Resilient Distributed Energy Management for Systems of Interconnected Microgrids

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***Abstract*— In this paper, distributed energy management of interconnected microgrids, which is stated as a dynamic economic dispatch problem, is studied. Since the distributed approach requires cooperation of all local controllers, when some of them do not comply with the distributed algorithm that is applied to the system, the performance of the system might be compromised. Specifically, it is considered that adversarial agents (microgrids) might implement control inputs that are different than the ones obtained from the distributed algorithm. By performing such behavior, these agents might have better performance at the expense of deteriorating the performance of the regular agents. This paper proposes a methodology to deal with this type of adversarial agents such that we can still guarantee that the regular agents can still obtain feasible, though suboptimal, control inputs in the presence of adversarial behaviors. The methodology consists of two steps: (i) the robustification of the underlying optimization problem and (ii) the identification of adversarial agents, which uses hypothesis testing with Bayesian inference and requires to solve a local mixed-integer optimization problem. Furthermore, the proposed methodology also prevents the regular agents to be affected by the adversaries once the adversarial agents are identified.**

***Index Terms*— Economic dispatch, distributed MPC, distributed optimization, resilient algorithm**

I. Introduction

In order to face the increasing penetration of distributed generation units, either dispatchable or non-dispatchable ones, and energy storages, such as batteries, supercapacitors, and fuel cells, in electrical networks, distributed approaches for energy management system currently gain a lot of attention, e.g., as discussed in [1]–[4]. The advantages of employing a distributed approach for this task include avoiding significant increase of information, communication, and modeling resources used for a centralized dispatch as well as distributing high computational burden [1].

In a distributed scheme, a distribution electrical network can be viewed as a system of interconnected microgrids [1], [5], each of which is a controllable entity that has its own local controller. Therefore, the economic dispatch problem of the network must be decomposed and assigned to the local controllers. A distributed optimization approach can then be formulated and applied to solve the problem. In this regard, Model Predictive Control (MPC) strategy, with receding horizon principle, is suitable, particularly when the dynamics of the storages are considered, since the decisions/control inputs are always updated at each sampling time according to the measurement of the states. Distributed MPC (DMPC) methods that have been proposed to solve economic dispatch problems include those that are based on dual decomposition [4], alternating direction method of multipliers [2], optimality condition decomposition [3] and population dynamics [6]. These approaches are suitable since they obtain an optimal solution if the related optimization problem is convex.

One of the important features in such distributed approaches is the cooperation of the agents to apply the algorithm and to comply with the decisions obtained from the distributed algorithm. In this work, we deal with the problem of agent compliance, in which some of the agents do not always implement the decision obtained from the distributed algorithm. Instead, they may implement a different decision that is more beneficial for them but compromise the performance of the other agents and hence the entire system. Agents with such adversarial behaviors are identified in [7] as *liar agents* or in [8] as *misbehaving agents*. The authors of [7] propose a secure dual-decomposition-based DMPC, in which the agents that provide extreme control input values are monitored and disregarded, to deal with this issue. Furthermore, [8] addresses a cyber-attack problem of a consensus-based distributed control scheme for distributed energy storage systems. The proposed approach in [8] includes a fuzzy-logic-based detection and a consensus based leader-follower distributed control scheme.

The contributions of this paper is as follows. We study the impact of an adversarial behavior in the distributed energy management system that is based on a DMPC scheme and propose to actively use the storage system and the possibility to establish/disestablish connections between agents to deal with this behavior. To this end, we propose an approach that consists of two main steps. The first step is the robustification of the economic dispatch problem. By considering the robust reformulation, we ensure that the regular agents always obtain a solution that satisfies all the constraints defined in the economic dispatch problem even though there are some agents that do not comply with the decisions. In the

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second step, we propose an active strategy to identify the adversarial agents that is based on hypothesis testing using Bayesian inference (e.g., [9]). In this method, each regular agent must solve a local mixed-integer problem to decide the connections with its neighbors at each time instant. By actively connecting/disconnecting with neighbors, regular agents can then assess their hypothesis.

Notations: The set of real numbers and integers are denoted by $\mathbb{R}$ and $\mathbb{Z}$, respectively. Moreover, $\mathbb{R}_{\geq a}$ denotes all real numbers in the set $\{b : b \geq a,\ b, a \in \mathbb{R}\}$. A similar definition can be used for $\mathbb{Z}_{\geq a}$ and the strict inequality case. For column vectors v_i with $i \in \mathcal{L} = \{l_1, \ldots, l_{|\mathcal{L}|}\}$, the operator $[v_i^\top]_{i\in\mathcal{L}}^\top$ denotes the column-wise concatenation, i.e., $[v_i^\top]_{i\in\mathcal{L}}^\top = [v_{l_1}^\top, \cdots, v_{l_{|\mathcal{L}|}}^\top]^\top$. The vector $\mathbb{1}_n$ denotes $[1\ 1\ \cdots\ 1]^\top \in \mathbb{R}^n$. The set cardinality and Euclidean norm are denoted by $|\cdot|$ and $\|\cdot\|_2$. Furthermore, $\mathbb{P}(\cdot)$ denotes the probability measure. Finally, discrete-time instants are denoted by the subscript k.

II. Problem Formulation & Distributed Approach

A. Dynamic Economic Dispatch Problem

Consider a network of interconnected microgrids, which can be represented as an undirected graph $\mathcal{S} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, |\mathcal{N}|\}$ denotes the set of microgrids and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ denotes the set of physical links among the microgrids. In this regard, the link $(i, j) \in \mathcal{E}$ implies that it is possible to exchange energy between microgrids i and j. Furthermore, denote the set of neighbors of microgrid i by $\mathcal{N}_i$, i.e., $\mathcal{N}_i = \{j : (i, j) \in \mathcal{E}\}$. Each microgrid $i \in \mathcal{N}$ consists of an aggregated local load, denoted by $p_{i,k}^{\mathrm{d}} \in \mathbb{R}_{\geq 0}$, a set of dispatchable distributed generators, denoted by $\mathcal{G}_i$, and a storage system from which electrical energy can be stored and retrieved. Each microgrid can also obtain power by buying it from the main grid. In this economic dispatch problem, optimal power generation of the generators and storage usage are sought by considering their economical costs such that the loads are satisfied. Additionally, $p_{i,k}^{\mathrm{d}}$ is assumed to be bounded as follows:

$$|p_{i,k}^{\mathrm{d}} - \hat{p}_{i,k}^{\mathrm{d}}| \leq d_i^{\max}, \tag{1}$$

where $\hat{p}_{i,k}^{\mathrm{d}}, d_i^{\max} \in \mathbb{R}_{\geq 0}$ denote the forecast and the upper bound, respectively, which are assumed to be known a priori.

The power balance equations for each microgrid $i \in \mathcal{N}$ at each time instant $k \in \mathbb{Z}_{\geq 0}$ are as follows [2], [3]:

$$\hat{p}_{i,k}^{\mathrm{d}} - p_{i,k}^{\mathrm{G}} - p_{i,k}^{\mathrm{st}} - p_{i,k}^{\mathrm{im}} - \sum_{j\in\mathcal{N}_i} p_{ji,k}^{\mathrm{t}} = 0, \tag{2}$$

$$p_{ij,k}^{\mathrm{t}} + p_{ji,k}^{\mathrm{t}} = 0, \quad \forall j \in \mathcal{N}_i, \tag{3}$$

where $p_{i,k}^{\mathrm{G}} = \sum_{m\in\mathcal{G}_i} p_{m,k}^{\mathrm{g}} \in \mathbb{R}_{\geq 0}$ denotes the total power generation in microgrid i, with $p_{m,k}^{\mathrm{g}}$ being the power generation of distributed generator m; $p_{i,k}^{\mathrm{st}} \in \mathbb{R}$, $p_{i,k}^{\mathrm{im}} \in \mathbb{R}_{\geq 0}$, and $p_{ji,k}^{\mathrm{t}} \in \mathbb{R}$ denote the power delivered by or to the storage, the imported power from the main grid, and the power flows between microgrids i and $j \in \mathcal{N}_i$, respectively. Furthermore, (3) ensures that there is an agreement between two neighboring microgrids in terms of the power exchanged between them.

The dynamics of the storage system, for each $i \in \mathcal{N}$, is represented as follows:

$$x_{i,k+1} = a_i x_{i,k} + b_i p_{i,k}^{\mathrm{st}}, \tag{4}$$

where $x_{i,k}$ denotes the state-of-charge (SoC) of storage i, $a_i \in (0, 1]$ denotes the the efficiency of the storage and $b_i = -\frac{T_{\mathrm{s}}}{e_{\mathrm{cap},i}}$, where T_{s} and $e_{\mathrm{cap},i}$ denote the sampling time and the maximum capacity of the storage, respectively.

Additionally, for each microgrid $i \in \mathcal{N}$, some local operational constraints are also considered as follows:

$$x_i^{\min} \leq x_{i,k} \leq x_i^{\max}, \tag{5}$$

$$-p_i^{\mathrm{ch}} \leq p_{i,k}^{\mathrm{st}} \leq p_i^{\mathrm{dh}}, \tag{6}$$

$$p_i^{\mathrm{G},\min} \leq p_{i,k}^{\mathrm{G}} \leq p_i^{\mathrm{G},\max}, \tag{7}$$

$$p_{i,k}^{\mathrm{im}} \leq p_i^{\mathrm{im},\max} \tag{8}$$

$$-p_{ji}^{\mathrm{t},\max} \leq p_{ji,k}^{\mathrm{t}} \leq p_{ji}^{\mathrm{t},\max}, \quad \forall j \in \mathcal{N}_i, \tag{9}$$

where $x_i^{\min}, x_i^{\max} \in \mathbb{R}_{\geq 0}$ denote the minimum and the maximum SoC of the storage of microgrid i, respectively. Moreover, $p_i^{\mathrm{ch}} \in \mathbb{R}_{\geq 0}$ and $p_i^{\mathrm{dh}} \in \mathbb{R}_{\geq 0}$ denote the maximum charging and discharging power of the storage. Furthermore, $p_i^{\mathrm{G},\min}, p_i^{\mathrm{G},\max} \in \mathbb{R}_{\geq 0}$ denote the minimum and the maximum power generated by the distributed generators of microgrid i, respectively, $p_i^{\mathrm{im},\max}$ denotes the maximum imported power from the main grid, and $p_{ji}^{\mathrm{t},\max}$ denotes the maximum energy that can be transferred between microgrid i and j.

Now, denote the control input vector of microgrid i by $\boldsymbol{u}_i = [p_{i,k}^{\mathrm{st}}\ p_{i,k}^{\mathrm{G}}\ p_{i,k}^{\mathrm{im}}\ \boldsymbol{u}_{i,k}^{\mathrm{c}\top}]^\top \in \mathbb{R}^{3+|\mathcal{N}_i|}$, where $\boldsymbol{u}_{i,k}^{\mathrm{c}} = [p_{ji,k}^{\mathrm{t}}]_{j\in\mathcal{N}_i}^\top$ is the vector of coupled control input variables. We denote h_p as the prediction horizon and consider the quadratic cost function

$$J_{i,k} = \boldsymbol{u}_{i,k}^\top R_i \boldsymbol{u}_{i,k}, \tag{10}$$

where $R_i = \mathrm{diag}([c_i^{\mathrm{st}}\ c_i^G\ c_i^{\mathrm{im}}\ c_i^{\mathrm{t}}\mathbb{1}_{|\mathcal{N}_i|}^\top]) > 0$, in which $c_i^{\mathrm{st}},\ c_i^G,\ c_i^{\mathrm{im}}, c_i^{\mathrm{t}} \in \mathbb{R}_{>0}$ denote the per-unit cost of storage operation, producing energy, buying energy from the main grid, and transferring energy to/from the neighbor due to losses, respectively [2]. Thus, the finite-time optimization problem that underlies an MPC strategy for the dynamic economic dispatch of this system can be written as

$$\underset{\{\{\boldsymbol{u}_{i,\ell|k}\}_{i\in\mathcal{N}}\}_{\ell=k}^{k+h_p-1}}{\text{minimize}} \sum_{i\in\mathcal{N}} \sum_{\ell=k}^{k+h_p-1} J_{i,\ell}(\boldsymbol{u}_{i,\ell|k}) \tag{11a}$$

$$\text{subject to} \quad \boldsymbol{F}_i \boldsymbol{u}_{i,\ell|k} \leq \boldsymbol{f}_{i,\ell},\ \forall i \in \mathcal{N}, \tag{11b}$$

$$\boldsymbol{u}_{i,\ell|k}^{\mathrm{c}} + \sum_{j\in\mathcal{N}_i} \boldsymbol{G}_{ij} \boldsymbol{u}_{j,\ell|k}^{\mathrm{c}} = \boldsymbol{0},\ \forall i \in \mathcal{N}, \tag{11c}$$

for all $\ell \in \{k, \ldots, k + h_p - 1\}$, where the local constraints (11b) that only include local control inputs are constructed from (2), (4)-(9), while the coupled constraints (11c) are constructed from (3). Problem (11) is convex since the inequality constraints form a polyhedron, the coupled equality constraints are affine, and the cost function (10) is strictly convex. Furthermore, the following assumption, which is related to the island mode, is considered.

Assumption 1: For Problem (11), there exists a nonempty set of feasible solutions and it includes a subset in which $p^{\mathrm{t}}_{ij,k} = p^{\mathrm{t}}_{ji,k} = 0$, for any $(i,j) \in \mathcal{E}$ and $k \in \mathbb{Z}_{\geq 0}$. □

Remark 1: Without loss of generality, $p^{\mathrm{G}}_{i,k}$ is considered as one of the control input instead of $p^{\mathrm{g}}_{m,k}$, for all $m \in \mathcal{G}_i$. Considering $p^{\mathrm{g}}_{m,k}$, for all $m \in \mathcal{G}_i$, is also straightforward and only increases the dimension of $\boldsymbol{u}_i$. □

B. Distributed Energy Management based on Dual Decomposition

In general, many distributed optimization algorithm can be applied as a DMPC strategy to solve Problem (11). However, for the clarity of the explanation, a DMPC algorithm based on dual decomposition is considered in this paper. In order to design the mentioned algorithm, the Lagrangian function associated to Problem (11) is derived and its dual problem [10] is decomposed into smaller problems that are assigned to the agents (microgrids). The DMPC strategy based on dual decomposition is stated in Algorithm 1, where $\boldsymbol{\lambda}_{i,\ell} \in \mathbb{R}^{|\mathcal{N}_i|}$, for all $\ell \in \{k, \ldots, k+h_p-1\}$ and all $i \in \mathcal{N}$, are the Lagrange multipliers associated to the coupled constraints (11c). Finally, denote the optimal decisions obtained by the DMPC strategy for time k by $\boldsymbol{u}^{\star}_{i,k|k}$, for all $i \in \mathcal{N}$.

Algorithm 1 DMPC algorithm based on dual decomposition, for each agent $i \in \mathcal{N}$

Set $r = 1$, $\varepsilon \in \mathbb{R}_{>0}$, and initialize $\boldsymbol{\lambda}^{(r)}_{i,\ell}$

while $\left|\left|\begin{bmatrix}\boldsymbol{\psi}^{\top}_{i,k} & \cdots & \boldsymbol{\psi}^{\top}_{i,k+h_p-1}\end{bmatrix}\right|\right|_2 > \varepsilon$ **do**

Solve the local optimization problem:

$$\begin{aligned}\underset{\{\boldsymbol{u}_{i,\ell|k}\}_{\ell=k}^{k+h_p-1}}{\text{minimize}} \quad & \sum_{\ell=k}^{k+h_p-1} \left(J_{i,\ell}(\boldsymbol{u}_{i,\ell|k}) + \boldsymbol{y}^{\top}_{i,\ell} \boldsymbol{u}^{c}_{i,\ell|k} \right) \\ \text{subject to} \quad & \text{(11b)}, \quad \forall \ell \in \{k, \ldots, k+h_p-1\},\end{aligned}$$

where $\boldsymbol{y}^{\top}_{i,\ell} = \boldsymbol{\lambda}^{(r)\top}_{i,\ell} + \sum_{j \in \mathcal{N}_i} \boldsymbol{\lambda}^{(r)\top}_{j,\ell} \boldsymbol{G}_{ji}$

Update $\boldsymbol{\lambda}_{i,\ell}$ for all $\ell \in \{k, \ldots, k+h_p-1\}$ as

$$\boldsymbol{\lambda}^{(r+1)}_{i,\ell} = \boldsymbol{\lambda}^{(r)}_{i,\ell} + \gamma \left(\boldsymbol{u}^{\mathrm{c}}_{i,\ell|k} + \sum_{j \in \mathcal{N}_i} \boldsymbol{G}_{ij} \boldsymbol{u}^{\mathrm{c}}_{j,\ell|k} \right),$$

where $0 < \gamma < 1$

$r \leftarrow r + 1$

end while

C. Adversary Model

The agents are classified as regular and adversarial agents based on the following definitions.

Definition 1: Agent i belongs to the set of regular agents, denoted by $\mathcal{R}$, if it always implements its control input $\boldsymbol{u}_{i,k}$ according to the decision obtained from the DMPC strategy, i.e., $\boldsymbol{u}_{i,k} = \boldsymbol{u}^{\star}_{i,k|k}$, for all $k \geq 0$. Otherwise, agent i belongs to the set of adversarial agents, denoted by $\mathcal{A}$. □

Definition 2: An attack is defined as the event at one time instant when at least one adversarial agent implements its control input that is different than the decision obtained from the DMPC strategy. □

Definition 3 ([11]): The set of adversarial agents is f-local if $|\mathcal{A} \cap \mathcal{N}_i| \leq f$, for $f \in \mathbb{Z}_{\geq 1}$ and all $i \in \mathcal{N}$. □

Assumption 2: Each agent has at most one adversarial neighbor. □

Assumption 3: Regular agents do not have prior knowledge of the attack occurrences, but they have an expectation on the attack probability, denoted by $P_{\mathrm{at}} \in (0,1]$. □

In this paper, the case is restricted for $f = 1$, as stated in Assumption 2. The adversarial agents may try to gain advantage by implementing a different decision that benefits these agents. For instance, the adversarial agents may get benefit if they decide to reduce the energy production and/or store more energy to their storages and ask their neighbors to provide the power deficiency. Although it leads to a global suboptimal solution, the adversarial agents gain an advantage locally. In other words, the adversarial agents are not willing to cooperate for their own interest.

III. Proposed Approach

A. Robustification Against Attacks

Regular agents might be affected negatively from the attacks of their adversarial neighbors. Due to the coupled constraints (3), regular agents must conform with the actions taken by their adversarial neighbors. In this regard, the existence of a storage unit at each microgrid could help to mitigate this issue without affecting the operation of the distributed generators. Additionally, uncertain loads might have similar effect to all microgrids and we consider that the deviation between the forecast and the actual load is compensated by the storage units.

In order to meet the power balance (2) when an attack occurs, more power from the storage ($p^{\mathrm{st}}_{i,k}$) is taken. However, it implies that the evolution of the SoC is different than the one that is predicted by the dynamic model (4). To ensure that there is no violation on the related constraints, a formulation that robustifies Problem (11) against such attacks as well as the uncertainty of the load is proposed. To this end, we consider the attack as disturbance, denoted by $w^{\mathrm{a}}_{i,k}$, and denote the load disturbance by $w^{\mathrm{d}}_{i,k}$. These disturbances affect the power balance (2) as follows:

$$\hat{p}^{\mathrm{d}}_{i,k} - p^{\mathrm{G}}_{i,k} - p^{\mathrm{st}}_{i,k} - p^{\mathrm{im}}_{i,k} - w^{\mathrm{d}}_{i,k} - w^{\mathrm{a}}_{i,k} - \sum_{j \in \mathcal{N}_i} p^{\mathrm{t}}_{ji,k} = 0. \quad (12)$$

Although $w^{\mathrm{d}}_{i,k}$ and $w^{\mathrm{a}}_{i,k}$ are uncertain, they are bounded by (1) and (9), respectively. Therefore, agent $i \in \mathcal{R}$ might consider the worst case of the total disturbance, denoted by $w_{i,k} = w^{\mathrm{a}}_{i,k} + w^{\mathrm{d}}_{i,k}$, which is stated as follows:

$$w^{\max}_{i,k} = \max_{j \in \mathcal{N}_i} \left(2 p^{\mathrm{t,max}}_{ji} \right) + d^{\max}_i, \quad (13)$$

due to (9) and Assumption 2. Since $w_{i,k}$ is compensated by the power delivered by/to the storage $p^{\mathrm{st}}_{i,k}$, the constraints related to $p^{\mathrm{st}}_{i,k}$, i.e., (5) and (6), might be violated. Therefore, these constraints are tightened to accommodate the worst case disturbance $w_{i,k}^{\max}$ as follows:

$$x_i^{\min} - b_i w_{i,k}^{\max} \le a_i x_{i,\ell} + b_i p_{i,\ell}^{\mathrm{st}} \le x_i^{\max} + b_i w_{i,k}^{\max}, \quad (14)$$

$$-p_i^{\mathrm{ch}} + w_{i,k}^{\max} \le p_{i,\ell}^{\mathrm{st}} \le p_i^{\mathrm{dh}} - w_{i,k}^{\max}, \quad (15)$$

for all $\ell \in \{k, \dots, k+h_p-1\}$. Hence, the robust reformulation of Problem (11) is stated as follows:

$$\underset{\{\{\boldsymbol{u}_{i,\ell|k}\}_{i\in\mathcal{N}}\}_{\ell=k}^{k+h_p-1}}{\text{minimize}} \sum_{i\in\mathcal{N}} \sum_{\ell=k}^{k+h_p-1} J_{i,\ell}(\boldsymbol{u}_{i,\ell|k}) \quad (16a)$$

$$\text{subject to } \boldsymbol{F}_i^{\mathrm{r}} \boldsymbol{u}_{i,\ell|k} \le \boldsymbol{f}_{i,\ell}^{\mathrm{r}},\ \forall i \in \mathcal{N}, \quad (16b)$$

$$\boldsymbol{u}_{i,\ell|k}^{\mathrm{c}} + \sum_{j\in\mathcal{N}_i} \boldsymbol{G}_{ij} \boldsymbol{u}_{j,\ell|k}^{\mathrm{c}} = \boldsymbol{0},\ \forall i \in \mathcal{N}, \quad (16c)$$

for all $\ell \in \{k, \dots, k+h_p-1\}$, where (16b) with the appropriate $\boldsymbol{F}_i^{\mathrm{r}}$ and $\boldsymbol{f}_{i,\ell}^{\mathrm{r}}$ is defined according to (2), (4), (7)-(9), and (13)-(15).

Proposition 1: Suppose that Assumption 1 holds. Problem (16) has feasible solutions if and only if

$$w_{i,k}^{\max} \le \min\left(\frac{1}{2}\left(p_i^{\mathrm{ch}} + p_i^{\mathrm{dh}}\right), -\frac{1}{2b_i}\left(x_i^{\max} - x_i^{\min}\right)\right). \quad (17)$$

Furthermore, suppose that both Assumption 2 and (17) hold. Then, any feasible solution of Problem (16) does not violate operational constraints (2)-(9) even though an attack, which is defined in Definition 2, occurs. □

Proof: The proof is provided in [12]. ■

Assumption 4: Condition (17) holds true, implying the existence of feasible solutions of Problem (16). □

If the condition of $w_{i,k}^{\max}$ stated in Proposition 1 is not satisfied, then $p_{i,k}^{\mathrm{G}}$ and/or $p_{i,k}^{\mathrm{im}}$ must also be involved in compensating $w_{i,k}$. In this regard, the constraints related to $p_{i,k}^{\mathrm{G}}$ and $p_{i,k}^{\mathrm{im}}$ must be tightened with similar procedure as that previously explained. For the remaining of the paper, suppose that Assumption 4 holds. Therefore, the DMPC method presented in Algorithm 1 can then be applied to solve Problem (16) by simply substituting (11b) with (16b) in the local optimization step.

B. Attack Identification and Mitigation

The attack identification and mitigation is an active detection strategy, where regular agents test their hypothesis to find their adversarial neighbors by deciding to open/close their connections with their neighbors. Firstly, a regular agent, $i \in \mathcal{R}$, detects an attack performed by one of its neighbors by evaluating its own SoC as follows:

$$\Delta_{i,k} = \left| x_{i,k} - \left(x_{i,k-1} + \boldsymbol{b}_i^\top \boldsymbol{u}_{i,k-1}^\star + b_i \hat{p}_{i,k-1}^{\mathrm{d}} \right) \right|, \quad (18)$$

where $\boldsymbol{b}_i = b_i[0\ \ -\mathbb{1}_{2+|\mathcal{N}_i|}^\top]^\top$. If $\Delta_{i,k} > b_i d_i^{\max}$, then at k, agent i detects an attack.

Remark 2: An attack $w_{i,k}^{\mathrm{a}}$ such that $|w_{i,k}^{\mathrm{a}} + w_{i,k}^{\mathrm{d}}| \le d_i^{\max}$ is undetectable since the regular agents cannot distinguish it from the load disturbance. However, such an attack is tolerable since the agents consider the bound of load disturbance as $d_i^{\max}$ in the first place. □

Although an attack can be detected, for $|\mathcal{N}_i| > 1$, it is not possible to determine which neighbor is the adversarial one by only evaluating (18). Therefore, in order to identify the adversarial neighbors, we apply a hypothesis testing method that is based on Bayesian inference [9].

Each agent, $i \in \mathcal{R}$, considers the following set of hypotheses, $\mathcal{H}_i = \{\mathbf{H}_i^0, \mathbf{H}_i^j : j \in \mathcal{N}_i\}$, where the hypotheses are defined as follows:

- $\mathbf{H}_i^0$: There is no attack,
- $\mathbf{H}_i^j$: Neighbor j is an adversarial agent,

for all $j \in \mathcal{N}_i$. The Bayesian inference is used as the model to update the probability of the hypotheses as follows:

$$\mathbb{P}_{k+1}(\mathbf{H}_i^j) = \frac{\mathbb{P}_k(\mathbf{H}_i^j)\mathbb{P}_k(\Delta_{i,k}|\mathbf{H}_i^j)}{\mathbb{P}_k(\Delta_{i,k})}, \quad (19)$$

for all $\mathbf{H}_i^j \in \mathcal{H}_i$, where $\mathbb{P}_k(\mathbf{H}_i^j)$ denotes the probability of hypothesis $\mathbf{H}_i^j$ at time instant k, $\mathbb{P}_k(\Delta_{i,k})$ denotes the the marginal likelihood of $\Delta_{i,k}$, and $\mathbb{P}_k(\Delta_{i,k}|\mathbf{H}_i^j)$ denotes the probability of observing $\Delta_{i,k}$ given hypothesis $\mathbf{H}_i^j$ and is formulated as follows:

$$\mathbb{P}_k(\Delta_{i,k} \le b_i d_i^{\max}|\mathbf{H}_i^j) = \begin{cases} 1, & \text{for } j = 0, \\ 1 - v_{i,k}^j P_{\mathrm{at}}, & \text{for all } j \in \mathcal{N}_i, \end{cases}$$

$$\mathbb{P}_k(\Delta_{i,k} > b_i d_i^{\max}|\mathbf{H}_i^j) = \begin{cases} 0, & \text{for } j = 0, \\ v_{i,k}^j P_{\mathrm{at}}, & \text{for all } j \in \mathcal{N}_i, \end{cases}$$

where $v_{i,k}^j \in \{0,1\}$, for all $j \in \mathcal{N}_i$, denote the decision whether agent i connects to and negotiates with neighbor j, i.e., $v_{i,k}^j = 1$ implies agent i connects to neighbor j, whereas $v_{i,k}^j = 0$ implies agent i does not connect to neighbor j. Note that $\mathbb{P}_{k+1}(\mathbf{H}_i^j)$ is the a posteriori probability of $\mathbf{H}_i^j$ given the event $\Delta_{i,k}$, i.e., $\mathbb{P}_{k+1}(\mathbf{H}_i^j) = \mathbb{P}(\mathbf{H}_i^j|\Delta_{i,k})$. The initial hypothesis probabilities are defined as

$$\mathbb{P}_0(\mathbf{H}_i^j) = \begin{cases} 1 - P_{\mathrm{at}}, & \text{for } j = 0, \\ P_{\mathrm{at}}/|\mathcal{N}_i| & \text{for all } j \in \mathcal{N}_i, \end{cases} \quad (20)$$

implying that it is initially considered that each neighbor is equally likely to be adversarial.

In order to decide the connection that a regular agent $i \in \mathcal{R}$ will have with its neighbors at each time instant, each agent $i \in \mathcal{R}$ solves a local mixed-integer optimization problem of the form:

$$\underset{\boldsymbol{v}_{i,k}, \{\boldsymbol{u}_{i,\ell|k}\}_{\ell=k}^{k+h_p-1}}{\text{minimize}} \sum_{\ell=k}^{k+h_p-1} J_{i,\ell}(\boldsymbol{u}_{i,\ell|k}) + J_i^{\mathrm{v}}(\boldsymbol{v}_{i,k}) \quad (21a)$$

$$\text{subject to } \boldsymbol{F}_i^{\mathrm{lc}} \boldsymbol{u}_{i,\ell|k} + \boldsymbol{F}_{\mathrm{v},i}^{\mathrm{lc}} \boldsymbol{v}_{i,k} \le \boldsymbol{f}_{i,\ell}^{\mathrm{lc}}, \quad (21b)$$

$$\boldsymbol{v}_{i,k} \in \mathcal{C}_i \cup \{\mathbb{1}_{|\mathcal{N}_i|}\}, \quad (21c)$$

where $\boldsymbol{v}_{i,k} = [v_{i,k}^j]_{j\in\mathcal{N}_i}^\top$. Here, the cost function $J_i^{\mathrm{v}}(\boldsymbol{v}_{i,k}) : \mathbb{R}^{|\mathcal{N}_i|} \to \mathbb{R}$ penalizes the decision of having a connection with the neighbors and is expressed as $J_i^{\mathrm{v}}(\boldsymbol{v}_{i,k}) = \gamma n_{\mathrm{at}} \sum_{j\in\mathcal{N}_i} \mathbb{P}_k(\mathbf{H}_i^j)(v_{i,k}^j)^2$, where $\gamma \in \mathbb{R}_{>0}$ denotes a predefined weight and n_{at} denotes the number of attacks that agent i has received, i.e., the number of time instants at which $\Delta_{i,k} > b_i d_i^{\max}$. By having n_{at} as a weight, establishing a connection with a neighbor is penalized more if the number of received attacks increases. Furthermore, notice that we penalize $v_{i,k}^j$, for each $j \in \mathcal{N}_i$, proportionally to $\mathbb{P}_k(\mathbf{H}_i^j)$. Moreover, (21b) is obtained from (2), (4), (7), (8), (14), and (15) as well as from the following expressions:

$$w_{i,k}^{\max} = \max_{j \in \mathcal{N}_i} \left(2 p_{ji}^{\mathrm{t},\max} v_{i,k}^j\right) + d_i^{\max}, \tag{22}$$

$$-p_{ji}^{\mathrm{t},\max} v_{i,k}^j \le p_{ji,\ell}^{\mathrm{t}} \le p_{ji}^{\mathrm{t},\max} v_{i,k}^j, \quad \forall j \in \mathcal{N}_i, \tag{23}$$

for all $\ell \in \{k, \ldots, k+h_p-1\}$, whereas, in the constraint (21c), $\mathcal{C}_i = \{\boldsymbol{z}_j = \mathbb{1}_{|\mathcal{N}_i|} - \boldsymbol{e}_j, j = 1, 2, \ldots, |\mathcal{N}_i|\}$, where $\boldsymbol{e}_j$, for all $j = 1, 2, \ldots |\mathcal{N}_i|$, are the standard basis vectors of $|\mathcal{N}_i|$-dimensional Euclidean space. Constraint (21c) implies that agent i only allows that it is disconnected from at most one neighbor. Problem (21) is a mixed-integer quadratic program (MIQP) due to the existence of $\boldsymbol{v}_{i,k}$.

Proposition 2: Suppose that Assumptions 1 and 4 hold. Then, Problem (21) has feasible solutions. □

Proof: The proof is provided in [12]. ■

Finally, suppose that the decision $\boldsymbol{v}_{i,k}^\star = [v_{i,k}^{j\star}]_{j\in\mathcal{N}_i}^\top$ is the solution obtained from solving Problem (21). Now, instead of using (13), each agent $i \in \mathcal{R}$ computes the worst case of the disturbance by plugging $\boldsymbol{v}_{i,k}^\star$ into (22). Thus, in the robust problem (16), the local constraints (16b) are switched by (21b) with $\boldsymbol{v}_{i,k} = \boldsymbol{v}_{i,k}^\star$, for all $i \in \mathcal{N}$.

C. Overall Scheme

The proposed method is summarized in Algorithm 2. Furthermore, we suppose that Assumption 5 holds. Assumption 5 implies that, although there exists a connection between agents i and j, either of them can block the influence by closing the connection.

Algorithm 2 Resilient distributed algorithm, for $i \in \mathcal{R}$

Initialize the hypothesis probabilities according to (20).
for $k = 1, 2, \ldots$ **do**
  Evaluate (18) to detect an attack.
  Update the probability value of the hypotheses according to (19).
  if $\mathbb{P}_k(\mathbf{H}_i^j) = 1,\ j \in \mathcal{N}_i$, **then**
    $v_{i,k}^{j\star} = \begin{cases} 0, & \text{for } \mathbb{P}_k(\mathbf{H}_i^j) = 1, \\ 1, & \text{for } \mathbb{P}_k(\mathbf{H}_i^j) = 0. \end{cases}$
    Compute $\boldsymbol{u}_{i,k|k}^\star$ by solving (16), considering (16b) is formed by (2), (4), (7), (8), (14), (15), (23) with $\boldsymbol{v}_{i,k} = \boldsymbol{v}_{i,k}^\star$, and $w_{i,k}^{\max} = d_i^{\max}$, using Algorithm 1.
  else
    Compute $v_{i,k}^{j\star}$, for all $j \in \mathcal{N}_i$, by solving (21)
    Compute $\boldsymbol{u}_{i,k|k}^\star$ by solving (16), considering (16b) is formed by (2), (4), (7), (8), (14), (15), (22) and (23), with $\boldsymbol{v}_{i,k} = \boldsymbol{v}_{i,k}^\star$, using Algorithm 1.
  end if
  Apply $\boldsymbol{u}_{i,k|k}^\star$ and $\boldsymbol{v}_{i,k}^\star$.
end for

Assumption 5: Any agent can temporarily disconnect the physical link between itself and its neighbors, respecting the decision of $\boldsymbol{v}_{i,k}^\star$. Two agents, i and j, where $(i,j) \in \mathcal{E}$, can only exchange energy if and only if $v_{i,k}^{j\star} = v_{j,k}^{i\star} = 1$. □

The decisions obtained by performing Algorithm 2 are characterized by the following Proposition 3. In addition, a sub-optimality certificate of the control inputs obtained by performing Algorithm 2 is presented in [12].

Proposition 3: Suppose that Assumptions 1-5 hold. If the regular agents, i.e. all $i \in \mathcal{R}$, apply Algorithm 2, then the obtained decision $\boldsymbol{u}_{i,k}^\star$, for all $i \in \mathcal{R}$, do not violate the operational constraints (2)-(9) under an attack that is defined by Definition 2, for all $k \in \mathbb{Z}_{\ge 0}$. □

Proof: The proof is provided in [12]. ■

Remark 3: The attack identification and mitigation methods can be implemented along with any distributed optimization algorithm that can solve Problems (11) and (16). □

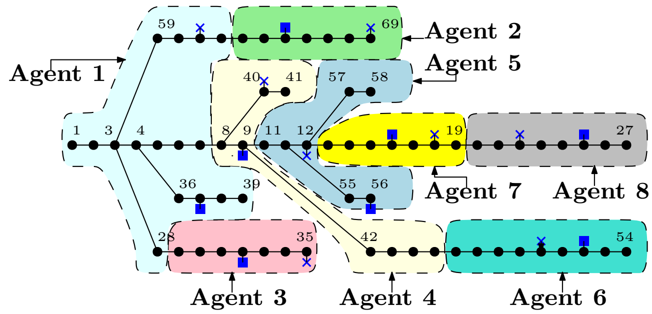

Fig. 1. The topology of the PG&E 69-bus distribution system and its 8-agent resulting partition. Blue crosses and squares indicate the distributed generators and storages, respectively.

TABLE I
PARAMETERS OF THE MICROGRIDS

Parameters	Value	Unit	Agent (i)
$x_i^{\min}$, $x_i^{\max}$, $x_{i,0}$	40, 70, 55	%	all
p_i^{ch}, p_i^{dh}, $p_i^{\mathrm{G,min}}$, $p_i^{\mathrm{G,max}}$	300, 300, 0,1500	kW	all
$p_i^{\mathrm{t,max}}$, $p_i^{\mathrm{im,max}}$	100, 2000	kW	all
$e_{\mathrm{cap},i}$	1000	kWh	all
a_i, c_i^{st}, c_i^{im}, c_i^{t}	1, 1, 250, 0.1	-	all
c_i^{g}	5	-	2, 3, 6, 7
c_i^{g}	10	-	1, 4, 5, 8

IV. CASE STUDY

As a case study, we use the PG&E 69-bus distribution network with additional distributed generators and energy storages [5], as depicted in Fig. 1. We follow the partition given by [5] to divide the network into eight interconnected microgrids (agents). The parameters of each microgrid are given in Table I. Furthermore, we consider two types of load profiles, which are industrial and residential, and assign each microgrid to one of the profiles randomly. In this case study, microgrids 2, 6, and 7 are adversarial and the probability of attacks is set to be 0.3. Furthermore, the prediction horizon of each agent is $h_p = 4$ steps and we consider one-day simulation with sampling time of 15 minutes.

TABLE II
TOTAL COST OF THE SYSTEM

Scenario	Dist. Strategy	Attack/Load Disturbance	Cost (Proportional)	Constraint Satisfaction
1	Nominal	No	1.00	Yes
2	Nominal	Yes	1.06	No
3	Alg. 3	Yes	1.91	Yes
4	Alg. 2	Yes	1.18	Yes

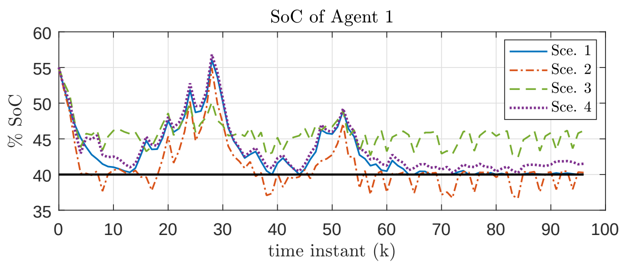

Fig. 2. The evolution of the SoC of agents 1. The black horizontal line indicates the minimum limit of SoC, $x_i^{\min}$.

Algorithm 3 Distributed robust scheme, for $i \in \mathcal{R}$

1: **for** $k = 1, 2, \ldots$ **do**
2: Compute $\boldsymbol{u}_{i,k}$ by solving Problem (16), considering (16b) is formed by (2), (4), (7)-(9), and (13)-(15), with Algorithm 1
3: Apply $\boldsymbol{u}_{i,k}$
4: **end for**

As shown in Table II, we consider four simulation scenarios, in each of which a different distributed strategy is applied. Table II also shows the overall performance of the network over the whole simulation time. The proposed approach achieves a better performance than the robustified approach while ensuring the satisfaction of the constraints. As shown in Fig. 2, in Scenario 2, the minimum limit of the SoC is violated. However, this violation does not occur in Scenarios 3 and 4. Moreover, Fig. 3 shows how agent 1 detects agent 2 as the adversarial neighbor in Scenario 4. Once detected, i.e., at $k = 8$, agent 1 disconnects from agent 2.

V. CONCLUSION AND FUTURE WORK

A distributed energy management for interconnected microgrid systems that is based on dynamic economic dispatch problem is studied. We analyze the case of having microgrids that perform an adversarial behavior, i.e., some microgrids do not comply with the decisions obtained from the distributed strategy. Furthermore, we propose a robustified formulation and an attack identification and mitigation method such that the distributed strategy can deal with such adversaries.

Future work includes extending the proposed approach such that the stochasticity of the loads is taken into account explicitly in order to improve the performance and assumptions on the number of adversarial neighbors are relaxed.

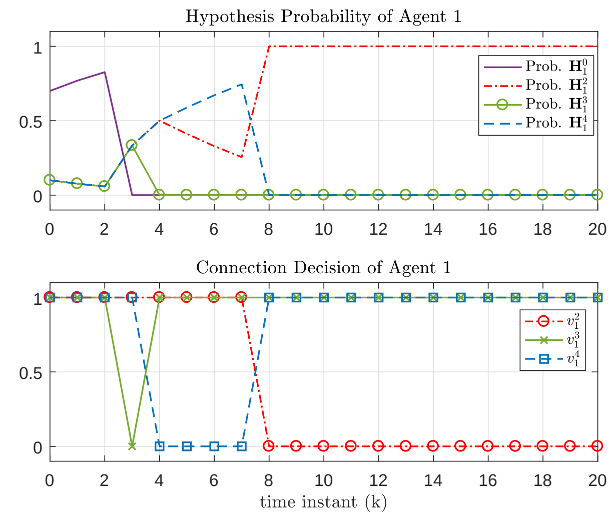

Fig. 3. The evolution of the hypothesis probability (top) and the connection decision (bottom) of agent 1. Note that the decision $\boldsymbol{v}_{1,k}^{\star}$ are the same for $k = 8, 9, \ldots, 96$, since the adversarial neighbor is detected at $k = 8$.

Furthermore, we will also explore the possibility to improve the detection strategy as well as the attack mitigation method.

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