Quadrotor Multi-model for Control Purposes

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Abstract. In this work, a multi-model of a quadrotor is developed in order to control this system. The kinematic model of each part of the quadrotor will be derived using the Euler angles, and also the dynamics model of the quadrotor will be calculated based on the first principles of a rigid body using the Newton-Euler formulation. Furthermore, the following assumptions are used :1) The structure is completely rigid and perfectly symmetric. 2) The center of mass is in the origin of the quadrotor fixed frame. 3) The thrusts are proportional to the square of the motors rotational speed. A state-space model (kinematics and dynamics) is developed by physical laws. But, this deduced model presents several no linearities that are produced by three factors: the orientation (Pitch, Roll and Yaw), the control action and the angular velocities. To be able to control the quadrotor system in simple, linear and manageable way, it is necessary to linearize the system. Two method are possible: a classical linearization around several set-points and a multi-model linearization. In this case, a multi-model linearization is proposed due to the obtained control model will be used to compute a multi-model controller using fuzzy techniques. Fuzzy control techniques are suitable for linear parameter varying systems with no linearities, as our quadrotor.

1. Introduction

Recent advances in sensor, in microcomputer technology, in control and in aerodynamics theory have made small Unmanned Aerial Vehicles (sUAV) a reality. The small size, low cost and maneuverability of these systems have made them potential solutions in a large class of applications. However, the small size of these vehicles poses significant challenges. The small sensors used on these Systems are much noisier than their larger counterparts. The compact structure of these vehicles also makes them more vulnerable to environmental effects. In this work, control strategies for a sUAV platform are developed. Simulation studies and experimental results are provided [1].

Usually, the control techniques used for this type of vehicles are linear methodologies (predictive control, robust control, etc. [2]). Therefore, linear control models are used to design these linear controllers. All the system information is not considered by the use of linear models [4]. This is because the quadrotors are non-linear systems with varying parameters. Currently there are control techniques that take into account these previous characteristics: Fuzzy Control and LPV (Linear Parameter Varying) Control [3]. These techniques require the formulation of a multi-model for control purposes. In this work, the authors develop a multi-model of a quadrotor with the objective to design a fuzzy control in future works. All the parameters have been extracted from a real quadrotor that authors have designed and developed in the Vision and Intelligent System research group at the Technical University of Catalonia.

2. Description of the System

A very simple and realistic way to model the quadrotor based solely in the explanation of their movements is a simple solid rigid which have four rotors with asymmetric rotation of adjacent propellers. This means that one pair of the propellers rotates clockwise, while the other pair rotates counterclockwise, as it is shown in Figure 1, but all the propellers generate an airflow for lifting the quadrotor. Moreover, this configuration helps to remove side propeller needed in standard helicopter.

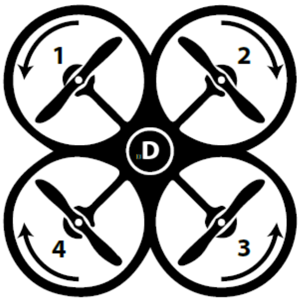

Figure 1. Simplified schematic of a quadrotor

Another interesting property of a quadrotor is that it is an underactuated system. This means that the number of degrees of freedom of the system (in total 6: 3 of position and 3 of rotation) are more than the number of actuators (4 rotors). Therefore, it is not possible achieve the desired state for all degrees of freedom. Despite this, the particular geometric configuration of a quadrotor helps to decouple control variables and selecting a simple controller. The basic movements are related to the way in which the spacecraft rotates, that is, Euler angles. Figure 2 shows the assignment of each Euler angle to each axis of quadrotor: pitch, roll and yaw.

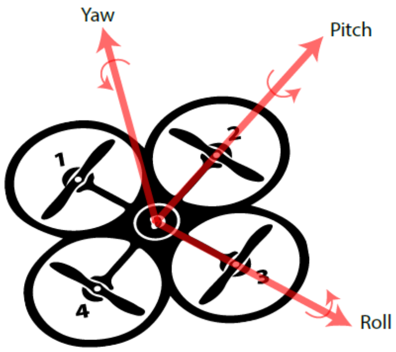

Figure 2. Euler angles for quadrotor

The design is based in a quadcopter that carries a gimbal with two cameras and an airsoft marker. So, the size of quadrotor is given by the rotors with the propellers that give enough thrust to move the quadrotor with the charge. This kind of quadrotor needs that the whole weight will be in the range 30-40% of the maximum thrust, because it is designed for carrying charge, not for speed races. In this case, it was calculated that the quadcopter was going to measure 1m x1m. The most of the structure is made of aluminium which is light but resistent at the same time. Only the blades of the propellers are made in carbon fibre which is less harmful than metal.

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3. Mathematical model

In this section the mathematical model of the quadrotor is explained. The kinematic model of each part of the quadrotor will be derived using the Euler angles, and moreover the dynamics model of the quadrotor will be calculated based on the first principle of a rigid body using the Newton-Euler formulation. Furthermore, the following assumptions are used:

- The structure is completely rigid and perfectly symmetric
- The center of mass is in the origin of the quadrotor fixed frame.
- The thrusts are proportional to the square of the motors rotational speed.

The State space of the quadrotor is formulated as:

$$X=[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T$$

which is mapped to the degrees of freedom of the quadrotor as the following:

$$X = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}]$$

where: $r = [x,y,z]^T$ is the quadrotor position, and (φ,θ,ψ) are the Euler angles (roll, pitch and yaw).

The control inputs are defined as the following: $U = [U_1, U_2, U_3, U_4,]$ where

$$U_1 = \omega_1^2, U_2 = \omega_2^2, U_3 = \omega_3^2, U_4 = \omega_4^2$$

The variable ω_{MAX} is the maximum velocity achieved by the motors and it is used to normalize the inputs, so control inputs U_i are ranged between 0 and ω^2_{MAX}.

Finally, the state space representation is the following:

$$\dot{x_1} = x_7$$
$$\dot{x_2} = x_8$$
$$\dot{x_3} = x_9$$
$$\dot{x_4} = x_{10}$$
$$\dot{x_5} = x_{11}$$
$$\dot{x_6} = x_{12}$$
$$\dot{x_7} = K_F(U_1 + U_2 + U_3 + U_4)(sin(x_4)sin(x_6) + cos(x_4)sin(x_5)cos(x_6))/m_q$$
$$\dot{x_8} = K_F(U_1 + U_2 + U_3 + U_4)(sin(x_4)cos(x_6) - cos(x_4)sin(x_5)sin(x_6))/m_q$$
$$\dot{x_9} = -g + K_F(U_1 + U_2 + U_3 + U_4)(cos(x_4)cos(x_5))/m_q$$
$$\dot{x_{10}} = \frac{x_{11}x_{12}(I_{yy} - I_{zz}) - x_{11}\omega_r(J_r) + lK_F(-U_2 + U_4)}{I_{xx}}$$
$$\dot{x_{11}} = \frac{x_{10}x_{12}(I_{zz} - I_{xx}) + x_{10}\omega_r(J_r) + lK_F(U_1 - U_3)}{I_{yy}}$$
$$\dot{x_{12}} = \frac{x_{10}x_{11}(I_{xx} - I_{yy}) + lK_M(U_1 - U_2 + U_3 - U_4)}{I_{zz}}$$

Parameters and constants	Meaning	Values (experimentally obtained)
g	gravitational acceleration	9.8 m/s^2
I_{xx}	inertia with respect the pitch axis of quadrotor frame	287,885.411 Kg mm^2
I_{yy}	Inertia with respect the roll axis of quadrotor frame	500,747.953 Kg mm^2
I_{zz}	inertia with respect the yaw axis of quadrotor frame	292,895.098 Kg mm^2
J_r	Inertia of the rotors	645 Kg mm^2
K_F	constant of the force (determined experimentally for each motor)	6.19x10^{-7} N/rpm^2
K_M	constant of the moment (determined experimentally for each motor)	10^{-8} N/rpm^2
m_q	quadrotor mass	4.23 Kg
l	distance between rotor axes	332 mm

Table 1. Quadrotor parameters

4. Multi-model Linearization

This linearization step aims to transform the nonlinear system into a linear system wich depends on some measurable scheduling variables. Therefore, the system can be formulated as:

$$\dot{x} = A(\rho(t))x + B(\rho(t))u$$

where $\rho(t)$ are the variable parameters.

The variable parameters $\rho(t)$ are function of the scheduling variable $\rho(t)$, which is recommended to be external to the system. The task of finding such functions for building a multi-model which makes a global model (with all the information of the system) is a complex task. The typical method is to hide the nonlinearities in parameters, known as the non-linear embedding. As a consequence, the nonlinear system is equivalent to a multi-model system which depends on the variation of the parameters. The variation of the parameters $\rho(t)$ is delimited using a bounding box that sets the maximum and the minimum of a function.

Finally, the formulation in the space state of the equations of the quadrotor is obtained:

$$A(\rho(t)) = \begin{pmatrix} 0_{6x6} & Id_{6x6} \\ 0_{6x6} & A_1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{9,10} & a_{9,11} & a_{9,12} \\ 0 & 0 & 0 & 0 & a_{10,11} & 0 \\ 0 & 0 & 0 & a_{11,10} & 0 & 0 \\ 0 & 0 & 0 & a_{12,10} & 0 & 0 \end{pmatrix}$$

$$a_{9,10} = {}^{-g}/_{3(\max(|x_{10}|,\varepsilon)sign(x_{10}))}$$
$$a_{9,11} = {}^{-g}/_{3(\max(|x_{11}|,\varepsilon)sign(x_{11}))}$$
$$a_{9,12} = {}^{-g}/_{3(\max(|x_{12}|,\varepsilon)sign(x_{12}))}$$
$$a_{10,11} = {}^{1}/_{I_{xx}}\left[x_{12}\left(I_{yy} - I_{zz}\right)\right]$$
$$a_{11,10} = {}^{1}/_{I_{yy}}\left[x_{12}\left(I_{zz} - I_{xx}\right)\right]$$
$$a_{12,10} = {}^{1}/_{I_{zz}x_{11}}\left[\left(I_{xx} - I_{yy}\right)\right]$$

$$B\big(\rho(t)\big) = \begin{pmatrix} 0_{6x4} \\ B_1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} b_{7,1} & b_{7,2} & b_{7,3} & b_{7,4} \\ b_{8,1} & b_{8,2} & b_{8,3} & b_{8,4} \\ b_{9,1} & b_{9,2} & b_{9,3} & b_{9,4} \\ b_{10,1} & b_{10,2} & b_{10,3} & b_{10,4} \\ b_{11,1} & 0 & b_{11,3} & 0 \\ b_{12,1} & b_{12,2} & b_{12,3} & b_{12,4} \end{pmatrix}$$

$$b_{7,1} = b_{7,2} = b_{7,3} = b_{7,4} = {}^{K_F}/_{m_q}\left[\sin x_4 \sin x_6 + \cos x_4 \sin x_5 \cos x_6\right]$$
$$b_{8,1} = b_{8,2} = b_{8,3} = b_{8,4} = {}^{K_F}/_{m_q}\left[\sin x_4 \cos x_6 - \cos x_4 \sin x_5 \sin x_6\right]$$
$$b_{9,1} = b_{9,2} = b_{9,3} = b_{9,4} = {}^{K_F}/_{m_q}\left[\cos x_4 \cos x_5\right]$$
$$b_{10,1} = \frac{-J_r x_{11}}{I_{xx}\sqrt{u_1}}$$
$$b_{10,2} = \frac{J_r x_{11}}{I_{xx}\sqrt{u_2}} - {}^{lK_F}/_{I_{xx}}$$
$$b_{10,3} = \frac{-J_r x_{11}}{I_{xx}\sqrt{u_3}}$$
$$b_{10,4} = \frac{J_r x_{11}}{I_{xx}\sqrt{u_4}} + {}^{lK_F}/_{I_{xx}}$$
$$b_{11,1} = {}^{lK_F}/_{I_{yy}}$$
$$b_{11,3} = {}^{-lK_F}/_{I_{yy}}$$
$$b_{12,1} = b_{12,3} = {}^{lK_M}/_{I_{zz}}$$
$$b_{12,2} = b_{12,4} = {}^{-lK_M}/_{I_{zz}}$$

5. Conclusions

In this work a quadrotor is designed and built (mechanical part, hardware, software, etc.). A complet mathematical model is formulated. But, for the design of the controller a more simple model is needed. To preserve of the information of the quadrotor (non-linearities) a multi-model that is variable according to some parameters is developed. This obtained control model will be very useful for fuzzy control techniques.

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