Notes on the measure of the structural similarity of OD matrices

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Origin to Destination (OD) matrices describing the mobility patterns across a road network are the main input to most traffic models, namely the dynamic traffic models implementing Dynamic Traffic Assignment (DTA) or Dynamic User Equilibrium (DUE) approaches. The estimation of OD matrices still represents a research challenge since OD matrices are not yet observable, or at least not fully observable, since the advances in Information and Communication Technologies (ICT) traffic measurements, as those available from Smartphone or GPS devices, are starting to provide partial estimates that must be fused with data from OD sources. These fusion processes are the domain of research on how better adjust or correct an initial estimate of an existing OD matrix, X^H, usually called the "historical or target " OD matrix, from a set of traffic measurements, usually traffic volumes $\{\hat{Y}_l\}$, measured in a subset of links of the road network, Ros-Roca et al. (2018).

The bi-level optimization approach is usually considered the most efficient formulation adjustment or correction of OD matrices combining available sources of data, because it takes explicitly into account the congestion effects that influence the use of paths between OD pairs. The most common formulation considers that the available sources of data are:

- An historical OD matrix X^H usually available from a household survey or a former demand model.
- Indirect traffic volumes measured, link flow counts, $\{\hat{Y}_l\}$ measured by detection stations, link count posts, at a subset of links $l \in \hat{L} \subseteq L$ in the network, where L is the set of links.

The problem is then formulated as the optimization problem:

$$\begin{aligned} Min\ Z(X,Y) = w_1 F_1(X, X^H) + w_2 F_2[Y, \tilde{Y}] \\ Y = Assignmt(X) \\ X \geq 0 \end{aligned} \qquad (1)$$

where F_1 and F_2 are distance functions between estimated and observed values, w_1 and w_2 are weighting factors reflecting the uncertainty of the information contained in X^H and $\tilde{Y}$ respectively. The underlying hypothesis is that $Y(X) = \{y_l\}$ are the link flows predicted by an assignment of the demand matrix X onto the network, which can be expressed by a proportion of the OD demand flows x_{nr} passing through the count location at link l, in terms of an assignment matrix $A(X)$, function of the demand X, that is:

$$Y = A(X)X \qquad (2)$$

Where $A(X) = \{a_{nr,l}(X)\}$, is the assignment matrix whose entries $a_{nr,l}$ are the fractions of the OD demand x_{nr} of (n,r) OD-pair that passes link l.

Usually the quality of the results is assessed in terms of the correlation between the measured link flows $\{\hat{Y}_l\}$ and the estimated link flows Y, provided by the matrix X resulting from the adjustment. In most cases the correlation coefficient is big enough to think that the result is significantly good. However, a more detailed insight reveals that this could not be the case, since the adjustment process (1) behaves as a meta-regression model, balancing the flows among the implied OD pairs by the structure of (2) induced by the detection layout. Table 1, presents the results of a set of experiments (with different initializations), according to which the adjusted matrices would be acceptable.

These experiments have been done in a synthetic exercise with a real network where the real flow counts have been generated by simulation, using a Ground Truth matrix. Therefore, using the Ground Truth OD matrix, 4 different OD matrices have been built in order to address the behavior of the dynamic extension of Spiess (1990) adjustment procedure, Barceló et al. (2018), in different realistic situations:

- Incremental +: Incrementing all the OD values by a fixed percentage: $Inc^+ = X^{GT}(1+\delta),\ \delta = 0.25.$
- Incremental -: Decrementing all the OD values by a fixed percentage: $Inc^+ = X^{GT}(1-\delta),\ \delta = 0.25.$
- Chaos+Inc +: Equidistributing all the OD values by rows and incrementing all by the same fixed percentage.
- Chaos+Inc -: Equidistributing all the OD values by rows and decrementing all by the same fixed percentage.

These experiments contemplate different situations, from similar-structure matrices with different number of trips to non-similar-structure matrices. The objective of the experiments is to analyze the solution of the OD estimation procedures.

Table 1: $\boldsymbol{R^2}$ results of OD matrix estimation using Dynamic Spiess method

		Chaos+Inc -	Chaos+Inc +	Incremental -	Incremental +
Dynamic Spiess without X^H	R_0^2	0.22001	0.34554	0.86299	0.93068
	R_f^2	0.98922	0.99070	0.98977	0.99002
Dynamic Spiess with X^H	R_0^2	0.22001	0. 34554	0.86299	0.93068
	R_f^2	0.98667	0.98759	0.98952	0.99008

where R_0^2 and R_f^2 are the fittings between the real counts and the simulated counts before and after the OD estimation process.

But, to make sure of the quality an additional question must be answered: is the structure of the adjusted matrix X similar to the structure of the original one X^H, or the process has unacceptably distorted the structure, just to adjust the values of the volumes? Ones should not forget the underlying physical nature of the problem, a transportation problem in which the OD trip pattern corresponds to a socioeconomic process whose nature cannot be substantially altered by a numerical procedure. A key aspect in the analysis of the quality of the results provided by the adjustment process is thus to measure how structurally similars are the original matrix X^H and the adjusted matrix X.

Classical distances between vectors can be applied to matrices by considering both matrices $\boldsymbol{M}, \boldsymbol{N} \in \mathcal{M}_n(\mathbb{R})$ as vectors of $\boldsymbol{M}, \boldsymbol{N} \in \mathbb{R}^{n \times n}$. Then, Euclidean, Manhattan and other vector distances can be used in the second component of the OD estimation problem, formulated as a minimization problem. However, these distances do not capture differences and similarities in many aspects such as the structure of the OD matrix and then, the spatio-temporal similarities of the OD matrices are not captured by these measures, Djukic (2014). It seems clear that alternatives to these vector measures must be taken into account for the comparison between OD matrices.

A new measurement of similarity is suggested in Chapter 6 of Tamara Djukic PhD Thesis. This measure is borrowed from Image quality assessment to compare two different images. Wang et al. (2004) presents SSIM - Structural SIMilarity- for a matrix of pixels as a product of three different comparison components: luminance, contrast and structure. Luminance corresponds to the intensity of illumination, which is, indeed, the mean of the different pixels in a sub-matrix. Contrast is the squared average between pixels once the luminance is removed, so that is the standard deviation, and finally, the structure comparison is done by using the covariance between the two matrices. These three factors are firstly transformed with the aim to adjust them to the interval $[0,1]$, where 1 means perfect match and 0 means totally different. SSIM therefore is a similarity measure independent of the magnitude of the values in the matrix. The formula summarizing this explanation is below:

$$SSIM(\boldsymbol{x}, \boldsymbol{y}) = l(\boldsymbol{x}, \boldsymbol{y})^{\alpha} c(\boldsymbol{x}, \boldsymbol{y})^{\beta} s(\boldsymbol{x}, \boldsymbol{y})^{\gamma}$$

where luminance, contrast and structure are defined as:

$$\begin{cases} l(\boldsymbol{x}, \boldsymbol{y}) = \dfrac{2\mu_x\mu_y + C_1}{\mu_x^2 + u_y^2 + C_1} \\ c(\boldsymbol{x}, \boldsymbol{y}) = \dfrac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2} \\ s(\boldsymbol{x}, \boldsymbol{y}) = \dfrac{\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3} \end{cases}$$

and $\mu_x, \sigma_x, \mu_y, \sigma_y, \sigma_{xy}$ are the mean, the standard deviation and the covariance of the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$. C_1, C_2, C_3 are stability constants to avoid numerical problems, typically set to $C_1 = C_2 = 2 \cdot C_3 = 1$ and α, β, γ are weighting coefficients, typically set to 1 [Wang et al. (2004)]. In image comparison, the MSSIM is computed as the mean of the SSIM of all the sub-matrices of dimension N, because pixels proximity is crucial in image patterns recognition.

In the case of OD matrices, MSSIM is very useful as suggested in Djukic (2014) but we understand it would be more explicative by averaging by rows or columns, instead of by sub-matrices, that is using rectangular sliding rules corresponding either to rows or columns of the OD matrix, as shown in the Figure 1. One row in an OD matrix is the distribution of trips departing from a single origin zone and, analogously, one column is the distribution of trips arriving to a single destination zone. Therefore, SSIM will capture the similarity between these distributions described, considering the mean, the variance and the structure of departing and arriving distributions, which correspond to a structural property of the trip patterns described by the OD.

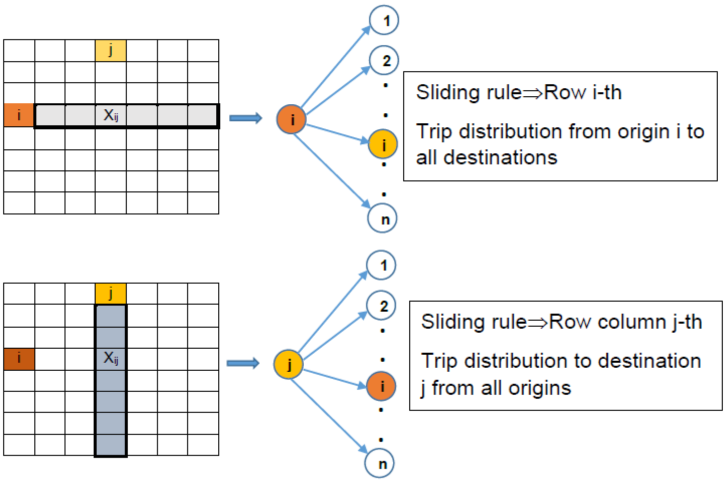

Figure 1: OD matrix distributions by rows and by columns

Table 2: MSSIM results of OD matrix estimation using Dynamic Spiess method

		Chaos+Inc -	Chaos+Inc +	Incremental -	Incremental +
Dynamic Spiess without X^H	$MSSIM_0$	0.35883	0.48398	0.94096	0.96418
	$MSSIM_f$	0.64158	0.65896	0.95322	0.96673
Dynamic Spiess with X^H	$MSSIM_0$	0.35883	0.48398	0.94096	0.96418
	$MSSIM_f$	0.54365	0.58238	0.96323	0.97692

Table 2 presents the MSSIM estimates for the same experiments in Table 1, Barceló et al. (2018). Where $MSSIM_0$ and $MSSIM_f$ stand for the MSSIM measure before and after the process of the OD matrix with respect to the Ground Truth OD matrix. The discrepancies shown indicate that further research is necessary to fully accept the adjustment results for their application to transportation analysis. A critical question to be further investigated is whether the distance function $F_1(X, X^H)$, in the formulation of the adjustment problem (1), is properly representing the difference between the historical and the adjusted OD or not. Some experimental evidences indicate that this distance could not be the most appropriate, the research question to address is, which is then the appropriate $F_1(X, X^H)$ term for the objective function in (1).

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