

The Clay Public Lecture and Conference on the Poincaré Conjecture, Paris, 7–9 June 2010

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When in 1904 Henri Poincaré wrote in [P] the sentence “mais cette question nous entraînerait trop loin”, as he referred to the problem of knowing whether S^3 could be the only (up to homeomorphism) closed simply connected manifold in dimension 3, he did not know that it was going to take almost 100 years to solve this problem and that the history of its resolution would involve the striving of different research groups in topology and geometry.

The Poincaré conjecture is one of the “one million dollar prize” problems in Clay’s list of Millennium Prizes. This list was announced on 24 May 2000 at the Collège de France. The other problems (still unsolved) in this list are the Birch and Swinnerton-Dyer Conjecture, the Hodge Conjecture, the Navier-Stokes problem, the P versus NP problem, the Riemann Hypothesis and the Quantum Yang-Mills Theorem. The fact that the Poincaré Conjecture was one of the Millennium Prizes probably incentivised many mathematicians to attack this problem but it also gave a social value and put a symbolic price to a continuous and longstanding effort to prove or disprove the conjecture. When in 2003 Grigoriy Perelman posted his 7-page paper on the arxiv “Finite extinction time for the solutions to the Ricci flow on certain three-manifolds”, the experts understood that the proof of the technical “lemma” about finite extinction time could really close a cycle of almost one century in search of a proof. Many different teams of researchers all around the world struggled to verify the proofs and check whether the arguments were complete.

The story of the resolution of this conjecture has led to the development of top research groups all around the world but it was a Russian Mathematician, Grigoriy Perelman, who solved the Poincaré Conjecture, following Hamilton’s programme to the extreme and including the study of collapse and singularities into the scene. On the way, he also proved the ambitious Geometrization Conjecture of Thurston’s programme of which the Poincaré Conjecture is a special case. What is more surprising in this progression to the proof is that Grigoriy Perelman solved a problem with a sheer topological statement using hard methods in differential geometry and geometric analysis (including the Ricci flow techniques). Therefore, the resolution of the Poincaré Conjecture was really an example of the principle “les maths ne sont qu’une histoire des groupes” [mathematics is nothing but the story of groups].

Public awareness of the proof of the Poincaré Conjecture arrived at the International Conference of Mathematicians in 2006 in Madrid when Grigoriy Perelman was awarded the Fields Medal. The mathematical community and even the general public were shocked

when it was understood that Grigoriy Perelman would not accept this prestigious prize. Four years later, on 18 March 2010, James Carlson (President of the Scientific Advisory Board of the Clay Mathematics Institute, CMI) announced that the Clay Prize was awarded to Grigoriy Perelman. The CMI Scientific Advisory Board (James Carlson, Simon Donaldson, Gregory Margulis, Richard Melrose, YumTong Siu and Andrew Wiles) appointed a Special Advisory Committee, formed of Simon Donaldson, David Gabai, Mikhail Gromov, Terence Tao and Andrew Wiles, to consider the correctness and attribution of the proof of the conjecture. After this announcement and out of the blue, the world was staring back at the mathematical community and was wondering whether Grigoriy Perelman would accept this prize. The media informed everyone that some institutions in Russia were urging Grigoriy Perelman to accept it.

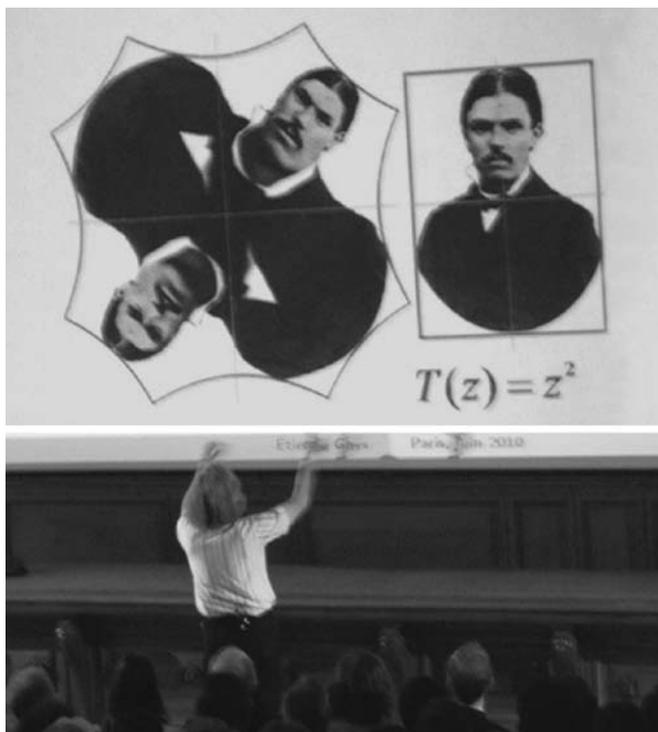
The conference

Also on 18 March 2010, James Carlson announced that the 2010 Clay Research Conference would take place in Paris at the Institut Henri Poincaré. The official ceremony for the prize would take place during this conference.

One could not imagine a better place to do that than the Institut Henri Poincaré, IHP, in Rue Pierre et Marie Curie in the beautiful 5^{ème} arrondissement of the city of lights. The IHP is located close to ENS-Ulm where many talented mathematicians have been scientifically born. These streets have registered in their pavement the steps of hesitation for many conjectures but this summer the resolution of a big conjecture was going to be celebrated there. It was the end of an era and the Institut Henri Poincaré was ready for that.

The Clay Institute, with the collaboration of the Institut Henri Poincaré, had overseen two big events to celebrate the Poincaré Conjecture. The first was a Clay Public Lecture and the second a Clay Research Conference on the resolution of the Poincaré Conjecture. The Clay Public Lecture was delivered by Étienne Ghys on 7 June at the Institut Océanographique. The Clay Research Conference took place on 8–9 June at the Institut Océanographique and the Institut Henri Poincaré (salle Darboux and salle Hermite).

The conference of Étienne Ghys was a grand public lecture beautifully organised in the historical Salle of the Institut Océanographique. According to Florence Lajoinie, who was responsible for the practical organisation of this event at the IHP, 450 people registered for the event. Among the participants, we recognised the faces of many well-known mathematicians but also many young “mathematicians-to-be”, together with the Animath students.



Étienne Ghys looking at Henri Poincaré through a Fuchsian perspective

Étienne Ghys' talk¹ was focused on the research of Henri Poincaré concerning Fuchsian groups. The talk, nicely prepared and energetically delivered, ended with the big conclusion that, in parallel to the individual efforts, mathematics is a story of groups. The title of his talk "Les maths ne sont qu'une histoire des groupes" was declared as a general principle in the work in mathematics at the end of the talk. The motto had the double meaning of the story of Fuchsian groups and the common striving in mathematics work.

The speakers on 8–9 June were certainly some of the composers of the key scores that led to the proof of this conjecture. They wrote several pages in the history of the Poincaré Conjecture and led, in one way or another, to the big endeavour of the proof.

The talks on 8 June were still taking place at the Institut Océanographique and were devoted to a general mathematical audience who wanted to know about the proof of Perelman and the history and influence of this result. After the opening by James Carlson and the director of the IHP Cédric Villani, there were talks by Sir Michael Atiyah and John Morgan about the history of the Poincaré Conjecture and related open problems. These talks were followed by the impressive talk of Curtis McMullen on "The evolution of geometric structures on 3-manifolds". The early afternoon was conducted by two of the fathers of the main streams in the history of the proof William Thurston and Steven Smale. The talk by William Thurston was about "The mystery of 3-manifolds" and contained some pedagogical scenifications of the eight Thurston's geometries. Steven Smale, who had proven

¹ The video of the conference is available under the link: <http://www.poincare.fr/evenements/item/18-les-maths-ne-sont-qu'une-histoire-de-groupes.html>



James Carlson, President of the Clay Institute, at the opening ceremony of the Clay Institute Conference at the Institut Océanographique



John Morgan on the blackboard. John Morgan together with Gang Tian wrote a book on the proof of the Poincaré Conjecture entitled "Ricci Flow and the Poincaré Conjecture".



McMullen and his vision on the wilderness of 3-geometry

the Poincaré conjecture in dimension 5 and higher, talked about topological problems post-Perelman. The last talk of 8 June was delivered by Simon Donaldson who spoke about invariants of manifolds and the classification problems with a special emphasis on manifolds with additional structures like symplectic manifolds in dimension 4. After the talks a reception took place at ENS-Ulm.

The second day of the conference was more specialised and tackled different aspects of the proof of Perelman. Speakers in the morning session were David Gabai (*Volumes of hyperbolic 3-manifolds*) and Mikhail Gromov, who gave a wonderful talk that had been announced with great simplicity under the title of "What is a manifold?". After Gromov came the talk of Bruce Kleiner on "Collapsing with lower curvature bounds". (Kleiner together with Lott maintain a webpage with material on the Poincaré Conjecture and the Geometrization Conjecture and its proof by Perelman [KL].) The speakers of the afternoon were Gérard Besson and Gang Tian. Gérard Besson talked about an alternative approach for the last step in Perelman's proof of Thurston's Geometrization Conjecture (joint work with L. Bessières, M. Boileau, S. Maillot and J. Porti). Gang Tian talked about counterparts of Perelman's techniques and the Ricci flow in dimension 4.

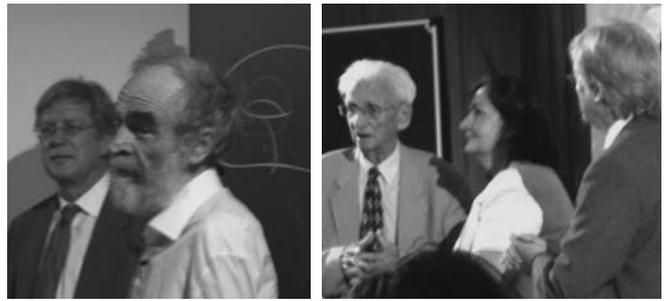
The ceremony

The ceremony took place immediately after noon on 8 June. It was now the time for the award of the prize and the “laudatio” for Grigoriy Perelman.² Andrew Wiles’ laudation described the importance of the problem and solution of the Poincaré Conjecture and thanked Perelman as well as his predecessors, especially Richard Hamilton, for that. Michael Atiyah used the parallelism of mathematics and landscape and pointed out that “*Grigoriy Perelman is the mountaineer who reached this pinnacle of the 3-dimensional world*”. William Thurston explained the mathematical importance of Perelman’s contribution and ended up with a touching paragraph about Perelman’s attitude to what he denominated “*public spectacle*”, closing his speech with the mindful sentence: “*Perhaps we should also pause to reflect on ourselves and learn from Perelman’s attitude toward life.*” Simon Donaldson described the work of Perelman and his originality as a cut of a Gordian knot in the problem and ended up his laudation with the following meaningful sentence: “*Perelman’s achievement is a testament to the continued power of the individual human mind in bringing about the most fundamental advances in mathematics.*” Mikhail Gromov used a metaphor to describe the work in three manifolds as a sea expedition to discover new lands. “*It will probably take a decade or so for the mathematical community to build up new edifices on the land discovered by Perelman,*” he pointed out at the end of his speech.

After Laudatio, it was time for the awarding of the prize. It had been well-known that Grigoriy Perelman would probably not show up to the Clay Conference in Paris. But when Cédric Villani walked slowly out of the room at that precise moment, the idea that Perelman could walk into the majestic room of the Institut Océanographique to pick up the prize really crossed the mind of some of us. But Perelman did not show up. Needless to say, one of the highlights of the ceremony was when Landon Clay offered the award symbolising the Millennium Prize looking at “infinity” in the big room of Océanographique and relaxed the formal atmosphere of the act by pronouncing the words “*to award this prize – silence – to whoever takes it*” before pronouncing the formal attribution: “*The Clay Mathematics Institute hereby awards the Millennium Prize for resolution of the Poincaré Conjecture to Grigoriy Perelman.*”

Probably one of the most emotive moments of the conference took place when Lavinia Clay stood up and walked to the centre of the main room at Océanographique with François Poincaré, one of the grandsons of Henri Poincaré. François Poincaré was holding an honorary plaque for the resolution of the Poincaré Conjecture (which will remain at the IHP) as he thanked the mathematical community for spreading the ideas of his grandfather and pointed out that he had been involved with his work as a linguist since he translated many of the letters of Felix Klein to Henri Poincaré.

² The transcript of laudations is available at the webpage of the CMI [C] under the link <http://www.claymath.org/poincare/laudations.html>.



Mikhail Gromov on the laudatio One of the grandsons of Poincaré, François Poincaré, in the ceremony.

Conclusions

Clay Research Conference was a big event marking the end of an era but also the beginning of a new one. The Poincaré Conjecture is now Perelman’s theorem but another six conjectures still remain unsolved in the Millennium problems list.

References:

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