

# Spectrum Trading in Virtualized Multi-Tenant 5G Networks

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**Abstract**—In this research work, we analyze the problem of spectrum trading in virtualized multi-tenant 5G networks using principles from matching theory. More specifically, we deal with the matching problem among the Mobile Network Operators (MNOs) and the Mobile Virtual Network Operators (MVNOs) and we propose a matching scheme that takes into account the preferences of each entity in terms of different utility variables. Our proposal includes a many-to-many matching scheme, that is an extension of the deferred acceptance algorithm, where each MNO and MVNO can cooperate with one or more MVNOs and MNOs, respectively. The performance of our proposed scheme is finally investigated by comparing it with various schemes and some useful conclusions are drawn.

**Keywords**—spectrum trading, virtualization, multi-tenancy, matching theory, many-to-many.

## I. INTRODUCTION

The concept of multi-tenancy, which is a result of the virtualization in the future wireless mobile networks, appears as a promising solution to the problem of spectrum underutilization due to the static allocation of the scarce spectrum resources. The virtualized business model implies the existence of two different types of operators: the Mobile Network Operators (MNOs), who own the infrastructure and spectrum resources, and the Mobile Virtual Network Operators (MVNOs), who lease these resources for certain period of time if becoming available from the MNOs. In contrast with the traditional model approach, where the MNOs do not offer their available unutilized spectrum resources, in spectrum trading, it is considered that the MNOs can cooperate with multiple MVNOs and lease their bandwidth to them. In this way, not only the allocation of spectrum resources becomes more efficient, but also the MNOs have monetary gain and the MVNOs have bandwidth resources gain.

During the last years, the spectrum trading model has been studied using concepts from different theories such as auction theory [1], game theory [2], etc. However, recently, also matching theory, which is a mathematical framework that describes the creation of mutually beneficial relations between two different sets of agents [3], has started to be employed by many researchers for the analysis of wireless communication networks [4]. In particular, any network communication problem is modeled as a matching game and a stable matching of this game is found. However, the majority of the references

that can be found in the literature deals with the user-cell association [5] and spectrum sharing procedure in cognitive radio networks [6], where one or multiple primary users have to choose one or multiple secondary users, known as one-to-one matching (O2OM) or one-to-many matching (O2MM) models. The matching problem in the wireless virtualization field has not been thoroughly examined yet. There are only just a few research works [7] that analyze the simplified case where one MNO can form a partnership with multiple MVNOs, while each MVNO can only form a partnership with one MNO, which is actually an O2MM model. However, in reality, MVNOs can cooperate with one or more MNOs. So far, many-to-many matching (M2MM) theory has been implemented only in research works of different fields such as biology [8], economic sciences, object recognition [9], caching [10]. From the best of our knowledge, our research work is the first that proposes the more complicated M2MM scheme for spectrum trading in virtualized multi-tenant 5G networks, where one MNO can form a partnership with multiple MVNOs and one MVNO can form a partnership with multiple MNOs.

In this research work, we study the problem of spectrum trading in virtualized multi-tenant 5G networks considering the M2MM model approach. In Section II, the system model under consideration along with the users' utilities are indicated, whereas in Section III, the corresponding matching problem is defined and the proposed matching scheme is presented. Finally, in Section IV, the simulation results are shown, while conclusions and future work are given in Section V.

## II. SYSTEM MODEL & PROBLEM DEFINITION

### A. Virtualized Multi-Tenant Model

The system model under consideration is presented in Fig. 1. There are considered to be two main entities in our model:

- MNOs are the traditional mobile network operators, which currently own all the resources (infrastructure, spectrum) and may be interested in making their resources available for resale in the market.
- MVNOs are virtual mobile network operators and act as new market entrants, which most of the time require additional resources to satisfy the demand of their own customers.

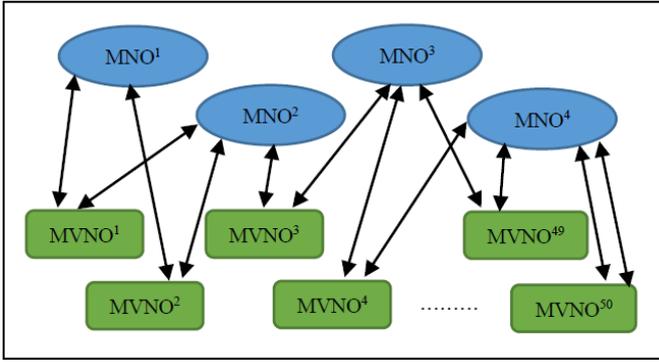


Fig. 1. System model under consideration.

We examine the case where each MNO and MVNO can cooperate with multiple MVNOs and MNOs, respectively. Thus, more economic benefits and accessing opportunities are provided to MNOs and MVNOs, respectively. It is also assumed that the total number of MNOs is  $O=4$ , while the total number of MVNOs is  $V=50$ . These numbers are the average ones in the European region, taking into consideration the allocation of MNOs and MVNOs in [11]. A sample of this heterogeneous allocation over the European countries, including the corner cases, is shown in Table I.

TABLE I. MNOs / MVNOs ALLOCATION OVER DIFFERENT COUNTRIES

	MNOs	MVNOs
United Kingdom	4	76
Germany	3	129
France	4	49
Spain	4	35
Greece	3	1

### B. Trading Market Definition

The initial stage of the trading market between these two entities needs to be the generation of the market demand and supply. This means that the MNOs should advertise their available bandwidth portions and relevant price, and the MVNOs should advertise their required bandwidth portions and relevant price willing to pay.

Let  $MNO = \{1, 2, \dots, O\}$  be the set of  $O$  participating MNOs and  $MVNO = \{1, 2, \dots, V\}$  be the set of  $V$  participating MVNOs.

1) Mobile Network Operators: Each MNO defines its advertised bandwidth and price, i.e. the available portion of spectrum to lease and the minimum payment that a MNO requires to satisfy its services. Both the bandwidth (BW) and price (PR) level are assumed to be a uniformly distributed integer in the range  $[1,5]$ . That is, the bigger the integer the more the bandwidth or price, and reversely.

$$BW_{MNO}(i) = U(1,5) \quad \forall i \in MNO$$

$$PR_{MNO}(i) = U(1,5) \quad \forall i \in MNO$$

The MNOs can be characterized using two more parameters: their reputation (REP) to the MVNOs and their quality of

service (QOS) provided to possible MVNOs' customers. Both the reputation and quality of service level are assumed to be a uniformly distributed integer in the range  $[1,3]$ . That is, the bigger the integer the better the reputation or quality of service, and reversely.

$$REP_{MNO}(j,i) = U(1,3) \quad \forall i,j \in MNO, MVNO \text{ respectively}$$

$$QOS_{MNO}(j,i) = U(1,3) \quad \forall i,j \in MNO, MVNO \text{ respectively}$$

2) Mobile Virtual Network Operators: Each MVNO defines its advertised bandwidth and price, i.e. the minimum required portion of spectrum to gain and the price it is willing to pay for it. Both bandwidth and price level are assumed to be a uniformly distributed integer in the range  $[1,5]$ . That is, the smaller the integer the less the bandwidth or price, and reversely.

$$BW_{MVNO}(j) = U(1,5) \quad \forall j \in MVNO$$

$$PR_{MVNO}(j) = U(1,5) \quad \forall j \in MVNO$$

The MVNOs can be characterized using one more parameter: their reputation to the MNOs. The reputation level is assumed to be a uniformly distributed integer in the range  $[1,3]$ . That is, the smaller the integer the worse the reputation, and reversely.

$$REP_{MVNO}(i,j) = U(1,3) \quad \forall i,j \in MNO, MVNO \text{ respectively}$$

### C. Utility Function of MNOs and MVNOs

The ultimate goal of the matching scheme is to find the best possible matching that satisfies both sides, MNOs and MVNOs, although they have conflicting interests. On the one side, a MNO would like to find the most appropriate MVNOs to host, that is the MVNOs which maximize its own revenues. On the other side, a MVNO would like to find the most appropriate MNOs to be hosted, that is the MNOs which offer enough bandwidth and the lowest price. Thus, each side would like to find its perfect pair in order to maximize its own utility function. In other words, the utility function should be expressed in such a way that efficiently represents the preferences of each side.

Therefore, the utility function of each entity should include a weighting vector, whose different elements characterize the importance level of utility function's parameters. More specifically, each entity builds its own strategy plan by selecting the appropriate weight values for the parameters of the opposite matching entity. The weighting factors of the MNOs and MVNOs and their relevant equation as well, are clearly demonstrated below in the Equations (1) and (2), respectively:

$$W_{MNO} = [W_{MVNO,BW}, W_{MVNO,PR}, W_{MVNO,REP}]$$

$$W_{MVNO,BW} + W_{MVNO,PR} + W_{MVNO,REP} = 1 \quad (1)$$

where  $W_{MVNO,BW}$  corresponds to the MNO's weight for the MVNO's bandwidth demand,  $W_{MVNO,PR}$  corresponds to the MNO's weight for the MVNO's willing price to pay, and  $W_{MVNO,REP}$  corresponds to the MNO's weight for the MVNO's reputation.

$$W_{MVNO} = [W_{MNO,BW}, W_{MNO,PR}, W_{MNO,QOS}, W_{MNO,REP}]$$

$$W_{MNO,BW} + W_{MNO,PR} + W_{MNO,QOS} + W_{MNO,REP} = 1 \quad (2)$$

where  $W_{MNO,BW}$  corresponds to the MVNO's weight for the MNO's bandwidth supply,  $W_{MNO,PR}$  corresponds to the MVNO's weight for the MNO's selling price,  $W_{MNO,QOS}$  corresponds to the MVNO's weight for the MNO's received quality of service, and  $W_{MNO,REP}$  corresponds to the MVNO's weight for the MNO's reputation.

As a result, the maximum utility functions of the MNOs and MVNOs are expressed in the Equations (3) and (4), respectively:

$$U_{MNO,max} = 5(W_{MVNO,BW} + W_{MVNO,PR}) + 3W_{MVNO,REP} \quad (3)$$

$$U_{MVNO,max} = 5(W_{MNO,BW} + W_{MNO,PR}) + 3(W_{MNO,QOS} + W_{MNO,REP}) \quad (4)$$

where number five terms correspond to the highest level that the bandwidth or price can be assigned, and number three terms correspond to the highest level that reputation or quality of service can be assigned.

Finally, taking into account all the assumptions made so far, the utility functions of the MNOs and MVNOs are defined below in the Equations (5) and (6), respectively, as:

$$U_{MNO}(i,j) = |W_{MVNO,BW} (5 + BW_{MVNO}(j) - BW_{MNO}(i)) + W_{MVNO,PR} (5 + PR_{MVNO}(j) - PR_{MNO}(i)) + W_{MVNO,REP} REP_{MVNO}(i,j) - U_{MNO,max}| \quad \forall i,j \in MNO, MVNO \text{ respectively} \quad (5)$$

where  $U_{MNO}(i,j)$  tends to zero when both differences  $BW_{MVNO}(j) - BW_{MNO}(i)$  and  $PR_{MVNO}(j) - PR_{MNO}(i)$  tend to zero as well, and  $REP_{MVNO}(i,j)$  is maximum. This means that the MNO better prefers to match with a MVNO having bandwidth-price advertisements close to its own ones and the highest possible reputation.

$$U_{MVNO}(j,i) = |W_{MNO,BW} (5 + BW_{MNO}(i) - BW_{MVNO}(j)) + W_{MNO,PR} (5 + PR_{MNO}(i) - PR_{MVNO}(j)) + W_{MNO,QOS} QOS_{MNO}(j,i) + W_{MNO,REP} REP_{MNO}(j,i) - U_{MVNO,max}| \quad \forall i,j \in MNO, MVNO \text{ respectively} \quad (6)$$

where  $U_{MVNO}(j,i)$  tends to zero when both differences  $BW_{MNO}(i) - BW_{MVNO}(j)$  and  $PR_{MNO}(i) - PR_{MVNO}(j)$  tend to zero as well, and both  $QOS_{MNO}(j,i)$  and  $REP_{MNO}(j,i)$  are maximum. This means that the MVNO better prefers to match with a MNO having bandwidth-price advertisements close to its own ones and the highest possible reputation and quality of service.

Thus, it becomes clear that the less the utility function value the better the preference of one entity for the relevant opposite matching entity.

### III. PROPOSED MATCHING SCHEME

#### A. Matching Theory

The matching between MNOs and MVNOs is implemented utilizing the extension of the deferred acceptance algorithm [12] for the M2MM case. This means that a MNO can form a partnership with at most  $n$  different MVNOs, where  $n$  equals to MNO's capacity size, i.e. how many available bandwidth sets each MNO can sell to the market. This also means that a MVNO

can form a partnership with at most  $m$  different MNOs, where  $m$  equals to MVNO's request size, i.e. how many bandwidth sets each MVNO can buy from the market. In our case, where the total number of MNOs and MVNOs is  $O=4$  and  $V=50$ , respectively, the values of  $n$  and  $m$  have been set to 25 and 2, respectively. As a consequence, this value selection results in 100 total available bandwidths sets ready to sell from the MNOs' side and 100 total required bandwidth sets ready to buy from the MVNOs' side. Thus, a necessary condition for complete matching of two sets, known as the supply-demand balance, is fulfilled.

In particular, the market demand consists of the bandwidth needed by the involving MVNOs to fulfill the traffic demand of their end customers and the maximum price they are willing to pay for these resources. Then, each MNO should gather information about the resource demand of each MVNO and advertise its own available bandwidth-price pairs.

It was mentioned before that the supply-demand balance is a necessary condition for complete matching of two sets. In order this complete matching to be also successful, one more condition should be fulfilled: the preference lists of both sets should be full. Therefore, both MNOs and MVNOs, set their preference list according to their own utility function value, i.e. higher preference for lower utility function value. The preference list of a MNO or MVNO consists of the ranking order list of the opposite set of MVNOs or MNOs, respectively.

It is important to note that we consider the case in which the MVNOs propose a partnership first. That is, MVNOs send their preference list to MNOs, and then MNOs, after taking into consideration their own preference list, select their match. Finally, the output of this matching game is a complete and successful matching of MNOs and MVNOs.

#### B. Matching Algorithm

The many-to-many matching algorithm is described below:

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#### *Many-to-many matching algorithm (M2MM)*

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##### Step 1 (Initialization):

- Each MNO broadcasts its available bandwidth  $BW_{MNO}(i)$  and relevant cost  $PR_{MNO}(i)$ . Each MNO also prepares and keeps a list including the reputation level of the MVNOs.
- Each MVNO broadcasts its required bandwidth  $BW_{MVNO}(j)$  and price willing to pay  $PR_{MVNO}(j)$ . Each MVNO also prepares and keeps a list including the reputation level of the MNOs and a list including the quality of service level provided by the MNOs (according to the received signal strength).

##### Step 2 (Preference list):

- Each MNO constructs its own preference list according to its utility function  $U_{MNO}(i,j)$ .
- Each MVNO constructs its own preference list according to its utility function  $U_{MVNO}(j,i)$ .

##### Step 3 (Matching mechanism):

- Each MVNO proposes to the MNO ranked first in its own preference list.

- Each MNO collects the offers from the MVNOs, and puts them into the same list with its already existing matches (if any).
- If this MNO's list size does not exceed its capacity  $n$ , then the MNO accepts all the offers and there is match.
- If this MNO's list size exceeds its capacity  $n$ , then the MNO consults its own preference list, accepts the  $n$  most preferred MVNOs and rejects the others.
- If any MVNO remains unmatched, then proposes to the MNOs ranked lower in its own preference list, until the moment it gets matched.
- The algorithm terminates, when all MVNOs find their matching MNO.

#### IV. SIMULATION RESULTS

We investigate the performance of the proposed M2MM scheme developing a simulation framework in MATLAB. We have considered a virtualized multi-tenant 5G network with  $O=4$  MNOs with  $n=25$  and  $V=50$  MVNOs with  $m=2$ , as mentioned in Section III. The simulations account for a daily market, which consists of 3 spectrum trading interactions. Hence, our yearly results correspond to average results from 3 daily interactions multiplied by 30 days, then multiplied by 12 months, i.e. 1080 iterations. Also, it is assumed that the MNOs give more importance to the price parameter, as their goal is to gain some revenues, which is then followed by the bandwidth and reputation parameter. Unlike MNOs, MVNOs give more importance to the bandwidth parameter, as their goal is to get some bandwidth, which is then followed by the quality of service, price and reputation parameter. So, the weighting vectors are the following:

$$W_{MNO} = [0.3, 0.5, 0.2]$$

$$W_{MVNO} = [0.4, 0.2, 0.3, 0.1]$$

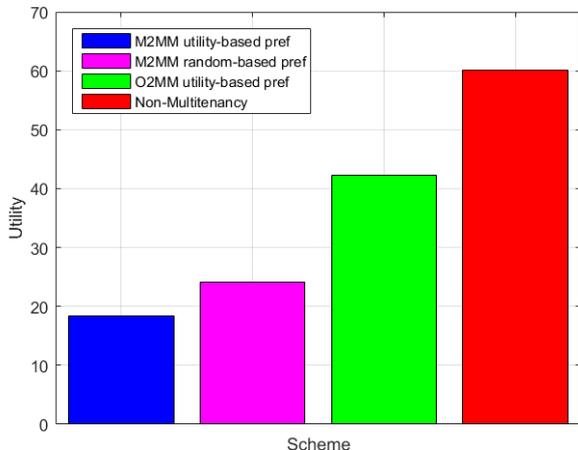


Fig. 2. Average MNOs' utility for four different approaches.

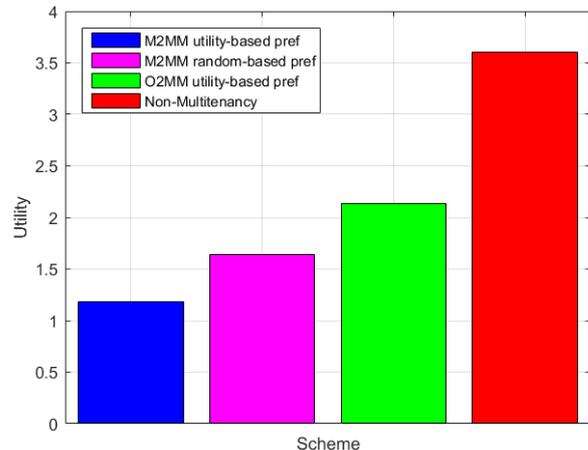


Fig. 3. Average MVNOs' utility for four different approaches.

Our proposed matching scheme can be called as M2MM scheme with utility-based preferences. In order to evaluate this scheme, we compare it with the following schemes:

- M2MM scheme with random-based preferences, where the preference list of both MNOs and MVNOs is constructed randomly.
- O2MM scheme with utility-based preferences, where one MNO can form a partnership with multiple MVNOs, but one MVNO can only form a partnership with one MNO.
- Non—multi-tenancy scheme, where there is no cooperation between MNOs and MVNOs, no spectrum sharing even if there is unused bandwidth.

Fig. 2 depicts the average MNOs' utility for the four different schemes. As explained in Section II, the less the utility function value the better the preference of one entity for the relevant opposite matching entity. Therefore, it can be seen that the cooperation between MNOs and MVNOs leads to MNOs' performance improvement compared with the non-cooperative (non—multi-tenancy) scheme, since the unused spectrum dynamically becomes available at the market for sale and the spectrum efficiency increases. Furthermore, it can be observed that the two M2MM schemes perform better than the O2MM matching scheme. This happens because in the O2MM case the MVNOs are restricted to collaboration rules, thus there is a part of the bandwidth, which is advertised for sale by the MNOs, that remains still unused from the MVNOs. Also, the M2MM with utility-based preferences seems to clearly outperform the M2MM with random-based preferences, thus proving the superiority of our proposed matching scheme over all the comparing schemes. Finally, the outperformance of the M2MM with utility-based preferences becomes obvious in Fig. 3, which demonstrates the average MVNOs' total utility for the four different schemes, as well. It is worth to mention that the MVNOs' utility value is smaller than the MNOs' one because MVNOs are first proposed to MNOs, thus having an advantage, and also because MVNOs consider one more parameter, i.e. QOS, in their utility function compared to MNOs. Our proposed matching scheme not only improves the spectrum utilization, but

also increases the interest of each entity involved to the spectrum trading.

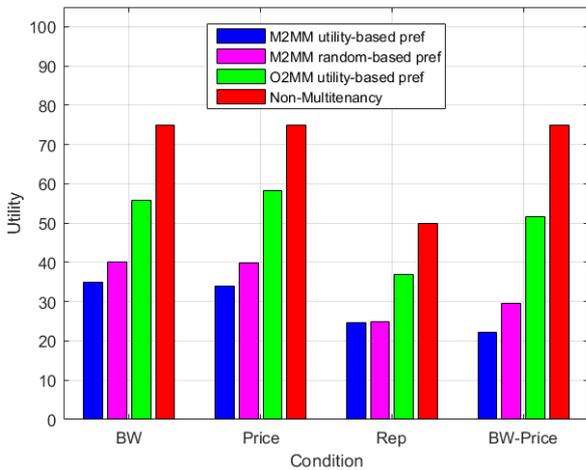


Fig. 4. Average MNOs' partial utility for four different approaches.

Fig. 4 and Fig. 5 present the average MNOs' and MVNOs' partial utility for the four different schemes, respectively. More specifically, partial utility is the total utility if we take out some involved parameters. Thus, unlike the total utility that consists of all parameters: bandwidth, price, reputation (and quality of service in MVNO's case), the partial utility consists of the bandwidth parameter only, or the price parameter only, or the reputation parameter only, or the quality of service parameter only, or both the bandwidth and price parameters only. As it can be seen, for all the schemes, the more the parameters included in the utility function, the better the matching performance, i.e. the lower the utility function value. It results in the fact that the applied weighting factors, when multiple parameters exist, not only define the strategy of a specific entity, but also help it maximize its own utility.

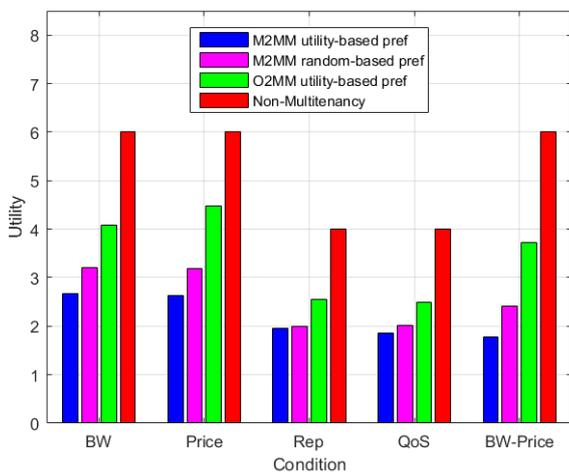


Fig. 5. Average MVNOs' partial utility for four different approaches.

First of all, we assume that the MNO decides for the bandwidth resource and the MVNO decides for the price resource. Taking into account this assumption, we conclude that

the MNO's request is satisfied when its matching pair pays same or more money than the advertised selling price, otherwise the MNO's request is unsatisfied. Respectively, the MVNO's request is satisfied when its matching pair provides same or more bandwidth than the advertised requesting bandwidth (thus giving the opportunity to offer a bigger variety of services to its end customers, e.g. video streaming services), otherwise the MVNO's request is unsatisfied. Thus, Fig. 6 demonstrates the average number of satisfied/unsatisfied requests from individual entities over the two M2MM schemes. Furthermore, the union of these conditions results in the total request satisfaction or not of a matching pair. Therefore, Fig. 7 depicts the average number of satisfied/unsatisfied requests from matching pairs over the two M2MM schemes.

Then, in Fig.6, it can be seen that the average number of satisfied requests from individual MNOs or MVNOs is larger than the number of unsatisfied ones in the utility-based preference scenario (U). However, it is also clear that the utility-based preference scenario does not show any significant improvement comparing with the random-based preference scenario (R). In fact, the number of satisfied/unsatisfied requests from individual MNOs or MVNOs is almost same in both M2MM schemes. Finally, in Fig. 7, it can be observed that the average number of satisfied matching pair's requests in the M2MM with utility-based preferences is larger than the corresponding number in the M2MM with random-based preferences. Similarly, the average number of total unsatisfied matching pair's requests in the M2MM with utility-based preferences is smaller than the corresponding number in the M2MM with random-based preferences. Note that, the term total unsatisfied matching pair's requests means the number of unsatisfied matching pair's requests plus the number of matching pair's requests where one entity's request is satisfied and the other entity's request is unsatisfied. This is expected, as this is the reason of utility function implementation, i.e. to increase the performance. However, the number of satisfied matching pair's requests is less than the number of total unsatisfied ones (MNO, MVNO, or both), which means that further modifications in the proposed utility function need to be studied to overcome this drawback.

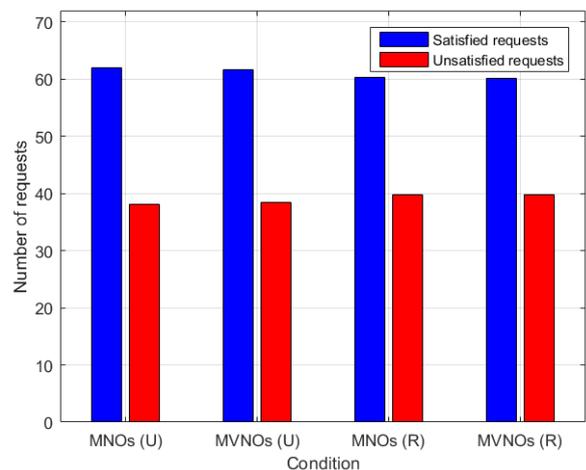


Fig. 6. Average number of satisfied or unsatisfied individual MNO/MVNO requests.

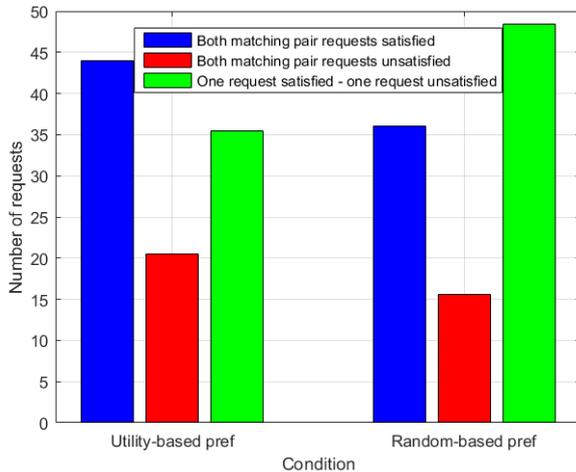


Fig. 7. Average number of satisfied or unsatisfied matching MNO-MVNO pair's requests.

## V. CONCLUSION AND FUTURE WORK

This research work has presented a M2MM scheme for the spectrum trading problem among the MNOs and the MVNOs of a virtualized multi-tenant 5G network. Indeed, a detailed description of this matching scheme was provided in order to help the better understanding of our proposal. This scheme has been compared with the corresponding random matching mechanism, the O2MM scheme and the non—multi-tenancy scheme, and evaluated for various system parameters to investigate its performance. Simulation results indicated that the M2MM algorithm with utility-based preferences leads to more efficient spectrum utilization and, at the same time to better performance for both MNOs and MVNOs.

As a future work, we plan to improve the performance of the proposed mechanism by renovating the utility function that orders the construction of the preference lists. Thus, the matching pairs will better serve both interests, maximizing the revenues from MNOs' side and the gained spectrum from MVNOs' side, and also reducing the number of unsatisfied entities' requests. Additionally, we are interested in exploring how the variety of advertisements from both sides can influence the efficiency of the matching process. This combination of

perspectives may allow us to shed light on previously unaddressed details behind the development of matching mechanisms suitable to be fostered by 5G networks.

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