# Health-aware LPV-MPC based on a Reliability-based Remaining Useful Life Assessment

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**Abstract:** One of the relevant information provided by the prognostics and health management algorithms is the estimation of the Remaining Useful Life (RUL). The prediction of the expected RUL is very useful to decrease maintenance cost, operational downtime and safety hazards. This paper proposes a new strategy of health-aware Model Predictive Control (MPC) for a Linear Parameter Varying (LPV) system that includes as an additional goal extending the system RUL via their estimation using reliability tools. In this approach, the RUL maximization is included in the objective function of the LPV-MPC controller. The RUL is included in the MPC model as an extra parameter varying equation that considers the control action as scheduling variable. The proposed control approach allows the controller to accommodate to the parameter changes. Through computing an estimation of the state variables during prediction, the MPC model can be modified to the estimated state evolution at each time instant. Moreover, for solving the optimization problem by using a series of Quadratic Programs (QP) in each time instant, a new iterative approach is exhibited, which improves the computational efficiency. A pasteurization plant control system is used as a case study to illustrate the performance of the proposed approach.

Keywords: Remaining Useful Life (RUL), Model Predictive Control, Linear Parameter Varying, Reliability

## 1. INTRODUCTION

During the last decade, the improvement in safety, performance, availability, and effectiveness of industrial systems has been achieved through prognostics and health management (PHM) paradigm (Pecht, 2008). PHM is a systematic strategy that is utilized to assess the reliability of a system in its actual life-cycle conditions, predict failure progression, and decrease damage via control actions. There are two roles in PHM, specifically, "*prognostics*" and "*health management*" (Si et al., 2013). Prognostic is now identified as a principal process in maintenance strategies based on the remaining useful life of the equipment, which makes it possible to avoid critical damages and reducing costs. The Remaining Useful Life (RUL) is the useful life that remains on an asset at a particular time of operation. Its estimation is fundamental to condition-based maintenance, health management and prognostics. RUL is generally random and unknown, and as such it must be estimated from available sources of information such as the information obtained in condition and health monitoring (Si et al., 2013). Therefore, it can be noted that the reliability estimation of equipment as well as its RUL prediction is necessary to establish if the mission goals can be achieved. Since the prediction of RUL is critical to operations and decision making, it is imperative that the RUL is determined accurately (Sankararaman et al., 2013).

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In recent years, the problem of actuator lifetime and system reliability and RUL prediction in service has received increasing attention. Gokdere et al. (2005) incorporated the actuator lifetime as a controlled parameter to reduce maintenance cost. The control of actuator lifetime is achieved by implementing a linear quadratic optimal controller. Li et al. (2000) proposed a method to estimate RUL of a bearing based on its defect growth while, the fatigue crack propagation is then compared to the estimation from the diagnostic model. On the other hand, Model Predictive Control (MPC) has been recently proved as an adequate strategy for implementing health-aware control schemes because the MPC can predict the appropriate control actions to achieve optimal performance according to physical constraints and multi-objective cost functions. Pereira et al. (2010) designed a MPC techniques that employed to distribute the loads among redundant actuators while imposing constraints to ensure that the accumulated actuator degradation will not reach an unsafe level at the end of the mission. Karimi Pour et al. (2016) designed a health-aware MPC controller with fatigue-based prognosis incorporated into MPC to minimize the damage of components while still keeping the performance of the system managed suitably.

The reliability are an exponential form of control input (Salazar et al., 2017). On the other hand, the expected RUL depends on the reliability evaluation assessment. Consequently, the RUL has an exponential relation with the control input that induces a nonlinear behavior. One major drawback of the previous approaches to reliability-based MPC is that they do not consider this issue inside the MPC loop. One way to deal with non-linear MPC is to represent the process behavior by means of Linear Parameter Varying (LPV) models (Bumroongsri and Kheawhom, 2012). LPV models are a class of linear models whose state-space matrices depend on a set of time-varying parameters. The main advantage of LPV models is that the system nonlinearities are embedded into the varying parameters, which make the nonlinear system become a linear-like system with varying parameters (Karimi et al., 2017).

This paper presents a health-aware LPV-MPC controller on the basis of PHM information and the RUL integration into the control algorithm using an LPV framework. The non-linear system is modelled using a LPV model where the scheduling parameters at each time instant are updated with the state vector value at that time. The main contribution of this paper consists in designing an improved health-aware LPV-MPC strategy in order to formulate an optimization problem that exploits the functional dependency of scheduling variables and state vector to develop a prediction strategy with numerically attractive solution. This attractive solution is iteratively forced to an accurate solution, thereby avoiding the use of non-linear optimization. Finally, the proposed algorithm for health-aware LPV-MPC strategy based on the quasi-LPV is tested in a simulation of a small-scale pasteurization plant that presents nonlinear behavior.

The remainder of the paper is organized as follows: in Section 2, the formulation of MPC based on quasi-LPV and iterative prediction scheme is introduced. Then, the LPV-MPC approach for EMPC is presented in Section 2. The health-aware controller scheme based on an LPV-MPC algorithm is the RUL integration into the control algorithm are presented in Section 3. In Section 5, results of applying the proposed control strategy to the pasteurization system as a case study are summarized. Finally,in Section , the conclusion of this work are drawn and some research lines for future work are proposed.

## 2. LPV-MPC APPROACH

### *2.1 Problem formulation*

Lets consider that the non-linear system to be controlled can be represent by the following discrete-time LPV system representation

$$x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k), \tag{1a}$$
$$y(k) = C(\theta(k))x(k), \tag{1b}$$

where the discrete-time variable is denoted by $k \in Z_{\geq 0}$. $x(k) \in \mathbb{R}^{n_x}$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the vector of manipulated variables, $y(k) \in \mathbb{R}^{n_y}$ is the system output and $\theta(k) \in \Theta \, \forall k \geq 0$ is the system vector of scheduling parameters, where $\Theta \in \mathbb{R}^{n_p}$ is a given compact set. This means that $A$ and $B$ are bounded on $\Theta$. Throughout this paper it is assumed that $(A(\theta), B(\theta))$ is stabilizable $\forall \theta \in \Theta$.

The MPC controller design is based on minimizing the finite horizon cost

$$J(k) = \sum_{i=0}^{N_p} \|x(i|k)\|_{p,w_1} + \sum_{i=0}^{N_p-1} \|u(i|k)\|_{p,w_2}), \tag{2}$$

where $N_p$ is the prediction horizon. Furthermore, the subindex $p$ denotes the norm used (for this paper, the 2-norm) and the weighting matrices $w_1 \in \mathbb{R}^{n_x \times n_x}$ and $w_2 \in \mathbb{R}^{n_u \times n_u}$ are used to establish the priority of the different control objectives. The value of $x(0|k)$ and $u(0|k-1)$ are known at each time instant, and the optimization problem

$$\min_{\mathbf{u}(k)} J_k(\mathbf{u}(k)) \tag{3a}$$

subject to:

$$x(i+1|k) = A(\theta(i|k))x(i|k) + B(\theta(i|k))u(i|k), \tag{3b}$$
$$\theta(i|k) = f(x(i|k), u(i|k)), \tag{3c}$$
$$u(k), u_{k+1}, ..., u_{k+N_p-1} \in \mathbb{U} \tag{3d}$$
$$x(k), u_{k+1}, ..., x_{k+N_p} \in \mathbb{X} \tag{3e}$$
$$\theta(i|k) = \theta(i|0), \tag{3f}$$
$$x(0|k) = x(k), \tag{3g}$$

is solved online for all $i \in \mathbb{Z}_{[0,N_p-1]}$, where $\mathbf{u}(k) = [u(k), u(k+1), ..., u(k+N_p-1)]^T$ is the decision sequence of controlled inputs. $\mathbb{X}$ and $\mathbb{U}$ define the set of acceptable states and inputs and it is assumed $f(\mathbb{X} \times \mathbb{U}) \subset \Theta$. The control law is applied in a receding horizon manner, that is, at time $k$ control input $u(0|k)$ is applied, whilst at time $k+1$ the minimization of $J(k+1)$ is solved for $\mathbf{u}(k+1)$ then the newly computed control input $u(0|k+1)$ is applied. Also, $x(i|k)$ is the predicted state at time $i$, with $i = 0, ..., N_p$, obtained by starting from the state $x(0|k) = x(k)$.

The LPV model can not be evaluated before solving the optimization problem (3), because the future state sequence are not known. Indeed $x(i|k)$ depend not only on the future control inputs $\mathbf{u}(k)$, but also on the future scheduling parameters $\theta(k)$, where for a general LPV system are not assumed to be known a priori but only to be measurable online at current time $k$.

### *2.2 Proposed solution*

In this section, a MPC scheme is presented in order to solve the optimization problem of a LPV system with varying parameters into the prediction horizon. In fact, the structure (3a) is linear but because of the (3c), the problem becomes nonlinear. Actually, this issue makes the problem (3) not easy to solve. The idea is to find a solution to the problem (3) by solving an online optimization problem as a QP problem. In this paper, the solution for this problem is to transform the exact LPV-MPC to an approximate one. This approximation is based on using an estimation of $\hat{\theta}$ instead of using $\theta$. It means that the scheduling variables in the prediction horizon are estimated and used to update the matrices of the model used by the MPC controller. In fact, for solving this problem, the sequence of the control input is used to modify to system matrices of the model used in the prediction horizon. Thus, from the optimal control sequence $\mathbf{u(k)}$, the sequence of states and predicted parameters can be obtained

$$\mathbf{x}(k) = \begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N_p) \end{bmatrix} \in \mathbb{R}^{N_p, n_x}, \quad \Theta = \begin{bmatrix} \hat{\theta}(k) \\ \hat{\theta}(k+1) \\ \vdots \\ \hat{\theta}(k+N_p-1) \end{bmatrix} \in \mathbb{R}^{N_p, n_\theta}. \tag{4}$$

Therefore, with slight abuse of notation $f$ can be defined as: $\Theta(k) = f([x^T(k) \quad \tilde{\mathbf{x}}^T(k)], \mathbf{u}(k))$. The vector $\Theta(k)$ includes parameters from time $k$ to $k + N_p - 1$ whilst the state prediction is considered for time $k+1$ to $k+N_p$.

Hence, by using the definitions (4), the predicted states can be simply formulated as follows

$$\tilde{\mathbf{x}}(k) = \mathcal{A}(\Theta(k))x(k) + \mathcal{B}(\Theta(k))\mathbf{u}(k), \tag{5}$$

where $\mathcal{A} \in \mathbb{R}^{n_x \times n_x}$ and $\mathcal{B} \in \mathbb{R}^{n_x \times n_u}$ are given by (6) and (7). By using (5) and augmented block diagonal weighting matrices $\tilde{w_1} = diag_{N_p}(w_1)$ and $\tilde{w_2} = diag_{N_p}(w_2)$, the cost function (2) can be rewritten in vector form as

$$J(k) = \sum_{i=0}^{N_p-1} \|x(i+1|k) - x_{ref}(i+1)\|_{p,\tilde{w}_1} + \|u(i+1|k)\|_{p,\tilde{w}_2}, \tag{8}$$

Since the predicted states $\Theta(k)$ in (5) are linear in control inputs $\mathbf{u}(k)$, the optimization problem can be solved as a QP problem, that is significantly further easier than solving a nonlinear optimization problem. To simplify the discussion the next assumption is considered. This idea leads to the following iterative approach at each discrete time instant $k$:

- In the first iteration, the problem (3) is solved as a linear problem due to the quasi-LPV model (1) is replaced by the LTI model that is obtained considering $\theta(0|l) \simeq \theta(1|l) \simeq \theta(2|l) \simeq ... \simeq \theta(N_p - 1|l)$ along the prediction horizon $N_p$.
- The sequence of the scheduling variables $\Theta(k)$ is repetitively steered to its optimal amount $\Theta^*(k) = f(\tilde{\mathbf{x}}^*(k), \mathbf{u}^*(k))$, whence $\tilde{\mathbf{x}}^*(k)$ and $\mathbf{u}^*(k)$ refer the input and state sequences related to the optimal solution.
- The optimal amount $\Theta^*(k)$ obtained by solving the optimization problem at iteration step $i$ such that $\Theta(k)$ is replaced by $\Theta_i(k)$, and by creating a new sequence from the result of the optimal state sequence $\tilde{\mathbf{x}}_i(k)$ as $\Theta_{i+1}(k) = f(\tilde{\mathbf{x}}_i(k), \mathbf{u}_i(k))$.
- The scheduling variables for the next iteration $\Theta_0(k+1)$ are determined when using $\tilde{\mathbf{x}}_i(k)$ and $\mathbf{u}_i(k)$, i.e., $\Theta_0(k+1) = f(\tilde{\mathbf{x}}_i(k), \mathbf{u}_i(k))$.

## 3. HEALTH-AWARE LPV-MPC FOR PRESERVING THE RUL

### 3.1 Reliability assessment

One of the motivation in this work is to integrate the information about actuator health in the controller design. The life time will be estimated by means of the RUL computed using an approach based on the system reliability. Reliability is the ability of a system or component to perform its expected functions and can be formally defined as follows.

*Definition 3.1.* (Gertsbakh, 2013). Reliability is characterized as the probability that components, units, types of equipment and systems will perform their predesignated function for a certain period of time under some operating conditions and specific environments.

More precisely, it is the probability of success in performing a task or reaching a desired property in the process, based on the availability of required components. Mathematically, reliability $R(k)$ is the probability that a system will be successful in the interval from time 0 to time $k$:

$$R(k) = P(T > k), \qquad k \geq 0 \tag{9}$$

where $T$ is a nonnegative random variable which represents time-to-failure or failure time.

The reliability of a system with the $j$-th component can be assessed by using the exponential function

$$R_j(k) = exp\Big(-\int_0^k \lambda_j(s)\, ds\Big), \qquad j = 1, 2, ..., m \tag{10}$$

where $\lambda_i(k)$ is the failure rate and the form of $R_j(k)$ displayed on Fig 1. Component's lifetime changes according to control strategies and/or system's operating points. Consequently, engineering systems are designed to support varying amounts of loads that can be measured in terms of usage frequency or busy period (Salazar et al., 2017). Results from literature have established that the function load strongly affects the component failure rate. Hence, it is important to consider the load versus failure rate relationship when evaluating system reliability. In the considered study, failure rates are obtained from actuators under different levels of load depending on the applied control input. One of the most used relations between is based on assuming that actuator fault rates changes with the load through the following exponential law (Salazar et al., 2017):

$$\lambda_j(k) = \lambda_j^0 exp\Big(\beta_j u_j(k)\Big), \qquad j = 1, 2, ..., m \tag{11}$$

where $\lambda_j^0$ represents the baseline failure rate (nominal failure rate) and $u_j(k)$ is the control action at time $k$ for the $j$-th actuator. $\beta_j$ is a constant parameter that depends on the actuator characteristics.

### 3.2 RUL computation via reliability assessment

Once the reliability function is calculated for each component, a method to evaluate *Rul* function is introduced.

***Proposition** 3.1. Recalling that* Rul *function is defined as the conditional expected time to failure given the current working time* (Banjevic, 2009):

$$\mathcal{A}(\Theta(k)) = \begin{bmatrix} I \\ A(\hat{\theta}(k)) \\ A(\hat{\theta}(k+1))A(\hat{\theta}(k)) \\ \vdots \\ A(\hat{\theta}(k+N_p-1))A(\hat{\theta}(k+N_p-2))\dots A(\hat{\theta}(k)) \end{bmatrix} \tag{6}$$

and

$$\mathcal{B}(\Theta(k)) = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ B(\hat{\theta}(k)) & 0 & 0 & \dots & 0 \\ A(\hat{\theta}(k+1))B(\hat{\theta}(k)) & B(\hat{\theta}(k+1)) & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ A(\hat{\theta}_{k+N_p-1})\dots A(\hat{\theta}(k+1))B(\hat{\theta}(k)) & A(\hat{\theta}_{k+N_p-1})\dots A(\hat{\theta}(k+2))B(\hat{\theta}(k+1)) & \dots & B(\hat{\theta}_{k+N_p-1})) & 0 \end{bmatrix} \tag{7}$$

$$Rul(k) = E(T-k|T>k), \tag{12}$$

*the expected* Rul *is given by*

$$Rul(k) = \frac{exp(-\lambda_j k)}{\lambda_j} \tag{13}$$

*in case of using the reliability function* (9).

**Proof.** According to the $Rul$ function definition (12) and considering reliability function (9), the expected $Rul$ can be computed as follows

$$Rul(k) = \int_0^\infty R(k+z|k)dz = \int_k^\infty R(z|k)dz,$$

Then,

$$\begin{aligned} Rul(k) &= \int_k^\infty exp\Big(-\int_0^z \lambda_j(s)\,ds\Big)\,dz, \quad t \le z < \infty \\ &= \int_t^\infty exp(-\lambda_j(z))dz, \\ &= -\frac{exp(-\lambda_j z)}{\lambda_j}\Bigg|_k^\infty, \\ &= \lim_{b\to\infty}\Big(-\frac{exp(-\lambda_j(b))}{\lambda_j} - (-\frac{exp(-\lambda_j(k))}{\lambda_j})\Big), \\ &= \frac{exp(-\lambda_j(k))}{\lambda_j} \end{aligned}$$

■

Actually, in the useful period of life, the component can be characterized at a given time $k$ by a baseline remaining useful life measure $Rul(k)$. In the following, $Rul(k)$ will be assigned to the remaining useful life of system that is obtained under nominal operating conditions such as:

$$Rul(k) = \frac{exp(-\lambda_j^0 k)}{\lambda_j^0}. \tag{14}$$

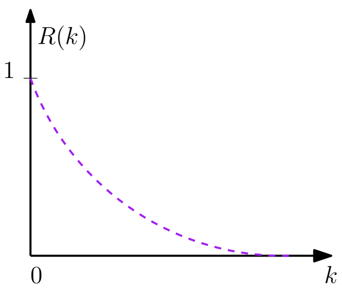


Fig. 1. Behaviour of the reliability.

Thus, the remaining useful life $Rul(k+1)$ can be estimated from the baseline of the remaining useful life $Rul(k)$ as follows:

$$Rul(k+1) = Rul(k)\frac{exp(-\lambda_j)}{\lambda_j}. \tag{15}$$

Using the reliability function (10) and the effect of the control input (11), the $Rul$ function is obtained as a exponential function of $\lambda_j$ that depends on the control input $u_j(k)$.

### *3.3 Health-aware LPV-MPC*

In order to integrate the $Rul$ in the linear MPC model as an additional state variable, a transformation is required that allows to compute $Rul$ in a linear-like form. The proposed transformation is based on using the logarithm of (13)

$$\log(Rul(k+1)) = \log\Big(Rul(k)\frac{exp(-\lambda_j)}{\lambda_j}\Big), \tag{16}$$

that leads to

$$\log(Rul(k+1)) = \log\big(Rul(k)\big) - \lambda_j - \log(\lambda_j). \tag{17}$$

Then, by renaming (17), the remaining useful life model of each actuator is obtained as

$$h_j(k+1) = h_j(k) - \xi(u_j(k)) - \zeta_j(k), \tag{18}$$

where $h_j$ is the logarithm of the remaining useful life, $\zeta_j$ is the logarithm of $\lambda_j$ at each time instant $k$ and $\xi(u_j(k))$ is function of control action of each actuator $u_j(k)$, with $j = 1, 2, ..., m$ as

$$\xi(u_j(k)) = \lambda_j^0 exp\Big(\beta_j u_j(k)\Big) \qquad j = 1, 2, ..., m. \tag{19}$$

Using this approach, the MPC model is augmented with (18) that is a LPV model that has as scheduling variable the control action $u_j(k)$ associated to teach actuator. Moreover, a new additional objective based on the new state variable $h$ is included into the LPV-MPC cost function (2) that aims to maximize the $Rul$ of the system. Thus, the problem formulation of the health-aware controller is similar to (3) but including $Rul$ objective and model:

$$\min_{\mathbf{u}_k} \sum_{i=0}^{N_p-1} \|x(i+1|k) - x_{ref}(i+1)\|_{p,\tilde{w}_1} + \|u(i+1|k)\|_{p,\tilde{w}_2} - \|h(i+1|k)\|_{p,w_3} \tag{20a}$$

subject to:

$$\tilde{\mathbf{x}}(k) = \mathcal{A}(\Theta(k))x(k) + \mathcal{B}(\Theta(k))\mathbf{u}(k) \tag{20b}$$

$$h_j(k+1) = h_j(k) - \xi(u_j(k)) - \zeta_j(k),\ j = 1, \cdots, m \tag{20c}$$

$$u(k), u_{k+1}, ..., u_{k+N_p-1} \in \mathbb{U} \tag{20d}$$

$$x(k), u_{k+1}, ..., x_{k+N_p-1} \in \mathbb{X} \tag{20e}$$

$$x(0|k) = x(k), \tag{20f}$$

for all $i \in \mathbb{Z}_{[0,N_p-1]}$. The health-aware objective with the corresponding weight $w_3$ is appended to the LPV-MPC cost function to maximize the *Rul*. According to Section 3.2, there is a direct relation between the reliability and *Rul* of the system, hence by increasing the *Rul* in (20), the reliability of each system component is preserved. Moreover, according to the nonlinearity term of the *Rul* in (20e) and the dependence on the control action that is not known in the predication horizon, the *Rul* for next time instant $h_{i+1}(k)$ into the prediction horizon can be calculated from the previous control action, $h_{i+1}(k) = f(u_i(k))$, similarly to what is proposed for the LPV parameters in Section 2.2.

## 4. APPLICATION TO THE PASTEURIZATION PLANT

### *4.1 Case study description*

To illustrate the proposed approach a pasteurization process is used. The pasteurization process considered is the utility-scale plant PCT-23MKII, manufactured by Armfield (UK) (Armfield, 2015). This laboratory system is the small version (1.2m, 0.6m, 0.6m) of real-time industrial pasteurization procedure. The system represents an industrial High-Temperature Short-Time (HTST) process. In this process, the goal is to heat and maintain the product at a prearranged temperature for the minimum time. This procedure is accomplished by flowing the heated fluid through a holding tube (Karimi Pour et al., 2017). During the pasteurization process, the fluid is pumped at a prearranged flow speed from storage tank to the heat exchanger. The heat is transported to the product inward of the first section of the heat exchanger, which is named regenerator. By applying energy to the pasteurized product, the raw product is heated to an average temperature. Later, in the second section, while utilizing a hot-water flow $F_h$ arising from a closed circuit with a heater, the product is heated from that intermediate temperature to the complete pasteurization temperature. The $T_{past}$ temperature is related to the output of the holding tube to monitor the temperature of the product after the pasteurization procedure. Eventually, the product temperature is reduced in the third section of the heat exchanger, where the remaining heat is recuperated to the incoming produce.

Figure 3 includes a block diagram of the pasteurization simulation model, containing the feedback loops corresponding to the hot-water flow and power of the hot-water tank. For the modeling purpose, the whole pasteurization system can be classified into three subsystems that are a heat exchanger, holding tube and hot water tank. To model the whole pasteurization plant, models of these subsystems are obtained and expressed in terms

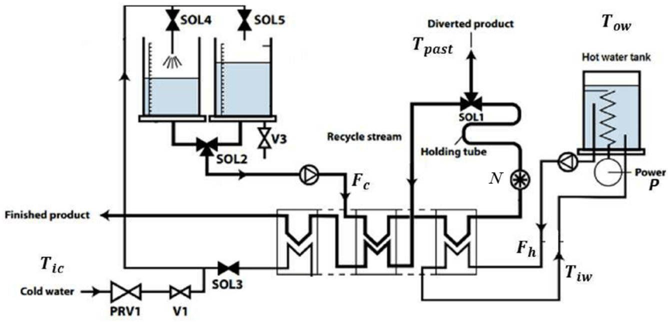


Fig. 2. Pasteurization plant scheme.

of behavioral equations of each subsystem. The mathematical models of the subsystems are collected from the experimental data reported in (Ibarrola et al., 2002). In addition, $G_{i,j}$ represents the first order transfer function of each subsystem. Accordingly, models obtained as transfer functions are suitably stated by their equivalent controllable realizations in state space, with varying parameters according to the hot-water flow, $F_h$, as a state of the system and the hot/cold-water flow ratio, $(R = F_h/F_c)$ that is a function of hot water flow $F_h$. Therefore, the state-space LPV model of the pasteurization plant can be expressed in the form (1), where the state vector including hot-water flow, $F_h$, hot-water tank temperature, $T_{ow}$ and pasteurization temperature, $T_{past}$. The system input is the vector of manipulated variables that includes the electrical power of the heater $P$ and the pump rotational speed $N$. Finally, the output is the vector of controlled variables that include the temperature of the hot water tank and pasteurization temperature, denoted by $T_{ow}$ and $T_{past}$, respectively. The state-space matrices in (1) for this system are (21), where $\tau$ and $K$ are time constant and static gain of the transfer functions of subsystems, respectively. The indices of $\tau$ and $K$ are linked to the transfer functions for each subsystem of the complete pasteurization plant (see Figure 3). The pasteurization system has four actuators that includes the electrical actuator, pump actuator and two actuators related to the valves of system.

The most important objective of the pasteurization procedure is to ensure that the pasteurization temperature is attained and preserved as close as to the set-point amount for a pre-established time. At the same time, the maximiz-

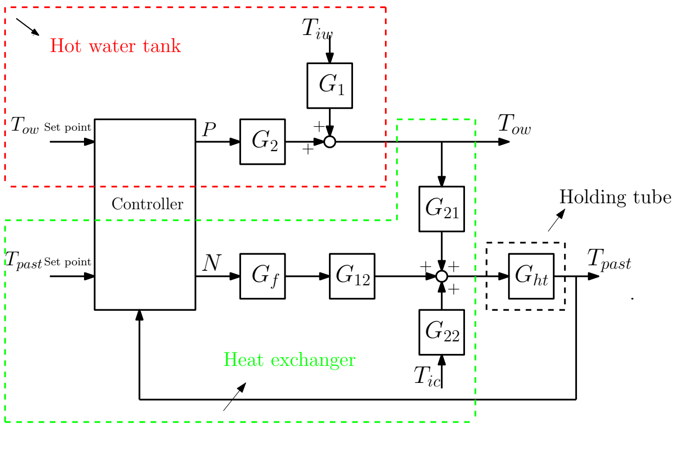


Fig. 3. Control block diagram.

$$
A = \begin{bmatrix}
1+\frac{-T_s}{\tau_1(F_h(t))} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1+\frac{-T_s}{\tau_2(F_h(t))} & 0 & 0 & 0 & 0 & 0 \\
\frac{T_sK_{21}(R(t))}{\tau_{21}(F_h(t))} & \frac{T_sK_{21}(R(t))}{\tau_{21}(F_h(t))} & 1+\frac{-T_s}{\tau_{21}(F_h(t))} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1+\frac{-T_s}{\tau_{12}(F_h(t))} & 0 & \frac{T_sK_{12}(R(t))}{\tau_{12}(F_h(t))} & 0 \\
0 & 0 & 0 & 0 & 1+\frac{-T_s}{\tau_{22}(F_h(t))} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1+\frac{-T_s}{\tau_f} & 0 \\
0 & 0 & 0 & \frac{T_sK_{ht}}{\tau_{ht}} & \frac{T_sK_{ht}}{\tau_{ht}} & \frac{T_sK_{ht}}{\tau_{ht}} & 1+\frac{-T_s}{\tau_{ht}}
\end{bmatrix}, \tag{21}
$$

$$
B = \begin{bmatrix}
0 & 0 & \frac{T_sK_1(R(t))}{\tau_1} & 0 \\
\frac{T_sK_2(R(t))}{\tau_2(F_h(t))} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{T_sK_{22}(R(t))}{\tau_{22}} \\
0 & 0 & \frac{T_sK_f}{\tau_f} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},
$$

ing of actuator RUL allows to raise the reliability of the system. The input temperature of hot-water tank $T_{iw}$ and the cold temperature $T_{ic}$ are maintained constant at $40°C$ and $30°C$, respectively. Furthermore, the power of the electrical heater $P$ and the speed of pump can take values in the range $P \in [0, 1.5]kW$ and $N \in [10, 80]m^3/s$, respectively. The states are constrained to be $[0, 0, 0, 0, 0, 0, 0]^\top \leq x_k \leq [120, 120, 120, 120, 120, 800, 120]^\top$. The initial state considered is $x_0 = [28, 0, 0, 0, 0, 155, 22]^\top$ and the prediction horizon has been selected as $N_p = 120$.

### *4.2 Results and Discussion*

All tests were done using the same weights, initial condition and perdition horizon as mentioned above. All simulation and computations have been carried out using a computer with i7 2.40-GHz Intel core processor with 12 GB of RAM running MATLAB R2016b. The optimization problem is solved by using the linear and nonlinear programming solvers available in YALMIP (Lofberg, 2004).

Figure 4 shows the evolution of the output temperature results that are obtained using health-aware LPV-MPC based on the LPV system model that considers the Rul objective and model compared against an implementation based on non-linear MPC (NMPC). Figure 4 presents the pasteurization temperature, $T_{past}$ and hot-water tank temperature, $T_{ow}$ obtained using the proposed approach tracking the predetermined appropriate setpoint with similar results to those of NMPC algorithm but with less computational effort. While the average time of optimization in each iteration of the proposed approach is on average two times faster than the NMPC. Figure 5 provides the power of the electrical heater and pump control action of the proposed approach with the Rul objective. The results of the Rul prediction that are obtained using the health-aware LPV-MPC with and without the health-aware objective are presented in Fig 6. According to these results, it can be observed that the performance the proposed is almost the same as the NMPC one. Moreover, results form Fig 6 show that the Rul is maximized about 21.16% in the LPV-MPC controller with the Rul objective. Due to a strong relationship between Rul and reliability established in the paper, when the controller can be increase the Rul, the actuator reliability improves accordingly

## 5. CONCLUSION

This paper has proposed a health-aware MPC strategy in the LPV framework based on the maximization of the *Rul* of the system components. The *Rul* is obtained as a function of control action via the reliability assessment. The model of the RUL is obtained as a function of con-

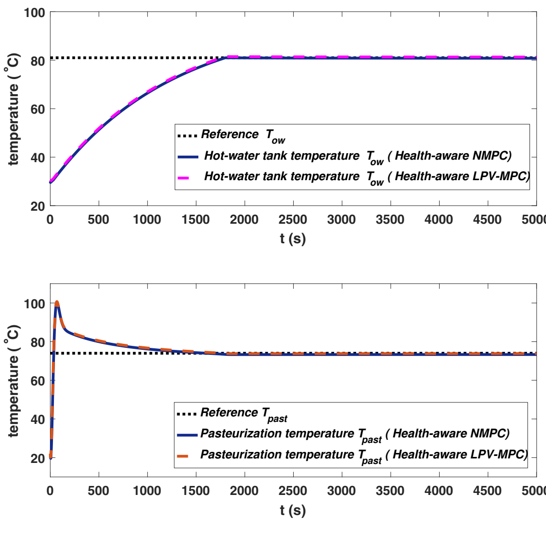


Fig. 4. Evolution of controlled temperature of health-aware NMPC strategy and the proposed algorithm.

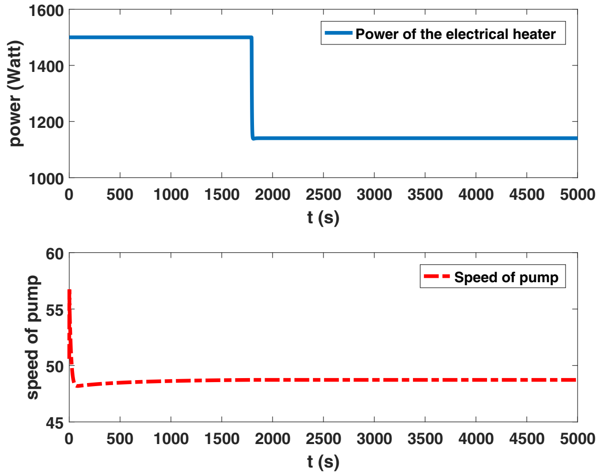


Fig. 5. Evolution of the control action of health-aware LPV-MPC.

trol action with a nonlinear term that is transformed into a linear-like form via the LPV framework. Then, the maximizing the *Rul* has achieved by its inclusion in the objective function and as an additional state in the MPC model. The new health-aware LPV-MPC approach is efficiently solved iteratively by a series of QP problems that uses an update MPC model updated via the scheduling parameters calculated at each time instant. The model prediction in the MPC horizon is obtained using the previous sequence of scheduling variables. The results obtained show that the *Rul* of the components is maximized with the MPC controller and the proposed approach is attractive and less computationally demanding that NMPC implementation that implies non-linear programming algorithms. Finally, the pasteurization process was used to assess the proposed health aware LPV-MPC scheme for extending the *Rul*. Future research will be directed to incorporate the whole system reliability as state in the control model of controller. Moreover, it is interesting to investigate and analyze the effect of proposed approach based on the complex nonlinear system with more nonlinearities and redundancy of actuators.

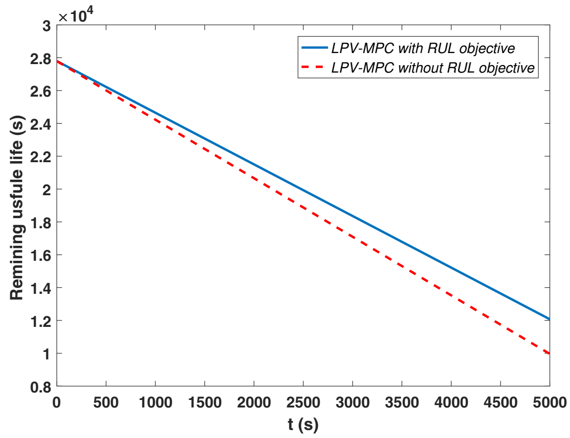


Fig. 6. Evolution of the RUL with and without health-aware objective in the MPC.

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