The Music of Sant Fructuós' Measures

Francisco Javier González Pérez[1(✉)] and Antonio Millán-Gómez[2(✉)]

[1] Universidad Politécnica de Cataluña, Escuela Técnica Superior de Arquitectura de Barcelona (ETSAB), Barcelona, Spain
fco.javier.gonzalez@upc.edu
[2] Departamento de Representación Arquitectónica, Universidad Politécnica de Cataluña, Escuela Técnica Superior de Arquitectura del Vallès, Barcelona, Spain
antonio.millan@upc.edu

Abstract. A reflection on the possible acquaintance of two texts, the treatises De Institutio Arithmetica and De Musica compiled by Boethius, that might have been tools of ideation used by the architect in the process of project for Sant Fructuós de Tàrraco's northern basilica is presented. A system of dimensions in accordance with arithmetic elaborations compiled by Boethius is proposed for the graphic survey of the basilica's excavation. A new reading of the archaeological floor rehearses a dimensional scheme for other architectures that sprung from texts such as the De Institutio Arithmetica and De Musica and its subsequent applications. The arithmetic and music compiled by Boethius was one of the rules used to shape Sant Fructuós at Tarraco's northern basilica. As noted by González Pérez (2016), some dimensionsadsdas of the floor of the basilica and the atrium of Sant Fructuós generate chords, though in different octaves.

Keywords: Northern basilica of sant fructuós · Ravenna · Roman foot
Symmetria · Treatises De Institutio Arithmetica and De Musica of Boethius

1 Introduction

Some cumbersome information seems necessary to inform the reader. From the year 474 AD up to 526 AD the Ostrogoth King Theodoric ruled, among others, on the Italian and Iberian peninsula, establishing the capital in Ravenna—notwithstanding the legitimate sovereignty of his grandson Amalaric in Hispania Visigoth kingdom. It was probably at this time—beginnings of 6th century—when Anicius Manlius Severinus Boethius compiled both treatises, De Institutio Arithmetica and De Musica, as relevant figure within the cultural and administrative policy of the Ostrogoth king. And, although we have no testimonies of the texts' presence in the Tàrraco of 6th century, it becomes evident that some arithmetical and musical compendium existed already, since Boethius produces a translation and a re-elaboration of the other authors' work, among them quite possible that of Nichomachus, new-Pythagorean philosopher and mathematician from the beginning of the 2nd century.

It will be precisely around the knowledge of arithmetic and music in the 5th century, that might have existed an ideation tool used by the architect for the project of Sant Fructuós in Tàrraco's northerm basilica. López (2004, p. 267) dated its

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construction between the end of the 4th century and the beginning of the 5th century—research published later by López (2006), completing earlier research.

Thus, this paper aims a first approach to a compositional research on the basilica, by proposing a proportional system of dimensions that uses some of the arithmetic elaborations compiled by Boethius superimposed on the building plan.

The methodology revises the archaeological graphic survey, tracing dimensional schemes for other buildings elaborated from the knowledge compiled by texts such as the De Institutio Arithmetica and De Musica, and its subsequent application as inception rule for the floor of the basilica.

2 Discussion

Only part of the basilica's floor survives and exclusively at the foundation level, located inside the Parc Central shopping center, two and a half meters below the original level. The structure was disassembled and moved, and although a complete documentation of sector V was carried out before the disassembly during the IV archaeological campaign—between 26th October 1995 and 1st February 1996—carrying out a new graphic survey at scale 1/20 (López 2006, p. 27), at the moment it has not been possible to consult it. Here, we will adopt those measures verified in situ during the course of the visit made by the authors on 25th March 2017, and verified also on the archaeological graphic survey published by López (2006, vol I Plate 108). And, as it is referred, it was redrafted by Òscar Curulla and Josep M. Puche from the graphic survey carried out by the first one, Moisés Díaz and Ricardo Mar (López 2006, p. 16).

The basilica, oriented in the east-west direction, and preceded by an atrium, is resolved in a floor with three naves with a transept that does not protrude, an apse and a counter-apse. The atrium is formed by a central courtyard with a perimeter ambulatory, that both in the north and the south has a series of rooms. It should be pointed out that the ambulatory is interrupted by the counter-apse.

López (2006, p. 30) identified a Roman foot of 0.296 m as the unit of measure used in the composition of the floor—and that corresponds with an approximation to a foot of 0.2957 m—the most prestigious and extended Roman foot, and that we adopt. The text raised the discussion on the advisability of taking interior measures, to axis of structural or exterior elements, that is to say, considering the thickness of the walls. And he also referred that, to establish measures and proportions in Rome, one had to split interior dimensions, whilst in Ravenna one had to consider exterior measures, according to De Angelis d'Ossat (1970, p. 391). Thus, he reformulated the floor, both of the basilica and the atrium, within a grid of 10 by 10 ft, 1 by 1 perches. Noting that the proposed location in the grid for some walls is not entirely perfect with respect to the construction of the southern aisle's width, attributing this deviation to an error in the fabric execution (López 2006, pp. 126–134, vol. I Plate 140). This deviation is quite evident in the axis of the apse, which, despite not being strictly in an east-west direction, the approach is much closer than the final longitudinal axis that ends up defining the general layout. It cannot be established owing to the axis correction; however, as the construction progressed, the axis must have been modified further

towards the north-east/south-west direction, but respecting the width defined in the easternmost area of the basilica's transept.

Before De Angelis d'Ossat—and in another context—Conant (1963, Plate III) measured the relation of dimensions of 25 ft in the plan of Cluny III, measuring, among others, the distance between outer faces of subsequent columns. Giner (2012, pp. 110–111) identified in Sant Pere de Rodes arithmetic proportions or musical chords: the width of the naves is half their length, 1/2—double proportion, chord of octave—; the length of the transept and the total of the building is of 2/3—*sesqualtera* proportion, chord of fifth—; and that the one between the maximum height of the vault and the length of the nave to the transept is 3/4—*sesquitercia* proportion, chord of fourth. What Boethius enunciated as the maximum and perfect harmony that extends in the three dimensions and that he exemplified through numbers 6, 8, 9 and 12 (Fig. 1) is thus achieved. Consequently, it is possible to establish simultaneously an arithmetical, geometric and harmonic proportion (Boèce 1995 [500], p. 174). Chords verified by Giner (2014, pp. 317–318) and again for the church of Sant Quirze de Pedret: the interior dimension of the width of the north aisle, and also the diameter of the north and south apsidioles (a); the interior dimension of the apse measured on the longitudinal axis, and also the average width (b); the interior width of the nave (c); and the interior length of the nave (d):

$$b - a = c - b \tag{1}$$

$$\frac{a}{b} = \frac{c}{d} \tag{2}$$

$$\frac{c - d}{b} = \frac{b - c}{d} \tag{3}$$

The 8/9 tone relation appeared inside the nave in Sant Pere de Rodes, and in the longitudinal direction, between the dimension of the supports and the distance between them in its longitudinal direction. Also in the cross section, between the difference of the height of opposite columns, and in a position of honor, the major semitone, 2187/2048 (Giner 2012, pp. 131–132).

The floor of the northern basilica of Sant Fructuós in Tàrraco presents a singular constructive solution in the apse: a double foundation—internal and external—where two different construction techniques were used, to which López (2006, p. 111) refers that the external foundation presents fits, that support on the interior foundations, trying to link them. Double foundation may also be identified in the old church of Sant Pere d'Ègara from the Episcopal period—middle of the 5th century to the beginning of the 6th century—where, parallel to the south aisle, another foundation can be observed, braced to the first through buttresses (Garcia et al. 2009, pp. 149–152).

López (2006, p. 121) stated that it is difficult to find an easy solution to the convenience of two different foundations. And that the final floor of the basilica is so well structured that it seems to have been planned this way from the beginning. Idea that will be the base on which we work, i.e., that of a unitary project.

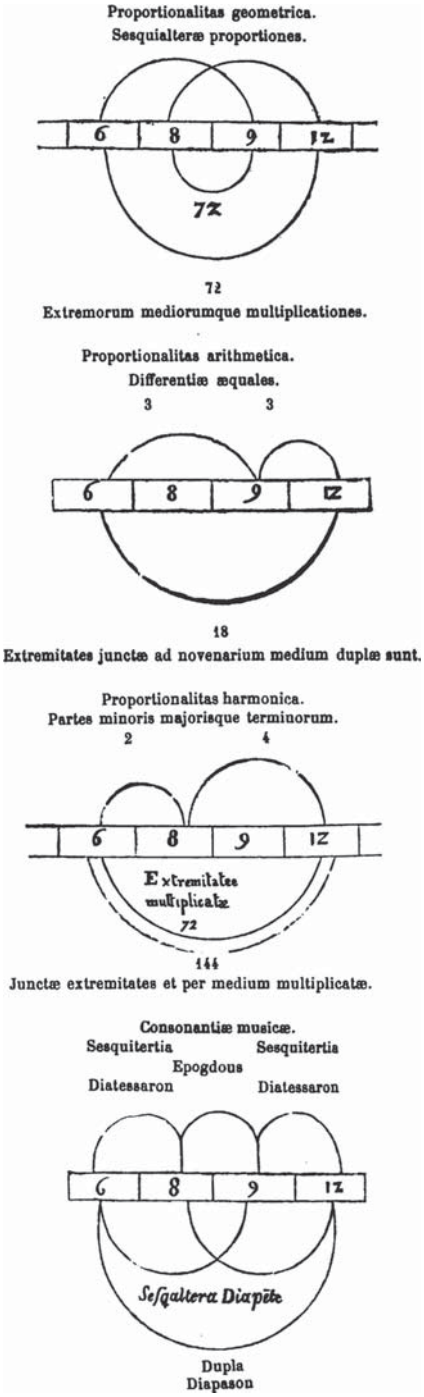

Fig. 1. Maximum and perfect harmony that extends in three dimensions (Migne 1882, pp. 1164–1168)

From this assumption, the plan of the basilica and the atrium has been redrawn from measurements verified in situ, adopting as a base the graphic survey published by López (2006, Vol. 1 Plate 140), taking partial and total distances with all the accuracy possible. And always considering exterior measures, compared with Roman feet of 0.2957 m, and rounding to the unit, half foot, a quarter or three quarters of foot. After rounding each measure, they were superimposed on the drawing to evaluate possible deviations, verifying that they were sufficiently adapted to the graphic survey plan base.

The specific width of the aisle north of the basilica, from the most external face of the foundation of the colonnade to the outer face of the foundation of the north wall, is 3.77 m, i.e., 12.75 ft of 0.2957 m; the total width of the transept—in the midday zone—is 5.03 m, 17 ft; the width of the central nave (considering the width of both colonnades) is 7.54 m, 25.50 ft, as well as from the outermost face of the foundation of the interior apse, to the outer face of the western foundation that defines the transept; the partial transversal width of the building considering the width of the central nave and northern aisle is 11.31 m, 38.25 ft, measure that corresponds exactly to that verified considering the width of the central and southern aisle in the easternmost part of the transept.

The total length of the transept, and therefore the width of the building—at the easternmost point—is 15.08 m, 51 ft, as well as from the western side of the foundation of the transept to the outer face of the foundation of the west wall that defines the nave; the total length of the basilica from the outermost face of the foundation of the interior apse, to the outer face of the foundation of the west wall of the nave is 22.62 m, 76.50 ft; and the total length of the basilica now from the outermost face of the foundation of the exterior apse, to the outer face of the foundation of the west wall of the nave is around 23.83 m, 80.59 ft (Fig. 2).

Amongst these measures (12.75, 17, 25.50, 38.25, 51, 68, 76.50, and 80.59 ft) there are ratios of:

1/2 (between the width of the north aisle, 12.75 ft, and the length measured from the outermost face of the foundation of the interior apse, to the outer face of western foundation that defines the transept, and also the width of the central nave, 25.50 ft); equally so between the width of the central nave, 25.50 ft, and the length measured from the westernmost face of the foundation of the transept to the outer face of the foundation of the west wall that defines the nave, and the total width of the building, 51 ft;

2/3 between the length measured from the outermost face of the foundation of the interior apse, to the outer face of the western foundation that defines the transept, 25.50 ft, and the width of the central nave and one aisle, 38.25 ft; 2/3 between the total width of the building, 51 ft, and the total length of the basilica from the outermost face of the foundation of the interior apse, to the outer face of the foundation of the west wall of the nave, 76.50 ft;

3/4 between the width of the north aisle, 12.75 ft, and the width of the transept, 17 ft; and again 3/4 between the width of the central nave and one side, 38.25 ft, and the length measured from the westernmost side of the foundation of the transept to the outer face of the foundation of the west wall that defines the nave 51 ft.

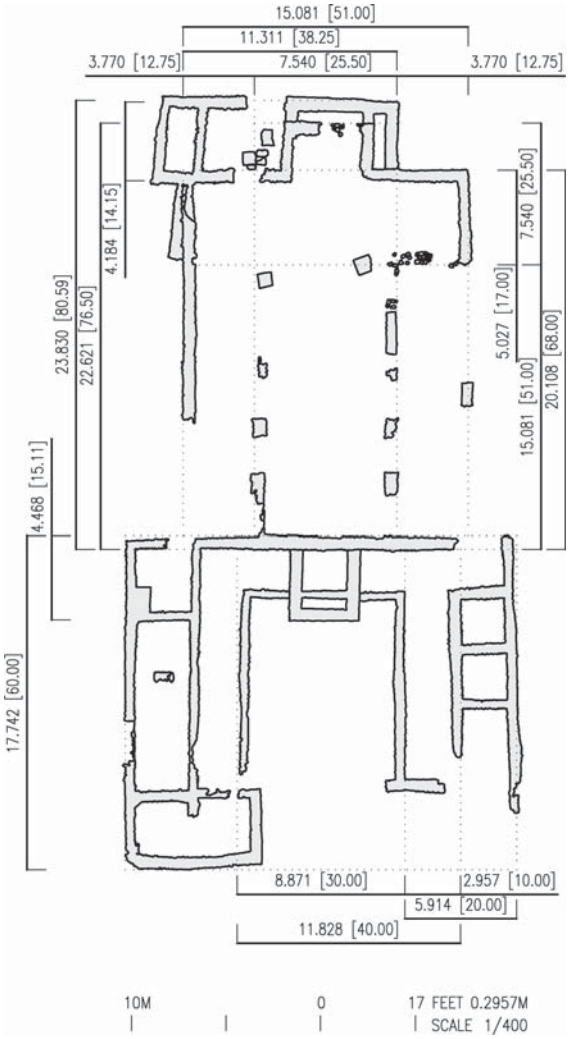

Fig. 2. Floor of the northern basilica of Sant Fructuós in Tàrraco from measurements verified in situ by González Pérez on 25th March 2017, and from the planimetry published by López (2006, Vol I Plate 140), (measures between square brackets in feet of 0.2957 m)

The total length of the basilica measured from the outermost face of the foundation of the exterior apse, to the outermost face of the foundation of the west wall of the nave, 80.59 ft, is in a minor semitone proportion, 256/243, in relation to the total length of the basilica but now from the outermost face of the foundation of the interior apse, 76.50 ft. The total length of counter-apse, verified between the outer face—east—of the foundation that defines the separation between the nave and the atrium, and the exterior face of the foundation that defines the atrium inside the courtyard, 4.47 m, 15.11 ft, and the measure found on the southern side of the apse, here, from the outer face of the

foundation of the exterior apse, to the interior face of the foundation that defines the transept, 4.18 m, 14.15 ft, and in one position of honor, is in a major semitone, 2187/2048.

Thus, we can state that the supreme and perfect chord is found here in the three dimensions, found here between 12.75, 25.50, 38.25 and 76.50 ft.

Supreme and perfect chords can also be found in the atrium, between: 2.96 m, that is, 10 ft of 0.2957 m, which corresponds to the depth of the south cloister of the atrium, from the outer face of the foundation of the colonnade to the outer face of the foundation that separates the cloister from the lateral room, 10 ft is also the measure of the southern side of the apse, from the outer face of the foundation of the interior apse to the interior face of the foundation that defines the transept; 5.91 m, 20 ft, is the depth of the south cloister of the atrium, from the outer face of the foundation of the colonnade to the outer face of the foundation that closes the lateral room; 8.87 m, 30 ft, is the width of the courtyard of the atrium considering the width of the foundation of the gallery of columns; 17.74 m, 60 ft, is the total length of the atrium verified between the outer face—east—of the foundation that defines the separation between the nave and the atrium, and the outer face of the foundation of the northern room that closes in the western area the atrium (Fig. 2).

Fernie (1985, p. 383) pointed to an old Roman perche equivalent to 17 ft, traditionally attributed to Isidoro of Sevilla, and equivalent to 5.03 m—17 ft of 0.2957 m—17 ft of 0.289 m is also the internal distance between supports of the central nave and the basic module that defines the church of Sant Pere de Rodes, and serves to fix the total length, that is, the distance between the inner face of the pilaster of the façade wall —on the south side—and the one facing the other end—in the head—is exactly 5 times 17 ft (Giner 2012, p. 124).

Bofill (1993, p. 45) in the chapter on the division of the monochord, assured that "The ignorance of the fraction prevents Boethius, especially in the more complicated divisions, from granting the unit value to the greater length and dividing it in parts […] He needs to take a smaller length as a unit, a smaller length, which fits a certain number of times in the largest. And the same ignorance of the fraction forces him to make this unit fit an exact number of times in all those cases that it is convenient to divide a certain distance by a certain number, that is to say, he needs to find the greatest common divisor of all the divisions of a system of two scales of scope."

Accordingly, this type of fractions can be identified in the northern basilica of Sant Fructuós, that of A/*n* on a unit, also identified by Giner (2012, pp. 125–129) for the church of Sant Pere de Rodes—establishing a notation that relates these previous values, always using as unit 1 perche of 17 ft (Table 1).

Such distances have been ordered, starting with the basilica, from the least to the greatest, 12.75, 17, 25.50, 38.25, 51, 68, 76.50, and 80.59 ft. And without introducing any measure of the part of the atrium, the series can be completed based on the hypothesis that the scale is in the major third, that is, 3.77 m, 12.75 ft of 0.2957 m, is in Mi and not in Do (Fig. 3). In this way if we go back 2 tones until the Do we would obtain a measure of 2.98 m, very close to the verified one of 2.96 m, 10 ft, in the cloister.

From the theoretical Do of 2.98 m, a scale of 3 octaves has been built in which dimensions of both the atrium and the basilica can be found, existing a perfect

Table 1. Fractions of type A/*n* over a unit of 17 ft

	9/12 x 17: Width of the north aisle.
	9/9 x 17: South transept width.
(2048/2187) x (8/9) x	9/9 x 17: Total lenght of the apse.
(8/9) x	9/9 x 17: Length of the counter-apse.
	9/6 x 17: Width of the central nave.
	9/4 x 17: Width of the central nave and aisle.
	9/3 x 17: Total width of the basilica.
	9/2 x 17: Basilica length (interior apse).
(8/9) x	9/2 x 17: Lenght of the nave and transept.
(256/243) x	9/2 x 17: Total lenght of the basilica.
(64/81) x	9/12 x 17: Width of the cloister.
(64/81) x	9/6 x 17: Width of the cloister and south room.
(64/81) x	9/4 x 17: Width of the courtyard of the atrium.
(64/81) x	9/3 x 17: Wide courtyard and cloister.
(64/81) x	9/2 x 17: Total lenght of the atrium.

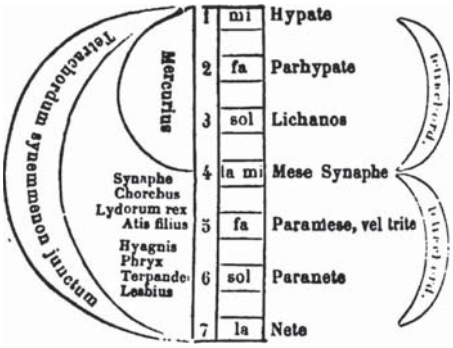

Fig. 3. Nicomachus refers that music was simple at first and was constituted by four rib, a fifth string was added later by Torebo. Hiagnis, added to these a sixth rib. And a seventh nerve was attached by Terpandro, like, of the seven planets: *Hypate* (Mi), *parhypate* (Fa), *lichanos* (Sol), *mese* (La), *paramese o trite* (Si), *paranete* (Do), *nete* (Re). (Migne 1882, p. 1184)

correspondence with those verified for the basilica—12.75, 15.11, 17, 25.50, 38.25, 51, 68, 76.50, and 80.59 ft (Table 2).

However, we note here that there is a tolerance in the system (ranging from 22 to 131 mm). That is, the difference in the floor between the width of the cloister after rounding to 10 ft the distance tested, 2.957 m, and the theoretical distance obtained from backing 2 tones up to the Do from the distance of 3.77, 2.979 m, is 22 mm. Tolerance that increases in the verified distance between the total length of the atrium between the outer face—east—of the foundation defining the separation between the nave and the atrium, and the outer face of the foundation of the northern room that closes the atrium in the west area, 17.742 m, and the theoretical distance obtained for our scale of 17.873 m, is 131 mm.

Table 2. Scale of 3 octaves (in the upper part, measured in meters, and in the lower one in feet of 0.2957 m). In bold the dimensions of both the atrium and the basilica

Do	Re	Mi	Fa	Sol	La	Si	Do	Re	Mi	Fa	Sol	La	Si	Do	Re	Mi	Fa	Sol	La	Si	Do
2.98	3.35	**3.77**	3.97	**4.47**	**5.03**	5.66	**5.96**	6.70	**7.54**	7.94	**8.94**	10.05	**11.31**	**11.92**	13.41	**15.08**	15.89	**17.87**	**20.11**	**22.62**	**23.83**
10.07	11.33	**12.75**	13.43	**15.11**	**17.00**	19.13	**20.15**	22.67	**25.50**	26.86	**30.22**	34.00	**38.25**	**40.30**	45.33	**51.00**	53.73	**60.45**	**68.00**	**76.50**	**80.59**

3 Conclusions

Some rules concerning the form of the northern basilica of Sant Fructuós in Tarraco can be isolated by checking the measuring unit, a foot of 0.2957 m, the most prestigious and extended of Roman feet; following a dimensional sequence that handles multiples and submultiples of 1 perche of 17 ft, expressed in a notation of A/*n* type over the unit of 17 ft. With such a process, amongst measures found in the basilica and the atrium, some of the most beautiful proportions of the arithmetic theory and the musical chords of Boethius can be found, including the supreme and perfect harmony that extends into the three dimensions; and all of them can be identified on a scale of 3 octaves.

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Francisco Javier González Pérez is an Architect (2010), Technical Architect (2002), Master in Theory and History of Architecture (2012), by the Polytechnic University of Catalonia, and Professor of the Department of Architectural Representation (since 2017).

Currently he is a pre-doctoral researcher in the programme of Patrimonio arquitectónico, civil, urbanístico y rehabilitación de construcciones existentes of the ETSAB. His research is taking place in the area of the monumental complex of Sant Pere de Terrassa and is directed by Ph.D. Antonio Millán-Gómez and co-directed by Ph.D. Josep Giner i Olcina.

Since 2016 he has participated in different congresses: Rehabend, Euro-American Congress, Burgos, 2016; International Symposium, Obra Congrua, 1416. 600 Aniversario de la consulta de la Catedral de Girona, Girona 2016; Territories and frontiers of representation. 39° International Meeting of Teachers of Representations Disciplines. XIV Congress of the Italian Union for Drawing. Naples 2017; and in Puig i Cadafalch. Arquitecte de Catalunya, Barcelona 2017. fco.javer.gonzalez@upc.edu

Antonio Millán-Gómez is a Ph.D. architect by the Polytechnic University of Catalonia, and Full Professor of Architectural Representation.

He has been Senior Lecturer at the ILA&UD Laboratory, International Laboratory of Architecture and Urban Design (1985, 1986 and 1992).

Awarded in 1991 the First Prize of the AEEA/EAAE, European Association of Architectural Education.

Currently coordinates at UPC_Barcelona TECH the Ph.D. program of the research group AR&M/RA_UPC on Patrimonio arquitectónico, civil, urbanístico y rehabilitación de construcciones existentes. antonio.millan@upc.edu