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# MODELING FRAMEWORK FOR COMPARING TAXI OPERATIONAL MODES: CASE STUDY IN BARCELONA

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## Abstract

This paper presents an aggregated mathematical model for the estimation of key performance indicators of the taxi market based on the system's generalized cost function, which is calculated using the expected statistical values of customers' trip distance, waiting/access time and the cost of the involved actors, including externalities, who are the taxi drivers, the taxi customers and the city represented by the rest of the drivers and the citizens. Optimum values for the taxi supply are obtained from mathematical formulations depending on the demand level and the size of the city. The model is developed for stand, hailing and dispatching taxi markets and the results are compared, presenting conclusions for the best type of market for each demand level and city size. The model is applied in the city of Barcelona, presenting useful conclusions on the performance indicators of the taxi services and the impact of the applied policies as well as the optimum number of taxis for each operational mode, ranging between 30 and 40 vehicles per hour and km<sup>2</sup>.

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*Keywords:* Taxi modeling, aggregated taxi model, taxicab problem, transport on demand, Barcelona

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## 1. Introduction

After the era of the empires it came the era of the countries and now the era of the cities, in which more than 80% of the world's population is expected to live in urban settlements by 2030, creating large cities with high densities and

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mobility needs, especially in the Central Business Districts. In order to satisfy this increase in the mobility needs and due to the lack of free urban space, there is a need for increasing the use of Public Transport, including the use of individual public transport means, such as taxis. Purportedly, fulfilling mobility needs with taxis will continue growing in the future, especially when considering the oversaturation of transport networks and the associated costs of owning and driving private vehicles as well as the policies applied in order to ban the presence of cars in the city centers. To this end, taxi markets, conventionally regulated by restraint of issued licenses and uncontrolled tariffs, arguably to the drivers' and customers' benefit, are lately liberalized following the economic austerity driven global trend. Yet, this associated market de-regulation is often more of challenge than merely an application; the relation between regulated and de-regulated taxi markets has been explored in Salanova et al. (2011), where the authors laid out a global city-based review on the prevailing taxi market (de-/regulated) and its implication, concluding that the de-regulation can have positive or negative aspects depending on how it is applied. It is thus pivotal for decision makers to develop the right policies that will guarantee both a sufficient Level of Service (LoS) to customers and an adequate income to taxi license holders while providing infrastructure for the provision of taxi services (due to the crisis more taxi stands are requested) in a already tight and congested urban space. In addition, innovative mobility services and business schemes are arising, which in the long run may modify the traditional taxi market, but which nowadays are seen negatively by the taxi market players, who are asking solutions/protection to the policy makers. Therefore, the provision of adequate tools to decision makers is crucial for the provision of the right/optimum mobility services/solutions aiming at reducing the system cost.

The aim of this paper is to estimate the performance indicators and the cost incurred in the different taxi operational modes and identify the optimum operational mode for each type of city from a theoretical point of view, achieved by developing a new analytical model based on the generalized system cost. The proposed model uses the different mathematical formulations for estimating the optimum fleet size related to each operational mode, city size and demand level. The correspondent generalized cost and waiting/access time of the customers are also obtained, comparing the performance of the three operational modes for the same city. These results can be used by the policy makers in order to define the taxi operation mode for the different areas of the city and time intervals of the day, which even if it may be a combination of various modes, it can favor the one having the smaller system unitary cost.

The remaining of the paper is organized as follows: the different models presented in the literature are briefly reviewed in the section 2; the formulation of the model for taxi services is presented in section 3, while section 4 contains its application to the three operational modes; section 5 deals with the application of the proposed methodology to the Barcelona taxi sector using real-world data and concluding in the comparison of the performance of the three operational modes.

## 2. Taxi modeling review

The first econometric model developed for evaluating the performance of the taxi services were presented by Douglas (1972) and De Vany (1975). Later, Manski and Wright (1976) and Schroeter (1983) developed models for a taxi stand and a dispatching service respectively. Cairns and Liston-Heyes (1996) redefined the model proposed by Douglas (1972) while Arnott (1996) was the first to talk about subsidization for covering the costs difference between the first and the second best solutions. Yang and Wong and Yang et al. (1998, 2000, 2005 and 2010) developed a set of equilibrium models taking into account the spatial distribution of demand and supply in the city. More recently, Bai and Wang (2012) presented a mixed model combining the different taxi operation modes. Some of the presented models have been applied in real world cases using various data sources. Beesley (1973), Beesley and Glaster (1983) collected data related to the taxi sector through questionnaires in London; Schroeter (1972) used data from taximeters in Mineapolis; Schaller (2007) used interviews and questionnaires in the USA; Kattan et al. (2010) developed regression models for work trips made by taxi in 25 Canadian cities. A detailed review of the aggregated and equilibrium models of taxi services can be found in Salanova et al. (2011). Few contributions were able to model the different operation modes and compare them obtaining the most adequate to each type of city.

## 3. Problem formulation

In order to define the optimum fleet size, all costs (monetary and time costs) of the involved stakeholders are added into a unique objective function. The most important decision variable of the model is the number of vehicles per unit

of area and surface. The demand is supposed to be inelastic to variations in the cost and the quality of the service offered and homogeneously distributed with regards to both the spatial and temporal dimensions (in addition, the temporal dimension is fixed to one hour in order to simplify the formulations). This somehow limits the model, but the authors already extended it in Salanova and Estrada (2017) where the fare is also a decision variable. The effect of the taxi fee charging on the demand is not analyzed in this paper (Salanova 2013 presents the results of the models taking into account demand elasticity). The hypothesis of demand homogeneity may sound restrictive and unrealistic and for this reason the results have been compared to ones obtained by the agent-based model developed by the authors and able to simulate heterogeneous demand (section 4.5), showing the error of this hypothesis for different demand heterogeneous distributions. The objective function is optimized, presenting the results in terms of minimum and optimum fleet size for each operation mode, demand level and city size. The proposed formulation is a multi-objective system, where the decision variable is the number of taxis in the network. This variable is regulated by the responsible authority in an attempt to reduce the system cost of the taxi services, subject to the constraints that need to be respected.

### 3.1. The objective function

The objective function is aimed at minimizing the system unitary cost calculated in equivalent user time units as shown in Equation (1). It can be translated into system monetary units per unit of time and area (for example, €/h-km<sup>2</sup>) just multiplying each term of Equation (1) by the taxi demand density and the perceived passenger value of time ( $\lambda_u \cdot VoT_u$ ). The objective function is composed by the costs of the involved actors.

$$\min_{\lambda_d} Z = Z_d + Z_u + Z_c + G \quad (1)$$

$$Z_u = \left[ \alpha_A \cdot T_A + \alpha_W \cdot T_W + \alpha_{IV} \cdot T_{IV} + \frac{\bar{c}}{VoT_u} \right] \quad (2)$$

$$Z_d = \frac{\lambda_d}{\lambda_u VoT_u} [-\bar{n} \cdot \bar{c} + (\bar{d} \cdot C_{km} + C_h)] \quad (3)$$

$$Z_c = \lambda_v \cdot \frac{\Delta T_v \cdot VoT_v}{\lambda_u VoT_u} + \frac{\lambda_d \cdot C_E \cdot E_d}{\lambda_u VoT_u} + \frac{\lambda_v \cdot C_E \cdot \Delta T_v \cdot E_d}{\lambda_u VoT_u} \quad (4)$$

$$\bar{n} \cdot \bar{c} - (\bar{d} \cdot C_{km} + C_h) \geq B \quad (5)$$

$$\alpha_A \cdot T_A + \alpha_W \cdot T_W < T_{max} \quad (6)$$

The cost component  $Z_u$  captures the average total travel time of a single user. In Equation (2), this term  $Z_u$  is estimated as the sum of the in-vehicle time ( $T_{IV}$ ), waiting time ( $T_W$ ), access time ( $T_A$ ) and the monetary trip cost ( $\bar{c}$ ) expressed in time units. Three weighting parameters ( $\alpha_A$ ,  $\alpha_W$  and  $\alpha_{IV}$ ) are proposed in order to use a unique Value of Time for the taxi users,  $VoT_u$ . These take into account the customers' perception of time in each step of their trip. There is the need for calibrating the parameters for each city and customers segment although Kittelson et al. (2003) proposed default values. The term  $Z_c$  determines the externalities caused by the provision of a taxi service to the city as a whole. It considers the travel time increase of other drivers using the same street network as taxis ( $\Delta T_v$ ) as well as the associated Green House Gases and local emissions ( $E_d$ ) monetized ( $C_E$ ). Finally, the taxi driver component ( $Z_d$ ) is evaluated as the difference between the operational cost and the fare income associated to one trip. The driver trip cost is calculated based on the average distance travelled by the taxi,  $\bar{d}$ , and the unit fee per distance ( $C_{km}$ ) and time ( $C_h$ ).

A set of constraints is proposed for reflecting physical or temporal restrictions of the policy decisions and economic market equilibrium. The values that should be within acceptable thresholds are the benefit of taxi drivers (equation 5) as well as the level of service provided to the users by means of maximal access and waiting time of customers (Equation 6).

The decision variable of this optimization problem is the provision of taxis per unit of area and time ( $\lambda_d$ ). Although the fare structure can be considered as a decision variable, it has been assumed as a given parameter in this paper. The other variables, such as the area or the value of time depend on the city in which the model is applied. The rest of cost components and parameters are presented in Table 1. A parameter that deserves mentioning is the term  $\bar{c}$ , the average fare of a single trip. This parameter does not affect the objective function of this problem since it appears in both the

customers’ and the drivers’ cost components with opposite signs. However, it is an important factor to be aware when the profitability of taxi drivers is analyzed by means of equation (5). In order to be consistent, the average number of trips per hour and driver should be equal to  $\bar{n} = \frac{\lambda_u}{\lambda_d}$ .

3.2. Formulations of the customer level of service and operating cost

Various formulations have been developed in order to estimate the variables of the equations (1) - (4) to the parameters of the model and the decision variable  $\lambda_d$ . The spatial density of taxis can be estimated as the sum of the vacant taxis and the taxis in service divided by the area of service. Salanova et al. (2014) presents an extended review of these formulations for different assumptions. The main operation assumptions in this paper are the following: in the dispatching and stand markets, taxis wait at predefined taxi stands, distributed homogeneously within the network while in the hailing market the taxis circulate continuously looking for new customers. The demand is considered homogeneously distributed both spatially and timely. Salanova and Estrada (2017) analyses the effect of the variation of the demand during the day and proposes an optimum shifts distribution in order to reduce the differences between the real offer based on a shift system and the optimum offer for each time of the day.

For the purpose of this paper, the in-vehicle time formulation is common for the three operational modes since the cruising speed is considered to be homogeneous. On the other hand, the formulations of the average hourly distance per driver,  $\bar{d}$ , as well as the waiting and access times ( $T_W$ ,  $T_A$  respectively) are different for each operational mode. Finally, the externalities have been formulated only for the hailing mode since only in this model the taxis circulate empty looking for customers. Details about the formulation of the problem variables and the optimization process can be found in Salanova 2013 and Salanova et al. (2014).

4. Comparison among the hailing, stand and the dispatching taxi models

In order to identify the optimum operation mode for each range of demand and to identify the most significant variables and parameters that define these ranges, a comparison of the three developed models is presented below (for the comparison only one mode exists at the same time). Reference values for the demand, the supply and the area (which defines the average trip distance) are used for generating the comparative analyses: Demand for taxi trips: 25 - 50 customers per hour and km<sup>2</sup>; Area of service: 100 - 400 km<sup>2</sup>; Supply: 15 - 35 taxis per hour and km<sup>2</sup>.

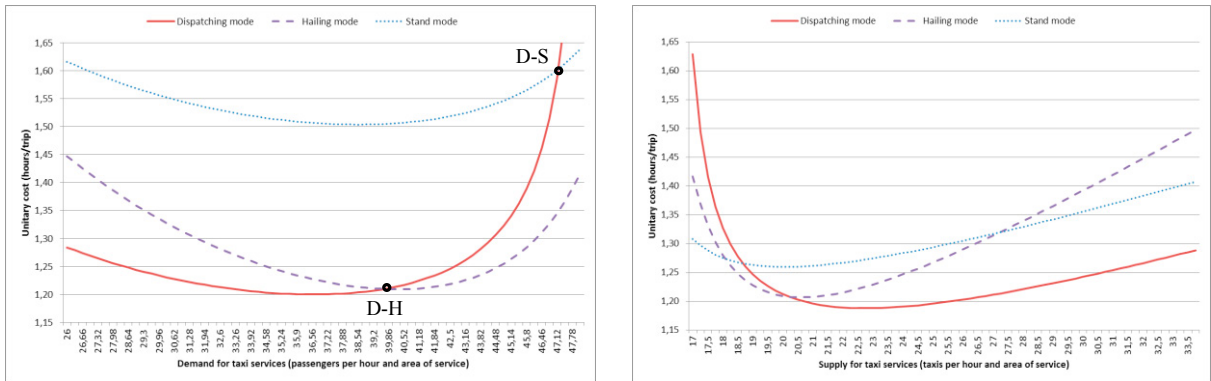


Figure 1 Unitary costs for various demand levels and operation modes (left); Unitary costs for various fleet sizes and operation modes (right)

The analyses conducted below are based on the variation of one of the three variables within the whole range, while the other two values remain constant and equal to the average value of the above ranges. It is important to highlight that three parameters have significant influence on the above comparisons: the operational cost, the average speed and the VoT. The relation between these three parameters defines the demand ranges where each operation mode has the minimum unitary cost. Using indicative values for the parameters of the dispatching and hailing formulations, the demand ranges where the dispatching market is more attractive than the hailing market are below 45 to 90 trips per

hour and km<sup>2</sup>, while the commercial speed should be smaller than 6-7 km/h for being the stand market more attractive than the dispatching market. Figure 1 defines the ranges for the demand and the supply where each operation mode provides the minimum unitary cost. The analytical formulations of the intersection points of Figure 1 can be found in Salanova 2013.

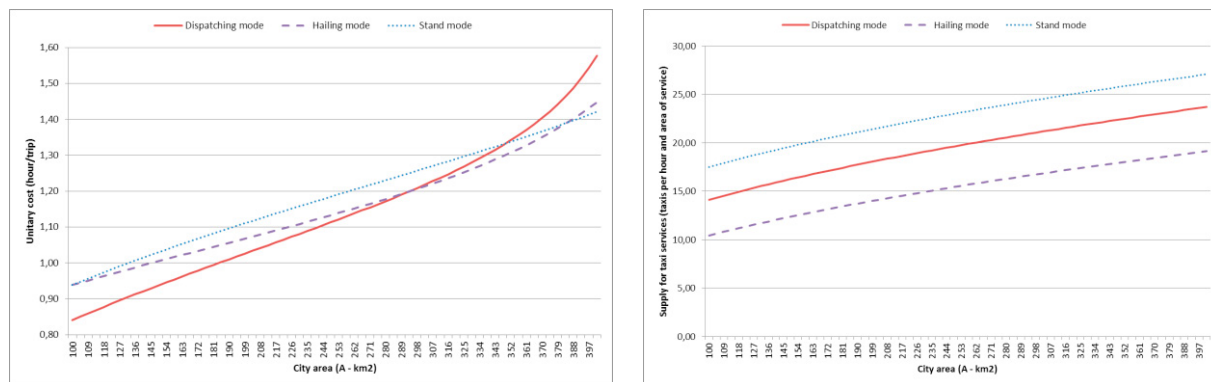


Figure 2 Unitary costs (left) and optimum fleet size (right) for various area levels and operation modes

For fixed demand ( $\lambda_u = ct$ ) and area ( $A = ct$ ) levels, unitary cost of operation is at its minimum value for small and medium fleets for the stand and hailing modes respectively (**Error! Reference source not found.**). The dispatching mode has the lowest unitary cost if the taxi fleet is large. It should be taken into account that there are technological barriers since a dispatching center and the respective software should be implemented and maintained, which has not been taken into account in the model.

For fixed demand ( $\lambda_u = ct$ ) and supply ( $\lambda_d = ct$ ) levels, the dispatching mode has the minimum unitary cost of operation in small to medium regions, the hailing mode has the lowest unitary cost in medium to large regions while the stand mode has the lowest unitary cost in very large regions (Figure 2 left). In the case of fixed demand level and independently of the area, the hailing optimum fleet is the smallest one, while the stand optimum fleet is the largest, as observed in Figure 2 right.

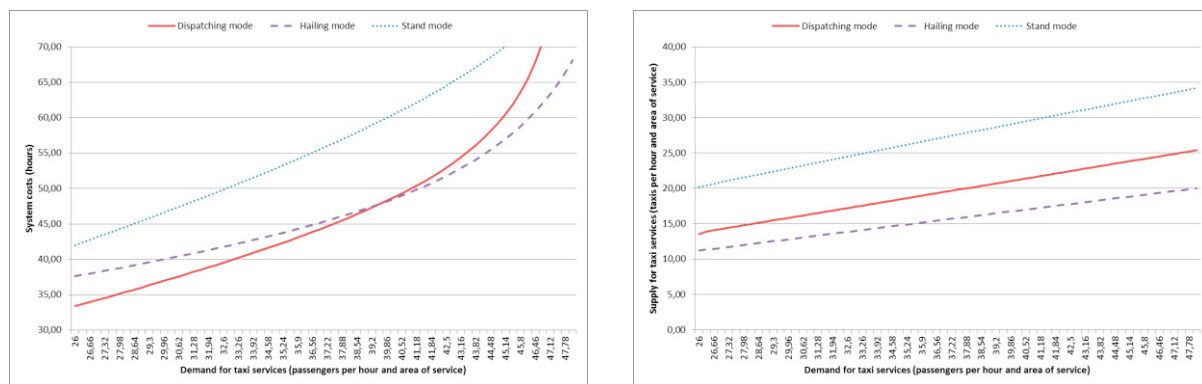


Figure 3 System costs (left) and optimum fleet size (right) for various demand levels and operation modes

For fixed supply ( $\lambda_d = ct$ ) and area ( $A = ct$ ) levels, the stand mode has the minimum unitary cost of operation for low demand levels, the dispatching mode has the lowest unitary cost for low to medium demand levels while the hailing mode has the lowest unitary cost for high demand levels (Figure 3 left). In the case of fixed area, the dispatching optimal fleet is the smallest one for low demand regions, while the hailing optimal fleet is the smallest one for high demand regions. The stand optimal fleet is the largest, independently of the demand level, as observed from Figure 3 right.

### 5. The case of Barcelona

The model has been applied to the city of Barcelona, using real data from taximeters recorded during the period 2009 and 2012 for obtaining the optimum fleet size for the three operation modes. Details about the evolution of the taxi market in Barcelona and the database can be found in Salanova 2013 and Salanova and Estrada al 2017.

#### 5.1. Application of the model

Figure 4 presents the results of the application of the hailing mode in Barcelona.

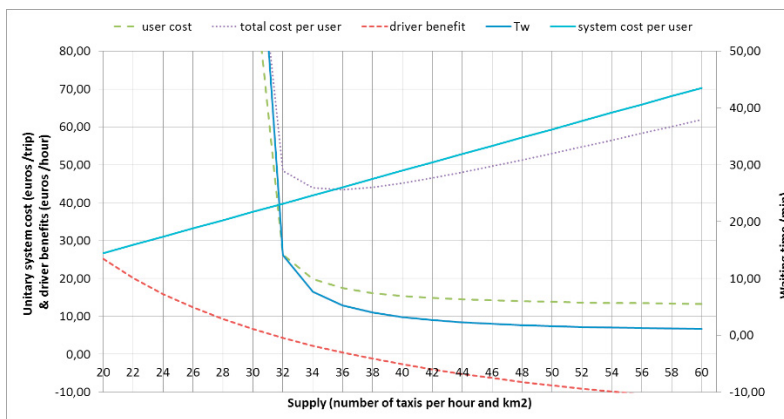


Figure 4 Waiting time, driver benefits and unitary costs for each fleet size obtained by the aggregated model (hailing operation mode)

It shows that the number of taxis per hour and km<sup>2</sup> should be higher than 31 since the waiting time for smaller fleets is very high due to the low number of free taxis. The strict minimum fleet size is 29 taxis/hour\*km<sup>2</sup>, which means that with this taxi fleet size, the number of demanded customer hours are equal to the offered vehicle hours. It can also be observed that the maximum number of taxis with positive benefits is 36 taxis/hour\*km<sup>2</sup> since more taxis than this value will generate losses to the drivers. Therefore, 36 is the second best solution. The optimum number of taxis taking into account the system costs is 34 –38 taxis/hour\*km<sup>2</sup> (first best solution). In the hailing market the first and the second best solutions are equal, which is due to the presence of externalities such as congestion and pollution, which increase linearly with the number of taxis. These externalities reduce the optimum number of taxis of the first best solution to levels where the taxi drivers have benefits. Similar results can be obtained from Figure 5 for the dispatching and stand models, but without the presence of externalities, the first best solution is higher than the second best solution in both cases. First best solution can be profitable for taxi drivers if each taxi trip is subsidized by the state by an amount of 3-4 euros per trip for the dispatching market, while in the stand case this value is sensibly larger (10 euros).

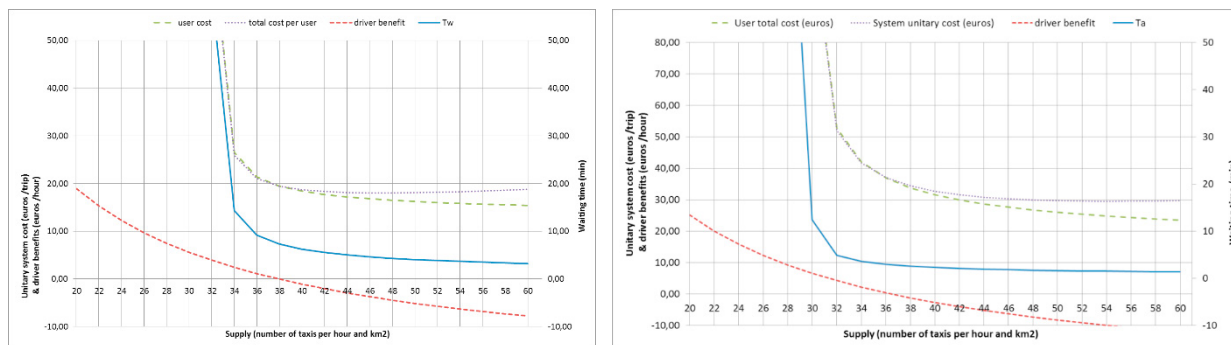


Figure 5 Waiting time, driver benefits and unitary costs for each fleet size obtained by the aggregated model (dispatching and stand modes)

In the stand model, due to the access cost, the stand network is of almost 5.000 taxi stands, distributed along the whole region with a spacing of 150 meters more or less, which significantly increases the number of taxis needed as well as the operating costs. Table 1 summarizes the results of the hailing, dispatching and stand model applications.

Table 1 Results of the application of the aggregated hailing, dispatching and stand models in the city of Barcelona

	Hailing	Dispatching	Stand
Minimum number of taxis	29 taxis/hour*km <sup>2</sup>	29 taxis/hour*km <sup>2</sup>	29 taxis/hour*km <sup>2</sup>
First best solution	34 – 38 taxis/hour*km <sup>2</sup>	44 – 46 taxis/hour*km <sup>2</sup>	52 – 54 taxis/hour*km <sup>2</sup>
Subsidy	-	3 – 4 euros per trip	10 euros per trip
Second best solution	36 taxis/hour*km <sup>2</sup>	38 taxis/hour*km <sup>2</sup>	37 taxis/hour*km <sup>2</sup>

## 5.2. Comparison of the three operation modes

The operation mode with the minimum optimum number of taxis is the hailing mode, due to the impact of the externalities and congestion created by the circulating taxis, while the operation mode with the maximum optimum number of taxis is the stand mode. The maximum number of taxis with non-negative profits (second best) is very similar for the three operation modes. Finally, the subsidization of the stand mode is the highest one, since the first best and the second best present the largest difference. There is no need for subsidization in the hailing market, since the inclusion of the externalities has reduced the optimum number of taxis below the second best value. This subsidization is the amount of money that taxi drivers lose if the number of taxis is equal to one of the social optimum. It can also be seen as the amount of money that should be provided to drivers per trip in order to provide the waiting times to customers of the first best solution and avoid losses to taxi drivers.

## 6. Conclusions

A formulation for the taxi market has been studied and presented, concluding in minimum and optimum number of taxis for different city sizes, demand levels and operation modes. While the minimum taxi fleet ensures that all trips will be served, the optimum fleet ensures a satisfactory LoS to travelers, while providing benefits to taxi drivers. The optimum fleet size is directly proportional to the customer value of time and inversely proportional to the speed and hourly operation costs of taxi drivers. The formulation has been applied to the three taxi markets, comparing the obtained results for obtaining guidelines on the implementation of taxi services to different cities (size and demand level). The presented results demonstrate that small cities or cities with low demand (less than 45-90 trips per hour and square kilometer depending on geometric and operational parameters) for taxi services must have dispatching taxi market, rather than hailing market. At the same time, if the taxi fleet is large, it is recommended to have a hailing market rather than a dispatching one. For highly congested cities or areas (commercial speed smaller to 6-7 km/h), the stand market is recommended rather than the dispatching market.

Regarding the impact of the main hypothesis of the model (homogeneous demand), the same waiting time can be related to different fleet sizes for the same demand, depending on the spatial distribution of the demand, so the spatial distribution of the demand for taxi services is therefore a significant factor when defining the optimum taxi fleet size. The authors developed also simulation models, which are able to better capture the spatial dimension of the taxi market. From the comparison it can be stated that the aggregated model underestimates the waiting time of the uniform demand distribution while overestimates it for non-uniform demand distributions (Salanova 2013).

The proposed model has been applied in the city of Barcelona using real data from both the network and the taxi trips. The model has concluded that the system optimum number of taxis in Barcelona is 34 – 40 vehicles per hour and km<sup>2</sup>, while the drivers optimum is slightly lower (30 – 36 vehicles per hour and km<sup>2</sup>), always depending on the operation mode. The model has also provided the subsidization values needed for obtaining the first best solution and having the lowest unitary cost without causing losses to taxi drivers, obtaining values between 3 and 4 euros for the dispatching market that should be afforded by the central administration. The latter is unfeasible which means that the market will converge to the second best solution. These results can be used by decision makers when planning policies for the taxi sector. The demand used is considered homogenous and inelastic, which is a strong limitation for testing

taxi policies. The authors developed a bi-level model (Salanova and Estrada 2017) where the optimum supply is obtained in the upper level while the demand is calculated in the lower level. The effects of non-homogeneous demand can be modeled in time by taking into account demand fluctuations through its standard deviation over time and in space by adding more taxis in order to account for decentralization using its standard deviation over space.

Further research direction lay in the field of optimally identifying market composition (stand, hailing, dispatching) for a given context based on the model results, which may need the development of mixed simulation models since the hypotheses of the aggregated models may not apply to heterogeneous fleets. In addition, the impact of the introduction of new transport on demand and shared mobility schemes (such as uber) may be modeled and analyzed. Finally, the stand model may be improved by including waiting time as a consequence of an unbalance on the taxi distribution or by a reduction of the number of stops and therefore queues larger than one vehicles in the ones having the most demand (in the case of heterogeneous demand).

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