k(t) = \frac{1}{T} \int_{t-T}^t x(\tau) e^{-jk\omega\tau} d\tau, \quad (11)$$

with  $\omega = 2\pi/T$  and  $k \in \mathbb{Z}$ . The time functions  $\langle x \rangle_k$  are known as index- $k$  averages or  $k$ -phasors. Notice that  $\langle x \rangle_0$  is just  $\langle x \rangle$ .

Under standard assumptions about  $x(t)$ , one gets, for  $\tau \in [t-T, t]$  with  $t$  fixed,

$$x(\tau) = \sum_{k=-\infty}^{+\infty} \langle x \rangle_k(t) e^{jk\omega\tau}. \quad (12)$$

If the  $\langle x \rangle_k(t)$  are computed with (11) for a given  $t$ , then (12) just reproduces  $x(\tau)$  periodically outside  $[t-T, t]$ , so it does not yield  $x$  outside of  $[t-T, t]$  if  $x$  is not  $T$ -periodic. However, the idea of GSSA is to let  $t$  vary in (11) so that we really have a kind of “moving” Fourier series:

$$x(\tau) = \sum_{k=-\infty}^{+\infty} \langle x \rangle_k(t) e^{jk\omega\tau}, \quad \forall \tau.$$

A more mathematically advanced discussion is presented in [9].

In order to obtain a dynamical GSSA model we need the following two essential properties:

$$\frac{d}{dt} \langle x \rangle_k(t) = \left\langle \frac{dx}{dt} \right\rangle_k(t) - jk\omega \langle x \rangle_k(t), \quad (13)$$

$$\langle xy \rangle_k = \sum_{l=-\infty}^{+\infty} \langle x \rangle_{k-l} \langle y \rangle_l. \quad (14)$$

Using (13) and (6) one gets

$$\begin{aligned} \frac{d}{dt} \langle x \rangle_k &= \left\langle \frac{dx}{dt} \right\rangle_k - jk\omega \langle x \rangle_k \\ &= \langle [\mathcal{J}(S, x) - \mathcal{R}(S, x)] \nabla H(x) + g(S, x)u \rangle_k - jk\omega \langle x \rangle_k. \end{aligned} \quad (15)$$

Assuming that the structure matrices  $\mathcal{J}$  and  $\mathcal{R}$ , the hamiltonian  $H$ , and the interconnection matrix  $g$  have a series expansion in their variables, the convolution formula (14) can be used and an (infinite) dimensional system for the  $\langle x \rangle_k$  can be obtained. Notice that, if we restrict ourselves to the dc terms (and without taking into consideration the contributions of the higher order harmonics to the dc averages), then (15) boils down to (10) since, under these assumptions, the zero-order average of a product is the product of the zero-order averages.



Notice that  $\langle x \rangle_k$  is in general complex and that, if  $x$  is real,

$$\langle x \rangle_{-k} = \overline{\langle x \rangle_k}.$$

We will use the notation  $\langle x \rangle_k = x_k^R + jx_k^I$ , where the averaging notation has been suppressed. In terms of these real and imaginary parts, the convolution property (14) becomes (notice that  $x_0^I = 0$  for  $x$  real)

$$\begin{aligned} \langle xy \rangle_k^R &= x_k^R y_0^R + \sum_{l=1}^{\infty} \{ (x_{k-l}^R + x_{k+l}^R) y_l^R - (x_{k-l}^I - x_{k+l}^I) y_l^I \} \\ \langle xy \rangle_k^I &= x_k^I y_0^R + \sum_{l=1}^{\infty} \{ (x_{k-l}^I + x_{k+l}^I) y_l^R + (x_{k-l}^R - x_{k+l}^R) y_l^I \} \end{aligned} \quad (16)$$

**Proposition 1** *Let  $\Sigma$  be the PCH system defined by*

$$\dot{x} = (A(x, S)) \nabla H + f(x, S)$$

where  $A(x, S) = J(x, S) - R(x, S)$  and  $f(x, S) = g(x, S)u$ ,  $x \in \mathbb{R}^n$ ,  $S \in \mathbb{R}^m$ ,  $u \in \mathbb{R}^p$  is a constant input and  $H \in C^\infty(\mathbb{R}^n, \mathbb{R})$  is a hamiltonian function. Let  $\Sigma_{PH}$  be the phasor system associated to  $\Sigma$ :

$$\frac{d}{dt} \langle x \rangle_k = -jk\omega \langle x \rangle_k + \langle A \nabla H \rangle_k + \langle f \rangle_k, \quad k \in \mathbb{Z} \quad (17)$$

Let  $\xi \equiv \langle \nabla H \rangle$ . Assume that there exists a phasor hamiltonian  $H_{PH}(x_0, x_k^R, x_k^I)$  such that

$$2 \frac{\partial H_{PH}}{\partial x_0} = \xi_0, \quad \frac{\partial H_{PH}}{\partial x_k^R} = \xi_k^R, \quad \frac{\partial H_{PH}}{\partial x_k^I} = \xi_k^I, \quad k > 0, \quad (18)$$

and symmetric matrices  $F_k$  for each  $k > 0$  such that

$$F_k \frac{\partial H_{PH}}{\partial x_k^R} = x_k^R, \quad F_k \frac{\partial H_{PH}}{\partial x_k^I} = x_k^I.$$

Then the phasor system can be written as an infinite dimensional hamiltonian system

$$\frac{d}{dt} \langle x \rangle = A_{PH} \nabla H_{PH} + f_{PH}$$

with  $A_{PH} = J_{PH} - R_{PH}$  for some matrices  $J_{PH}$  skew-symmetric and  $R_{PH}$  symmetric and satisfying  $(\nabla H_{PH})^T R_{PH} \nabla H_{PH} \geq 0$ .

**Proof.** Splitting the phasors into real and imaginary parts, ordering the terms as

$$\langle x \rangle = (x_0, x_1^R, x_1^I, x_2^R, x_2^I, \dots)$$

and using (15) and (16), it is immediat to obtain

$$\begin{aligned} \dot{x}_0 &= \tilde{A}_{00} \partial_{x_0} H_{PH} + \sum_{l=1}^{\infty} \tilde{A}_{0l} \begin{pmatrix} \partial_{x_l^R} H_{PH} \\ \partial_{x_l^I} H_{PH} \end{pmatrix} + f_0, \\ \frac{\partial}{\partial t} \begin{pmatrix} x_k^R \\ x_k^I \end{pmatrix} &= \tilde{A}_{k0} \partial_{x_0} H_{PH} + \sum_{l=1}^{\infty} \tilde{A}_{kl} \begin{pmatrix} \partial_{x_l^R} H_{PH} \\ \partial_{x_l^I} H_{PH} \end{pmatrix} + \begin{pmatrix} f_k^R \\ f_k^I \end{pmatrix}, \end{aligned}$$

where, using also the above notation for the averaged elements of  $A$ ,  $\tilde{A}_{00} = 2A_0$  and

$$\tilde{A}_{00} = 2A_0, \quad \tilde{A}_{0l} = \begin{pmatrix} 2A_l^R & 2A_l^I \end{pmatrix}, \quad \tilde{A}_{k0} = \begin{pmatrix} 2A_k^R \\ 2A_k^I \end{pmatrix},$$

$$\tilde{A}_{kl} = \begin{pmatrix} A_{k-l}^R + A_{k+l}^R & -A_{k-l}^I + A_{k+l}^I + \delta_{kl}k\omega F_k \\ A_{k-l}^I + A_{k+l}^I - \delta_{kl}k\omega F_k & A_{k-l}^R - A_{k+l}^R \end{pmatrix},$$

with the  $F_k$  terms coming from the  $-jk\omega\langle x \rangle_k$  parts in (17) and contributing to  $J_{PH}$ . Using  $A_k = \langle J \rangle_k - \langle R \rangle_k$  and  $A_{-k}^R = A_k^R$ ,  $A_{-k}^I = -A_k^I$ , it is immediat to check the skew-symmetry of the structure matrix and the symmetry of the dissipation matrix of the phasor system. Notice that  $R_{PH}$  is not, in general, semi-positive definite. However, it can be proved that  $R_{PH} \geq 0$  on the subspace formed by gradients of phasor hamiltonians (18). Since for passivity-based control  $R_{PH}$  is used only in this setting, this is not a problem.  $\square$

As an example, if terms up to the second harmonic are kept, the structure+dissipation matrix becomes  $(5 \times n)$ -dimensional and is given by

$$A(x, S) = \begin{pmatrix} 2A_0 & 2A_1^R & 2A_1^I & 2A_2^R & 2A_2^I \\ 2A_1^R & A_0 + A_2^R & A_2^I + \omega F_1 & A_1^R + A_3^R & A_1^I + A_3^I \\ 2A_1^I & A_2^I - \omega F_1 & A_0 - A_3^R & -A_1^I + A_3^I & A_1^R - A_3^R \\ 2A_2^R & A_1^R + A_3^R & -A_1^I + A_3^I & A_0 + A_4^R & A_4^I + 2\omega F_2 \\ 2A_2^I & A_1^I + A_3^I & A_1^R - A_3^R & A_4^I - 2\omega F_2 & A_0 - A_4^R \end{pmatrix} \quad (19)$$

where each entry is  $n \times n$ .

Notice that generalized quadratic hamiltonians defined as

$$H(x) = \frac{1}{2}x^T W x + D x$$

satisfy the hypothesis of Proposition 1, with  $F_k = W^{-1}$ ,  $\forall k$ . Even if  $W$  is singular, matrices  $J_{PH}$  and  $R_{PH}$  can still be found [4].

### 3.2 GSSA model of a full-bridge boost rectifier

In [4] a GSSA model for a full-bridge rectifier is given. In that case, a variable transformation is used to linearizing and decoupling the system. In this paper we present a GSSA-PCHS model with the originals variables using the method proposed in the previous subsection.

Following proposition 1 we can build the  $\mathcal{J}(x, S) - \mathcal{R}$  matrix from matrix  $A(x, S)$ , equation (19). Considering only the terms under the second harmonic and from definitions of  $F$  and  $A_k$

$$F_1 = \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix} \quad A_0 = \begin{pmatrix} -r & -\langle S \rangle_0 \\ \langle S \rangle_0 & 0 \end{pmatrix}$$

$$A_1^R = \begin{pmatrix} 0 & -\langle S \rangle_1^R \\ \langle S \rangle_1^R & 0 \end{pmatrix} \quad A_1^I = \begin{pmatrix} 0 & -\langle S \rangle_1^I \\ \langle S \rangle_1^I & 0 \end{pmatrix}$$

$$A_2^R = \begin{pmatrix} 0 & -\langle S \rangle_2^R \\ \langle S \rangle_2^R & 0 \end{pmatrix} \quad A_2^I = \begin{pmatrix} 0 & -\langle S \rangle_2^I \\ \langle S \rangle_2^I & 0 \end{pmatrix}.$$

Notice that  $\langle r \rangle_k = 0 \forall k > 0$  because  $r$  is a constant. Then the  $A(x, S)$  matrix yields

$$A(x, S) = \begin{pmatrix} -2r & -2\langle S \rangle_0 & 0 & -2\langle S \rangle_1^R & 0 & -2\langle S \rangle_1^I \\ 2\langle S \rangle_0 & 0 & 2\langle S \rangle_1^R & 0 & 2\langle S \rangle_1^I & 0 \\ 0 & -2\langle S \rangle_1^R & -r & -\langle S \rangle_0 - \langle S \rangle_2^R & \omega_s L & -\langle S \rangle_2^I \\ 2\langle S \rangle_1^R & 0 & \langle S \rangle_0 + \langle S \rangle_2^R & 0 & \langle S \rangle_2^I & \omega_s C \\ 0 & -2\langle S \rangle_1^I & -\omega_s L & \langle S \rangle_2^I & -r & -\langle S \rangle_0 + \langle S \rangle_3^R \\ 2\langle S \rangle_1^I & 0 & -\langle S \rangle_2^I & -\omega_s C & \langle S \rangle_0 - \langle S \rangle_3^R & 0 \end{pmatrix}.$$

If we consider the first harmonic Fourier components for  $\lambda$  and the zero component for  $q$  (taking into account the harmonic contents of states in steady-state and a large value of  $C$ ),

$$A(x, S) = \begin{pmatrix} 0 & 2\langle S \rangle_1^R & 2\langle S \rangle_1^I \\ -2\langle S \rangle_1^R & -r & \omega_s L \\ -2\langle S \rangle_1^I & -\omega_s L & -r \end{pmatrix}.$$

Finally we obtain a PCHS with the GSSA variables

$$\langle x \rangle^T = (\langle q \rangle_0, \langle \lambda \rangle_1^R, \langle \lambda \rangle_1^I),$$

hamiltonian function (18),

$$H(\langle x \rangle) = \frac{1}{4C} \langle q \rangle_0^2 + \frac{1}{2L} \langle \lambda \rangle_1 \langle \lambda \rangle_1^T,$$

structure and dissipation matrices,

$$\mathcal{J}(x, S) - \mathcal{R} = \begin{pmatrix} 0 & 2\langle S \rangle_1^R & 2\langle S \rangle_1^I \\ -2\langle S \rangle_1^R & -r & \omega_s L \\ -2\langle S \rangle_1^I & -\omega_s L & -r \end{pmatrix},$$

and port connections

$$g(x, S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \langle u \rangle = \begin{pmatrix} \langle i_l \rangle_0 \\ \langle v_i \rangle_1^R \\ \langle v_i \rangle_1^I \end{pmatrix}.$$

Adding the Dirac structure of the inverter subsystem (equation 3) the port connections become

$$g(x, S) = \begin{pmatrix} f^T & 0 & 0 \\ O_{1 \times 3} & 1 & 0 \\ O_{1 \times 3} & 0 & 1 \end{pmatrix} \in \mathbb{R}^{3 \times 5} \quad \langle u \rangle = \begin{pmatrix} \langle i_a \rangle_0 \\ \langle i_b \rangle_0 \\ \langle i_c \rangle_0 \\ \langle v_i \rangle_1^R \\ \langle v_i \rangle_1^I \end{pmatrix} \in \mathbb{R}^5.$$

## 4 GSSA variable extraction and experimental Setup

### 4.1 GSSA variable extraction

The theoretical development of the digital filters for the extraction of the required GSSA variables are based in the time dependent Fourier Transform concept [6], that is beyond

the scope of this report. However a short explanation is enough for the understanding of the underneath process. The basic idea is that each GSSA variable can be computed by means of a narrow band filtering process centered at the desired harmonic frequency, followed by a modulation process which translates the desired frequency to zero, as it is implicit in this selective averaging technique. Figure 5 shows graphically this fact, which corresponds to the concept of a filter bank followed by modulation, a variant of the more known modulation followed by a filter bank approach as in [7]. In Figure 5,  $x[n]$  corresponds to a physical sampled signal,  $N$  is the number of discrete-time samples in a period  $T$ ,  $0 < k < N - 1$  is an integer corresponding to the order of the harmonic to compute,  $h[n, k]$  is the impulse response of the selective filter for the  $k$ -harmonic frequency, and  $X[n, k]$  is the complex value that corresponds to the GSSA variable at the sampled time  $n$ . Figure. 6 shows the digital filter designed which efficiently fulfill the schema in Figure 5 for the extraction of the GSSA variables associated to the  $k$  harmonic, where the one sample period shifted output of the filter,  $\tilde{X}_s[n]$  and the corresponding shifted modulation procedure are present only to allow some reduction in the computational effort.

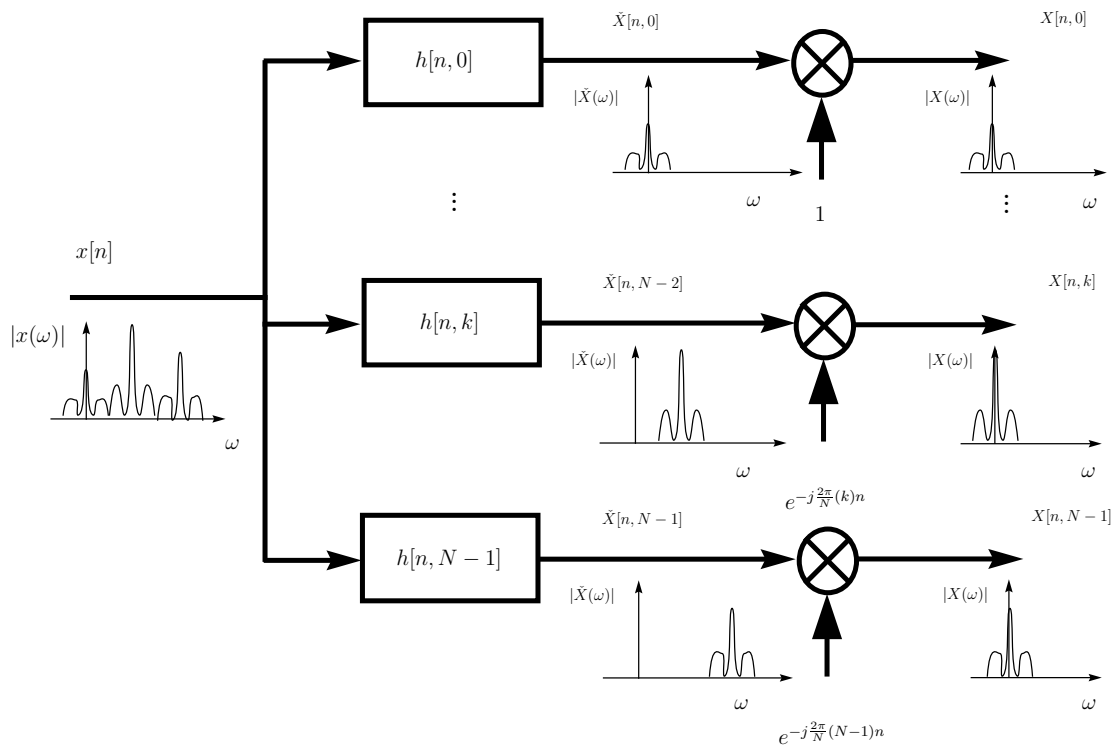


Figure 5: GSSA derived from filters bank followed by modulation.

## 4.2 Experimental setup

The experimental setup described here has been partially used in [4], but several improvements are possible. A difficulty of the model is the parameter  $r$ , which has a complex

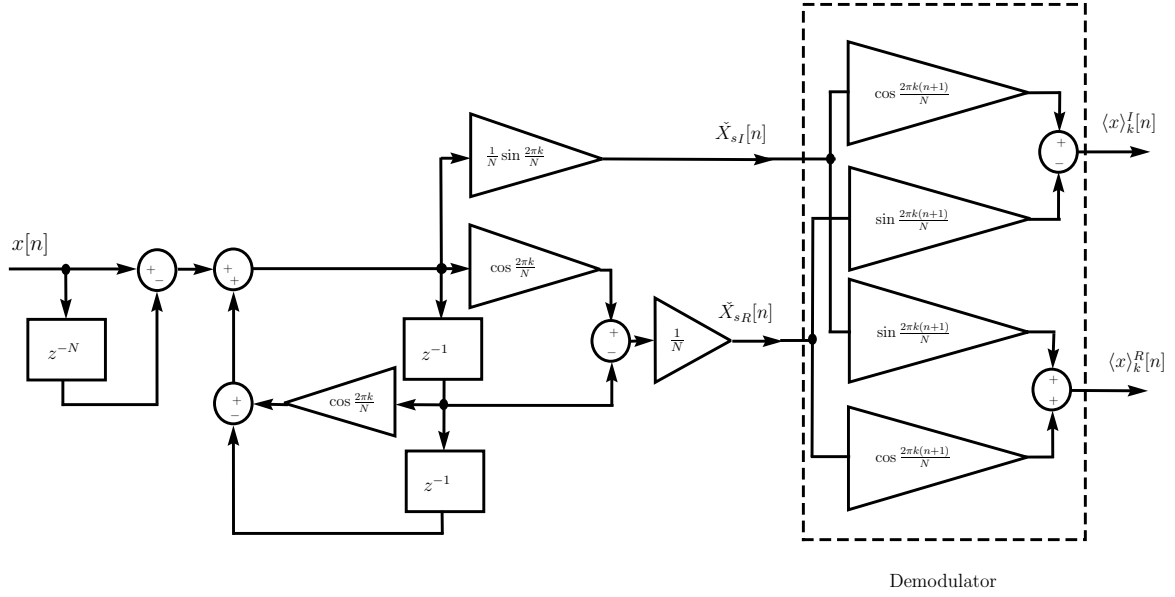


Figure 6: Recursive digital filter for the real time extraction of GSSA components associated to the  $k$  harmonic.

behavior depending on the commutation frequency of the PWM and the frequency of the grid. To account for this, a new robust control law should be designed or a way to estimate  $r$  should be considered. Also, to improve the performance of the controller a new microprocessor is needed to work with floating-point instead of fixed-point.

The experimental setup available to work has the following parts:

- Full-bridge boost converter with IGBT switches (1200 V, 100 A) and parameters:  $r = 0.092\Omega$ ,  $L = 2.7\text{mH}$ ,  $C = 1400\mu\text{F}$ . The switching frequency of the converter is 20 KHz and a synchronous centered-pulse single-update pulse-width modulation strategy is used to map the controller's output to the IGBT gate signals.
- 3-phase DC/AC inverter with a set of IGBT switches (1200 V, 100 A). The switching frequency of the inverter is 20 KHz and a synchronous centered-pulse single-update pulse-width modulation strategy is used to map the controller's output to the IGBT gate signals.
- Analog circuitry of sensors: the AC main source, PMW and DC bus voltages and currents are sensed with isolation amplifiers. All the signals from the sensors pass through the corresponding gain conditioning stages to adapt their values to A/D converters.
- Control hardware and DSP implementation: the control algorithm can be implemented using the Analog Devices DSP-21116 and DSP-21992 processors. The processing core of this device runs at 100MHz and has a 32bit floating-point unit. The sampling rate of the A/D channels has been selected at 20KHz, the same as the switching frequency of the full-bridge system.

- The nominal RMS AC mains voltage is  $V_s = 48.9\text{V RMS}$  and its nominal frequency is 50 Hz.
- Control Objectives: For the full-bridge boost rectifier the desired regulated DC voltage and the power factor should be  $v_l^* = 130\text{V}$  and near unity, respectively. For the inverter the 3-phase voltages  $v_{abc}$  should be a suitable PWM to feed the electrical machine. It is important to note that the IGBT switches are oversized for this particular application, resulting in undesirable power losses and harmonic distortion. These power losses can be taken into account increasing the series resistance  $r$  by a switch resistance  $r_{sw} = 0.3\Omega$ . The system performance could be improved replacing these IGBT switches with lower power ones.

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