

Different reheating mechanisms in quintessence inflation

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Different well-know ways to reheat the universe such as instant preheating, the creation of particles nearly or conformally coupled with gravity, or from the decay products of a curvaton field, are revisited and discussed in detail in the framework of quintessence inflation, where the inflaton field at the end of inflation, instead of oscillating, rolls monotonically towards the infinite to drive the universe to a kination regime. For any kind of these preheating (particle creation) mechanisms, in order to calculate the reheating temperature, we point out the importance of the Big Bang Nucleosynthesis bounds and the decay process of the massive fields involved in the theory, whose decay products form a relativistic plasma in thermal equilibrium with energy density that eventually will dominate over the one of the background after the phase transition, yielding a universe in thermal equilibrium which matches with the hot Friedmann universe.

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1. INTRODUCTION

A reheating mechanism, as pointed out by A. Guth in his seminal paper [1], although the one presented there did not work, is an essential part of the inflationary paradigm in order to match inflation with the hot Friedmann universe, because particles existing before the beginning of this period are completely diluted at the end of it. The most popular way to reheat the universe in standard inflation, is via particle production due to the oscillations of the inflaton field after the end of the inflationary period [2]. Things changed after the discovery of the current cosmic acceleration [3, 4] and the subsequent introduction of pioneering quintessence inflation models to unify the early inflation period with the late time acceleration of the universe [5–7], because the potentials in these models do not contain a deep well and, thus, the inflaton field rolls down without oscillations, so the inflaton field cannot release its energy to produce particles. For these models, the various existing mechanisms of particle production are completely different: The first one used in quintessence inflation was the gravitational particle production studied long time ago in [8–11] and at the end of nineties in [12, 13], and applied to quintessence inflation in [5, 6, 14, 15]. The second mechanism was the so-called *instant preheating* introduced in [16], applied for the first time to quintessence inflation in [17] and recently in [15, 18] in the context of α -attractors in supergravity. The last one, that we study in this work, is the *curvaton reheating* applied to brane-world inflation in [19] and to quintessence inflation in [20] and [21].

Dealing with the reheating temperature, the general convention is that it is the temperature of the universe at the beginning of the radiation-dominated era as it is mentioned in many papers (see for example the introduction of [22, 23]). In standard inflation, i.e., when the potential contains a deep well, as we have already explained, the inflaton field begins to oscillate creating a relativistic plasma of light particles, and assuming as usual that they thermalize instantaneously, the reheating temperature coincides with the temperature of the universe when the inflaton field decays in the minimum of the potential, which happens when the Hubble parameter is of the order of the decaying rate, because all the energy density of the inflation vanishes when it reaches the minimum of the potential [24]. However, this does not happen anymore in quintessence inflation where the energy density of the inflaton field at the end of inflation does survive immediately after the phase transition, which could lead to a misunderstanding in the definition of the reheating temperature. For example, in curvaton reheating, the reheating temperature is sometimes calculated at the moment when the curvaton field has completely decayed [20, 21]) although the energy density of the curvaton field was subdominant at the end of its decay, or in instant preheating some authors identify the reheating temperature as the temperature of produced particles when they are created, whose energy density is clearly subdominant at that moment [25, 26]. Here we will use the usual convention, and we will denote by T_R the temperature of the relativistic plasma in thermal equilibrium at the beginning of the radiation-domination era.

To obtain bounds for the reheating temperature, first of all one has to notice that the radiation-domination era is previous to the Big Bang Nucleosynthesis (BBN) epoch which occurs in the 1 MeV regime [27], meaning that the

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reheating temperature has to be greater than 1 MeV. On the other hand, many supergravity and superstring theories contain particles, such as the gravitino or a modulus field, with only gravitational interactions, and consequently the thermal production of these relics and its late time decay may jeopardize the success of the standard BBN [28]. This problem can be solved by assuming sufficiently low reheating temperature, of the order of 10^9 GeV [29]. Moreover, in [30] the authors realized, using heuristic arguments, that a non-thermal production of these gravitational relics is possible during the inflationary phase, which imposes upper bounds on the reheating temperature as low as 100 GeV. In the present work, we will take a prudent viewpoint and we will restrict the reheating temperature to be between 1 MeV and 10^9 GeV.

For some of the presented mechanisms, preheating consists in the creation, after the phase transition to a kination regime, of heavy massive particles that have to decay into lighter ones to form a relativistic plasma whose energy density, after the plasma achieves the thermal equilibrium, will eventually dominate ahead of the energy density of the background. Then, one of the main targets of the present work is to show the importance of the parameters appearing in the definition of the decaying rate, as the parameters involved in the particle production process such as the bare mass of the heavy particles, the coupling constant associated to the interaction between the inflaton and the quantum field, the coupling constant associated to the interaction of the quantum field with gravity or the curvaton mass, which must satisfy several algebraic constraints due to physical conditions such as the BBN success, the negligible effect of the vacuum polarization during inflation or the fact that the energy density of the massive produced particles was subdominant before its decay, which has to be imposed in order to obtain a viable reheating temperature.

Our study is based in an improved version of the well-know Peebles-Vilenkin model [6], which leads to theoretical values such as the power spectrum and the ratio of tensor to scalar perturbations agreeing with the recent observational data obtained in [31]. For our model of quintessence inflation, which contains an abrupt phase transition from inflation to kination where the adiabatic regime is broken producing particles, we study with great detail four well-known different mechanisms to reheat the universe. Moreover, we will find the algebraic constraints satisfied for the parameters involved in the reheating process obtaining explicit bounds for all these parameters and, for the allowed values of them, we calculate analytically the corresponding reheating temperature for any mechanism. We also show that, choosing properly the parameters appearing in the theory, the overproduction of gravitational waves during the phase transition does not affect the BBN process. At this point, it is important to realize that in our study we do not consider the possible existence of dark radiation, e.g., sterile neutrinos, supported by the BBN-predicted relic lithium abundance, which would alter the evolution of the universe by ceasing to be in a radiation regime, and thus modifying the BBN bounds (see for instance [32] for a review).

The present work is structured as follows. In Section II we start discussing the instant preheating formalism in non-oscillatory models showing that, for this reheating mechanism, the particle production is equivalent to the well-known Schwinger's effect, i.e., the particle production in a constant electric field. Once we have obtained the energy density of these massive particles, we study its decay into light products and find the constraints that the parameters involved in the theory must satisfy. For the viable values of these parameters we calculate analytically the reheating temperature of the universe, which is around a million of TeV. Section III is devoted to the study of the production of massless particles nearly conformally coupled with gravity. In this case, the calculation of the vacuum modes is perturbative, so using the first Picard's iteration we will show that the reheating temperature is in the TeV regime. The production of heavy massive particles conformally coupled with gravity (particles with masses greater than the one of the inflaton) is considered in Section IV, where to calculate analytically the vacuum modes one can use the WKB approximation. The reheating temperature, which could be calculated as a function of the inflation mass, the mass of the quantum field and the decay rate, is lower than in the previous cases, and depending on the values of the parameters involved in the decaying rate it could be in the GeV or TeV regime. Finally, in last Section we deal with the particle creation due to the decay of the curvaton field. We show that, in the case that the curvaton decays when its energy density is subdominant, in order to obtain temperatures compatible with the BBN success the mass of the curvature field must be smaller than $10^{-7}M_{pl}$.

The units used throughout the paper are $\hbar = c = 1$, and we denote the Planck's and reduced Planck's mass respectively by $m_{pl} \equiv \frac{1}{\sqrt{G}} \cong 1.2 \times 10^{19}$ GeV and $M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \cong 2.4 \times 10^{18}$ GeV. We also denote, as we have already pointed out, by T_R the temperature of the universe at the beginning of the radiation epoch, which is the reheating temperature of the universe. Finally we will denote by Γ the thermalization rate and by $\bar{\Gamma}$ the decay rate of a process.

2. INSTANT PREHEATING IN NO-OSCILLATORY MODELS

In this Section we review and discuss in detail the work of Felder, Kofman and Linde [17] where a mechanism called *instant preheating*, which was introduced in [16] in the framework of standard inflation, was applied to the so-called non-oscillatory models, i.e., to models where the inflaton field, instead of oscillating, moves monotonically towards ∞ .

The full Lagrangian density of the model can be split as follows: $\mathcal{L}_{tot} = \mathcal{L}_{grav} + \mathcal{L}_{\varphi} + \mathcal{L}_{\chi} + \mathcal{L}_{int}$ where $\mathcal{L}_{grav} = \frac{M_{pl}^2}{2}R$, being R the Ricci or scalar curvature, is the gravitational part, $\mathcal{L}_{\varphi} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$ corresponds to the Lagrangian energy density of the inflaton field φ which is minimally coupled to gravity, $\mathcal{L}_{\chi} = \frac{1}{2}(-\partial_{\mu}\chi\partial^{\mu}\chi - m_{\chi}^2\chi^2 - \frac{1}{6}R\chi^2)$ [33] is the Lagrangian density of the massive quantum field χ , with mass m_{χ} , conformally coupled with gravity, and $\mathcal{L}_{int} = -\frac{1}{2}g^2\varphi^2\chi^2$, with g a coupling constant, is the interacting part of the Lagrangian density.

Then, the party responsible for the instant preheating is the interacting part of the Lagrangian density, because due to a phase transition from inflation to a kination regime [34], the adiabatic behavior is broken and the quantum field χ ceases to be in the vacuum state to produce heavy massive particles. These created particles will decay into light ones forming a relativistic plasma which will reach the thermal equilibrium and whose energy density eventually dominates those of the background, leading to a radiation-dominated universe. However, we want to stress that to obtain particle creation, instead of considering an interaction between the inflation and the quantum field, one can deal with a massive quantum field with a quartic self-interaction and coupled with gravity as has been recently done in [35].

Once we have presented the idea behind instant preheating, dealing with the flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, and working in Fourier space, the dynamical equation of the field χ coming from the variation of the action with respect χ is given by

$$\chi_k'' + 2\mathcal{H}\chi_k' + \left(k^2 + (m_{\chi}^2 + g^2\varphi^2)a^2 + \frac{a''}{a}\right)\chi_k = 0, \quad (1)$$

where the derivative is with respect the conformal time τ , and \mathcal{H} is the conformal Hubble parameter.

To better understand this equation it is useful to perform the change of variable $\bar{\chi} = a\chi$, because (1) becomes then the equation of a time dependent harmonic oscillator

$$\bar{\chi}_k'' + \omega_k^2(\tau)\bar{\chi}_k = 0, \quad (2)$$

with a frequency equal to

$$\omega_k(\tau) = \sqrt{k^2 + a^2(\tau)(m_{\chi}^2 + g^2\varphi^2(\tau))}. \quad (3)$$

To clarify and simplify ideas about preheating, we will consider, although the reasoning will serve for a general class of quintessence inflation models, a simple model based in an improved version of the well-known Peebles-Vilenkin model [6]:

$$V(\varphi) = \begin{cases} \frac{1}{2}m^2(\varphi^2 + M^2) & \text{for } \varphi \leq 0 \\ \frac{1}{2}m^2\frac{M^6}{\varphi^4 + M^4} & \text{for } \varphi \geq 0, \end{cases} \quad (4)$$

where m is the mass of the inflaton, which as we will immediately see is of the order $m \sim 5 \times 10^{-6}M_{pl}$, and M is a very small mass compared with the reduced Planck's mass M_{pl} , whose numerical value is determined in order that, at the present time, the ratio of the energy density of the inflaton field φ to the critical energy density is approximately 0.7 [31], i.e., $\frac{\rho_{\varphi,0}}{3H_0^2 M_{pl}^2} \cong 0.7$, where the sub-index 0 means "at present time" and $\rho_{\varphi} = \frac{\dot{\varphi}^2}{2} + V(\varphi)$. In fact, heuristically the value of M could be calculated as follows: during the radiation and matter domination epoch the inflation field is all the time of the order M_{pl} (see [6] for a detailed discussion). Then, in our model the field will dominate at late time when

$$\frac{m^2 M^6}{M_{pl}^4 + M^4} \sim \frac{m^2 M^5}{M_{pl}^4} \sim H_0^2 M_{pl}^2 \implies M \sim \left(\frac{H_0}{m}\right)^{\frac{1}{3}} M_{pl} \sim 10^{-18} M_{pl} \sim 1 \text{ GeV}, \quad (5)$$

where we have used that the current value of the Hubble parameter is $H_0 \sim 10^{-61}M_{pl}$.

Remark 2.1 We can see that for $\varphi \leq 0$ the potential is quadric, i.e., it belongs in the class of large-field inflationary models [36], and for $\varphi \geq 0$ it is a inverse power law potential which leads to quintessence (see for instance [7]). On the other hand, in the original Peebles-Vilenkin model the inflationary piece of the potential was quartic, which implies that the theoretical values of spectral index and the ratio of tensor to scalar perturbations do not enter in the marginalized joint confidence contour in the plane (n_s, r) at 2σ C.L., without the presence of running [37]. For this reason, we have changed it by a quadratic potential, because the spectral values provided by a quadratic potential enter in this contour [37].

The dynamics of the model is not difficult to understand. When $\varphi \ll -M_{pl}$ the field slow-rolls, and thus the universe inflates, after inflation there is a phase transition from inflation to kination [34] which occurs about $\varphi \cong 0$ and particles are produced. Since in a kination regime the energy density of the background decays as a^{-6} , this allows a relativistic plasma in thermal equilibrium, whose energy density evolves as a^{-4} , to eventually become dominant, and the universe is thus reheated. Finally, at the present time, the potential energy of the scalar field φ becomes dominant once again and the universe accelerates, depicting the current cosmic acceleration.

Choosing $\tau = 0$ as the phase transition time, writing $\varphi(\tau) \cong \varphi'(0)\tau$ for values of τ close to zero, we can approximate the frequency $\omega_k(\tau)$ by $\sqrt{k^2 + a^2(0)(m_\chi^2 + g^2(\varphi'(0))^2\tau^2)}$, where near the phase transition we do not have taken into account the expansion of the universe, and thus we have approximated $a(\tau)$ by $a(0)$.

Taking in mind all these approximations, what we have obtained is the well-known over-barrier problem in scattering theory [38], where the β_k -Bogoliubov coefficient is related with the reflection coefficient via the formula (see [39–41] and [42, 43] for a mathematical explanation)

$$|\beta_k|^2 = e^{-Im \int_\gamma \omega_k(\tau) d\tau}, \quad (6)$$

with γ a closed path in the complex plan containing the two turning points $\tau_\pm = \pm i \frac{\sqrt{k^2 + a^2(0)m_\chi^2}}{a(0)g|\varphi'(0)|}$.

A simple calculation shows that the number of particles in the k -mode is given by [41]

$$n_k \equiv |\beta_k|^2 = e^{-\frac{\pi(k^2 + a^2(0)m_\chi^2)}{a(0)g|\varphi'(0)|}}. \quad (7)$$

Remark 2.2 What is important in this approach is that, strictly speaking, n_k will be the number of created particles at late times. However, as the authors argue in [17], high wavelength particles are produced when the effective mass of the field χ starts to change non-adiabatically, $g|\dot{\varphi}| \geq g^2\varphi^2$, which happens at the phase transition. In fact, as one can see from formula (7), high frequency modes are exponentially suppressed.

Remark 2.3 Essentially, instant preheating is the same as the Schwinger's effect [44], i.e., the particle production in a constant electric field [45–47], because in both processes the time dependent frequency has the same particular form $\omega(t) = \sqrt{A^2 + B^2t^2}$, where A and B are constant.

Then, the number density of produced particles is given, in terms of the cosmic time t , by [33]

$$n_\chi(t) \equiv \frac{1}{2\pi^2 a^3(t)} \int_0^\infty k^2 |\beta_k|^2 dk = \frac{(g|\dot{\varphi}(0)|)^{\frac{3}{2}}}{8\pi^3} \left(\frac{a(0)}{a(t)}\right)^3 e^{-\frac{\pi m_\chi^2}{g|\dot{\varphi}(0)|}}, \quad (8)$$

where the dot means the derivative with respect the cosmic time, and we have chosen $t = 0$ as the phase transition time. Analogously, the energy density of the produced particles is given by [33]

$$\rho_\chi(t) \equiv \frac{1}{2\pi^2 a^4(t)} \int_0^\infty \omega_k(t) k^2 |\beta_k|^2 dk. \quad (9)$$

From these formulas, one can see that at the phase transition

$$\rho_\chi(0) = \frac{g^2 \dot{\varphi}^2(0)}{4\pi^4} e^{-\frac{\pi m_\chi^2}{g|\dot{\varphi}(0)|}}, \quad (10)$$

and at late times $\rho_\chi(t) \cong \sqrt{m_\chi^2 + g^2 \varphi^2(t)} n_\chi(t)$, meaning that χ -particles acquire a effective mass $m_{eff} \equiv \sqrt{m_\chi^2 + g^2 \varphi^2(t)}$.

Three constraints must be imposed [17]:

1. To avoid an exponential suppression of the energy density, the bare mass has to satisfy $m_\chi \leq \sqrt{g|\dot{\varphi}(0)|}$. For the sake of simplicity, in this work we will take $m_\chi = 0$.
2. Recall that the effective mass of the field χ is now $g|\varphi|$ and taking into account that for masses greater than the Hubble parameter the vacuum polarization energy density due to the field χ , which could be calculated using the WKB approximation, is of the order $\frac{H^6}{g^2 \varphi^2}$ [48], as this quantity is smaller than the energy density of the background ($\sim H^2 M_{pl}^2$), when there is a classical picture of the universe, which happens for $H \leq 10^{-2} M_{pl}$ (see Section IV), one can conclude that imposing the condition $H \ll g|\varphi|$, the polarization effects will not affect the dynamics of the universe during inflation. On the other hand, during the slow roll regime $H \cong \sqrt{\frac{V}{3M_{pl}^2}}$, thus for a quadratic potential the condition $H \ll g|\varphi|$ is accomplished imposing $g \gg \frac{m}{M_{pl}}$.
3. The energy density of the produced χ -particles cannot dominate before decaying into light particles, which will form the relativistic plasma, because if so, the force driving the inflation back to $\varphi = 0$ will not disappear and the inflaton field could not continue its movement forward up to ∞ . The interaction term $\frac{1}{2}g^2\varphi^2\chi^2$ effectively entails that, after the phase transition, the inflaton field satisfies the equation

$$\ddot{\varphi} + 3H\dot{\varphi} = -g^2\chi^2\varphi. \quad (11)$$

Then, when the energy density of the χ -particles is sub-dominant, the right hand side of (11) is negligible and the field rolls towards ∞ , but when it is dominant the right hand side of (11) ceases to be negligible, meaning that the inflaton field is under the action of the quadratic potential $\frac{1}{2}g^2\varphi^2\chi^2$, and, thus, it will roll down to zero, which may produce a new inflationary phase.

To avoid this situation, first of all we have to calculate the energy densities of the background and of the field χ at the equilibrium, that is, when they are of the same order

$$\rho(t_{eq}) \sim \rho_\chi(t_{eq}) \iff 3H^2(t_{eq})M_{pl}^2 \sim g\varphi(t)n_\chi(t_{eq}). \quad (12)$$

To obtain these quantities, first of all we use that, for the model presented here, whose potential is given by (4), after the phase transition the universe enters in a kination regime and the evolution of universe is given by

$$\dot{H} = -3H^2 \iff H(t) = \frac{H(0)}{3H(0)t + 1} \iff a(t) = a(0)(3H(0)t + 1)^{\frac{1}{3}}. \quad (13)$$

Secondly, using the Raychauduri equation $\dot{H} = -\frac{\dot{\varphi}^2}{2M_{pl}^2}$, one gets

$$\varphi = M_{pl} \int_0^t \sqrt{-2\dot{H}(s)} ds = M_{pl} \int_{H(t)}^{H(0)} \sqrt{\frac{-2}{\dot{H}(H)}} dH, \quad (14)$$

which for $t > 0$, leads to

$$\varphi(t) = \sqrt{\frac{2}{3}} M_{pl} \ln(3H(0)t + 1). \quad (15)$$

Taking into account this results, and using that $\rho_\chi(t) \cong g\varphi(t)n_\chi(t)$, one arrives at

$$\rho_\chi(t) \cong \frac{6^{1/4}}{4\pi^3} g^{5/2} M_{pl}^{5/2} H^{3/2}(0) \ln(3H(0)t + 1) \left(\frac{a(0)}{a(t)} \right)^3, \quad (16)$$

and $\rho(t) = 3H^2(0)M_{pl}^2 \left(\frac{a(0)}{a(t)} \right)^6$. Then, both quantities are of the same order when

$$t \sim t_{eq} \equiv \frac{2\pi^3}{g^{5/2} \sqrt{H(0)M_{pl}}}, \quad (17)$$

which yields an important constraint for this theory: in order that the back-reaction is subdominant and the inflaton field rolls monotonically towards ∞ , the decaying time, i.e., when the particles decay in a relativistic plasma, must be smaller than the equilibrium time t_{eq} .

On the other hand, assuming as usual that there is no substantial drop of energy between the end of inflation and the beginning of the kination regime, and using that the value of the power spectrum of the curvature fluctuation in co-moving coordinates when the pivot scale leaves the Hubble radius is given by [36] $\mathcal{P}_\zeta \cong \frac{H_*^2}{8\pi^2 M_{pl}^2 \epsilon_*} \sim 2 \times 10^{-9}$, where $\epsilon = \frac{M_{pl}^2}{2} \left(\frac{V_\varphi}{V} \right)^2$ is the main slow roll parameter and the "star" means that the quantity is evaluated when the pivot scale leaves the Hubble radius, one obtains

$$m^2 \sim 3 \times 10^{-9} \pi^2 (1 - n_s)^2 M_{pl}^2, \quad (18)$$

where we have used that, for our model, since $-\varphi_* \gg M_{pl}$, one has $\epsilon_* = \frac{2M_{pl}^2 \varphi_*^2}{(\varphi_*^2 - M_{pl}^2)^2} \cong \frac{2M_{pl}^2}{\varphi_*^2}$ and thus $\epsilon_* \cong \frac{1-n_s}{4}$, where n_s denotes the spectral index. Then, since recent observations constraint the value of the spectral index to be $n_s = 0.968 \pm 0.006$ [31], taking its central value one gets $m \cong 5 \times 10^{-6} M_{pl}$, and as a consequence, $H(0) \sim H_{end} \cong \frac{m \sqrt{\varphi_{end}^2 - M_{pl}^2}}{\sqrt{6} M_{pl}} = \sqrt{\frac{1+\sqrt{3}}{6}} m \cong 3 \times 10^{-6} M_{pl}$ (recall that inflation ends when $\epsilon = 1 \implies \varphi_{end} = -\sqrt{2 + \sqrt{3}} M_{pl}$). So, for values of $g \leq 10^{-2}$ one obtains $t_{eq} \geq 10^8 M_{pl}^{-1}$ and $\varphi(t_{eq}) \sim M_{pl}$. Thus, for times $t \in [10^6 M_{pl}^{-1}, t_{eq}]$ the value of the inflation field remains close to M_{pl} , and the effective mass of the χ -field will be gM_{pl} .

Now, assuming that the χ -field interacts with fermion particles via a Yukawa type interaction $h\psi\bar{\psi}\chi$, the decaying rate will be $\bar{\Gamma} = \frac{h^2 g \varphi(t)}{8\pi}$, where h is a coupling constant (see [50] and references therein). As we have shown, the inflaton field spends most of the time prior to t_{eq} at $\varphi \sim M_{pl}$. Therefore one can safely take $\bar{\Gamma} = \frac{h^2 g M_{pl}}{8\pi}$, and, thus, the condition $t_{dec} < t_{eq}$, where $t_{dec} \cong \frac{8\pi}{3} \frac{1}{h^2 g} M_{pl}^{-1}$ is the time when the field χ decayed, i.e., $H(t_{dec}) \sim \bar{\Gamma}$, leads to the relation $g < 3 \times 10^2 h^{4/3}$, which together with the condition $g \gg \frac{m}{M_{pl}} \cong 3 \times 10^{-6}$, constraints the value of h to fulfill $h \gg 10^{-6}$.

As we have already pointed out, the χ -field must have completely decayed before the equilibrium, then the temperature at the equilibrium time, namely \bar{t}_{eq} , i.e., when $\rho(\bar{t}_{eq}) \sim \rho_\chi(\bar{t}_{eq})$, (note that we write \bar{t}_{eq} instead of t_{eq} , because the equilibrium time is different depending on whether the χ -field decays before or after the equality of both energy densities), is calculated as follows: choosing $h^2 g < 8 \times 10^{-6} \iff t_{dec} > 10^6 M_{pl}^{-1}$, which means $\varphi(t_{dec}) \sim M_{pl}$, since the corresponding energy densities evolve as

$$\rho(\bar{t}_{eq}) = \rho(t_{dec}) \left(\frac{a(t_{dec})}{a(\bar{t}_{eq})} \right)^6 \quad \text{and} \quad \rho_\chi(\bar{t}_{eq}) = \rho_\chi(t_{dec}) \left(\frac{a(t_{dec})}{a(\bar{t}_{eq})} \right)^4, \quad (19)$$

one has $\left(\frac{a(t_{dec})}{a(\bar{t}_{eq})} \right)^2 = \frac{\rho_\chi(t_{dec})}{\rho(t_{dec})}$, and thus, the temperature at the equilibrium is given by

$$T_{eq} \sim \rho_\chi^{1/4}(\bar{t}_{eq}) = \rho_\chi^{1/4}(t_{dec}) \sqrt{\frac{\rho_\chi(t_{dec})}{\rho(t_{dec})}}. \quad (20)$$

Taking into account that

$$\rho(t_{dec}) = 3\bar{\Gamma}^2 M_{pl}^2 \quad \text{and} \quad \rho_\chi(t_{dec}) = g M_{pl} n_\chi(t_{dec}) \cong 10^{-2} g^{5/2} \sqrt{H(0) M_{pl}} M_{pl}^2 \bar{\Gamma}, \quad (21)$$

one gets

$$T_{eq} \sim 2 \times 10^{-2} g^{15/8} H^{3/8}(0) M_{pl}^{7/8} \bar{\Gamma}^{-1/4} \cong 5 \times 10^{14} \frac{g^{13}}{\sqrt{h}} \text{ GeV}. \quad (22)$$

In the case of a instantaneous thermalization, the reheating time t_R coincides with \bar{t}_{eq} , and the reheating temperature will be $T_R = T_{eq}$, which means that $10^{-11} h^{\frac{4}{13}} \leq g \leq 3 \times 10^{-4} h^{\frac{4}{13}}$. Then, one can conclude that there is a narrow range of values of the parameters h and g accomplishing all the requirements:

1. $5 \times 10^{-6} \ll g < 3 \times 10^2 h^{4/3}$. (The back-reaction is not important during inflation and the χ -field decays before equilibrium)
2. $g < 8 \times 10^{-6} h^{-2}$. (The decay ends when the value of the inflaton field is of the order of the reduced Planck mass).
3. $10^{-11} h^{\frac{4}{13}} \leq g \leq 3 \times 10^{-4} h^{\frac{4}{13}}$ (Reheating temperatures guaranteeing the BBN success).

Finally, the choices ($h = 10^{-1}$, $g = 10^{-4}$) or ($h = 10^{-2}$, $g = 5 \times 10^{-5}$), which satisfy the above conditions, lead to the same reheating temperature $T_R \sim 5 \times 10^8 \text{ GeV}$.

3. REHEATING VIA GRAVITATIONAL PRODUCTION OF MASSLESS PARTICLES

In this section, we do not consider any interaction between the inflaton and the quantum field χ , and we assume that the particles are nearly conformally coupled with gravity, i.e., the coupling constant with gravity, namely ξ , is approximately equal to $\frac{1}{6}$. Then, the Klein-Gordon equation for the quantum χ -field is given by

$$\bar{\chi}_k''(\tau) + \left(k^2 + \left(\xi - \frac{1}{6} \right) a^2(\tau) R(\tau) \right) \bar{\chi}_k(\tau) = 0, \quad (23)$$

where once again $\bar{\chi} = a\chi$.

To define the vacuum modes before and after the phase transition, we assume that at early and late time the term $a^2 R$ will vanish. Then the behavior at early and late times is respectively

$$\bar{\chi}_{in,k}(\tau) \simeq \frac{e^{-ik\tau}}{\sqrt{2k}} \quad (\text{when } \tau \rightarrow -\infty), \quad \bar{\chi}_{out,k}(\tau) \simeq \frac{e^{-ik\tau}}{\sqrt{2k}} \quad (\text{when } \tau \rightarrow +\infty). \quad (24)$$

Therefore, the vacuum modes at early ("in" modes) and late times ("out" modes) (exact solutions of (23)) are given by [51]

$$\begin{aligned} \bar{\chi}_{in,k}(\tau) &= \frac{e^{-ik\tau}}{\sqrt{2k}} - \frac{\xi - 1/6}{k} \int_{-\infty}^{\tau} a^2(\tau') R(\tau') \sin(k(\tau - \tau')) \bar{\chi}_k(\tau') d\tau', \\ \bar{\chi}_{out,k}(\tau) &= \frac{e^{-ik\tau}}{\sqrt{2k}} + \frac{\xi - 1/6}{k} \int_{\tau}^{\infty} a^2(\tau') R(\tau') \sin(k(\tau - \tau')) \bar{\chi}_k(\tau') d\tau'. \end{aligned} \quad (25)$$

On the other hand, since we are considering particles nearly conformally coupled to gravity, we can consider the term $(\xi - 1/6)a^2(\tau)R(\tau)$ as a perturbation, and we can approximate the "in" and "out" modes by the first order Picard's iteration, i.e., inserting (24) in the right hand side of (25), as

$$\begin{aligned} \bar{\chi}_{in,k}(\tau) &\cong \frac{e^{-ik\tau}}{\sqrt{2k}} - \frac{\xi - 1/6}{k\sqrt{2k}} \int_{-\infty}^{\tau} a^2(\tau') R(\tau') \sin(k(\tau - \tau')) e^{-ik\tau'} d\tau', \\ \bar{\chi}_{out,k}(\tau) &\cong \frac{e^{-ik\tau}}{\sqrt{2k}} + \frac{\xi - 1/6}{k\sqrt{2k}} \int_{\tau}^{\infty} a^2(\tau') R(\tau') \sin(k(\tau - \tau')) e^{-ik\tau'} d\tau', \end{aligned} \quad (26)$$

which will represent, respectively, the vacuum before and after the phase transition.

After the phase transition, we could write the “in” mode as a linear combination of the “out” mode and its conjugate as follows

$$\bar{\chi}_{in,k}(\tau) = \alpha_k \bar{\chi}_{out,k}(\tau) + \beta_k \bar{\chi}_{out,k}^*(\tau), \quad (27)$$

and imposing the continuity of $\bar{\chi}$ and its first derivative at the transition time we obtain, up to order $(\xi - 1/6)^2$, that the value of these coefficients [52, 53] will be

$$\alpha_k \cong 1 - \frac{i(\xi - \frac{1}{6})}{2k} \int_{-\infty}^{\infty} a^2(\tau) R(\tau) d\tau, \quad \beta_k \cong \frac{i(\xi - \frac{1}{6})}{2k} \int_{-\infty}^{\infty} e^{-2ik\tau} a^2(\tau) R(\tau) d\tau, \quad (28)$$

where the integral of the β -Bogoliubov coefficient (28) is convergent because, at early and late times, the term $a^2(\tau)R(\tau)$ converges fast enough to zero.

The energy density of the produced particles due to the phase transition is given by [33]

$$\rho_\chi = \frac{1}{2\pi^2 a^4} \int_0^\infty k^3 |\beta_k|^2 dk, \quad (29)$$

where, if at the transition time, namely once again $t = 0$, the first derivative of the Hubble parameter is continuous one has $\beta_k \sim \mathcal{O}(k^{-3})$, which means that this energy density is not ultra-violet divergent, and it approximately becomes [51]

$$\rho_\chi(t) \cong \left(\xi - \frac{1}{6}\right)^2 \mathcal{N} H^4(0) \left(\frac{a(0)}{a(t)}\right)^4, \quad (30)$$

where \mathcal{N} is a dimensionless numerical factor.

To understand this formula, we take $\tau = 0$ as the value of the conformal time at the transition time, and we assume that at that time the second (or greater) derivative of the Hubble parameter is discontinuous. Then, since $\frac{d^n(a^2 R)}{d\tau^n}(0) = C(n)a^{2+n}(0)H^{2+n}(0)$, where $C(n)$ is a dimensionless constant that only depends on n , integrating by parts one gets

$$\beta_k = \left(\xi - \frac{1}{6}\right) \sum_{n=1}^{\infty} (-i)^n C(n) \frac{a^{2+n}(0)H^{2+n}(0)}{(2k)^{2+n}} \equiv \left(\xi - \frac{1}{6}\right) f\left(\frac{k}{a(0)H(0)}\right), \quad (31)$$

where f is some function. Thus, inserting this expression in (29) and performing the change of variable $s = \frac{k}{a(0)H(0)}$ one gets the expression (30), with $\mathcal{N} = \frac{1}{2\pi^2} \int_0^\infty s^3 |f(s)|^2 ds$.

Remark 3.1 *The number \mathcal{N} is clearly model dependent. For the example proposed by Ford in [51] mimicking a transition from de Sitter to a matter domination modeled by $a^2(\tau)R(\tau) \equiv \frac{12}{\tau^2 + \tau_0^2}$, \mathcal{N} could be calculated analytically giving as a result $\frac{9}{8}$. However, note that in this case reheating is impossible because the energy density of the produced particles decrease faster than those of the background. We have calculated numerically this factor for some simple models that have a transition from a de Sitter regime to a deflationary one, and in all cases \mathcal{N} is of the order 1 (see for instance [54]).*

Note also that reheating via particle production of massless particles suffers the overproduction of gravitational waves [6] which could destabilize the BBN process. To avoid this challenge one has to impose that the *heating efficiency*, namely Θ , and defined as the ratio of the energy density of the produced particles to the energy density of the background at the beginning of the kination epoch $\left(\Theta = \frac{\rho_\chi(0)}{\rho(0)}\right)$, satisfies the constraint [55] (see also [14])

$$\Theta \geq 6 \times 10^{-3} \left(\frac{H(0)}{M_{pl}}\right)^2 \implies \left(\xi - \frac{1}{6}\right)^2 \geq 2 \times 10^{-2} \implies \left(\xi - \frac{1}{6}\right) \sim 10^{-1}. \quad (32)$$

On the other hand, and contrary to the previous Section, here we will assume that thermal equilibrium of the produced particles is not an instantaneous process [49]. Instead, we will consider the following thermalization rate $\Gamma = n_\chi(0)\sigma_{2\rightarrow 3}$, where the most important processes for the chemical equilibrium are $2 \rightarrow 3$ scatterings with gauge boson exchange in the t-channel, whose typical energy energy is $E \sim H(0) \left(\frac{a(0)}{a(t)}\right)$ (see Section V of [6]), and whose cross section is given by $\sigma_{2\rightarrow 3} = \frac{\alpha^3}{E^2}$ (see for instance the Section IV of [49]), where as usual, $\alpha^2 \sim 10^{-3}$ [5].

Since

$$n_\chi(t) = \frac{1}{2\pi^2 a^3(t)} \int_0^\infty k^2 |\beta_k|^2 dk = \left(\xi - \frac{1}{6}\right)^2 \mathcal{M} H^3(0) \left(\frac{a(0)}{a(t)}\right)^3, \quad (33)$$

where, for many models, one finds [56]

$$\mathcal{M} \equiv \frac{1}{16\pi a^3(0) H^3(0)} \int_{-\infty}^\infty a^4(\tau) R^2(\tau) d\tau \sim 1, \quad (34)$$

the thermalization rate will acquire the form

$$\Gamma = \alpha^3 \left(\xi - \frac{1}{6}\right)^2 \mathcal{M} H(0) \left(\frac{a(0)}{a(t)}\right). \quad (35)$$

The relativistic fluid reaches the thermal equilibrium at $t = t_{th}$ when $H(t_{th}) \sim \Gamma$ [5, 6], i.e., for $\left(\frac{a(0)}{a(t_{th})}\right)^2 \sim \alpha^3 \left(\xi - \frac{1}{6}\right)^2 \mathcal{M}$, meaning that the temperature at the thermalization time is of the order

$$T_{th} \sim \rho_\chi^{1/4}(t_{th}) \sim \alpha^{3/2} \left(\xi - \frac{1}{6}\right)^{3/2} \mathcal{N}^{1/4} \mathcal{M}^{1/2} H(0) \sim 5 \times 10^{-3} \left(\xi - \frac{1}{6}\right)^{3/2} \mathcal{N}^{1/4} \mathcal{M}^{1/2} H(0), \quad (36)$$

which for typical values $H(0) \sim 10^{-6} M_{pl}$ and $\xi - \frac{1}{6} \sim 10^{-1}$ leads to the temperature $T_{th} \sim 4 \times 10^8$ GeV.

Finally, the equilibrium occurs when the energy density of the background and that of the created particles are of the same order ($\rho_\chi(t_{eq}) \sim \rho(t_{eq})$), which implies, for $H(0) \sim 10^{-6} M_{pl}$, a temperature at the equilibrium of the order

$$T_{eq} \sim \rho_\chi^{1/4}(t_{eq}) = \rho_\chi^{1/4}(0) \sqrt{\frac{\rho_\chi(0)}{\rho(0)}} \sim \left(\xi - \frac{1}{6}\right)^{3/2} \mathcal{N}^{3/4} \left(\frac{H(0)}{M_{pl}}\right)^2 M_{pl} \sim 2 \times 10^6 \left(\xi - \frac{1}{6}\right)^{3/2} \text{ GeV}, \quad (37)$$

which for $\xi - \frac{1}{6} \sim 10^{-1}$ leads to the temperature $T_{eq} \sim 6 \times 10^4$ GeV. Since $T_{eq} \leq T_{th}$, one can conclude that the thermalization occurs well before the equilibrium, and thus, the reheating temperature, i.e., the temperature of the universe when it is dominated by a relativistic plasma in thermal equilibrium, is $T_R = T_{eq}$.

4. REHEATING VIA GRAVITATIONAL PRODUCTION OF HEAVY MASSIVE PARTICLES

In this section we will consider the creation of heavy massive particles conformally coupled with gravity, disregarding any interaction with the inflaton field, although it is also possible to deal with the production of heavy massive particles minimally coupled with gravity whose mass depends on the inflaton field [55]. In that situation, the frequency of the particles in the k -mode is $\omega_k(\tau) = \sqrt{k^2 + m_\chi^2 a^2(\tau)}$, and during the adiabatic regimes, in order to calculate the k -vacuum mode one can use the WKB approximation [57]

$$\bar{\chi}_{n,k}^{WKB}(\tau) \equiv \sqrt{\frac{1}{2W_{n,k}(\tau)}} e^{-i \int^\tau W_{n,k}(\eta) d\eta}, \quad (38)$$

where n is the order of the approximation.

When some high order derivatives of the Hubble parameter are discontinuous, and thus the adiabatic regime breaks down, to obtain the evolution of the vacuum one has to match the k -vacuum mode (approximated by $\bar{\chi}_{n,k}^{WKB}$) before this

moment with a linear combination of positive and negative frequency modes (approximated by a linear combination of $\bar{\chi}_{n,k}^{WKB}$ and its conjugate), which is the manifestation of the gravitational particle production. Basically, this is Parker's viewpoint of particle creation in curved space-times [8].

What is important to keep in mind is when is it possible to apply the WKB approximation. It is well-known that at temperatures of the order of the Planck's mass quantum effects become very important and it is impossible to have a classical picture of the universe. However, at temperatures below $m_{pl} = \frac{1}{\sqrt{G}}$, for example $T \sim 10^{-1} M_{pl} \sim 10^{17}$ GeV, as has been explained in the introduction of [1], the beginning the hot big bang scenario is possible. Since, for the flat FLRW universe $T \sim \rho^{1/4} \sim \sqrt{HM_{pl}}$ one can safely deduce that a classical picture of the universe is possible at scales of the order $H \sim 10^{-2} M_{pl}$. Thus, since at the beginning of inflation the Hubble parameter is practically constant the universe is approximately in a de Sitter phase where, for a massive quantum field, the vacuum polarization, which was calculated in [58] showing that for masses less than the reduced Planck's one, it is subdominant with respect to the energy density of the background ($H^2 M_{pl}^2$). Therefore, if one wants these polarization effects to also be subdominant at the last stages of inflation, one could impose that $m_\chi \gg H_*$, where once again H_* denotes the value of the Hubble parameter when the pivot scale leaves de Hubble radius, because one can use the WKB approximation to calculate the vacuum modes obtaining a vacuum energy density of the order $\frac{H_*^6}{m_\chi^2}$ [48] which is also subdominant. On the other hand, particles with mass greater than the Planck's one (in fact, for masses satisfying $m_\chi \geq \frac{m_{pl}}{\sqrt{2}}$) have a Compton wavelength smaller than their Schwarzschild radius $\frac{2m_\chi}{m_{pl}^2}$. These created particles becomes micro or Planck-size Black Holes, whose physics is unknown [59] because the semiclassical thermodynamic description breaks down – Hawking's formulas about evaporation are not applicable – and it is not clear whether or how they radiate [60]. For this reason, and since for a quadratic potential $H_* \cong 2 \times 10^{-5} M_{pl}$, we have to consider massive quantum fields satisfying

$$2 \times 10^{-5} M_{pl} \ll m_\chi \leq 2\sqrt{\pi} M_{pl}. \quad (39)$$

In order to deal with an analytically solvable problem, i.e., having an analytic expression of the β -Bogoliubov coefficient, we consider a phase transition where the second derivative of the Hubble parameter is discontinuous, for example, the following model which is another improved version of the well-known Peebles-Vilenkin potential [6]

$$V(\varphi) = \begin{cases} \frac{1}{2} m^2 \left(\varphi^2 - M_{pl}^2 + M^2 \right) & \text{for } \varphi \leq -M_{pl} \\ \frac{1}{2} m^2 \frac{M^6}{(\varphi + M_{pl})^4 + M^4} & \text{for } \varphi \geq -M_{pl}, \end{cases} \quad (40)$$

with $m \cong 5 \times 10^{-6} M_{pl}$ and $M \sim 1$ GeV.

To study the dynamics given by this potential, we consider the effective Equation of State parameter which is equal to $w_{eff} = -1 - \frac{2\dot{H}}{3H^2} = -1 + \frac{2}{3}\epsilon$, where $\epsilon = -\frac{\dot{H}}{H^2} \cong \frac{M_{pl}^2}{2} \left(\frac{V_\varphi}{V} \right)^2$ is once again the mean slow-roll parameter. For $\varphi \ll -M_{pl}$, one has $\epsilon \ll 1$ (slow-roll period) and then $w_{eff} \cong -1$, what means that the universe is inflating. Immediately after the phase transition, which occurs near $\varphi = -M_{pl}$ due to the small value of M (the potential energy is negligible), all the energy becomes kinetic, and the universe enters in a kination [34] or deflationary [5] regime with $w_{eff} \cong 1$. After the kination the universe enters in the radiation regime, $w_{eff} \cong \frac{1}{3}$, when the energy density of the produced particles starts to dominate, and at the matter-radiation equality the universe enters in a matter domination regime with $w_{eff} \cong 0$. Finally, at the present time, the potential energy density of the inflaton fields dominates once again obtaining the current cosmic acceleration with w_{eff} near -1 .

Note that, for this model, the derivative of the potential is discontinuous at $\varphi = -M_{pl}$, which means, due to the conservation equation, that the second derivative of the inflaton field is discontinuous at the transition time, and consequently, from the Raychaudhuri equation $\dot{H} = -\frac{\dot{\varphi}^2}{2M_{pl}^2}$ one can deduce that the second derivative of the Hubble parameter is also discontinuous at this time.

In this case one only needs the the first order WBK solution to approximate the k -vacuum modes before and after the phase transition

$$\bar{\chi}_{1,k}^{WKB}(\tau) \equiv \sqrt{\frac{1}{2W_{1,k}(\tau)}} e^{-i \int^\tau W_{1,k}(\eta) d\eta}, \quad (41)$$

where [61]

$$W_{1,k} = \omega_k - \frac{1}{4} \frac{\omega_k''}{\omega_k^2} + \frac{3}{8} \frac{(\omega_k')^2}{\omega_k^3}, \quad (42)$$

because $W_{1,k}$ contains the first derivative of the Hubble parameter, and since the matching involves the derivative of the mode, the β -Bogoliubov coefficient does not vanish.

Before the transition time, namely $\tau = 0$, the vacuum mode is depicted by $\chi_{1,k}^{WKB}(\tau)$, but after the phase transition this mode becomes a mix of positive and negative frequencies of the form $\alpha_k \chi_{1,k}^{WKB}(\tau) + \beta_k (\chi_{1,k}^{WKB})^*(\tau)$. The β_k -Bogoliubov coefficient is obtained matching both expressions at $\tau = 0$, leading to

$$\beta_k = \frac{\mathcal{W}[\chi_{1,k}^{WKB}(0^-), \chi_{1,k}^{WKB}(0^+)]}{\mathcal{W}[(\chi_{1,k}^{WKB})^*(0^+), \chi_{1,k}^{WKB}(0^+)]}, \quad (43)$$

where $\mathcal{W}[f(0^-), g(0^+)] = f(0^+)g'(0^-) - f'(0^+)g(0^-)$ is the Wronskian of the functions f and g at the transition time, and $F(0^\pm) = \lim_{\tau \rightarrow 0} F(\pm|\tau|)$.

The square modulus of the β -Bogoliubov coefficient will be given approximately by [37]

$$|\beta_k|^2 \cong \frac{m_\chi^4 a^{10}(0) \left(\ddot{H}(0^+) - \ddot{H}(0^-) \right)^2}{256(k^2 + m_\chi^2 a^2(0))^5}, \quad (44)$$

with

$$\ddot{H}(0^+) - \ddot{H}(0^-) = -\frac{\dot{\varphi}(0)}{M_{pl}^2} (\ddot{\varphi}(0^+) - \ddot{\varphi}(0^-)) = -\frac{\dot{\varphi}(0)}{M_{pl}^2} V_\varphi(-M_{pl}^-) = \frac{m^2 \dot{\varphi}(0)}{M_{pl}} = m^3, \quad (45)$$

where, assuming that there is no substantial drop of energy density between the end of inflation and the beginning of kination, we have used that at the transition time all the energy at the end of inflation, which is approximately $\frac{1}{2} m^2 M_{pl}^2$ because $\varphi_{end} = -\sqrt{2} M_{pl}$, is converted into kinetic.

Then, for our model, the number density of produced particles and their energy density will be given by the following expressions

$$n_\chi(t) \sim 10^{-5} \left(\frac{m}{m_\chi} \right)^3 m^3 \left(\frac{a(0)}{a(t)} \right)^3, \quad \rho_\chi(t) \sim m_\chi n_\chi(t). \quad (46)$$

Remark 4.1 Using the second order WKB approximation, the number density of produced particles is corrected by a term of the order $10^{-5} \left(\frac{m}{m_\chi} \right)^5 m^3$ whose contribution is negligible due to the fact that $m \ll m_\chi$.

Remark 4.2 Contrary to the procedure used in [37, 56], we cannot take as a thermalization rate $\Gamma = n_\chi \sigma_{2 \rightarrow 3}$, because the created particles are very massive and this rate is only justified for light particles. Instead, first of all these particles must decay into lighter ones, which will interact by the exchange of bosons to reach the thermal equilibrium. In this Section, to simplify the calculations we will consider an instantaneous thermalization and we only will take into account the decaying process.

Considering the decay of the χ -field in fermions ($\chi \rightarrow \psi \bar{\psi}$), the rate will be $\bar{\Gamma} = \frac{h^2 m_\chi}{8\pi}$ [50], and the energy density of the background and the one of the relativistic plasma, when the decay is finished, i.e., when $\bar{\Gamma} \sim H(t_{dec}) = H(0) \left(\frac{a(0)}{a(t_{dec})} \right)^3 \cong \frac{m}{\sqrt{6}} \left(\frac{a(0)}{a(t_{dec})} \right)^3$, will be

$$\rho(t_{dec}) = 3\bar{\Gamma}^2 M_{pl}^2 \quad \text{and} \quad \rho_\chi(t_{dec}) \sim 2 \times 10^{-5} \left(\frac{m}{m_\chi} \right)^2 \frac{\bar{\Gamma}}{m} m^4. \quad (47)$$

Imposing that the end of the decay precedes the domination of the relativistic plasma formed by the decay products $\rho_\chi(t_{dec}) \leq \rho(t_{dec})$, one gets

$$h^2 \geq \frac{16\pi}{3} \times 10^{-5} \left(\frac{m}{m_\chi}\right)^3 \left(\frac{m}{M_{pl}}\right)^2, \quad (48)$$

which for the value of the inflaton mass $m \sim 5 \times 10^{-6} M_{pl}$ obtained in Section II, and taking the bare mass of the quantum field one order greater than H_* , i.e., $m_\chi \cong 2 \times 10^{-4} M_{pl}$, constrains the value of the coupling constant to satisfy $h \geq 2 \times 10^{-10}$.

Remark 4.3 *The end of the decay only happens after the domination of the relativistic plasma for abnormally small values of the parameter $h \leq 10^{-10}$. For this reason, we will disregard this situation here.*

Then, the reheating temperature, i.e., the temperature of the universe when the relativistic plasma in thermal equilibrium starts to dominate, will be

$$T_R \sim \rho_\chi^{\frac{1}{4}}(t_{dec}) \sqrt{\frac{\rho_\chi(t_{dec})}{\rho(t_{dec})}} \sim 2 \times 10^{-4} \left(\frac{m}{m_\chi}\right)^{3/2} \left(\frac{m}{\bar{\Gamma}}\right)^{1/4} \left(\frac{m}{M_{pl}}\right)^2 M_{pl}, \quad (49)$$

which for the above values of the inflaton mass and m_χ , is of the order

$$T_R \sim 7 \times 10^{-19} h^{-1/2} M_{pl} \sim h^{-1/2} \text{ GeV}. \quad (50)$$

Taking $h \sim 10^{-2}$, one gets reheating temperature in the GeV regime, and in the limit case $h \sim 10^{-9}$, the reheating temperature would be around 30 TeV.

To end the Section, we come back to the potential (4) which has a smoother phase transition than in the previous case, i.e., than the potential (40). For this potential the discontinuity appears in the third derivative of the Hubble parameter, thus, using the second order WKB approximation to obtain a non-vanishing β -Bogoliubov coefficient, one gets [56]

$$|\beta_k|^2 \cong \frac{m_\chi^4 a^{12}(0) (\ddot{H}(0^+) - \ddot{H}(0^-))^2}{1024(k^2 + m_\chi^2 a^2(0))^6}, \quad (51)$$

where, taking into account that there is no substantial drop of energy between the end of inflation and the phase transition to a kination regime, one has

$$\ddot{H}(0^+) - \ddot{H}(0^-) = -\frac{\dot{\varphi}(0)}{M_{pl}^2} (\ddot{\varphi}(0^+) - \ddot{\varphi}(0^-)) = -\frac{\dot{\varphi}^2(0)}{M_{pl}^2} V_{\varphi\varphi}(0^-) = \frac{m^2 \dot{\varphi}^2(0)}{M_{pl}^2} = 2m^4. \quad (52)$$

The number density of massive produced particles and their energy density are given by

$$n_\chi(t) \sim 8 \times 10^{-6} \left(\frac{m}{m_\chi}\right)^5 m^3 \left(\frac{a(0)}{a(t)}\right)^3, \quad \rho_\chi(t) \sim m_\chi n_\chi(t), \quad (53)$$

and for the same decaying rate as in the previous cases, the corresponding energy densities at the end of decay will be

$$\rho(t_{dec}) = 3\bar{\Gamma}^2 M_{pl}^2 \quad \text{and} \quad \rho_\chi(t_{dec}) \cong 10^{-5} \left(\frac{m}{m_\chi}\right)^4 \bar{\Gamma} m^3. \quad (54)$$

Assuming, once again, that the end of the decay occurs before the radiation-domination epoch ($\rho_\chi(t_{dec}) \leq \rho(t_{dec})$), one obtains the relation

$$h^2 \geq \frac{8\pi}{3} \times 10^{-5} \left(\frac{m}{m_\chi}\right)^5 \left(\frac{m}{M_{pl}}\right)^2, \quad (55)$$

which for the values $m \cong 5 \times 10^{-6} M_{pl}$ and $m_\chi \cong 2 \times 10^{-4} M_{pl}$ leads to the constraint $h \geq 3 \times 10^{-12}$.

Finally, if the thermalization of the relativistic plasma is instantaneous, the reheating temperature formula will be

$$T_R \sim \rho_\chi^{1/4}(t_{dec}) \sqrt{\frac{\rho_\chi(t_{dec})}{\rho(t_{dec})}} \sim 5 \times 10^{-4} \left(\frac{m}{m_\chi}\right)^{\frac{13}{4}} \left(\frac{m}{M_{pl}}\right)^2 h^{-1/2} M_{pl}, \quad (56)$$

which for the masses as above, leads to the following low reheating temperature $T_R \sim 7 \times 10^{-20} h^{-1/2} M_{pl} \sim 10^{-1} h^{-1/2}$ GeV. From this result, one can conclude that for $h \sim 10^{-4}$ the reheating temperature is in the GeV regime, and to obtain a temperature in the TeV regime one needs $h \sim 10^{-8}$. The maximum temperature is around 10 TeV.

An important final remark is in order: To avoid the problem of the overproduction of gravitational waves, following step by step the section V of [55], in the case of massive particle production whose energy density decreases as a^{-3} before decaying in a relativistic plasma (this process is previous to the equilibrium), the *heating efficiency* has to satisfy the constraint

$$\Theta \left(\frac{H(0)}{\bar{\Gamma}}\right)^{1/3} \geq 6 \times 10^{-3} \left(\frac{H(0)}{M_{pl}}\right)^2. \quad (57)$$

When reheating is via instant preheating there is no problem, because this constraint is satisfied for all the viable values of h and g . Dealing with reheating via production of heavy massive particles, for the potential (40), this constraint together with the bound (48) coming from the imposition that the decay preceded the equilibrium, bounds the value of the mass of the quantum field to satisfy $2 \times 10^{-5} M_{pl} \ll m_\chi \leq 7 \times 10^{-4} M_{pl}$, which implies $m_\chi \sim 2 \times 10^{-4} M_{pl}$ and leads to a value of h of the order $h \sim 10^{-9}$, obtaining a reheating temperature around 30 TeV. However, for the potential (4), the constraints (57) and (55) lead to the bound $m_\chi \leq 10^{-5} M_{pl}$, which is incompatible with the bound $m_\chi \gg 2 \times 10^{-5} M_{pl}$, meaning that in this case the amount of created particles is not big enough to prevent gravitational waves from influencing the BBN process.

5. CURVATURE REHEATING IN NON-OSCILLATORY MODELS

In this last Section we will review the so-called *curvature reheating* mechanism in quintessence inflation. To do that we will follow [20], but taking into account that the authors of that paper, contrary to our convention, define the reheating temperature as the temperature of the universe when the curvaton field has totally decayed into relativistic particles, regardless of whether its energy density was dominant or not.

Assume that the potential of the curvaton field, namely σ , is quadratic $V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2$, where the mass of the curvaton is chosen to be smaller than the value of the Hubble parameter at the end of inflation $m_\sigma \ll H_{end}$. This choice ensures that at the end of the inflation the curvaton field is in a slow-roll regime, because the condition $m_\sigma \ll H_{end} \iff V_{\sigma\sigma} \ll H_{end}^2$ means that the curvaton potential is flat enough at that time [62]. Then, in order to avoid a second inflationary stage, now driven by the curvaton, one has to impose that its energy density is subdominant when the curvaton starts to oscillate, which happens when $m_\sigma \cong H$ [62] (see also the section 5.5.1 of [63] for a detailed discussion of the quadratic potential). Then,

$$\rho_\sigma(t_{osc}) < \rho(t_{osc}) = 3H^2(t_{osc})M_{pl}^2, \quad (58)$$

where t_{osc} is the time when the curvaton starts to oscillate. Taking into account that $H(t_{osc}) \cong m_\sigma$ and using the virial theorem, which for a quadratic potential states that the average over time of the kinetic and potential energy density coincide, we will make the approximation $\rho_\sigma(t_{osc}) \cong m_\sigma^2 \sigma^2(t_{osc})$, obtaining the bound $\sigma^2(t_{osc}) < 3M_{pl}^2$.

Here, the potential of the inflaton field, in the improved versions of the Peebles-Vilenkin model (4) and (40), is chosen under the requirement that after the end of inflation the universe enters in a kination regime. This entails that, after the phase transition, the energy density of the background evolves as a^{-6} , and that of the curvaton as a^{-3} , because during the oscillatory regime the effective Equation of State parameter for a power law potential $V(\sigma) = V_0 \left(\frac{\sigma}{M_{pl}}\right)^{2n}$, is given by $w_{eff} \cong \frac{n-1}{n+1}$ [64].

Now, let $\bar{\Gamma}$ be the decay rate of the curvaton. There are two completely different situations:

1. The curvaton decays when it is subdominant.
2. The curvaton decays when the curvaton field dominates the universe.

In the first case, the curvaton decays in radiation (to simplify we assume that thermalization is instantaneous) at a time t_{dec} satisfying $H(t_{dec}) \cong \bar{\Gamma}$ (Note that the background energy density is the one of the inflaton). At that time we will have

$$\rho_\sigma(t_{dec}) < \rho(t_{dec}) \implies \rho_\sigma(t_{osc}) \frac{\bar{\Gamma}}{m_\sigma} < 3\bar{\Gamma}^2 M_{pl}^2, \quad (59)$$

where we have used that the energy density of the curvaton decays as a^{-3} , that the universe is in a kination phase (the Hubble parameter also decays as a^{-3}) and $H(t_{osc}) \cong m_\sigma$. Then, since $\rho_\sigma(t_{osc}) \cong m_\sigma^2 \sigma^2$ and $\bar{\Gamma} \cong H(t_{dec}) \leq H(t_{osc}) \cong m_\sigma$ one gets the constraint

$$\frac{\sigma^2(t_{osc})}{3M_{pl}^2} \leq \frac{\bar{\Gamma}}{m_\sigma} \leq 1. \quad (60)$$

To obtain the reheating temperature, which in this case coincides with the temperature at the equilibrium time t_{eq} ($\rho(t_{eq}) \sim \rho_\sigma(t_{eq})$), one has to take into account that

$$\rho(t_{eq}) = \rho(t_{dec}) \left(\frac{a(t_{dec})}{a(t_{eq})} \right)^6 \quad \text{and} \quad \rho_\sigma(t_{eq}) = \rho_\sigma(t_{dec}) \left(\frac{a(t_{dec})}{a(t_{eq})} \right)^4, \quad (61)$$

which leads to

$$T_R \sim \rho_\sigma^{1/4}(t_{eq}) \sim \rho_\sigma^{1/4}(t_{dec}) \sqrt{\frac{\rho_\sigma(t_{dec})}{\rho(t_{dec})}} \sim \frac{\rho_\sigma^{3/4}(t_{dec})}{\sqrt{3}M_{pl}\bar{\Gamma}} \sim \frac{m_\sigma^{3/4} |\sigma(t_{osc})|^{3/2}}{\sqrt{3}M_{pl}\bar{\Gamma}^{1/4}}, \quad (62)$$

where we have used that $\rho_\sigma(t_{dec}) = \rho_\sigma(t_{osc}) \left(\frac{a(t_{osc})}{a(t_{dec})} \right)^3 = \rho_\sigma(t_{osc}) \frac{H(t_{dec})}{H(t_{osc})} \cong m_\sigma \sigma^2(t_{osc}) \bar{\Gamma}$.

Then, using the bound (60) we can see that the reheating temperature is constrained to be in the range

$$\frac{m_\sigma^{1/2} |\sigma(t_{osc})|^{3/2}}{\sqrt{3}M_{pl}} \leq T_R \leq \frac{m_\sigma^{1/2} |\sigma(t_{osc})|}{3^{1/4} M_{pl}^{1/2}}. \quad (63)$$

On the other hand, if the decay of the curvaton occurs when it is subdominant, the power spectrum of the curvature fluctuation in co-moving coordinates is given by [19, 21] $\mathcal{P}_\zeta = \frac{1}{1296\pi^2} \frac{m_\sigma^2}{\Gamma^2} \frac{H_*^2 \sigma_*^2}{M_{pl}^4}$, which from the bound (60), is constrained to satisfy

$$\frac{1}{1296\pi^2} \frac{H_*^2 \sigma_*^2}{M_{pl}^4} \leq \mathcal{P}_\zeta \leq \frac{1}{144\pi^2} \frac{H_*^2}{\sigma_*^2}, \quad (64)$$

where we have used that before the oscillations the curvaton slowly rolls, and thus $\sigma(t_{osc}) \sim \sigma_*$. Now, taking into account the observational data $\mathcal{P}_\zeta \sim 2 \times 10^{-9}$, one gets the bounds

$$\frac{H_*}{|\sigma_*|} \geq 2 \times 10^{-3} \quad \text{and} \quad H_* |\sigma_*| \leq 5 \times 10^{-3} M_{pl}^2. \quad (65)$$

Then, choosing $H_* \sim 2 \times 10^{-5} M_{pl}$, which for $n_s \cong 0.968$ and without the presence of the curvaton field, is the value of the Hubble parameter when the pivot scale leaves the Hubble radius [54], one can safely take $|\sigma_*| \sim 10^{-6} M_{pl}$, which agrees with the bound $\sigma_*^2 \sim \sigma^2(t_{osc}) < 3M_{pl}^2$.

For these values, the equation (63) becomes

$$5 \times 10^{-9} \sqrt{m_\sigma M_{pl}} \leq T_R \leq 7 \times 10^{-7} \sqrt{m_\sigma M_{pl}}, \quad (66)$$

which means that, to get the bounds coming from BBN success, one has to choose curvaton masses satisfying

$$10^{-9} \text{ GeV} \cong 7 \times 10^{-27} M_{pl} \leq m_\sigma \leq 3 \times 10^{-7} M_{pl} \cong 7 \times 10^{11} \text{ GeV}, \quad (67)$$

and one has to avoid the effects of an overproduction of gravitational waves. Taking into account that the energy density of the curvaton at the beginning of the kination regime is approximately $m_\sigma^2 \sigma_*^2$, the inequalities (60) and (57) lead to

$$\frac{m_\sigma \sigma_*^2}{3M_{pl}^2} \leq 2 \times 10^5 \frac{m_\sigma^6 \sigma_*^6}{H^{11}(0)} \iff 10^{-5} \leq 6 \frac{m_\sigma^5 \sigma_*^4 M_{pl}^2}{H^{11}(0)}, \quad (68)$$

which is fulfilled for a wide range of viable values of the parameters. E.g., choose $H(0) \sim 3 \times 10^{-6} M_{pl}$, $\sigma_* \sim 10^{-6} M_{pl}$ and $m_\sigma \sim 10^{-7} M_{pl}$.

Finally, assuming once again instantaneous thermalization, we consider the situation where the curvaton decays while dominating, that is, $\rho_\varphi(t_{dec}) \leq \rho(t_{dec})$ (Now ρ denotes the energy density of the curvaton), and the reheating will occur at $t_R \equiv t_{dec}$. Let t_{eq} be once again the equilibrium time ($\rho_\sigma(t_{eq}) \sim \rho_\varphi(t_{eq})$), which will satisfy $t_{osc} \leq t_{eq} \leq t_{dec} = t_R$, where, once again, we have denoted by t_R the reheating time.

Following the same steps as in the previous case, now the combination of conditions

$$\rho_\varphi(t_{dec}) \leq \rho(t_{dec}) \quad \text{and} \quad \rho(t_{eq}) \sim \rho_\varphi(t_{eq}), \quad (69)$$

leads to the constraint [20]

$$\frac{\bar{\Gamma}}{m_\sigma} \leq \frac{\sigma^2(t_{osc})}{3M_{pl}^2} < 1. \quad (70)$$

Since, in this case, the reheating temperature is $T_R \sim \rho^{1/4}(t_{dec}) \sim \sqrt{\sqrt{3} M_{pl} \bar{\Gamma}}$ the constraint leads to the bound

$$T_R \leq \sqrt{\frac{m_\sigma}{\sqrt{3} M_{pl}}} |\sigma(t_{osc})|. \quad (71)$$

On the other hand, when the curvaton decays after its domination, the power spectrum of the curvature fluctuation is given by [62]

$$\mathcal{P}_\zeta \cong \frac{1}{9\pi^2} \frac{H_*^2}{\sigma_*^2} \sim 2 \times 10^{-9} \implies \frac{H_*}{|\sigma_*|} \sim 4 \times 10^{-4} M_{pl}, \quad (72)$$

and choosing, as in the previous case, $H_* \sim 2 \times 10^{-5} M_{pl}$, one gets $|\sigma_*| \sim 5 \times 10^{-2} M_{pl}$. Moreover, since the curvaton rolls slowly before the oscillations one can take $|\sigma(t_{osc})| \sim |\sigma_*| \sim 5 \times 10^{-2} M_{pl}$ which satisfies the bound (70).

From these values and the equation (71), one can conclude that only for curvaton masses satisfying $m_\sigma \leq 10^{-16} M_{pl} \sim 2 \times 10^2 \text{ GeV}$, are obtained reheating temperatures compatible with the nucleosynthesis success.

Finally, dealing with the overproduction of gravitational waves, since in this case the energy density of the curvaton decreases as a^{-3} , the constraint (57) will become

$$\Theta^{2/3} \geq 6 \times 10^{-3} \left(\frac{H(0)}{M_{pl}} \right)^2, \quad (73)$$

where once again, $H(0)$ is the value of the Hubble parameter at the beginning of the kination epoch. The *heating efficiency* will approximately be $\Theta \cong \frac{m_\sigma^2 \sigma_*^2}{3H^2(0)M_{pl}^2}$, then inserting it in (73) and taking $H(0) \sim 3 \times 10^{-6} M_{pl}$, one obtains the constraint $m_\sigma \sigma_* \geq 5 \times 10^{-16} M_{pl}^2$, which is never fulfilled. So, in that case the gravitational waves could affect the BBN success.

6. CONCLUSIONS

In the present work we have studied in detail four ways to reheat the universe in quintessence inflation via the production of particles, showing that each preheating mechanism, when the parameters involved in the theory are properly chosen, leads to different reheating temperatures compatible with the Big Bang Nucleosynthesis, in the sense that, the overproduction of Gravitational Waves do not affect its success.

The first one studied is the so-called *instant preheating* based in the interaction of the inflaton field with a quantum field. We have shown that for this kind of preheating the particles are produced as in Schwinger's effect, i.e., the Bogoliubov coefficient is calculated in the same way as for the over-barrier problem in scattering theory when the external electric field is constant. We have also shown that the relevant parameters of the theory, i.e., the coupling constant between the inflaton and the quantum field and the coupling constant between the quantum field and fermions, are constrained in a narrow range of values, and for such values we have obtained a high reheating temperature which is of the order of 10^9 GeV.

The second mechanism consists in the production of massless particles nearly conformally coupled with gravity. Due to the fact that the particles are nearly conformally coupled, the modes could be calculated in a perturbative way obtaining an analytic expression of the β -Bogoliubov coefficient. For some simple models depicting phase transitions from a de Sitter phase to a kination regime we have obtained, when the coupling coefficient between the quantum field and gravity ξ satisfies $\xi - \frac{1}{6} \sim 10^{-1}$, i.e. that the gravitational waves do not affect the success of the Big Bang Nucleosynthesis, a reheating temperature of the order of 10^5 GeV.

The third method, which is generally not used as much as the other two, is the reheating via the creation of very heavy massive particles ($m_\chi \sim 5 \times 10^{14}$ GeV) conformally coupled with gravity, which decay in lighter ones to produce a relativistic plasma. We point out that due to the high value of the mass of the quantum field, the modes could be calculated using the WKB approximation, obtaining for potentials whose first derivative is discontinuous, such as the improved version of the Peebles-Vilenkin model presented in this work, analytic formulas for the energy density of the produced particles, and thus, allowing the analytic calculation of the reheating temperature as a function of the inflaton mass, the mass of the quantum field and the decay rate (see formula (49)). Assuming that after the decay of these massive particles the products thermalize instantaneously, which is a well established fact (see the remark 3.2 of [65]), we have shown that the overproduction of gravitational waves does not disturb the Big Bang Nucleosynthesis process, obtaining a reheating temperature around 30 TeV.

Finally, we have studied the reheating in quintessence inflation via the decay of the curvaton field in very light relativistic particles, which we have assumed to thermalize instantaneously, showing that only for light masses of the curvaton field ($m_\sigma \leq 10^{11}$ GeV when the curvaton is subdominant at the decay, or $m_\sigma \leq 10^3$ GeV in the case that the energy density of the curvaton dominates that of the inflaton at the decay) the universe is reheated at a temperature compatible with the bounds coming from the Big Bang Nucleosynthesis.

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