Design methodology for a dc-dc power conversion system with EIS capability for battery packs

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Abstract

This paper presents a design methodology for a dc-dc power conversion system (PCS) for battery packs. The methodology provides with an optimal design of the PCS and the associated inductive-capacitive filter interfacing the battery pack with the PCS. The PCS adds superior capability over conventional designs, which is performing current and voltage perturbations at the battery terminals for the so-called Electrochemical Impedance Spectroscopy (EIS). This technique is an option for battery state-of-charge (SoC) and state-of-health (SoH) assessment. The design is optimal in the sense that it minimizes volume and system power losses. Such multi-objective optimization is addressed adopting the theory of weighted sum and Pareto front. The methodology is tested through a case study, addressing a lithium-ion battery pack. The offered analyses permit to identify the impact in system performance of diverse design variables such as dc-link voltage for the PCS and its switching frequency.

Keywords: Batteries, H-bridge, LC filter, Electrochemical Impedance Spectroscopy (EIS)

1. Introduction

The technological advance in battery-based energy storage systems is one of the key challenges for the modernization of many cornerstone fields of the society, such as the electrical networks and the electro-mobility, for instance.

The current catalog of commercially available electrochemistries is vast. Each type offers different performance in many aspects such as cost, cyclability, energy and power density, efficiency, charge and discharge current rates, and so on; hence each type is best intended for providing different applications [6]. In general terms, but specially for high performance battery types such as lithium-ion ones, continuous monitoring and protection is needed, and this is provided by the so-called battery management systems (BMS devices hereinafter). A BMS is an electronic board interfacing the battery itself (or the considered pack of batteries connected between them in series and/or parallel) with the power conversion system (PCS), which is a power electronics based module permitting an effective energy exchange between the batteries and the electrical system the energy storage solution is connected to. The PCS is usually composed by various power conversion steps. For grid connected solutions, a usual topology is that based on a front-end inverter interfacing with the AC grid. This inverter is then connected in series with a dc-dc converter that interfaces with the battery pack with a passive filter in between. Such dc-dc converter is the one in charge of actually managing the energy storage system. In turn, there are different types of BMSs and these can be classified in terms of their complexity including from centralized to distributed architectures. The BMS is the system in charge of carrying out several functions related to the battery data acquisition, state-of-charge (SoC) and state-of-health (SoH) monitoring and control, so as to ensure proper and safety operation during the battery lifetime [14, 20]. So the BMS is equipped with digital and analog inputs and outputs to read and evaluate external signals such as voltages and temperatures, and also to govern the power electronics of the battery to perform charge and discharge processes.

In regards of SoC, the BMS implements different algorithms for calculation. Basic ones are based on Coulomb counting [1], while sophisticated ones implements optimization routines including predictive and adaptive state estimation to fit battery parameters online [48]. While these technologies are widely employed in field, other powerful techniques, such as the so-called equivalent impedance spectroscopy (EIS) are still explored in laboratory environments. EIS is based on calculating the electrochemical impedance of the battery at different electrical frequencies [46, 43, 9, 4]. From this information, the SoC, SoH and temperature can be estimated [3, 37, 36]. EIS technique can be applied through two modes: the galvanostatic and potentiostatic modes. In the former, the transfer function for the equivalent impedance of the cell is deduced from the evaluation of the alternative voltage across the cell while applying a small alternative current through it. On the other way round, in the latter mode, is the voltage across the cell and not the current through it, what is imposed for the experiment. The frequency of such perturbation can be comprised in a very wide range including from $10^{-2}$ Hz to $10^{5}$ Hz. Such wide range is due to the need of representing a wide variety of
phenomena within. Conversely, the magnitude of perturbation should be small, reaching no more than a few tens of mV across cells with high impedance, and as low as less than 10 mV for low impedance ones [12]. According to [33], the galvanostatic mode is preferred for testing batteries by the fact that the current can be effectively driven and controlled using inexpensive transistors. In the same line, [12] indicated that galvanostatic mode is superior over potentiostatic mode because any small error in the applied voltage (even of about 1 mV), could be translated into undesirable currents through the cell.

The wide range for the electrical frequency of the voltage and current to be applied at the cell, along with the need of precise measuring of electrical magnitudes during the process, suggest the application of dedicated apparatus [2], bounding most of the applicability of EIS at laboratory environments, at the end. So the question is whether the potentialities of EIS for SoC and SoH estimation could be effectively applied in field.

This research question has been recently addressed in few articles [15, 31, 13, 8]. The work done in [8] developed a BMS with dedicated dc-dc converters per each battery cell so current and voltage perturbations can be applied for EIS technique also ensuring cell charge balancing. Conversely, the approaches in [15, 13] proposed the excitation of the battery cells through some additional analog circuitry connected at the battery terminals. In particular, in [15], the connection of a battery pack to a motor inverter through a capacitor (passive filter) is considered. An additional circuitry for excitation in between, which is not detailed in the paper, is managed so as to apply the required current perturbation at the battery terminals in a frequency range limited at 2 kHz. Such relatively low electrical frequency is enough so as to represent most of the phenomena in battery cells and thus to obtain enough information for SoC and SoH estimation. Altogether yields a low cost option for EIS implementation. The authors of the present paper would add that this, in turn, potentially reduces the sophistication of the BMS to be included in the system, and thus its cost.

As in previous literature, the present paper adopts the idea of exploring a low cost approach for EIS implementation. Here, a battery pack is connected to its corresponding PCS through a passive filter but, as a difference with [15, 13], no additional circuitry for excitation is included in between. Consequently, the current perturbation through the battery for EIS implementation is to be directly managed by the control loops of the PCS. This makes the system even simpler than in previous works but poses some constraints for the design of such PCS and the passive filter: i) the switching frequency of the PCS should be high enough to effectively synthesize current perturbations with the required waveform and in the range of few kHz at the battery terminals for EIS technique; ii) the selection of the cutoff frequency of the passive filter, acting as a low-pass filter for current and voltage fluctuations at the battery terminals, should be consistent with the frequency range for EIS technique.

The cutoff frequency of the filter and the switching frequency of the PCS have, in turn, direct implications in the performance of the system. In fact, these factors, along with the voltage magnitude across the switches of the PCS, greatly determine the volume and power losses of the system. Also, other factors affect the design of the filter, such as those related to the battery characteristics. Indeed, the battery impedance actually contributes to the filtering and this is also a third stream to consider while designing the filter size. Further, the battery imposes some limitations in the current ripple at end of charge condition, so this should also be addressed while designing the filter. Addressing all above mentioned factors, this paper proposes a design methodology for the PCS and passive filter so the system becomes optimal in terms of volume and power losses, while still being able to perform EIS technique and ensure the required power quality at battery terminals. This is the main contribution of the paper.

The design methodology innovates in different aspects. In regard of the design of the filter, conventional approaches are based on firstly designing an inductive configuration as long as the switching frequency is high enough. This could result into a bulky inductor though, with the associated cost and losses. So in the case it does not satisfy with the requirements, a LCL filter can be proposed so as to minimize the injection of harmonics, prioritizing volume and minimizing the capacitive reactive consumption. Such procedures are explained in [19, 21].

Finally, in regard of the design of the PCS, it is worth noting that focus is on the dc-dc H-bridge converter interfacing the battery pack with a dc-link. For sure, and as previously introduced, a front-end inverter is supposed to be also included for AC grid connected systems but this is not designed in the paper. This front-end inverter can be a state-of-the-art one since being in charge of just maintaining constant voltage at the above-mentioned dc-link. No advanced duties such as the management of the charging and discharging of batteries and the application of EIS technique are associated to this inverter.

The design methodology is presented in following contents as a step-by-step procedure in Section 2. Then, the applicability of the design methodology is proved in the particular case study of a lithium-ion pack, in Section 3. The conclusions and further research are introduced in Section 4.

2. Design methodology

This section presents the methodology for the design of the PCS for batteries. The main design variables are: i) the switching frequency of the PCS, which is based on the dc-dc H-bridge structure operated under unipolar pulse width modulation; ii) the dc-voltage of the H-bridge at the high-voltage side (i.e. not the battery side); iii) and the size of the inductive-capacitive filter (LC filter) interfacing the battery with the H-bridge. The phases for the design methodology are represented in Figure 1. As can be observed, modeling tasks includes the battery impedance, as well as the H-bridge and LC filter, since they are the main components affecting the performance of the system. The H-bridge and LC filter are modeled so as the average and alternative voltage across and current through them can be easily calculated from the operating conditions and some input data, which are usually available in manufacturer’s datasheets for the semiconductors and capacitive and inductive components. The modeling of the H-bridge and LC filter also includes
the mathematical expressions for the power losses calculation, as well as for the volume determination.

Using the modeling of the battery impedance and of the H-bridge and LC filter, it is possible to find out eligible designs (i.e. switching frequency ranges, dc-link voltage, and for the magnitude of the capacitance and inductance of the LC filter) that fulfill the design constraints (minimum ripple, EIS option and proper operation at end of charge condition). So at this point of the design methodology it lasts to apply a selection criterion (or a combination of criteria) to finally determine the best design among eligible ones. As can be observed in Figure 1, different criteria can be considered as the minimization of the power losses of the system and the minimization of the volume of the LC filter, yielding a multi-objective (non-linear) optimization problem. The determination of eligible designs and the selection of best configuration are included into a so-called decision algorithm, which is explained in detail throughout the paper.

Subsections 2.1 to 2.4 present the modeling of the main components of the system, i.e. the battery and the H-bridge and LC filter, thus answering the first steps of the step-by-step design methodology. Then subsection 2.5 describes the decision algorithm so as to determine the best solution among eligible.

2.1. Battery modeling

There are different options to model battery cells. These are usually adopted in simulation environments but also in real devices, to estimate SoC and SoH for cells. Model types can be classified in electrochemical, analytical [32][25], stochastic [16], and those based on equivalent electrical circuits [18]. Each model type is able to represent to a greater or lesser extent specific phenomena in the battery cell. Thus, none of them is accurate enough to represent all factors affecting battery performance by their own, yielding the need of adapting the modeling to the specificities and level of detail required for each case of study.

In general terms, electrochemical, analytical and stochastic models are able to reproduce specific phenomena with high accuracy, but they are hardly implementable in dynamic simulation environments. Conversely, electrical circuit-based models permit to reproduce current and voltage characteristics for battery cells while working, and accounting on reasonable computation cost. To do so, these models are based on controllable voltage and current sources, in combination with resistances and capacitors, that can present even variable characteristics. Diaz et al. (2016) [7] summarized some of the main types of electrical circuit-based models.

The performance of the model improves in parallel to its complexity, but also the difficulty in determining the associated parameters. There are different options to parametrize models and some of the most used ones are: i) from manufacturer datasheets (as for the case of Sheperd’s model, (1963) [40], along with its updates by [29], [44]); ii) from optimization algorithms minimizing the difference between empirical and simulated data [10]; iii) and through the previously introduced EIS technique [46, 43, 9, 4].

As pointed out by Jossen (2006) [17], the behavior of battery cells can be influenced by both internal and external aspects. Among internal ones, we find the SoC and aging, the dc and ac impedance, as well as other battery design parameters. Among external ones, the list includes the temperature, the dc current through the battery, as well as long and short term history. All these aspects determine the performance and life of battery cells and are influenced by different physical phenomena, which can be divided into: i) electric and magnetic phenomena; ii) mass transport and double layer phenomena; and iii) long term phenomena associated to the operation regimes history. Following subsections just briefly define such physical phenomena as a needed background knowledge so as to understand the model components and dynamic cell response.

2.1.1. Long term phenomena: SoC and SoH

Long term phenomena refer to variations in SoC and SoH, which are typically in a time domain in the range of minutes to hours (for the case of SoC); and in the range of months to years (for the case of SoH).

2.1.2. Medium term phenomena: Mass transport and double layer effects

These phenomena are in a time domain in the range of milliseconds to hours and are of major importance while determining cell time response. They are classified as mass transport and double layer effects, as explained in the following.

Mass transport effects: diffusion (and migration). Ions, which are products of electrochemical reactions in cells, are exchanged between cell electrodes through the cell, as opposed to the other type of products, i.e. the electrons, which are exchanged between electrodes through an external circuit to the cell (the load or power source connected to). Such movement of ions can be provoked by two mechanisms which are named diffusion and migration, being diffusion the most determining mechanism for ion exchange.

Diffusion is in a time domain in the range of seconds to minutes, so at frequencies below 1 Hz [42], [35]. It can be represented by an electrical impedance for the charge to move towards the active material of electrodes and react. Such electrical impedance is modeled by the so-called Warburg impedance.
The Warburg impedance is a series association of a resistive and a capacitive term, and their value is dependent on the operating frequency. The Warburg impedance is formulated as

\[ Z_{w} = \frac{\sigma}{\omega^{1/2}} \Delta(1 - j) \]  

being \( \sigma \) the Warburg coefficient and \( \omega \) the angular frequency in rad/s. In a Nyquist plot, this impedance is represented by a straight line at -45 degrees. At high frequencies, \( Z_w \) tends to zero. Conversely, at low frequencies, \( Z_w \) increases, representing the increasing importance of diffusion—the movement of reactants—within the cell.

Double layer effects: batteries resembling capacitors. At each of the electrode-electrolyte interfaces (charge-transfer zones), an structure resembling to a capacitor is naturally built. Having two electrodes in the cell, at the end there is a double electrical layer within. The difference of charge between the two sides of each layer is translated into an electric potential, i.e. the potential across the above mentioned capacitor named hereinafter as \( C_{ct} \). The over-potential contribution of charge-transfer zones can be modeled by the parallel association of \( C_{ct} \), along with the so-called charge transfer resistance \( R_{ct} \). This R-C association acts as a low pass filter for the charge-transfer reactions with a time constant yielded by \( R_{ct}C_{ct} \), so at high frequencies (typically above 100 Hz [45, 42] for lead-acid and lithium-ion cells, for instance).

Thus, at high frequencies, batteries mimic capacitors, and its behavior is not depicting a mass transfer phenomena. Instead, fast electric and magnetic phenomena will determine the cell behavior, as noted in the following section. For frequencies between 1 Hz and 100 Hz, cell current will be affected by the combined contribution of the capacitive and resistive branches modeling the charge-transfer zone. Some portions of the active material of electrodes will be actually holding electrochemical reactions, but some permitting a movement of charges within due to electrical fields. Finally, for very low frequencies below 1 Hz, the current through \( R_{ct} \) will dominate heat generation. Diffusion is fully operative in this frequency range and the whole active material of electrodes may be subjected to electrochemical reactions.

2.1.3. Short term phenomena: electric and magnetic

Typically above 100 Hz other phenomena different than mass transfer-related one dominates the performance and behavior of the cell. Among the diverse phenomena in the range of kHz [17, 47], one of the most characteristic one is that a lagging current with respect to voltage can be experienced, thus showing an inductive behavior for cells. Such inductance can be due to geometry of electrodes and the wiring associated. A figure of merit for this inductance for a lithium-ion cell is in the range of few hundreds of nH [47].

2.2. Model formulation

All previously explained phenomena in cells can be reflected in an equivalent circuit depicting the battery impedance. The circuit is presented in Figure 2.

The model parameters are: i) OCV, in Volts, is the open circuit voltage of the cell at rest conditions; ii) \( R_s \), in Ohm, is the Ohmic resistance of the cell, mostly accounting on electrolyte, current collector, active mass resistance and the resistance due to the transition between active mass and current collector; iii) \( L \), in Henries, represents the inductive behavior of the cell at high frequencies. The inductance is due to the geometry of the cell and the inductance of wiring; iv) \( C_{ct} \), in Farads, is the capacitance of the charge-transfer area; v) \( R_{ct} \), in Ohm, is the resistance of the charge-transfer area; vi) \( Z_w \), in Ohm is the Warburg impedance, associated to the phenomena of diffusion.

The representation of the impedance of the cell in a Nyquist plot can be very useful, since it permits to identify and relate model parameters with the diverse phenomena involved. A typical shape of the curve for a cell impedance in a Nyquist plot is shown in Figure 3.

2.3. The dc-dc H-bridge converter: operation and power losses

The aim of the dc-dc H-bridge converter is to step the voltage of the dc-link down to suitable levels for charging and discharging battery cells. The H-bridge topology has been selected because of the possibility of achieving very low average voltage levels (as low as zero Volts, in fact), so this can be exploited to test single battery cells. The achievement of such low voltage
levels is not easy using buck boost simple topologies [28]. Following contents briefly discusses on the operating principle and power losses modeling.

The operating principle of the converter varies with the adopted modulation scheme. The unipolar pulse width modulation (UPWM) switching scheme is adopted here. According to the UPWM technique, the switches in each leg are operated so that they are never ON at the same time. Otherwise, they will short-circuit the dc voltage source feeding the converter, \( V_{dc} \). The H-bridge topology operated under the UPWM can reverse the polarity of the output voltage and current, so it yields a 4-quadrant converter. In UPWM technique, a triangular waveform \( V_{tri} \) is compared with a control voltage \( V_{control} \), and this controls the leg composed by transistors \( T1 \) and \( T2 \), so leg A hereinafter. The other leg, the leg \( B \), is controlled by comparing the triangular waveform with \( -V_{control} \) (see Figure 4).

The average voltage output \( V_0 \) of the converter (at the battery side) can be expressed as

\[
V_0 = V_d \cdot (2 \cdot D1 - 1),
\]

being \( D1 \) the duty cycle for transistors \( T1 \) and \( T4 \) (see figure 4). According to the last expression, for \( D1 = 0 \), \( V_0 = -V_d \); for \( D1 = 0.5 \), \( V_0 = 0 \); and for \( D1 = 1 \), \( V_0 = V_d \). Therefore, for unipolar voltage \( V_0 \), \( D1 \) should vary between 0.5 and 1. A detailed explanation of H-bridge operation under UPWM can be found in [28].

The losses in the converter are caused by the switches and can be classified in conduction and switching losses. One straightforward option to calculate switching and conduction losses is based on the guidelines (and parameters in datasheets) provided by manufacturers. The application manual of the manufacturer Semikron proposes a relatively simple formulation to do so [39]. Adopting these formulas, switching and conduction losses for a single IGBT can be calculated and for each switching period. The collector-emitter voltage drop for the IGBT while conducting is estimated there and for different operating chip junction temperatures. Also, the on-state resistance is provided in terms of temperature. From the above voltage and resistance—and in terms of the output current of the converter—the conduction losses can be calculated.

The energy dissipated while turning on and off is also depicted in datasheets and in terms of the current through the IGBT and for specific chip junction temperature and voltage across. These curves can be easily incorporated into the corresponding formulas for power losses estimation by resembling them to second order polynomials.

So whenever a transistor is in ON state and/or changing state, it will incur in some losses that could be evaluated analytically and using the information provided by the manufacturer. But what is most important to correctly calculate power losses is to understand how the different transistors of the dc-dc H-bridge converter are switched. Answering to this, Figure 4 depicts possible current paths of the current for all states within a switching period.

### 2.3.1. Conduction losses for a single IGBT

While conducting, IGBTs can be modeled by a dc voltage source in series with a linear resistance. The voltage source is the saturation voltage for the IGBT and is representative of the threshold voltage level across—i.e. a collector-emitter voltage drop—, to switch to ON state, and also considering that the current through the device is zero. This voltage is dependent on the working temperature. The linear resistance models the internal resistance (usually named as terminal to chip resistance) of the IGBT, thus increasing the voltage drop across the device as proportional to the current through it. The value of this resistance is also dependent on the working temperature. Adopting the above modeling approach, the conduction power losses for a single IGBT can be calculated as [39],

\[
P_{con} = I_o \cdot (V_{ce}(25^\circ C) + T_{C_v} \cdot (T_j - 25^\circ C)) + I_j^2 \cdot (r_{ce}(25^\circ C) + T_{C_r} \cdot (T_j - 25^\circ C)) \cdot D_T,
\]

where: i) \( V_{ce}(25^\circ C) \) is the collector-emitter saturation voltage, in Volts; ii) \( r_{ce}(25^\circ C) \) is the terminal to chip resistance, in Ohm; iii) \( I_o \) is the current through the device, i.e. the output current of the converter, in Amperes; iv) \( T_j \) is the temperature of the junction, in Celsius degree; v) \( T_{C_v} \) is a thermal coefficient affecting the voltage drop at the IGBT, in V/deg. This coefficient is calculated as \( T_{C_v} = (V_{ce}(125^\circ C) - V_{ce}(25^\circ C))/(125^\circ C - 25^\circ C) \), being \( V_{ce}(125^\circ C) \) the collector-emitter voltage drop at 125°C; vi) \( T_{C_r} \) is a thermal coefficient affecting the resistance of the IGBT while conducting, in Ohm/deg. This coefficient is calculated as \( T_{C_r} = (r_{ce}(125^\circ C) - r_{ce}(25^\circ C))/(125^\circ C - 25^\circ C) \), being \( r_{ce}(125^\circ C) \) the terminal to chip resistance at 125°C; vii) and \( D_T \) is the duty cycle for the IGBT \( T \).

### 2.3.2. Switching losses for a single IGBT

The power losses over a switching period for an IGBT can be calculated from the sum of the energy dissipated in one switch ON event, \( E_{on} \), and another switch OFF event, \( E_{off} \). Such energy is derived from the graphs provided by the manufacturer function of the operating current (as discussed previously in the paper). To effectively adopt these graphs in a simple equation to compute power losses, the trends relating \( E_{on} \) and \( E_{off} \) with
the operating current are approximated to second order polynomials. Thus, the switching power losses over a switching period for an IGBT are formulated as [39],

\[ P_{sw} = f_{sw} \cdot E_{onoff_j}(I_d) \cdot \left( \frac{I_c}{I_{ref}} \right)^k \cdot \left( \frac{V_c}{V_{ref}} \right)^k \cdot (1 + T C_{sw} \cdot (T_s - T_{ref})), \]  

(4)

where: i) \( f_{sw} \) is the switching frequency, in Hz; ii) \( E_{onoff_j}(I_d) \) is the energy dissipated in a switching period, i.e. one turn on and one turn off transition, by an IGBT, in J; iii) \( I_{ref} \) is the reference value (current, in A) of the switching loss measurement taken from the datasheet; iv) \( V_{ref} \) is the reference value (collector-emitter voltage, in V) of the switching loss measurement taken from the datasheet; v) \( T_{ref} \) is the reference value (junction temperature, in Celsius degrees) of the switching loss measurement taken from the datasheet; vi) \( V_i \) is the input dc voltage for the converter, in V; vii) \( T C_{sw} \) is the temperature coefficient of the switching losses, estimated around 0.003 1/K; viii) \( k_i \) is the exponent for the current dependency of switching losses, estimated around 1.0; ix) \( k_v \) is the exponent for the voltage dependency of switching losses, estimated between 1.3 and 1.4.

2.3.3. Conduction and switching losses for the H-bridge

According to the previously presented formulation, and the operating states of the H-bridge, as presented in Figure 4, the average power losses due to switching and conduction in the converter in a switching period can be calculated as

\[ P_{con} = 2 \cdot [P_{cond,1} + P_{cond,2}] + 2 \cdot [P_{sw,1} + P_{sw,2}], \]  

(5)

where \( P_{cond,1} \) and \( P_{sw,1} \) are the conduction and switching losses for transistor T1 (and also for transistor T4). Analogously, terms \( P_{cond,2} \) and \( P_{sw,2} \) quantifies the losses for transistor T3 (and also for transistor T2).

2.4. The filter at the battery terminals

To ensure smooth dynamics in voltage and current levels at the battery terminals, an LC filter should be included between the converter and the battery.

The design of the filter depends on the admissible magnitude for voltage and current variations at battery terminals during a switching period. And the switching frequency, in turn, is also a decision variable for the problem. In general terms, the larger the filter, the lower the voltage and current variations, but large filter components means high volume. To reduce volume, switching frequency could be increased, but this implies to also increase switching losses at semiconductors. So the optimal design for the filter is the solution of an optimization problem accounting on diverse factors.

The design criterion for the capacitive and inductive components of the LC filter is based on the admissible magnitudes for the alternating components of voltage and current waveforms. The square profiled voltage waveform at the H-bridge can be intended as the sum of an alternating voltage with average value equal to zero, and a constant voltage with an average value equal to that for \( v(t) \), so \( V_0 \). Subtracting such constant value \( V_0 \) from \( v(t) \), and neglecting the voltage drop at the switches, the alternating component of the voltage \( v(t) \), named hereinafter as \( e(t) \) can be represented as in Figure 5.

![Figure 5: Waveform of the alternating components of the voltage at the terminals of the dc/dc H-bridge converter (battery side).](image)

This alternating voltage is that provoking the alternating charging and discharging of the LC filter. Note that because of the UPWM technique, the frequency of such alternating voltage is twice the switching frequency for the converter. This directly yields smaller LC filters than for the case of applying other switching techniques.

For the analysis of the magnitude of voltage and current waveforms after the filter, i.e. at the battery terminals, we could consider the equivalent electrical circuit shown in Figure 6. As can be observed the battery is simply represented by an impedance, since the dc voltage source previously included in the model does not intervene in the analysis of the alternating components of the current and the voltage.

![Figure 6: Equivalent circuit for the design of the LC filter.](image)

To easily obtain an expression for the magnitude of the current and voltage at the battery terminals in terms of the filter size, it is proposed now to analyze the system in the frequency domain—with phasors—, approaching the alternating voltage \( e(t) \) to a sinusoidal function, with a RMS value equal to the RMS value of \( e(t) \) and oscillation \( \omega = 2 \cdot \pi \cdot (2 \cdot f_{sw}) \).

The RMS value of \( e(t) \), named \( E \), can be calculated based on the profile shown in Figure 5. According to this figure, \( E \) results

\[ E = V_d \cdot \sqrt{1 - D^2}. \]  

(6)

This yields the formulation of the phasor \( \tilde{E} \) as \( \tilde{E} = E \angle 0^\circ = V_d \cdot \sqrt{1 - D^2} \angle 0^\circ \). Also in the frequency domain, the reactance
and the impedance of the capacitor and the inductor are

\[
Z_L = j \cdot \omega \cdot L, \quad (7)
\]
and

\[
Z_C = -j \frac{1}{\omega \cdot C}. \quad (8)
\]
So the equivalent impedance from the points 1 and 2 in the circuit (see Figure 6) is

\[
Z_{12} = \frac{Z_C \cdot Z_{bat}}{Z_C + Z_{bat}} + Z_L. \quad (9)
\]

Knowing the voltage \(E\) and the impedance \(Z_{12}\), the ac current consumed by the circuit is

\[
I_{ac,jot} = \frac{E}{Z_{12}} = \frac{V_{dc} \cdot \sqrt{1-D^2}}{Z_C + Z_{bat}} + Z_L. \quad (10)
\]
The magnitude of the ac current is

\[
|I_{ac,jot}| = \frac{Z_C}{Z_{12}} \cdot |Z_{bat}|, \quad (11)
\]
And the magnitude of the ac voltage across the battery terminals is

\[
V_{12} = L_{ac,jot} \cdot Z_{bat} = \frac{Z_C}{Z_{12}} \cdot |Z_{bat}| \cdot Z_{bat}. \quad (12)
\]
The above formulation relates the magnitude of the alternating components of the current and the voltage at the battery terminals with the size of the LC filter.

So as to configure an efficient power conversion system for the battery testing platform, it is important to also consider the losses in the LC filter as a design variable. The losses at the inductor dominate total losses in the filter; the Ohmic losses in the capacitor has great dependency with the electrical frequency and they are minimum for a frequency range between 10\(^2\) and 10\(^5\) Hz [11]. All current entering or drawn by the battery travel through the inductor, and due to the resistance of its coil (and its magnetic core), this is the main component of the filter dissipating energy. Following lines present the mathematical expressions weighting the losses at the inductor.

The first step is to find out the dimensions of both the coil and the magnetic core. To do so, the so-called “area-product” sizing criterion is adopted [27]. This method allows for an appropriate design of the inductor based on the following input data: i) The value of the inductance of the coil, \(L\), in Henries; ii) the peak flux density in the core, \(B_{max}\), in Tesla; iii) the peak current density in the conductors, \(j_{max}\), in A/(mm\(^2\); iv) the switching frequency, \(f_{sw}\), in Hz; v) the dc current through the inductor and the magnitude of the current ripple, \(I_{dc}\) and \(I\), respectively, in A; and vi) the assumed window fill factor of the core, \(k_w\), in per unit values.

Accounting now on assumed \(B_{max}\), \(j_{max}\) and \(k_w\), and the current shape and magnitude through the inductor, the area-product value is calculated. This value is obtained by multiplying the core cross-sectional area \(A_{core}\) with its window area \(A_{window}\), so

\[
A_p = \frac{L \cdot I_{pk} \cdot I_{rms}}{k_w \cdot j_{max} \cdot B_{max}}, \quad (13)
\]
and \(A_p\) results in m\(^4\) specifying all the parameters in the equation in units of the international system.

Being \(A_p\) an indicator of the desired dimensions of the inductor, the aim now is to find out the particular geometry of both the winding and the core so as the product of \(A_{core}\) and \(A_{window}\) is greater than the threshold value \(A_p\), thus fulfilling the requirements. To do so, the value of \(A_{window}\) can be expressed by

\[
A_{window} = \frac{N \cdot A_{cond}}{k_w}, \quad (14)
\]
whilst that for the core, \(A_{core}\), is

\[
A_{core} = \frac{L \cdot I_{pk}}{N \cdot B_{max}}, \quad (15)
\]
The area \(A_{cond}\) is the section of the conductors configuring the coil and it is determined by the quotient between the rms value of the current through and the admissible current density,

\[
A_{cond} = \frac{I_{rms}}{I_{max}}. \quad (16)
\]
The product between \(A_{window}\) and \(A_{core}\) will result greater than \(A_p\) for a determined number of turns \(N\), which should be found out by applying an iterative method, for instance.

So far, the type of the core did not have any influence in the calculation of the cross sectional areas. At this point of the procedure, we adopt the geometry double \(E\) for the core type. This is a widely used geometry for inductors and transformers, so this is pertinent for the present study case. Mohan et al. (2002) [28] provided an indication of the dimensions for this type of core function of a characteristic parameter \(a\), equal to the width of the central core column.

From the characteristic dimension \(a\), we can define the following geometries:

\[
A_{core} = 1.5 \cdot a^2, \quad (17)
\]
\[
V_{core} = 13.5 \cdot a^3, \quad (18)
\]
\[
l_c = 9.5 \cdot a, \quad (19)
\]
where \(V_{core}\) is the core volume, in m\(^3\), and \(l_c\) is the average length of a coil, in m.

From equation (17) and the value for the cross sectional area of the core achieved in equation (15), the characteristic parameter can be calculated and then, the rest of the geometrics \(V_{core}\) and \(l_c\). These dimensions are needed to calculate the losses in the coil and in the magnetic core.

2.4.1. Coil losses

The power losses in the coil are associated to the resistance of the copper wires while experiencing a current through. These power losses are translated into heat that should be effectively evacuated. Knowing the section of the conductors \(A_{cond}\) and the length of the coil \(l_c\), the resistance of the coil, in Ohm, can be calculated as

\[
R_{Cu} = \rho \cdot \frac{l_c}{A_{cond}}, \quad (20)
\]
being \(\rho\) the resistivity of the copper, in Ohm-m. From \(R_{Cu}\) and \(I_{rms}\), the power losses at the coil can be calculated by

\[
P_{Cu} = R_{Cu} \cdot I_{rms}^2. \quad (21)\]
2.4.2. Magnetic core losses

The power losses in the magnetic core are function of the magnitude and frequency of the ac magnetic flux through it (so twice the switching frequency for the converter). They concern losses due to the magnetic hysteresis cycle of the material and induced eddy currents. Such losses can be estimated from datasheets of core manufacturers. For the present document, we adopt the information and procedures suggested by the manufacturer Magnetics Inc [23]. According to this source, the core loss density, in mW/cm³, can be calculated using the following expression (the Steinmetz equation actually)

\[ P_{fc} = a \cdot B_{pk}^b \cdot (2 \cdot f_{sw})^c, \]

being \( a, b \) and \( c \), constants determined from curve fitting in core loss density curves.

To apply the formula above one should select a specific material for the core and calculate somehow the magnitude of the ac flux swing. Having selected this, the magnitude of the ac flux swing \( B_{pk} \) can be determined from the dc magnetization curve. This curve relates the magnitude of the flux density with the ac flux swing. Having selected this, the magnitude of the ac flux swing can be determined from the dc magnetization curve.

\[ H_{max} = \frac{N}{l_e} \cdot (I_{av} + I_{av} \cdot \frac{\Delta I}{2}), \]

and

\[ H_{min} = \frac{N}{l_e} \cdot (I_{av} - I_{av} \cdot \frac{\Delta I}{2}). \]

From these two values, \( H_{max} \) and \( H_{min} \), we can calculate the corresponding magnitudes of flux density \( B_{max} \) and \( B_{min} \), by accessing to the dc magnetization curve. Then, the ac flux swing can be calculated by

\[ B_{pk} = \frac{B_{max} - B_{min}}{2}. \]

Finally, using \( B_{pk} \) in equation (22), the core loss density is determined, in mW/cm³. The last step is to calculate the core loss \( P_{fc} \), in W, from the core loss density and the core volume, as determined in equation (18). Thus,

\[ P_{fc} = P_{fc} \cdot V_{core}. \]

2.5. The decision algorithm

After solving the modeling of the battery, the H-bridge and the LC filter in previous sections, following contents address the description of the decision algorithm, i.e. the proposed procedure for determining a set of eligible designs of the power conversion system in a first instance, and then come up with the best solution among eligible ones.

The main steps of the decision algorithm are detailed in Figure 7. As can be noted, the first step is to determine the dc-link voltage of the H-bridge module. A minimum voltage level at the high-voltage side of the H-bridge should be ensured so the converter can effectively synthesize the required voltage at the battery terminals for its proper charging and discharging. This dc-link voltage depends on the size of the considered battery pack and on the voltage drop due to the internal resistance of the IGBTs, so this is why the modeling done in previous steps is utilized at this point while determining the dc-link voltage. In addition to the model of the battery and H-bridge, there is a third affecting factor while determining the required dc-link voltage: the internal resistance of the LC filter, because of the involved voltage drop. At this point of the decision algorithm, the size of the LC filter is not determined yet, so an initial value is assumed (parameter \( r_0 \)).

\[ \text{dc-link voltage} \]

\[ = \lambda x \text{H-bridge & LC losses function (f)} \]

\[ + (2 - \lambda) x \text{Filter volume function (f)} \]

Once a first tentative value for the dc-link voltage is selected, the decision algorithm address the design of the LC filter from two different criterions: the fulfillment of minimum voltage and current ripple at the battery terminals, and the proper operation at end of charge condition of the battery. These procedures yield a set of eligible designs, depending on the switching frequency of the converter. Now, this set is narrowed addressing one of the principal design constraints, which is letting the power conversion system to apply the EIS technique so as to evaluate the battery impedance. After doing this, the following step of the decision algorithm is to deduce the resistance of the inductor of the LC filter (parameter \( r_I \) in Figure 7), all function of the switching frequency yet.

The immediate step forward of the design algorithm is check the convergence of, precisely, the resistance of the inductor of the LC filter. Not reaching convergence at this point means to go back to the determination of the dc-link voltage, now considering the last approximation for \( r_I \). Conversely, the iterative process is considered as finished, yielding a set of eligible designs of the LC filter function of the selected switching frequency.
The last step is now to select the best solution among eligible addressing the multi-objective optimization problem indicated in Figure 7.

Following contents deep in the calculations performed at each of the above described points of the decision algorithm.

2.5.2. LC filter design for minimum ripple

The AC component of the electrical current through the battery pack, $I_{ac, bat}$, can be now defined as the ratio between $L_{sw}$ and the average current through the battery pack $I_{av}$, in a certain operating conditions. Resembling $L_{sw}$ to a sinusoidal waveform, the calculation of $\gamma$ can be calculated by the following expression, and should result lower than a certain value $\gamma_{max}$.

$$\gamma = \frac{\left|I_{ac, bat}\right| \cdot \sqrt{3}}{I_{av}} = \frac{L_{sw} \cdot \frac{\sqrt{2}}{\sqrt{2} Z_{f}}}{I_{av}} \leq \gamma_{max}.$$  \hspace{1cm} (28)

Both $I_{ac, bat}$ and $I_{av}$ are function of the applied duty cycle $D1$, since it determines the magnitude of the voltage synthesized by the converter at the battery side of the circuit. In turn, the dependency of $I_{ac, bat}$ with $D1$ is depicted in equation (10), as well as that with the inductance $L$ of the LC filter. Further, the dependency of $I_{av}$ with $D1$ can be expressed as

$$I_{av} = \frac{V_d \cdot (2 \cdot D1 - 1) - 2 \cdot Vce - V_{cell} \cdot n_s}{2 \cdot R_{ce} + R_s + n_s / n_p},$$  \hspace{1cm} (29)

where all terms have been previously defined and the rated voltage of the battery cell is assumed ($V_{cell} = V_{cc})$, thus depicting the operating condition for maximum ripple. Setting now $I_{av}$ as the nominal current of the battery pack (so $I_{av} = I_{nom} \cdot n_p$) in the equation above, the corresponding duty cycle can be easily calculated. Using it in constraint (28), the inductance $L$ of the LC filter could be found so the current ripple does not overcome the maximum threshold level $\gamma_{max}$, provided that an estimation for the capacitance $C$ is also given.

Such estimation comes from an additional constraint related to the resonance frequency of the filter, named as $f_{LC}$. Indeed, it is considered that the resonance frequency of the filter should be placed at least at half of the frequency of the alternating voltage and current squared-profiled waveforms at the converter terminals, so they do not counteract. Bearing in mind that the frequency of the above waveforms is $2 \cdot f_{sw}$, the resonance frequency $f_{LC}$ results as

$$f_{LC} = f_{sw} = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}},$$  \hspace{1cm} (30)

where frequency is expressed in Hz, $L$ in Henries and $C$ in Farads. Isolating $C$ in the last equation and combining it with equation (28), it results an expression solely dependent on $L$, which can be found now for a certain $\gamma_{max}$. The set of determined LC values –one pair of values per each considered switching frequency–, are named $(L_{ripple}, C_{ripple})_{f_{sw}}$.

2.5.3. LC filter design for end-of-charge condition

The design procedure is analogous to that proposed for the design of the filter under the criterion of minimum current ripple. Equations (28) to (30) are adopted here and also in the sequence proposed in the previous section. However, the operating point for the battery assumed here is not that characterized by the rated cell voltage and current, but by the maximum cell voltage and minimum current entering into the battery, thus indicating the end-of-charge conditions. In practice, both maximum cell voltage $V_{cell} = V_{cc}$ and minimum charging current $I_{av} = I_{min} \cdot n_p$ are considered in equation (29) for $D1$ calculation.
At end-of-charge condition, the average voltage at the converter terminals is quite similar to the voltage at the battery terminals, so little current is entering into it. The term \( I_{\text{min}} \) is just the average magnitude of this current, but an alternating component (a ripple) should be added to so as to represent the actual current waveform. The amplitude of such ripple should not exceed \( I_{\text{min}} \). Otherwise, the battery will experience reverse (discharging) currents at each switching period. According to this, in equation (28), the design parameter \( \gamma_{\text{max}} \) should not exceed 1 pu.

Applying the above described procedure, a set of LC values, one pair of values per each considered switching frequency, can be calculated, and are named at this point as \((|L_{\text{oc}},|C_{\text{oc}}|)_{F_{\text{sw}}})

2.5.4. LC eligible designs: EIS option

The two previous procedures for LC filter design yield two different sets of eligible designs function of the switching frequency. Just part of the calculated eligible designs are to be selected at this point for further analyses addressing the third of the main design criteria for LC filter: the possibility of applying the so-called EIS technique at the battery terminals for pack monitoring and evaluation. This selection is done applying a two-step procedure. First, and for each considered switching frequency, the most stringent LC design should be selected. In practice, this means to select a set \((|L|,|C|)_{F_{\text{sw}}}, \) for each switching frequency, that design ensuring minimum current ripple both at normal operating conditions and at end of charge circumstances as well. So

\[
L_i = \max(\{\text{ripple}, L_{\text{oc}}\}) \quad \forall i \in F_{\text{sw}} = \{f_{\text{sw}1}, \ldots, f_{\text{sw}n}\}.
\]

(31)

being \( F_{\text{sw}} \) the set of considered switching frequencies. Per each \( L_i \), the corresponding \( C_i \) results directly determined by \( L_{\text{ripple}} \) or \( L_{\text{oc}} \).

The second step of the procedure for selecting the final set of eligible LC filter designs is to narrow \((|L|,|C|)_{F_{\text{sw}}}, \) by constraining the set of admissible switching frequencies. As presented in section 2.1, for EIS technique deployment, a sinusoidal voltage waveform with a frequency up to few kHz must be applied at the battery terminals. This will yield a sinusoidal current waveform through the battery, from which measurement one can derive the battery impedance for monitoring and protection purposes. So in practice, a current control loop must govern the dc-dc converter so such sinusoidal current waveform can be effectively applied. To synthesize such current waveform with admissible quality, the switching of the converter must be much higher (e.g. \( n \) times) than the frequency of the current waveform. Otherwise, the shape of the obtained current waveform would not be sinusoidal at all, neither of the desired amplitude. So for EIS deployment technique, the eligible set of switching frequencies \( F_{\text{sw}} \) is bounded by a minimum switching frequency which is \( n \) times higher than the maximum frequency of the current waveform to be applied at the battery terminals for EIS technique (named as \( f_{\text{EIS}} \)), so

\[
F_{\text{sw}} = [n \cdot f_{\text{EIS}}, \ldots, f_{\text{sw}n}].
\]

(32)

This yields the final set of eligible LC filter designs \((|L|,|C|)_{F_{\text{sw}}})

2.5.5. Selection of the best design: Multi-objective optimization function

The last step of the decision algorithm is to select the best design among eligible ones, so among the set \((|L|,|C|)_{F_{\text{sw}}}, \). The idea here is, at the end, selecting one switching frequency for the converter yielding an optimal solution in terms of minimum power losses of the whole system (i.e. dc-dc converter and LC filter), and volume. So this is a multi-objective optimization that aims to minimize the following function

\[
\min_{f_{\text{sw}}} J = \lambda \cdot \frac{(P_{\text{cu}} + P_{\text{Fe}})}{P_{\text{cu}}} + (1 - \lambda) \cdot \frac{V_{\text{core}}}{V_{\text{oc}}},
\]

(33)

being the first term of the sum that related to the power losses at the dc-dc module and the filter, and the second one that related to the volume of the filter. \( P_{\text{cu}} \) can be computed through equation 5; \( P_{\text{Fe}} \) through equation (21); and \( P_{\text{Fe}} \) through equation (26). The term \( \lambda \) is a weighting parameter bounded between [0, 1], while \( P^* \) and \( V^* \) are the unconstrained optimal solutions solely addressing the optimization criteria of minimizing the power losses and the volume, respectively. Their inclusion in function \( J \) permits to express losses and volume as normalized magnitudes, so they can be mathematically combined. The development of a case study in the following section solves such multi-objective optimization, contributing to the understanding of its formulation.

3. Case study: a lithium ion battery pack

The present case study aims to prove the validity of the design methodology presented throughout the paper. Following contents are distributed in three main subsections. Firstly, the input data for the execution of the design methodology are presented. Secondly, the results of the methodology are depicted stressing in the determination of the three main design variables: the dc-link voltage level of the converter, the LC filter and the switching frequency. And thirdly, some sensitivity analyses are performed so as to evaluate the influence of different aspects in the design.

3.1. Input data

The input data refer to the battery cell model parameters, the number of cells in series and in parallel of the pack, the adopted IGBTs for the H-bridge converter, and for the core material for the LC filter.

Tables 1 and 2 present the parameters for the battery cells and pack. For the sake of simplicity, tables are placed at the appendix. As can be noted, the pack is configured by 10 cells in series and 20 in parallel. This results in a pack with an open circuit voltage around 37 V and capacity around 37.6 Ah, making this representative of a battery pack for a small scooter, for instance. The Nyquist plot of the considered battery pack is presented in Figure 8. Note that the impedance of the battery pack is evaluated ten times per decade and the frequency range covers from \( 10^{-2} \) to \( 10^{4} \) Hz.
The parameters for the considered IGBT module and inductor magnetic core material are presented in Tables 3 and 4. Just note that the IGBT module has been selected as adequate to withstand the maximum expected ratings for the current exchanged with the battery pack (around 40 A). In regard of the magnetic core for the inductor, the powder cores from Magnetics Inc. manufacturer (Kool Mu cores family, 60µ (24)) are selected, under the geometry EI. These powder cores admit a wide range of operating frequencies along with a good saturation magnetic flux, so they are considered as suitable for the purposes of the present work. The design procedure will determine the geometry and operating frequency of the core, while the parameters characterizing the materials are input data (see Table 4). All above mentioned tables can be also found in the appendix.

3.2. Results of the design methodology

This section presents the results of the design methodology to the particular case of a power conversion system for a lithium-ion pack, as presented in subsection 3.1. Results presented here are with different aims, all following the different steps of the design methodology in Figure 7. Firstly, some notes on the selection of dc-link voltage for the H-bridge are presented. Then, the eligible designs for the filter at the battery terminals are presented, considering the different constraints on minimum admissible voltage and current ripple, as well as on the proper behavior at end-of-charge condition and for EIS technique application. Further, some notes on the wellness of the proposed iterative process for reaching the correct value of the resistance of the inductive filter are offered. And finally, a discussion on the obtained optimization objective, and thus on the final design of the system are offered, both in terms of the selected switching frequency and filter. Contributing to the validation of the system design, results are checked through simulations.

3.2.1. Selection of the dc-link voltage

The determination of the dc-link voltage directly comes from constraint (27). Under the circumstances of applying the maximum duty cycle \( D_1 = 0.95 \), the minimum admissible dc-link voltage so as to synthesize the required dc voltage at the filter terminals is \( V_d = 52.1 \) V. This is the dc-link voltage applied hereinafter since, in addition, this yields minimum power losses in the converter. Sensitivity analyses on the magnitude of this voltage will come in section 3.3 so as to evaluate the impact in system performance.

3.2.2. Eligible LC filter design function of switching frequency

According to the design methodology in Figure 7, after the modeling of the battery impedance (subsection 2.1); the H-bridge (subsection 2.3) and LC filter (subsection 2.4); and after selecting the dc-link voltage level as well, the following step is to select the eligible designs for the LC filter, still function of the switching frequency. Eligible designs are addressing different constraints on maximum admissible ripple at battery terminals (constraint (28), along with equations (29) and (30)); performance at end-of-charge condition (same constraint (28), along with equations (29) and (30) but now considering the maximum cell voltage and minimum charging current); and EIS capability (constraint (31) along with equation (32)).

Before addressing obtained results of such procedure, just note some comments on the wellness of the iterative process to adjust the value of the resistance of the inductive filter (see Figure 7). Starting from an arbitrary value of the resistance \( r_0 \) of 1 Ohm (resistance of the inductor for a particular switching frequency of 300 Hz), it took just 4 iterations for the decision algorithm to effectively converge to a final value about 0.0139 Ohm. Convergence is assumed while reaching a variation for \( r_0 \) lower than 10% (parameter \( \varepsilon \) in Figure 7) from one iteration to the subsequent one. As a summary, the calculated value of \( r_0 \) for each of the iterations was 1 Ohm (first iteration), 0.0289 Ohm (second iteration), 0.0140 Ohm (third iteration) and 0.0139 Ohm (fourth iteration). Convergence of such iterative process is ensured for any positive value of \( r_0 \) initially set by the designer, since it would keep as positive the selected dc-link voltage of the H-bridge converter through constraint (27), as well as for the rest of constraints calculating \( L \) and \( C \) values.

Now, the obtained results for LC eligible designs are summarized in Figure 9. At this point, two sets of LC pairs are determined: one set under the criterion of minimum admissible ripple, and the other one under the criterion of required performance at end-of-charge condition. Now, according to constraint (22), the most restrictive set in terms of current filtering is to be selected as it ensures proper power quality in normal operating conditions (i.e. minimum current ripple at battery terminals), and small current ripple at end-of-charge conditions, letting the battery to become fully charged. For the case evaluated in Figure 9, the set of LC filters under the criterion of proper performance at end-of-charge condition is chosen as the eligible one. In addition, just those filters within the set for switching frequency above 10 times the maximum frequency for EIS technique are to be selected (i.e. 10 times 1 kHz, so 10 kHz).
Complementing the description of filters design, the Bode plot for selected ones are depicted in Figure 10, and for some of eligible switching frequencies. As can be noted, the filters are tuned so resonance frequency is around half of the corresponding switching frequency. At switching frequency, the filters attenuate oscillations in voltage (negative gain), while for frequencies below half of the switching frequency the Bode plot shows a gain of 0 dB, so voltage oscillations are not attenuated, as required for EIS technique.

The performance of the filters is validated through simulations, which results are plotted in Figure 11. The particular case of the eligible filter corresponding to the switching frequency of 18 kHz is considered (L = 0.182 mH and C = 0.429 \( \mu \text{F} \)). This figure shows the current and voltage ripple at the battery terminals while: i) charging at nominal current and the battery voltage is also the rated one (upper subplot); ii) charging at end-of-charge conditions (subplot in the middle); iii) applying a sinusoidal current perturbation for EIS technique in galvanostatic mode (lower subplot). The RMS value for the sinusoidal current perturbation is 1 A. In the upper subplot, the average current is 38.2 A, which is near to the nominal current for the pack (36.7 A), and the average voltage is 38.03 V, which mainly corresponds to the nominal voltage for the pack, 37 V, plus the experienced overvoltage at charging due to the internal resistance, \( (r_f \cdot n_s/n_p) \cdot 36.84 \text{ A} = (0.055 \Omega \cdot 10/20) \cdot 36.84 \text{ A} = 1.013 \text{ V} \). Under these conditions and the magnitude of the current ripple is 1.5%, which matches the maximum admissible ripple and the voltage ripple does not exceed the value of 0.1%.

The subplot in the middle represents the waveforms at end-of-charge conditions. This situation can be noted by the application of a very small average charging current, 1.90 A, which is almost equal to the minimum charging current noted at the datasheet of battery cells, 0.094 A, times the number of cell strings in parallel, \( n_p = 20 \), so 1.88 A. The voltage of the pack reaches nearly its maximum value, around 42 V. The simulation shows that the designed filter attenuates enough the current ripple so it becomes lower than the average charging current (the magnitude of the ripple is 2.48 A - 1.90 A = 0.58 A). This ensures that the battery does not experience reverse (discharging) currents at each switching period.

Finally, the lower subplot proves that the H-bridge converter, along with the designed filter, effectively permits to perform EIS technique. The conditions for the simulation in this case are that the battery voltage is the nominal one (around 37.1 V) and little current is entering into the battery. The current applied is not for charging purposes but according to EIS requirements. In particular, a sinusoidal waveform for the duty cycle managing the switching of the H-bridge transistors is applied and this produces a sinusoidal voltage waveform at the converter terminals (and battery terminals also), which is translated into a sinusoidal waveform of current through the battery. The frequency of the above waveforms is 1 kHz. The computation of these two waveforms would permit to obtain the battery impedance, following EIS technique. The designed filter does not attenuate the fluctuations at 1 kHz so it can be applied at the battery terminals. The fluctuations at switching frequency (18 kHz)
kHz) are attenuated so resulting magnitudes can be appreciated as superimposed to the 1 kHz components.

Addressing the simulation results, it is worth mentioning a particular aspect around the need of including accurate measurement sensors for the battery voltage and current, because of its practical implication. According to Figure 11, for EIS technique, current perturbations around few Amps (3 A) would be enough for the considered battery pack (rated current 37.6 A at 1C). To have this, and because of the internal impedance of the battery, the amplitude of the voltage perturbation should not overcome 0.1 V (peak-to-peak). For battery impedance calculation, the registration of voltage waveform should be accurate enough and for this, measurement sensors should be properly selected.

3.2.3. Objective function

As described in section 2.5, once a set of eligible designs for the LC filter function of switching frequency is determined by the decision algorithm, the final step is to select the best configuration addressing the criterion of minimum losses, volume, or a combination of both criteria. The weighted sum method is adopted here so as to find out solutions for such a multi-objective optimization problem. As presented in the global objective function $J$ (see equation (33)), the two optimization criteria for the design of the system (power losses and volume) are combined in the objective function $J$ through the weighted factor $\lambda$. In addition, to be able to effectively sum losses and volume magnitudes these are normalized through $P^*$ and $V^*$. The selection of reasonable $P^*$ and $V^*$, as normalizing factors, is one of the key aspects for an effective application of the weighted sum method. In this work, these are selected as the unconstrained optimal solutions for the problem while solely addressing minimum losses or volume as the optimization criteria. The results of such exercise are presented in Figure 12.

![Figure 12: Unconstrained optimization solutions.](image)

As can be noted, the higher the switching frequency, the smaller the system becomes in volume. However, the power losses are not monotonically decreasing with frequency, but they reach a minimum value around 18 kHz. So for the frequency range between 10 kHz and 18 kHz there is no doubt that the optimal design of the system addressing the two optimization criteria at the same time (losses and volume) is reached at maximum frequency, 18 kHz. Please note that 10 kHz is identified here as the minimum eligible frequency for the range $f^*_sw$, so the minimum required switching frequency to be able to effectively deploy the EIS technique. However, in between 18 kHz and 30 kHz, losses are increasing and volume is decreasing, so there is a trade-off region, which is to be solved through the weighted sum method. For the application of the method, and according to Figure 12, the unconstrained minima of the two functions is

$$P^* = 144.5 \text{ W}$$

and

$$V^* = 103.7 \text{ cm}^3.$$  

Representing the value of volume function $J_2$ with respect to losses function $J_1$ in a two axis plane for each value of $\lambda$ and $f^*_sw$ would yield a map of points. Each of the points would represent a feasible solution for the problem in this case, so a feasible system design. The set of solutions that cannot be improved in any of the objectives, either $J_1$ and $J_2$, without degrading the other objective are located in the so-called Pareto front. In plane words, those solutions in the Pareto front are those minimizing all objective functions simultaneously. The construction of the Pareto front can be solved through different mathematical methods [5]. One of the straightforward methods to find it is solving a goal attainment problem, by setting different weights to functions $J_1$ and $J_2$. For the application of such method, both $J_1$ and $J_2$ should be explicitly formulated, and this can be done by fitting the two curves of Figure 12 from 18 kHz on, to second order functions, which have the following form

$$J_1 = 7.741 \cdot 10^{-11} \cdot f^2_{sw} - 2.566 \cdot 10^{-6} \cdot f^*_{sw} + 1.021, \quad (34)$$

$$J_2 = 1.360 \cdot 10^{-6} \cdot f^2_{sw} - 1.033 \cdot 10^{-4} \cdot f^*_{sw} + 2.878. \quad (35)$$

Altogether permits to plot the Pareto front, as presented in Figure 13. What shows the Pareto front is a set of optimal solutions for the design of the system, all function of the weighted factor $\lambda$. For $\lambda = 1$, function $J_1$ (so power losses) are minimum

![Figure 13: Pareto front for the optimization function $J$. The optimal switching frequency is noted per diverse values of $\lambda$.](image)
(144.5 W) and \( J_3 \) is maximum (152.1 cm\(^3\)). Conversely, for \( \lambda = 0 \), \( J_1 \) is maximum (146.4 W) and what is minimum is \( J_2 \) (103.7 cm\(^3\)). These two solutions directly correspond to the unconstrained minima for losses and volume in Figure 12, so it is clear that minimum losses are reached for switching frequency 18 kHz, while minimum volume is for 30 kHz. In between these two boundary values for \( \lambda \) though, the Pareto plot shows diverse solutions for which the optimal switching frequency is also shown in Figure 13. So, for instance, providing optimization objectives \( J_1 \) and \( J_2 \) with the same weight (so \( \lambda = 0.5 \)), the optimal switching frequency results as 24.3 kHz. For this point, \( J_1 \) is 1.004 pu and \( J_2 \) is 1.171 pu, and the magnitude of corresponding power losses and volume in W and cm\(^3\) can be done through the definition of the objective function \( J \) in equation (33). For this point, it results \( P_{\text{tot}} = 144.9 \text{ W} \) and \( V_{\text{core}} = 122.2 \text{ cm}^3 \). For the sake of completeness, the optimal switching frequency per diverse values of \( \lambda \) is noted in Figure 13.

### 3.3. Sensitivity analysis on the dc-link voltage level

Some figures are shown in the following so as to illustrate the impact of the dc-link voltage in converter losses. The voltage at the converter terminals (battery side) should be high enough to charge the battery pack (the open circuit voltage can reach 42 Vdc) at least at 1C. This, along with the internal ohmic resistance of the pack and for the LC filter in between the battery and the converter, determines the voltage level to be synthesized at the converter terminals. Such average voltage at the converter terminals will result by applying the maximum possible duty cycle, so around 0.95. By decreasing this duty cycle, the voltage synthesized at the converter terminals will also decrease and so the current injected into the battery. This will happen till applying a duty cycle low enough to yield a voltage at the converter terminals lower than the open circuit voltage of the battery, thus reversing the current. As a reminder, the minimum admissible duty cycle is 0.5, yielding a null average voltage at the converter terminals, i.e. yielding the battery at short-circuit condition. The impact in converter losses of the above explained exercise of progressively varying the duty cycle is plotted in Figure 14. This Figure also shows the result in varying the switching frequency of the converter. In this case, the dc-link voltage of the converter results around 52.1 V. The obtained results shows that the higher the switching frequency, the higher the converter losses, no matter the applied duty cycle, i.e. the current exchanged with the battery. In regard of the duty cycle, it can be observed that for the highest duty cycle (around 0.95 p.u.), the battery is being charged at nominal current and this maximizes the converter losses. By decreasing the duty cycle, the voltage difference between the converter terminals and battery terminals decreases and thus the current in between. This progressively decreases the converter losses. For a duty cycle between 0.81 and 0.85 approximately, the above mentioned voltage difference is minimum. Further decreasing the duty cycle, the voltage difference increases again reversing the current (discharging the battery).

A projection of the converter losses to the switching frequency / duty cycle plane is offered in Figure 15. This representation also helps to depict such dependency of converter losses with the applied duty cycle and switching frequency. Please note that the curves shown represent the losses in Watts. A question that may arise is the impact of the dc-link voltage in converter losses. To depict this aspect, Figure 16 repeat the previously presented results for a dc-link voltage around 120 V (so doubling the dc-link voltage of the previous case). The results show that converter losses are clearly higher than for the case case, reaching up to 220 W approximately). The duty cycle range becomes also narrower than for the first case (from 0.63 to 0.7 p.u., versus 0.76 to 0.91 approximately). Finally, note that such 220 W in power losses (maximum duty cycle point, rated current charging the battery) yields an energy efficiency for the converter around \( \eta = 1 - 220 \text{ W} / (1.88 \text{ A} \times 20 \text{ strings in parallel} \times 37 \text{ Vdc}) = 0.84 \text{ p.u.} \)
4. Conclusions and further research

This paper presented a methodology for the design of a PCS with EIS capability for battery packs. The methodology solves with an optimal dc-link voltage level and switching frequency for the converter, as well as with the magnitude of the associated inductive-capacitive filter interfacing the battery pack. The solution is optimal in terms of volume and power losses. It is based on data commonly found in component’s datasheet and can be systematized, so it results useful for end user. It considers also the characteristics of the battery to be connected so proper management and charging is ensured even at end-of-charge conditions, being this another important feature of the methodology. Further, the design methodology adds a new functionality to the PCS, which is the possibility of performing EIS technique for battery monitoring and evaluation. This is a feature potentially simplifying the associated BMS to the system, which would result into costs reduction. The methodology is tested through a case study, which permits to quantify the great impact of switching frequency and dc-link voltage level in system’s volume and losses, as well as the importance of properly assessing the accuracy of measurement sensors for the battery voltage and current perturbations for EIS technique.

As further research, the authors consider that one principal aspect to assess is the development of a prototype of the system to evaluate the applicability of the concept for different battery types and of various size, as well as to quantify the limitations and/or advantages of applying EIS technique by the PCS with respect to conventional approaches for BMSs while calculating SoC and SoH.

Acknowledgments

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Appendix

See Tables 1, 2 and 4.

Table 1: Input data - battery cell (model Sanyo UF103450P, [47])

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e$</td>
<td>Ohmic resistance</td>
<td>$55\cdot10^{-3}$ [Ohm]</td>
</tr>
<tr>
<td>$C_{ct}$</td>
<td>Capacitance charge transfer area</td>
<td>0.076 [F]</td>
</tr>
<tr>
<td>$R_{ct}$</td>
<td>Resistance charge transfer area</td>
<td>$30\cdot10^{-3}$ [Ohm]</td>
</tr>
<tr>
<td>$L$</td>
<td>Cell inductance</td>
<td>2.38-$10^{-1}$ [H]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Warburg coefficient</td>
<td>0.003963 [-]</td>
</tr>
<tr>
<td>$I_{nom}$</td>
<td>Rated current (1C)</td>
<td>1.88 [A]</td>
</tr>
<tr>
<td>$V_{nom}$</td>
<td>Rated open circuit voltage</td>
<td>3.7 [V]</td>
</tr>
<tr>
<td>$I_{eoc}$</td>
<td>Current at end of charge condition</td>
<td>0.0940 [A]</td>
</tr>
<tr>
<td>$V_{eoc}$</td>
<td>Voltage at end of charge condition</td>
<td>4.2 [V]</td>
</tr>
</tbody>
</table>

Table 2: Input data - battery pack

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_s$</td>
<td>Number of cells in series</td>
<td>10 [-]</td>
</tr>
<tr>
<td>$n_p$</td>
<td>Number of cells in parallel</td>
<td>20 [-]</td>
</tr>
</tbody>
</table>

Table 3: Input data - Magnetic core [24] and winding

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Steinmetz eq.</td>
<td>193 [-]</td>
</tr>
<tr>
<td>$b$</td>
<td>Steinmetz eq.</td>
<td>2.01 [-]</td>
</tr>
<tr>
<td>$c$</td>
<td>Steinmetz eq.</td>
<td>1.29 [-]</td>
</tr>
<tr>
<td>$a_m$</td>
<td>Magnetization curve</td>
<td>$1.658\cdot10^{-4}$ [-]</td>
</tr>
<tr>
<td>$b_m$</td>
<td>Magnetization curve</td>
<td>$2.301\cdot10^{-5}$ [-]</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Magnetization curve</td>
<td>$7.297\cdot10^{-5}$ [-]</td>
</tr>
<tr>
<td>$d_m$</td>
<td>Magnetization curve</td>
<td>$5.906\cdot10^{-5}$ [-]</td>
</tr>
<tr>
<td>$e_m$</td>
<td>Magnetization curve</td>
<td>$6.053\cdot10^{-5}$ [-]</td>
</tr>
<tr>
<td>$f_m$</td>
<td>Magnetization curve</td>
<td>0.5 [-]</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>Max. flux density</td>
<td>0.5 [T]</td>
</tr>
<tr>
<td>$k_w$</td>
<td>Window fill factor</td>
<td>0.5 [-]</td>
</tr>
<tr>
<td>$J_{max}$</td>
<td>Max. current density</td>
<td>3 [A/mm²]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Copper resistivity</td>
<td>$1.71\cdot10^{-8}$ [Ohm-m]</td>
</tr>
</tbody>
</table>

References

Table 4: Input data - IGBT module (model Semikron SK50GH065F [17])

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vce at 25 and 125°C</td>
<td>Sat. voltage</td>
<td>[1.2, 1.1] [V]</td>
</tr>
<tr>
<td>rce at 25 and 125°C</td>
<td>Resistance</td>
<td>[12, 22] [mOhm]</td>
</tr>
<tr>
<td>(Iref, Vce, Tref)</td>
<td>Ref. values</td>
<td>[40, 300, 125] [A, V, °C]</td>
</tr>
<tr>
<td>[k1, k2]</td>
<td>Exponents</td>
<td>[1,0,1,3] [-]</td>
</tr>
<tr>
<td>TCR</td>
<td>Temp. coef.</td>
<td>0.003 [°C⁻¹]</td>
</tr>
<tr>
<td>(Vref, rref)</td>
<td>Diodes</td>
<td>[1, 1, 12] [V, mOhm]</td>
</tr>
<tr>
<td>(Rlow, Ralph)</td>
<td>Thermal res.</td>
<td>[0.03, 0.03] [kW]</td>
</tr>
<tr>
<td>(a0, a1, a2)</td>
<td>Curves</td>
<td>[1.45, 0.19, 1.45] x 10⁻⁵</td>
</tr>
<tr>
<td>(b0, b1, b2)</td>
<td>Curves</td>
<td>[0.0176, 0.0165, 0.0176]</td>
</tr>
<tr>
<td>(c0, c1, c2)</td>
<td>Curves</td>
<td>[0.0507, 0.0954, 0.0507]</td>
</tr>
</tbody>
</table>


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