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Voltage recovery influence on three-phase grid-connected inverters under voltage sags

A. Rolán1,*, P. Giménez1, S. J. Yagüe1, S. Bogarra2, J. Saura2, M. Bakkar2

1Department of Industrial Engineering, I QS-URL, Via Augusta 390, Barcelona 08017, Spain
2Department of Electrical Engineering, ETSEIAT-UPC, Colom St. 1, Terrassa (Barcelona) 08222, Spain
*alejandro.rolan@iqs.url.edu

Abstract: Faults in power systems cause voltage sags, which, in turn, provoke large current peaks in grid-connected equipment. Then, a complete knowledge of the inverter behaviour is needed to meet fault ride-through capability. The aim of this paper is to propose a mathematical model that describes the behaviour of the currents that a three-phase inverter with RL filter inject to a faulty grid with symmetrical and unsymmetrical voltage sags. The voltage recovery process is considered, i.e., the fault is assumed to be cleared in the successive zero-cross instants of the fault current. It gives rise to a voltage recovery in different steps (discrete voltage sag), which differs from the usual model in the literature, where the voltage recovers instantaneously (abrupt voltage sag). The analytical model shows that the fault-clearing process has a strong influence on the injected currents. Different sag durations and depths have also been considered, showing that there exist critical values for these magnitudes, which provoke the highest current peaks. The analytical study is validated through simulations in MATLAB™ and through experimental results.

1. Introduction

Nowadays, there has been a noticeable increase in the penetration of renewable energy systems into the main grid, whose percentage could reach up to 80% by 2050 [1]. This will imply a new scenario as far as power generation is concerned: the traditional high power stations based on nuclear, thermal or hydro power will reduce their importance, as renewable energy systems will increase its contribution into power generation. This will cause a noticeable reduction in the CO₂ emissions due to the combustion of carbon-based fuels, but it has the drawback that the new power generation units will not be as robust as the traditional ones under grid disturbances. Certainly, high-power synchronous machines used for electricity generation in traditional power stations have strong inertia, so they can get over faults in power systems with relatively no malfunction. However, when a renewable energy system is connected to the grid, it becomes “weak” under electrical disturbances, as there exist no inertia. Moreover, the critical point is the three-phase inverter, as this device couples the DC-link (where the energy delivered from the renewable energy source is stored) with the main grid. In order to get over this shortage in renewable energy systems, control techniques have been proposed in the literature.

Among all grid disturbances, voltage sags are the most common ones [2]. They are mainly caused by faults in power systems, which can cause a reduction in the rms magnitude in one or two phases (unsymmetrical voltage sags) or in the three phases (symmetrical voltage sags). Studies in the literature reveal that large current and torque peaks appear on grid-connected equipment, such as transformers [3] or induction motors [4]-[5]. Protections in power systems are a good solution to tackle the problem [6], but they disconnect the equipment from the grid, thus power is not sent to the end-users. For this reason, it is important to propose analytical models that help in the understanding of grid-connected equipment under voltage sags with the aim of providing solutions to mitigate the problem without disconnecting the system from the grid (fault ride-through capability).

The aim of this paper is not to provide a robust or a sound control for grid-connected renewable energy systems (a summary of these control techniques, as well as the grid interconnection issues between wind and photovoltaic systems and the grid can be found in [7]). This paper focuses on the study that the voltage recovery process cause on grid-connected inverters, because it gives rise to less severe effects than when sags are assumed to be cleared instantaneously. Then, the instantaneous voltage recovery process (abrupt sag) overestimates the sag severity, while if the voltage recovery process takes into account the successive zero-cross instants of the fault current (discrete sag), the sag severity is weakened, which is what it happens in real applications.

This paper is structured as follows. Firstly, a description of voltage sags and the voltage recovery process is given. Secondly, the analytical model of a three-phase grid-connected inverter with an RL filter is carried out when subject to either symmetrical or unsymmetrical voltage sags. Thirdly, the sag parameters influence (duration and depth) is analyzed. Finally, the analytical study has been validated through simulations in MATLAB™ and experimental results.

2. Voltage sags

Voltage sags are the grid disturbances originated mainly by faults, which cause a decrease in the rms voltage with respect to the steady-state pre-fault voltage from 0.1 pu and 0.9 pu and the usual durations go from 0.5 cycles and 1
The voltage sag is defined as the instant in which the voltage crosses zero. It gives rise to a voltage recovery process, which can be done in one, two, or three steps, as indicated in Table 1.

Table 1 Voltage sags: types, phasors and sequence components (adapted from [2]) and voltage recovery process (adapted from [9])

<table>
<thead>
<tr>
<th>Type</th>
<th>Phases</th>
<th>Zero seq.</th>
<th>Positive seq.</th>
<th>Negative seq.</th>
<th>Voltage Recovery Process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>First recovery ((\omega t))</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>(V^0) = 0</td>
<td>(V^+ - \alpha_p + \psi + 90^\circ)</td>
<td>(V^- - \alpha_n + \psi + 90^\circ)</td>
<td>(A_1) (n) (180^\circ - \alpha_n + \psi - 90^\circ)</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>(V^0) = 0</td>
<td>(V^+ - \alpha_p + \psi + 30^\circ)</td>
<td>(V^- - \alpha_n + \psi + 30^\circ)</td>
<td>(A_1) (n) (180^\circ - \alpha_n + \psi - 90^\circ)</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>(V^0) = (-1\frac{1}{3}V)</td>
<td>(V^+ - \frac{2}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(B) (n) (180^\circ - \alpha_n + \psi - 90^\circ)</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>(V^0) = 0</td>
<td>(V^+ - \frac{1}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(C) (n) (180^\circ - \alpha_n + \psi)</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>(V^0) = 0</td>
<td>(V^+ - \frac{1}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(D) (n) (180^\circ - \alpha_n + \psi - 90^\circ)</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>(V^0) = (-1\frac{1}{3}V)</td>
<td>(V^+ - \frac{2}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(E_1) (n) (180^\circ - \alpha_n + \psi + 30^\circ)</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>(V^0) = (-1\frac{1}{3}V)</td>
<td>(V^+ - \frac{2}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(E_2) (n) (180^\circ - \alpha_n + \psi + 30^\circ)</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td>(V^0) = (-1\frac{1}{3}V)</td>
<td>(V^+ - \frac{2}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(F_1) (n) (180^\circ - \alpha_n + \psi + 120^\circ)</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td>(V^0) = (-1\frac{1}{3}V)</td>
<td>(V^+ - \frac{2}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(F_2) (n) (180^\circ - \alpha_n + \psi - 90^\circ)</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>(V^0) = 0</td>
<td>(V^+ - \frac{1}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(G_1) (n) (180^\circ - \alpha_n + \psi + 30^\circ)</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>(V^0) = 0</td>
<td>(V^+ - \frac{1}{3}V)</td>
<td>(V^- - \frac{1}{3}V)</td>
<td>(G_2) (n) (180^\circ - \alpha_n + \psi + 150^\circ)</td>
</tr>
</tbody>
</table>

With respect to the voltage recovery process, Table 1 shows that there are 14 types of discrete voltage sags: 5 symmetrical sags (A1, A2, A3, A4, and A5) plus 9 unsymmetrical sags (B, C, D, E1, E2, F1, F2, G1, and G2). However, the authors’ previous work [10] demonstrated that ...
when studying the behaviour of grid-connected equipment under voltage sags the following group of voltage sags cause the same behaviour in the electrical variables: $A_1-A_2$, $A_1-A_3$, $C-D$, $F_1-G_1$ and $F_2-G_2$. As a result, for the present paper only the following sags are considered: symmetrical sag types $A_1$ and $A_2$, and unsymmetrical sag types $C$, $F_1$ and $F_2$.

3. Three-phase grid-connected inverters

Fig. 1 shows the general scheme of a three-phase inverter connected to the main grid and its control. Generator-sign convention

![General scheme of a three-phase inverter with RL filter connected to the main grid and its control. Generator-sign convention](image)

when studying the behaviour of grid-connected equipment under voltage sags the following group of voltage sags cause the same behaviour in the electrical variables: $A_1-A_2$, $A_1-A_3$, $C-D$, $F_1-G_1$ and $F_2-G_2$. As a result, for the present paper only the following sags are considered: symmetrical sag types $A_1$ and $A_2$, and unsymmetrical sag types $C$, $F_1$ and $F_2$.

4. Analytical study

4.1. Injected current solution in complex form

The mathematical study is developed using the complex form of the $dq$ variables, as they provide a compact form for the electrical expressions, which eases the task to obtain an analytical solution to the problem. This is done by applying the $Ku$ transformation (see Appendix II for more details).

The mathematical expressions that model the system of Fig. 1 (considering the generator-sign convention) are:

$$
\begin{align*}
\mathbf{v}_{ia} &= \begin{bmatrix} R & 0 & 0 \end{bmatrix} \mathbf{i}_d + \frac{d}{dt} \begin{bmatrix} L & 0 & 0 \end{bmatrix} \mathbf{i}_q + \mathbf{v}_{ig} \\
\mathbf{v}_{ib} &= \begin{bmatrix} 0 & R & 0 \end{bmatrix} \mathbf{i}_d + \frac{d}{dt} \begin{bmatrix} 0 & L & 0 \end{bmatrix} \mathbf{i}_q + \mathbf{v}_{ig} \\
\mathbf{v}_{ic} &= \begin{bmatrix} 0 & 0 & R \end{bmatrix} \mathbf{i}_d + \frac{d}{dt} \begin{bmatrix} 0 & 0 & L \end{bmatrix} \mathbf{i}_q + \mathbf{v}_{ig} \\
\end{align*}
$$

(1)

If we apply the $Ku$ transformation in the synchronous reference frame (equation (19) in Appendix II) to the matrix system (1), we obtain the following expression:

$$
\mathbf{v}_g = [R + L(s + jo)] \mathbf{i}_d + \mathbf{v}_{ig}
$$

(2)

where $\mathbf{v}_g$ is the transformed voltage at the inverter output, $R$ and $L$ are the filter resistance and the filter inductance, respectively, $s = \frac{d}{dt}$ is the derivative operator, $\alpha = 2\pi f$ is the pulsation of the grid voltages ($f = 1/T$ is the grid frequency and $T$ is its period), $i_d$ is the transformed current that the inverter injects to the grid, and $\mathbf{v}_{ig}$ is the transformed grid voltage, which is given by the following equation (according to (22) in Appendix II) under unbalanced conditions:

$$
\mathbf{v}_{ig} = \mathbf{v}_{ig}^* + \mathbf{v}_{ig}^* e^{-j2\alpha t}
$$

(3)

and according to (25) in Appendix II, $\mathbf{v}_{ig}^*$ and $\mathbf{v}_{ig}^*$ are:

$$
\mathbf{v}_{ig}^* = \sqrt{2/3} \mathbf{i}_d^* \\
\mathbf{v}_{ig}^* = \sqrt{2/3} \mathbf{i}_q^*
$$

(4)

being $\mathbf{I}_d^*$ and $\mathbf{I}_q^*$ the positive- and negative-sequence components of sags shown in Table 1. Then, the differential equation of the transformed current can be obtained from (2) considering the transformed grid voltage (3), resulting in:

$$
i_d = \frac{1}{L} \left[ \mathbf{v}_g^* - (\mathbf{v}_{ig}^* + \mathbf{v}_{ig}^* e^{-j2\alpha t}) - (R + joL)i_d \right].
$$

(5)

In order to find an analytical solution for (5), the following assumption is made: the control systems are able to set the transformed inverter voltage ($\mathbf{v}_{ig}$) in the synchronous reference frame at its pre-fault steady-state value. By doing this, (5) is a first-order ODE with constant coefficients. Its solution will be the homogenous solution plus the particular solution. The homogenous solution is obtained by neglecting the excitations of (5), i.e. $\mathbf{v}_g = \mathbf{v}_{ig} = \mathbf{v}_{ig} e^{-j2\alpha t} = 0$, giving:

$$
i_{d,\text{homog}} = \mathbf{K} e^{-s \alpha L} e^{-j2\alpha t}.
$$

(6)

And the particular solution is the steady-state solution

$$
i_{d,\text{part}} = \frac{1}{L} \mathbf{v}_g^*.
$$

(7)
of (5), which can be expressed as:

\[ i_{t,\text{st}} = i_{t,\text{ini}} + i_{t,\text{part}} e^{-\omega \Delta t} \]  

where \( i_{t,\text{ini}} \) is the steady-state solution (\( s = 0 \)) of (5) with no negative-sequence voltage (\( \nu_{gf} = 0 \)), and \( i_{t,\text{part}} \) is the steady-state solution of (5) with no positive-sequence voltage (\( \nu_i = \nu_{gf} = 0 \)), which results in:

\[ i_{t,\text{part}} = \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} e^{-\omega \Delta t} \]  

Finally, the solution of the first-order ODE (5) is the addition of the homogeneous solution (6) plus the particular solution (7), resulting in:

\[ i_t = K e^{(k(t) - \omega \Delta t)} e^{-\omega \Delta t} \]

The complex constant \( K \) is obtained by means of the initial condition (at \( t = t_0 \)), the initial transformed current is \( i_{t_0} = i_{t_0}^* \):

\[ K = \left( i_{t_0} + \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} e^{-\omega \Delta t} \right) e^{(k(t) - \omega \Delta \Delta t)} \]  

Finally, substituting (10) in (9) gives:

\[ i_t = \left( i_{t_0} + \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} e^{-\omega \Delta t} \right) e^{(k(t) - \omega \Delta \Delta t)} \]

\[ + \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} e^{-\omega \Delta t} \]  

\[ i_{t_1} = \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} \quad (t < t_1) \]  

4.2. State 1: Before the sag (\( t < t_f \))

It is assumed that the system operates at its steady-state before the sag is originated at \( t = t_f \). So, the positive-sequence component for the transformed grid voltage equals its steady-state value (\( \nu_{gf} = \nu_{gf,a} \)) and there is no negative-sequence voltage (\( \nu_i = 0 \)). So, the steady-state expression for (11) is:

\[ i_{t_1} = \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} \quad (t < t_1) \]  

4.3. State 2: During the sag (\( t_1 \leq t < t_f \))

The voltage sag starts at \( t = t_1 \), and ends at \( t = t_f \). The initial transformed current for this time interval is (12), which substituted in (11) results in:

\[ i_{t_2} = K e^{(k(t_f) - \omega \Delta t)} e^{-\omega \Delta t} \]

where the complex constants \( K_1, K_2 \) and \( K_3 \) are given by:

\[ K_1 = \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} \]

\[ K_2 = \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} \]

4.4. State 3: After the sag (\( t \geq t_f \))

The fault is cleared at \( t = t_f \). The initial condition is then given by the (13) evaluated at \( t = t_f \). Moreover, when the sag ends the system goes back to its steady-state operation (\( \nu_{gf} = \nu_{gf,a} \)), so there is no negative-sequence voltage (\( \nu_i = 0 \)). Substituting these conditions in (11), it results in:

\[ i_{t_3} = \frac{\nu_i^* - \nu_{gf}^*}{j \omega L} + \frac{\nu_i + \nu_{gf}}{R+j \omega L} \]

The proposed analytical model consists of equations (12), (13), and (15), which describe the behaviour of a three-phase grid-connected inverter with RL filter under unsymmetrical voltage sags. These equations can be represented by the three voltage-controlled current sources depicted in Fig. 2. Note that the presented model is also valid for symmetrical sags (in this case, there is no negative-sequence component, so \( \nu_i = 0 \)).

5. Voltage recovery influence

Fig. 3 shows the voltage recovery influence (abrupt and discrete sags) on the injected currents from the inverter to the grid when faults cause symmetrical sags (type A) and unsymmetrical sag type F. All the simulated sags have a duration \( \Delta t = 5.5 \) cycles (i.e., 110 ms with \( f = 50 \) Hz) and a depth \( h = 0.8 \). Both transformed and real currents are represented in per unit (or pu) values, according to:

\[ i_{t_{pu}} (t) = \frac{i_t (t)}{I_{pu}} \]

\[ i_{t_{ab}} (t) = \frac{i_{t_{ab}} (t)}{I_{pu}} \]

where \( I_{pu} \) is the current base value (\( h = I_{pu} \), see Appendix I).

Regarding symmetrical voltage sags, when the sag starts at \( t = t_f \), note that the transformed current rotates at pulsation \( \omega \); the locus of the transformed current (Fig. 3a) has a circumferential shape and the time evolution of the transformed current (Fig. 3b) shows a pulsation \( \omega \) during the period of time when the fault occurs. This is due to the exponential term \( e^{j \omega (\Delta t)} \) that appears in (13). Note also that when the sag ends at \( t = t_f \) if the sag is modelled abrupt, then the voltage recovery process is more severe than if it is modelled discrete. Indeed, note from Fig. 3c that for type A1 (abrupt), the peak value of the real currents is 1.77 pu, while for type A3 (discrete with two-step voltage recovery) this peak value is 1.48 pu and for type A4 (discrete with three-step voltage recovery) the value is 1.47 pu. It means that as the number of steps in fault-clearing increases, the sag causes less severe effects on the injected currents.
Fig. 3 Voltage recovery influence on the currents injected from the inverter to the grid under symmetrical sag types A₁ and A₄ (abrupt and discrete) and under unsymmetrical sag type F₁ (abrupt and discrete). Sags characteristics: Δₜ = 5.5 cycles, h = 0.8 and ψ = 80°

a Locus of the transformed current
b Real and imaginary part of the transformed current
c Three-phase components (abc) of the current
This is a logical consequence if we look at the locus of the transformed current shown in Fig. 3a. Note from this figure that if the sag has an \textit{abrupt} voltage recovery process, the value of the transformed current when the sag ends, \(i_f(t_f)\), is located in a further position from its pre-fault steady-state value, \(i_f(t_i)\). However, when the sag has a \textit{discrete} voltage recovery, the transformed current in the second recovery, \(i_f(t_{f2})\), or in the third recovery, \(i_f(t_{f3})\), is closer to the pre-fault steady-state value, \(i_f(t_i)\).

Regarding unsymmetrical sags (type F1), note that when the sag starts at \(t = t_i\), neither the locus of the transformed current (Fig. 3a) has a circumferential shape, nor the time evolution of the transformed current (Fig. 3b) has a pulsation \(\omega\). This is explained by the exponential term that depends on twice the fundamental pulsation, \(e^{-j2\omega t}\), which appears in (13). The addition of this term and the term that depends on the fundamental pulsation, \(e^{-j\omega(t - t_i)}\), creates a distorted circumferential shape on the locus during the voltage sag (Fig. 3a). However, note that when the voltage sag ends at \(t = t_f\), the transformed current has again a circumferential shape in its locus (Fig. 3a) and the fundamental pulsation \(\omega\) is in the time response (Fig. 3b). This is because after voltage recovery, (15) shows that there is no exponential term that depends on twice the fundamental pulsation. Note also from Fig. 3a that, as it happened under symmetrical sags, when unsymmetrical sags are modelled \textit{discrete}, the transformed current after the second voltage recovery, \(i_f(t_{f2})\), is closer to its pre-fault steady-state value, \(i_f(t_i)\), than in the case of \textit{abrupt} sags. Then, it leads to a less severe peak values in the injected abc currents from the inverter. Indeed, the current peak value is 1.36 pu for \textit{abrupt} unsymmetrical sag type F1, while for \textit{discrete} sag type F1, the current peak value is 1.22 pu.

Finally, it should be noted that the aim of the paper is not to explain or to improve the control of a three-phase grid-connected inverter under voltage sags (as there is a lot of literature to this respect), but to show how the voltage recovery process have a strong influence on the injected currents by the inverter. Indeed, note from the results shown in Fig. 3 that if voltage sag is modelled abrupt, then the peak values of the injected currents after the voltage recovery are higher in the abrupt sag than in the discrete sag. It should also be noted that the filter’s inductor tries to soothe the voltage sag effects. If this inductor where not placed in the filter, the time index of the current expression, i.e., \(L/R\) in the exponential term in (11) will be zero, which implies that the injected current will never reach its steady state value during the sag.

6. Sag parameters influence

The maximum (or peak) value of the injected current from the inverter to the grid is chosen as the variable to analyze the sag parameters influence. This variable (in pu) is obtained as:

\[
I_{\text{max, pu}} = \max \left[ \frac{\left| i_a(t) \right|}{I_b} + \frac{\left| i_b(t) \right|}{\sqrt{2}I_b} + \frac{\left| i_c(t) \right|}{\sqrt{2}I_b} \right] \tag{18}
\]

where \(I_b\) is the current base value (\(I_b = I_{nc}\) see Appendix I).
and \(i_a(t), i_b(t)\) and \(i_c(t)\) are the abc (or real) components of the currents injected from the inverter to the grid.

Fig. 4 shows the sag depth influence. All the simulated sags have a duration \(\Delta t = 5.5\) cycles (i.e., 110 ms with \(f = 50\) Hz) and the sag depth is varied from \(0 \leq h \leq 1\), i.e., from the completely loss of voltage to the steady-state pre-fault voltage. Note that for both symmetrical sags (type A1 and A2) and unsymmetrical sags (type F1 and F2) the effects are less severe if the discrete voltage recovery process is considered. Certainly, the obtained curves for abrupt sags are always in a higher position than in the case of discrete sags, which means that the peak values of the injected currents will always be higher in the former than in the latter. Take the example of the sag depth \(h = 0\): for symmetrical sag types A1 and A2 the peak current is around 5 pu if sag is modelled abrupt, while it is around 4 pu if sag is modelled discrete; for unsymmetrical sag types F1 and F2, the peak current is around 4 pu for abrupt sags, while for discrete sags this value is around 3 pu. Note also that for unsymmetrical sag type C there is no discrete recovery process, as Table 1 shows that the fault-clearing for this sag type is done in just one step (instantaneously).

Fig. 5 shows the sag duration influence. All the simulated sags have a depth \(h = 0.8\) and the duration is varied from \(5T \leq \Delta t \leq 7T\), i.e., from 5 to 7 cycles, which corresponds from 100 ms to 140 ms, with \(f = 50\) Hz (\(T = 20\) ms). From this figure, two conclusions can be drawn. On the one hand, discrete voltage sags originate less severe current peaks, as happened when studying the sag depth influence (Fig. 4). On the other hand, the sag duration influence is periodical, i.e., the current peaks for the most unfavourable sag durations are repeated either every cycle or half cycle.

Table 2 shows the most unfavourable sag durations for each sag type, i.e., the durations that cause the highest current peaks, as happened when studying the sag depth influence (Table 1) shows that the fault-clearing for this sag type is done in just one step (instantaneously).

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**Table 2** Most unfavourable sag durations \((n = 0, 1, 2, \ldots)\)

<table>
<thead>
<tr>
<th>Symmetrical voltage sags</th>
<th>Unsymmetrical voltage sags</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta t = nT + 0.55T)</td>
<td>(\Delta t = (n/2)T + 0.30T)</td>
</tr>
<tr>
<td>(\Delta t = nT + 0.64T)</td>
<td>(\Delta t = nT + 0.63T)</td>
</tr>
</tbody>
</table>

---

**Fig. 6** Experimental setup used to validate the analytical model and the simulation results of the three-phase inverter with RL filter under voltage sags

This paper has proposed a mathematical model for the three-phase grid-connected inverters under voltage sags, which describes their behaviour when either abrupt or discrete fault-clearing process take place. The simulations results have shown that a voltage sag modelled with an abrupt voltage recovery process overestimates the sag’s severity, as the peak value of the injected currents is higher than in the real case (i.e. discrete voltage recovery process).

So, the authors propose the use of discrete voltage sags when studying the behaviour of three-phase grid-connected inverters under voltage sags. In the following section, the analytical model and simulation results are validated by means of experimental tests.

7. **Experimental results**

The analytical model and the simulations results have been validated by means of the experimental setup shown in Fig. 6a. It consists of the following parts: two 6.4 kVA three-phase AC voltage sources and a control panel (with contactors and protections) to generate the voltage sag (emulating the grid); a 10 kVA three-phase inverter of
Fig. 7 Three-phase voltage sag effects on the injected current by the inverter. Sag characteristics: \( \Delta t = 5 \text{ cycles}, h = 0.9 \) and \( \varphi = 86^\circ \)

a) Measured abc voltages (in pu)

b) Measured abc currents (in pu)

c) Simulated abc currents (in pu)

CINERGIA with a switching frequency of 20 kHz and DC link of 800 V with a capacitor of 1 mF; a three-phase RL filter with values 0.2 \( \Omega \) and 10 mH; a 6.4 kVA voltage source that rectifies AC into DC voltage to generate the required voltage for the inverter’s DC bus; voltmeters and ammeters connected to an oscilloscope, which registers the time evolution of the measured variables; and a dSPACE DS1104 of Texas Instruments for the double aim to send the measured voltages and currents to a PC (with the software MATLAB\textsuperscript{TM}) and to send the switching times for the switching devices.

The behaviour of the experimental setup can be explained by means of the electrical scheme depicted in Fig. 6b. The procedure to follow is explained below:

1) An adjustable three-phase AC source (\( V_s \)) is regulated to deliver 50 V (phase-to-phase voltage). This source emulates the grid.

2) An adjustable DC source (\( V_d \)) is regulated to deliver 180 V to the inverter’s DC-link.

3) The inverter with RL filter is connected to the grid. Note that this inverter is safeguarded by means of an overcurrent protection.

4) The measuring devices read the values of the grid voltages (\( v_{m} \)), the line currents (\( i_{m} \)), and the DC-link voltage (\( v_{dc} \), which are sent to an oscilloscope and to the dSPACE DS1104.

5) The dSPACE sends the measured real-time values to a PC with the MATLAB\textsuperscript{TM} software, by means of which a PLL obtains the voltage angle in order to synchronize the inverter with the grid. The inverter is controlled to deliver 95 V (phase-to-phase voltage). Note that this voltage is higher than the grid voltage, so the current flows from the inverter to the grid.

6) Once the whole system is operating in steady-state conditions, a three-phase voltage sag with depth \( h = 0.9 \) is applied by pressing a button (from the control panel), which acts on two contactors. The first contactor is now open, so the \( V_s \) source is disconnected from the system, while the second contactor is now closed, so the \( V_d \) source is connected to the system. This source has been previously regulated to create the voltage sag with depth \( h = 0.9 \) from 50 V.

Fig. 7 shows the measured voltage and currents when applying a three-phase voltage sag (i.e. type A sag) with depth \( h = 0.9 \) and duration \( \Delta t = 5 \text{ cycles} \). The measured abc components of the grid voltages are depicted in Fig. 7a. It should be noted that the grid is not perfectly balanced, so that the measured abc components of the current, which are depicted in Fig. 7b, are not perfectly balanced in steady state conditions. Finally, Fig. 7c shows the abc components of the currents in the simulated case. It is observed that when the voltage sag originates, the injected current increases in order to keep constant the active power. This current increase is not critical, as the protection system did not act in the experimental test. If we compare the results shown in Fig. 7b and in Fig. 7c we can conclude that they are very similar: there is one phase (c phase) that has the highest peak values (because at the time instant when the sag starts, it corresponds to the c phase of the voltage, as shown in Fig. 7a). Finally, when the sag finishes, the system goes back to the steady-state regime. So, it can be concluded that the experimental results and the simulated ones correspond each other.

8. Conclusion

The present paper has developed an analytical model to study the behaviour of three-phase grid-connected inverters with RL filter under symmetrical and unsymmetrical voltage sags. The analytical results have been validated by simulations in MATLAB\textsuperscript{TM} and by experimental results.

Voltage sags are usually modelled in the literature with abrupt or instantaneous voltage recovery, while in practice the voltage recovery process takes part in different steps (according to the natural zero crossing of the fault current), giving rise to discrete voltage sags. This work has shown that abrupt sags cause the most severe effects on three-phase
inverters, as they provoke the largest current peaks. In other words, abrupt sags overestimate the sag severity. Therefore, this paper proposes the use of discrete sags, because apart from being the real case, they are less severe than abrupt sags.

This paper has also shown that symmetrical sags are more severe than unsymmetrical sags, as they lead to the largest current peaks. Among unsymmetrical sags, the most severe is abrupt sag type F2; and the least severe is type C.

An in-depth analysis considering all sag depths and durations has been carried out, showing two effects. On the one hand, discrete sags are less severe than abrupt sags, as they cause the largest current peaks injected from the inverter. On the other hand, the sag duration effects are periodic, i.e., the largest current peaks are obtained periodically. Moreover, it has also been observed that the largest peak currents are obtained not during sag but after voltage recovery, as depending on sag duration the value of the current could be closer or further from its pre-fault steady-state value. So, each sag type has its most unfavourable sag durations, which should be taken into account when proposing a suitable control algorithm to tackle the problem.

Finally, this preliminary study could be used to propose more robust control algorithms, which would let three-phase inverters with more robust control algorithms, which would let three-phase inverters with"filter meet fault ride-through requirements.

9. References


10. Appendix 1: parameters of the studied system

Table 3 shows the parameters of the studied system, which consists of an AC-side (RL filter plus AC grid), and a DC-side (renewable energy source and DC-link).

11. Appendix 2: Ku transformation

The Ku transformation is used to obtain the transformed components of a given variable x from the abc components of this variable. Its power-invariant (or normalized) form is

$$\begin{bmatrix} 1 & e^{j\phi} & e^{-j\phi} \\ e^{j\phi} & 1 & e^{-j\phi} \\ e^{-j\phi} & e^{j\phi} & 1 \end{bmatrix} = \begin{bmatrix} a \\ a^2 e^{j\phi} \\ a e^{j\phi} \end{bmatrix} a = e^{j2\pi} \text{ (19)}$$

where x is the studied variable (voltage, current or flux), the subscripts a, b and c stand for the abc components of the variable x, the subscripts 0, f and b stand for the zero, forward and backward components of the transformed variable x, and Ψ is the transformation angle. If the synchronous reference frame is used, then Ψ = ωt, where ω = 2πf is the pulsation of the grid voltages (f is the grid frequency). Note that the backward (b) component is the complex conjugate of the forward (f) component, so only the latter is considered.

Let’s assume an unbalanced system of three-phase voltages:
where $V_a$, $V_b$ and $V_c$ are the phasors of the abc voltages, $V_a$, $V_b$ and $V_c$ are the modulus (rms values) of this phasors, $\phi_a$, $\phi_b$ and $\phi_c$ are the phase angles of the abc voltages and $\omega$ is the pulsation of the voltages. If the Ku transformation (19) is applied to (20), the transformed forward component results in:

$$V = e^{-j\omega}\left(V_a + aV_b + a^2V_c\right).$$  \hspace{1cm} (21)

Substituting (20) in (21) and taking into account the trigonometric relation $\cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$ we have:

$$V = V' + V'e^{-j2\omega}$$ \hspace{1cm} (22)

where $V'$ and $V''$ are given by:

$$V' = \frac{1}{\sqrt{3}}\left(V_a e^{j\phi_a} + aV_b e^{j\phi_b} + a^2V_c e^{j\phi_c}\right)$$

$$V'' = \frac{1}{\sqrt{3}}\left(V_a e^{-j\phi_a} + aV_b e^{-j\phi_b} + a^2V_c e^{-j\phi_c}\right).$$ \hspace{1cm} (23)

Now, if we apply the Fortescue transformation [18] to the phasors in (20) we obtain their zero-, positive- and negative-sequence components (the zero component is neglected):

$$V^+ = \frac{1}{3}\left(V_a e^{j\phi_a} + aV_b e^{j\phi_b} + a^2V_c e^{j\phi_c}\right)$$

$$V^- = \frac{1}{3}\left(V_a e^{j\phi_a} + a^2V_b e^{j\phi_b} + aV_c e^{j\phi_c}\right).$$ \hspace{1cm} (24)

If we compare (24) with (23), it results in:

$$V'^+ = \sqrt{\frac{2}{3}}V^+; \hspace{0.5cm} V'' = \sqrt{\frac{2}{3}}V^-.$$ \hspace{1cm} (25)

Note that if (20) were a set of sinusoidal balanced voltages (steady-state conditions), then there will be no negative-sequence voltage, i.e., $v'' = 0$, and $V^+$ would correspond to the steady-state phasor:

$$V_{steady\hspace{0.1cm}state} = \sqrt{\frac{2}{3}}V = \sqrt{\frac{2}{3}}\sqrt{V}e^{j\phi}. \hspace{1cm} (26)$$

Finally, the Ku forward component is related to the Park dq components as follows:

$$x_f = \frac{1}{\sqrt{2}}(x_q + jx_d) \rightarrow x_d = \sqrt{2}\text{Re}(x_f); \hspace{0.5cm} x_q = \sqrt{2}\text{Im}(x_f).$$ \hspace{1cm} (27)