

PREFACE OF
LLAVEFEST: A BROAD PERSPECTIVE ON FINITE AND
INFINITE DIMENSIONAL DYNAMICAL SYSTEMS

This special issue of DCDS is dedicated to Rafael de la Llave on the occasion of his 60-th birthday, and collects original articles by participants in the conference *LlaveFest: A Broad Perspective on Finite and Infinite Dimensional Dynamical Systems*, held at University of Barcelona, during June 12–16, 2017. This event was attended by 149 scientists from US, Spain, Netherlands, Italy, Portugal, Sweden, Switzerland, France, Denmark, Greece, Finland, Poland, UK, Russia, Israel, China, Mexico, Brazil, and Australia. Among the participants there were many graduate students and young researchers. There were featured 28 main talks, 20 contributed talks, and 28 posters. Support was generously provided by the NSF grant DMS-1700154, by the Alfred P. Sloan Foundation grant G-2016-7320, by the Centre de Recerca Matemàtica (CRM), and by the Institute of Mathematics of the University of Barcelona (IMUB).

The conference served as a forum for scientists throughout the world to present recent advances on various aspects of Dynamical Systems and Partial Differential Equations. The focus was on promoting interactions between these fields and others, from analysis and geometry to numerical computations and simulations. These interactions have proven to be fruitful and enlightening, and stimulated new research directions.

Rafael de la Llave was born in Madrid in 1957. He studied Physics at Universidad Complutense of Madrid, and earned his PhD in Mathematics at Princeton University in 1983. He held positions at Princeton University, University of Minnesota, IHES, the University of Texas at Austin, and he is currently at the Georgia Institute of Technology.

Rafael de la Llave's research interests expand over a wide range of problems related to dynamics, where he has been searching, by means of analytical and computational techniques, for *landmarks that organize the dynamics*: periodic and quasiperiodic orbits, invariant manifolds, homoclinic tangles. He has made fundamental advancements in several areas of dynamics, such as Arnold diffusion, KAM theory, hyperbolic dynamics, rigidity, perturbation theory, and variational methods. He has also been working on problems motivated by physics applications, such as convergence to equilibrium, stability of matter, semiclassical limits, magnetohydrodynamics, fluid dynamics, optics.

His research has been brilliantly intertwined with training of young scientists. Besides teaching a great variety of courses related to Dynamical Systems, Differential Equations and Numerical Analysis, he has been a fervent organizer of seminars, conferences and research programs, as well as a co-founder of Mathematical Physics Preprint Archive (MP_ARC) and Mathematical Physics Electronic Journal (MPEJ). Over the years, he has been devoting a lot of effort and enthusiasm to supervision of students; up to now he supervised 19 doctoral theses (some more in progress), and his current math genealogy lists 25 descendants.

The contents of this special issue of DCDS reflect the broad range of interests and ideas that Rafael de la Llave has promoted through his work. Below we summarize the main contributions of the included articles.

In the paper by B. Abdellaoui, A. Attar, A. Dieb and I. Peral, necessary and sufficient conditions to obtain the attainability of a fractional Hardy inequality are studied. Moreover, the mixed Dirichlet-Neumann boundary problem associated to the minimization problem, which turns out to be a semilinear elliptic problem for the fractional Laplacian, is also studied.

In the paper by P.G. Barrientos and A. Raibekas, the authors are able to construct symplectomorphisms in dimension $d \geq 4$ having a semi-local robustly transitive partially hyperbolic set containing C^2 -robust homoclinic tangencies of any codimension c with $0 < c \leq d/2$.

L. Caffarelli, L. Duque and H. Vivas study the two membranes problem for two different fully non-linear operators, providing a viscosity formulation for the problem and proving existence of solutions. They also prove a general regularity result and the optimal $C^{1,1}$ regularity when the operators are the Pucci extremal operators.

The paper by Delshams and Schaefer is devoted to the Arnold diffusion problem. It provides an explicit description of a geometric mechanism for global diffusion in the case of a system consisting of a pendulum and a rotor subject to a perturbation depending on two independent harmonics.

The paper by Dias Carneiro and Ruggiero obtains some generalizations of the celebrated Birkhoff graph theorem. They give sufficient conditions for an invariant Lagrangian manifold of the geodesic flow on the torus to be a graph.

S. Dipierro, A. Karakhanyan and E. Valdinoci provide an integral estimate for the second order elliptic equation in non-divergence (non-variational) form $a_{ij}u_{ij} = u^p$, $u \geq 0$, $p \in [0, 1)$, with bounded discontinuous coefficients a_{ij} having small BMO norm. As an application, they show that the free boundary corresponding to the obstacle problem (i.e. when $p = 0$) cannot be smooth at the points of discontinuity of the coefficients.

In the paper by J.-P. Eckmann and G. Wayne, metastable motions in weakly damped Hamiltonian systems are studied, which are believed to inhibit the transport of energy through Hamiltonian, or nearly Hamiltonian, systems with many degrees of freedom. The authors investigate this question in a very simple model where the breather solutions that are thought to be responsible for the metastable states can be computed perturbatively to an arbitrary order. Using a modulation hypothesis, they derive estimates for the rate at which the system drifts along this manifold of periodic orbits and verify the optimality of our estimates numerically.

The paper by Falcolini and Tedeschini-Lalli is concerned with the 2-parameter Henon family of maps in the weakly dissipative regime, that is, for parameters close to the conservative limit. In this context, they analyze families of coexisting attractive periodic orbits with diverging periods.

In the paper by M. Gidea and Y. Shmalo a combinatorial approach, based on the method of ‘correctly aligned windows’, is presented to rigorously show the existence of fixed points, periodic orbits, and symbolic dynamics in discrete-time dynamical systems, as well as to find numerical approximations of such objects.

The paper by E. Kostousova deals with a problem of target control synthesis for dynamical bilinear discrete-time systems under uncertainties (which describe disturbances, perturbations or unmodelled dynamics) and state constraints, using polyhedral (parallelotope-valued) solvability tubes.

The paper by J. Mierczynski, S. Novo and R. Obaya deals with the study of principal Lyapunov exponents, principal Floquet subspaces, and exponential separation for positive random linear dynamical systems in ordered Banach spaces. The main contribution lies in the introduction of a new type of exponential separation, called of type II, important for its application to nonautonomous random differential equations with delay.

In the paper by V. Millot, Y. Sire and H. Yu, the authors address the regularity issue for minimizing fractional harmonic maps of order $s \in (0, 1/2)$ from an interval into a smooth manifold. Hölder continuity away from a locally finite set is established for a general target, whereas Hölder continuity holds everywhere when the target is the standard sphere.

The paper by C. Simó raises more than 50 questions about open problems in 21 areas of Dynamical Systems related to KAM theory, Diophantine approximations, Gevrey expansions and chaos, Celestial Mechanics, Integrability and non-integrability, effective stability, . . . , with a plentiful bibliography, so it can be a great source of reference for future research.

The paper by X. Su and Y. Wei is concerned with a class of nonlinear elliptic PDE driven by the fractional Laplacian with asymptotically linear term. The authors obtained results on the existence and multiplicity of non-trivial solutions under some weak hypotheses, as well as some local and global existence of solutions to a class of non-local elliptic equations with combined nonlinearities. In the appendix, an L^∞ regularity result is also given based on the De Giorgi-Stampacchia iteration method.

In the paper by Y. Sun and X. Yuan, the existence of small-amplitude quasi-periodic solutions for the fifth-order KdV equation under quasi-linear Hamiltonian perturbation on the torus in presence of a quasi-periodic forcing is obtained.

In the paper by A. Zvyagin, by combining the approximating topological method with the theory of attractors of trajectory spaces, the global and pullback attractors for an incompressible non-Newtonian fluid, which models a weak aqueous polymer solutions motion, in 2D and 3D bounded domains is studied.

In the paper by V. Zvyagin and V. Orlov, the existence of weak solutions for a fractional Voigt type model of viscoelastic fluid, which takes into account memory along the motion trajectories, is established. The proof relies on the theory of regular Lagrangian flows, approximating the problem by a sequence of regularized Navier-Stokes systems.

Guest Editors:

Xavier Cabré, Amadeu Delshams, Marian Gidea and Chongchun Zeng