The hydraulic influence matrix of opening gate trajectories in canals based on the method of characteristics: concept, compilation and practical examples

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The hydraulic influence matrix

1.1 Introduction

A sluicegate trajectory is the sequence of positions that the sluicegate follows in a temporal horizon. In the same way, when a channel is regulated by more than one sluicegate, this is known as a set of sluicegate trajectories.

When determined sluicegate trajectories are applied to a channel, the flow that results (and which can be predicted using a simulation model) demonstrates a unique behaviour, which is a function exclusive to the set of trajectories applied. This univocal relation between the control action and the response of the flow will apply as long as there is no external disturbance. In such a case, different disturbances will give different results with the same set of trajectories.

The behaviour of the flow that results from the application of determined trajectories of sluice position can be expressed as a sequence of depths, velocities and/or discharges obtained at certain sections of the channel at determined instants.

However, if these trajectories are modified slightly, the response of the system (given by the simulation model) will be different to that obtained with the unmodified trajectories. And if the disturbance is smaller, the resemblance of the results will be greater.

Using the hydraulic influence matrix is a simple way to find any change in behaviour when a modification to the trajectory of sluice positions is introduced. If all the modifications to the trajectory are placed in a vector $\triangle U$, and then multiplying the hydraulic influence matrix by this vector, a new vector is obtained which includes all the disturbances the flow will suffer expressed in the form of another vector $\triangle X$:

$$\triangle X = \left[I_M\left(U\right)\right]_X \triangle U$$

where $[I_M(U)]_X$ is the hydraulic influence matrix. To obtain this matrix is by no means trivial and in this chapter I will try to go through the deduction of its terms. The hydraulic influence matrix is so called because its components represent the influence of a position of sluicegate on the hydrodynamic varible at all points of the channel and in all instants of time.

In this chapter, the physical significance that the hydraulic influence matrix represents is also explained. As the deduction of this is based on the equations of the simulation model, this chapter has been used to describe, also, the numerical simulation model employed, which is also the model of the characteristics.

On the other hand, the so-called sluicegate trajectory is defined mathematically starting from a sequence of sluicegate positions or sluicegate parameters.

Starting from the equations of Saint-Venant in their complete form (which describes free laminar water flow in channels) and starting, also, from the equations we will call point of control, a set of discrete equations has been established which allows the behaviour of flow to be calculated starting from the trajectories of the tested sluicegates. Taking this system of equations as a base and using an analytical process of direct derivation, the next step was to establish the calculation of the influence of a sluicegate position on flow at a determined instant. Later, the way of calculating the evolution the influences have when they travel along the channel and through time was fixed. Finally, everything which affects the objective of the chapter: the "hydraulic influence matrix" was compiled in a specific way.

1.2 The hydraulic influence on a channel section of a sluicegate movement at a given moment

1.2.1 Free laminar flow equations

The equations of Barré de Saint-Venant (1871) describe the free laminar flow of water in prismatic channels and are the result of the application of the principles of mass conservation and of the quantity of movement in a controlled volume of short length which is transversal to the direction of flow along the whole section of the channel. A fairly rigourous deduction of these equations and for prismatic channels can be found in Walker and Skogerboe (1987). The resulting set is made up of two partially derived first-order differential equations and is as follows:

$$\left.\begin{aligned} \frac{\partial y}{\partial t} + v\frac{\partial y}{\partial x} + \frac{A(y)}{T(y)}\frac{\partial v}{\partial x} = 0 \\ \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x} = g\left[S_0 - S_f\left(y, v\right)\right] \end{aligned}\right\} \quad (1.1)$$

where x and t are the independent variables space and time, y is the level of the free surface regarding the depth of the channel, v is the average velocity of all particles of a transversal section of the flow, $A\left(y\right)$ is the function area of wet section which depends on the depth, $T\left(y\right)$ is maximum width also dependent on the depth, S_0 is the gentle gradient of the channel bed, and finally, $S_f\left(y, v\right)$ is the friction gradient.

The set of equations (1.1) will be applicable to reality if one can suppose that:

- - the curvature of the laminar is small; with this:
- - the vertical accelerations are disregarded, and
- - the pressure distribution along an axis vertical to the liquid is the same as in hydrostatic conditions,
- - the slope is supposed sufficiently small that its sin is...
- - the energy dissipation term is be specified through Manning's equation which is used in stationary regime, that is to say,

  $$S_f\left(y, v\right) = n^2\frac{v|v|}{R_H^{\frac{4}{3}}} \qquad \text{with} \quad R_H = \frac{A(y)}{P(y)} \quad (1.2)$$

  where n is Manning's coeffiecient and $P\left(y\right)$ is the wetted perimeter which is a function of the water level, and

  the changes in flow conditions are not fast enough to generate wave fronts.

- These equations cannot be solved analytically, only numerically. Thus, a set of numeric methods exist which permit the set (1.1), to be resolved, which can be found, among others, in Gómez (1988). According to Wylie (1969) all the numerical methods of resolution, whether they are explicit, implicit or characteristics methods, present results which are similar when compared to reality, depending more on the exactness of the starting data than of the different methologies. Bearing this key fact in mind, this thesis has used the characteristics method as it helps physical comprehension of the underlying wave phenomenum in free laminar flow. The choice of a resolution method would not characterise that which is exposed in this chapter because it would be applicable in any other numerical method used.

- Usually, the axes upon which the set (1.1) are based are the classical ones of space and time (x/t) but if it is based on curves known as characteristic curves- expressed parametrically with $x^{+}(t)$ and $x^{-}(t)$— that locally fulfill the two following differential equations,

$$\left.\begin{aligned} \tfrac{dx^{+}}{dt} &= v + c(y) \\ \tfrac{dx^{-}}{dt} &= v - c(y) \end{aligned}\right\} \tag{1.3}$$

  thus, the set (1.1) is transformed into the following two fully derived equations

$$\left.\begin{aligned} \tfrac{dv}{dt} + \tfrac{g}{c(y)}\tfrac{dv}{dt} &= g\left[S_0 - S_f(y,v)\right] \\ \tfrac{dv}{dt} - \tfrac{g}{c(y)}\tfrac{dv}{dt} &= g\left[S_0 - S_f(y,v)\right] \end{aligned}\right\} \tag{1.4}$$

  where the first is valid along the length of the curve $x^{+}(t)$ and the second along $x^{-}(t)$ and where $c(y) = \sqrt{\frac{gA(y)}{T(y)}}$ is the speed of the wave.

The good thing about the transformation of the characteristic method is that the set of partially derived equations (1.1) becomes a totally derived set (1.4). The difficulty of the method lies in the fact that the ordinary differential equations (1.4) have to be solved along the characteristic curves or the local axes that are the solution of the set (1.3). As this last one is a set of non-linear equations it obliges us to solve the four equations simultaneously. Fortunately, the curves $x^{+}(t)$ and $x^{-}(t)$ always intersect, although they are not orthogonal, and therefore assure hyperbolicity.

In short, solving the set of two partially derived equations (1.1) is the same as solving the following set of four fully derived equations:

$$\left.\begin{aligned} \tfrac{dv}{dt} + \tfrac{g}{c(y)}\tfrac{dv}{dt} &= g\left[S_0 - S_f(y,v)\right] && \text{(a)} \\ \tfrac{dx^{+}}{dt} &= v + c(y) && \text{(b)} \\ \tfrac{dv}{dt} - \tfrac{g}{c(y)}\tfrac{dv}{dt} &= g\left[S_0 - S_f(y,v)\right] && \text{(c)} \\ \tfrac{dx^{-}}{dt} &= v - c(y) && \text{(d)} \end{aligned}\right\} \tag{1.5}$$

The mathematical process of transformation of set (1.1) to the equivalent (1.5) is found in many bibliographical references such as Gómez (1988), Solé

(1996), Duchateau and Zachmann (1998), and Ames (1977) so will not be discussed further here.

The set of equations (1.5) describes the conditions of flow in a channel in the same way as the set of equations (1.1) as it adds no new hypothesis in the transformation. However, set (1.5) is limited in the way it is applied. The variable x which was initially independent is now dependent on time t, as it is understood in (1.3); then, (1.5-(a)) will be true only along the curves which fulfill the EDO (1.5-(b)) and, in the same way (1.5-(c)) will be true along the solution curves of (1.5-(d)).

The set of equations (1.5) can be represented in the graph x/t as in figure 1.1 where, at the point of intersection R the four equations are verified and therefore the four unknowns x, t, y and v can be found theoretically. This way, if flow conditions at points P and Q are known, the position of point R can be found and integrated numerically, along with the flow conditions.

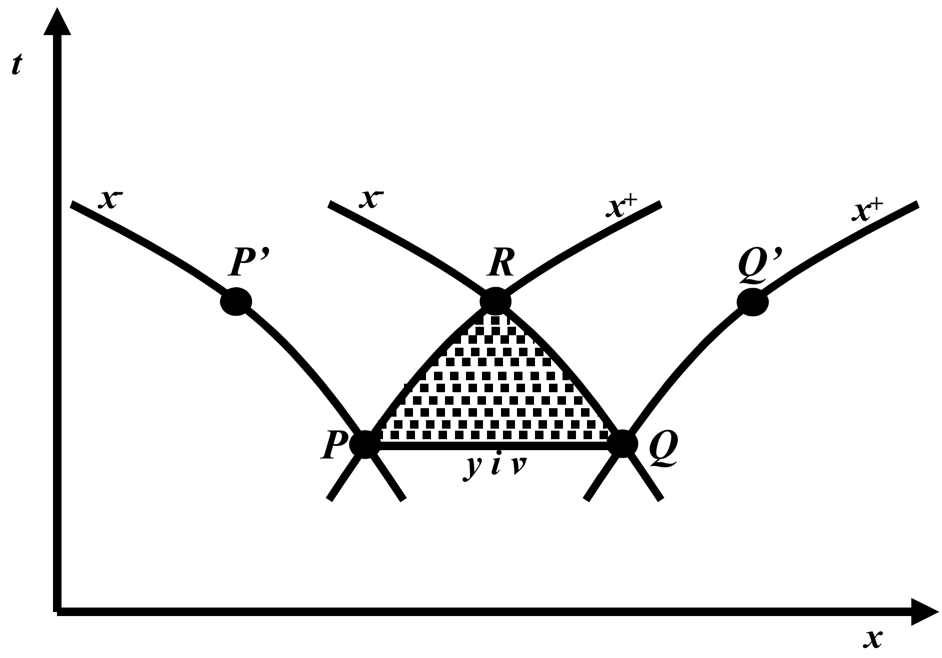

Figura 1.1: *The dependence domain of point R.*

This can be proved thanks to the first of the uniqueness theories developed by Crandall (1956) in which he shows that if on a curve on the graph x/t which is not a characteristic curve - as with the line PQ in Figure 1.1-

the conditions of flow y and v are known, then the set of equations (1.5) determines its solution with uniqueness in the zone marked PQR; and it is this zone that we call the dependence domain of point R because the solution at this point is determined exclusively by the conditions of flow produced at any point of this domain. That means that any disturbance introduced at any point of the dependence domain will affect the position of point R and the conditions of flow.

A complementary concept to the dependence domain is the influence domain. In Figure 1.2 you can see, for example the influence domain of point P, that is the set of points of the graph x/t (the area shaded with horizontal lines) which are seen to be affected by the conditions of flow present at this point. In the same way, the area shaded with vertical lines is the influence domain of point Q and obviously, the area shaded with both vertical and horizontal lines is the influence domain of point R.

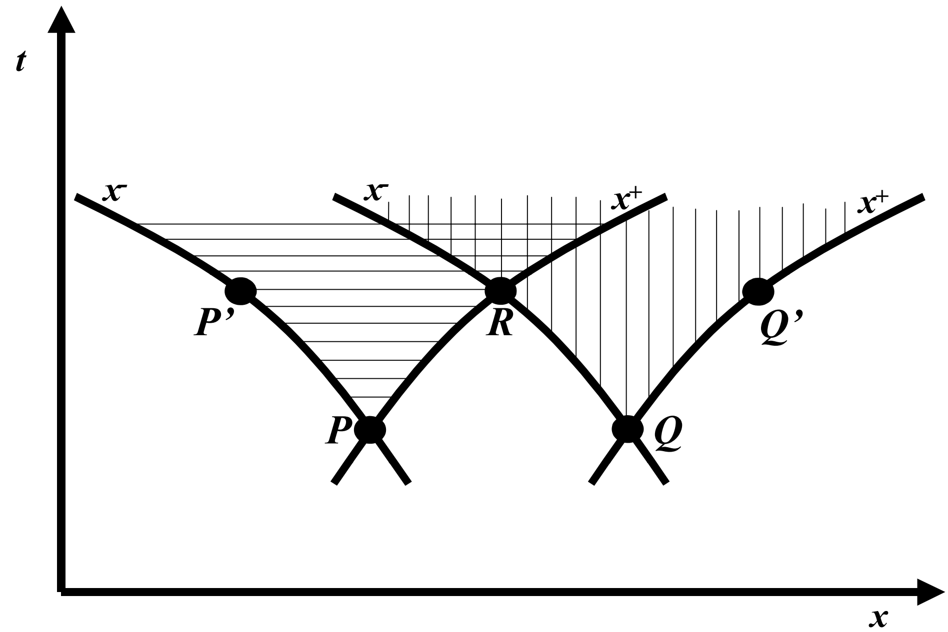

Figura 1.2: *The influence domains of points P, Q and R.*

The influence domain of point P is so-called because a variation in the conditions of flow —y_P and v_P— at the point x_P of the absciss canal at the moment t_P affects the conditions that are produced at the points represented by the domain, including the points P' and R. In the same way point Q

influences Q' and R.

Once the concepts of the influence and dependence domains have been introduced we are able to present the basic objective of the following part. When a movement is made to the sluicegate, a series of changes to the flow condition are produced. Firstly, near to the sluicegate itself and then further away. If the movement is considered a disturbance, then we can speak of the disturbance influence domain and thus the influence domain of the sluicegate movement. Therefore: " the presentation of a form for calculating and quantifying the tested variation in the conditions of point R **—that we will call $\triangle y_R$ and $\triangle v_R$— when disturbances are introduced to points P and Q —which we call $\triangle y_P$ and $\triangle v_P$ and $\triangle y_Q$ and $\triangle v_Q$, respectively- is the immediate objective of the following section."**

1.2.2 The discretization of the characteristic equations

As previously commented the set of equations (1.1) and the equivalent (1.5) have no known analytical solution, and therefore, the use of numeric techniques has, until present, been compulsary. There are many numerical methods that can be used, at least in a theoretical manner. With this, in this thesis I have preferred to use a specific plan of discretization and make the appropriate mathematical developments on the result of this discretization.

In order to have the longest possible integration time segments without loss of precision, I have opted for a **discretization in finite differences of second order**, called by Gómez 1988 as "the characteristic curves method." This is a partially implied method which considers parts of the value as parabolic. The need to use this wide ranging plan will be seen more clearly later. If the method is applied to equations (1.5) and the nomenclature presented in the diagram in 1.2 is taken into account, then:

$$\left.\begin{array}{r}
\frac{v_R-v_P}{t_R-t_P}+\left[\theta\frac{g}{c_R}+(1-\theta)\frac{g}{c_P}\right]\frac{y_R-y_P}{t_R-t_P}=gS_0-g\left[\theta S_{f_R}+(1-\theta)S_{f_P}\right]\\
\frac{x_R-x_P}{t_R-t_P}=\theta\left[v_R+c_R\right]+(1-\theta)\left[v_P+c_P\right]\\
\frac{v_R-v_Q}{t_R-t_Q}-\left[\theta\frac{g}{c_R}+(1-\theta)\frac{g}{c_Q}\right]\frac{y_R-y_Q}{t_R-t_Q}=gS_0-g\left[\theta S_{f_R}+(1-\theta)S_{f_Q}\right]\\
\frac{x_R-x_Q}{t_R-t_Q}=\theta\left[v_R-c_R\right]+(1-\theta)\left[v_Q-c_Q\right]
\end{array}\right\}\quad(1.6)$$

where $S_{f_R}=S_f(y_R,v_R)$, $S_{f_P}=S_f(y_P,v_P)$, $S_{f_Q}=S_f(y_Q,v_Q)$ and $0\leq\theta\leq1$ is the coefficient of average time that indicates the type of plan used. This is to say, when $\theta=1$ the set is implicit, and when $\theta=0$ the set is explicit, and when $\theta = \frac{1}{2}$ the plan is in **central differences or of characteristic curves**.

If the conditions of flow at points P and Q are known, as we have commented before, then from the set of four equations (1.6) x_P, t_P, y_P, v_P and x_Q, t_Q, y_Q, v_Q are known and x_R, t_R, y_R and v_R remain as unknowns which can be found by using any of the methods of solving non-linear equations, such as the Newton-Raphson method. The four equations of (1.6) can therefore be re-written

$$\left.\begin{array}{l}
f_1 \equiv x_R - x_P - \frac{1}{2}\left(t_R - t_P\right)\left[v_R + c_R + v_P + c_P\right] = 0 \\
f_2 \equiv \left(v_R - v_P\right) + \frac{g}{2}\frac{c_R + c_P}{c_R c_P}\left(y_R - y_P\right) + g\left(t_R - t_P\right)\left(\frac{S_{f_R} + S_{f_P}}{2} - S_0\right) = 0 \\
f_3 \equiv \left(v_R - v_Q\right) - \frac{g}{2}\frac{c_R + c_Q}{c_R c_Q}\left(y_R - y_Q\right) + g\left(t_R - t_Q\right)\left(\frac{S_{f_R} + S_{f_Q}}{2} - S_0\right) = 0 \\
f_4 \equiv x_R - x_Q - \frac{1}{2}\left(t_R - t_Q\right)\left[v_R - c_R + v_Q - c_Q\right] = 0
\end{array}\right\} \quad (1.7)$$

Once the set of equations has been solved, one can ask the following question: What would have been the solution if, instead of the conditions (y_P, v_P)at point P, we had had, for example, $(y_P + \triangle y_P, v_P)$, that is to say, when a change in depth is introduced? A question like this is easily answered and is closely related to the concept of *hydraulic influence*.

Before answering the question, a brief line is necessary regarding the significance of *hydraulic influence*. Therefore, I define *hydraulic influence of the conditions of depth and velocity of a point* (as for example point P in figure 1.1 or 1.2) *over the conditions of another point at another instant of time* (as for example point R in the same figures) as *the disturbance effect that is produced on the conditions of point R caused by a small modification to the conditions of point P*. It should be said that the grade of influence depends on both the conditions of point P and the conditions of point R. Therefore, the same modification of the conditions at point P will have a greater disturbance effect on R as the depth and the velocity at P become shallower and lower. For the time being, this definition can be considered valid as the concept is more general than that just exposed and I will return to this later on.

In order to answer the given question now that the definition of *hydraulic influence* is known, suppose that all the variables of the set (1.7) depend implicitly on y_P: $y_R(y_P)$, $v_R(y_P)$, $t_R(y_P)$ and $x_R(y_P)$ and also suppose that the theory of the implicit function is applied, then by solving the set 4×4:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_R} & \frac{\partial f_1}{\partial y_R} & \frac{\partial f_1}{\partial v_R} & \frac{\partial f_1}{\partial t_R} \\ \frac{\partial f_2}{\partial x_R} & \frac{\partial f_2}{\partial y_R} & \frac{\partial f_2}{\partial v_R} & \frac{\partial f_2}{\partial t_R} \\ \frac{\partial f_3}{\partial x_R} & \frac{\partial f_3}{\partial y_R} & \frac{\partial f_3}{\partial v_R} & \frac{\partial f_3}{\partial t_R} \\ \frac{\partial f_4}{\partial x_R} & \frac{\partial f_4}{\partial y_R} & \frac{\partial f_4}{\partial v_R} & \frac{\partial f_4}{\partial t_R} \end{bmatrix} \begin{pmatrix} \frac{\partial x_R}{\partial y_P} \\ \frac{\partial y_R}{\partial y_P} \\ \frac{\partial v_R}{\partial y_P} \\ \frac{\partial t_R}{\partial y_P} \end{pmatrix} = - \begin{pmatrix} \frac{\partial f_1}{\partial y_P} \\ \frac{\partial f_2}{\partial y_P} \\ 0 \\ 0 \end{pmatrix} \tag{1.8}$$

the values $\frac{\partial x_R}{\partial y_P}$, $\frac{\partial y_R}{\partial y_P}$, $\frac{\partial v_R}{\partial y_P}$ i $\frac{\partial t_R}{\partial y_P}$ can be found. Obviously the matrix of the set is assessed at x_P, t_P, y_P, v_P, x_Q, t_Q, y_Q and v_Q. Linearizing to the proximities of y_P we can write a first order Taylor approximation series.

$$y_R\left(y_P + \triangle y_P\right) = y_R\left(y_P\right) + \frac{\partial y_R}{\partial y_P}\triangle y_P + O\left(\triangle y_P^2\right) \tag{1.9}$$

This results in the desired altered values for y_R. Expressions similar to (1.9) can be found for x_R, v_R and t_R. It should be said that as the set (1.8) is the result of the application of the theory of the implicit function on (1.7) it needs to totally fulfill the condition where the matrix of (1.8) can be inverted. That is to say, that its determinant is different from zero. It can be shown that when a flow is present this condition is fullfilled and therefore the values of the implicit derivatives $\frac{\partial x_R}{\partial y_P}$, $\frac{\partial y_R}{\partial y_P}$, $\frac{\partial v_R}{\partial y_P}$ and $\frac{\partial t_R}{\partial y_P}$ always exist and are in a unique form. Sets like (1.8) can be formed by the remaining "disturbable" variables v_P, y_Q and v_Q. In general, and for future developments, I will refer to a general parameter ϕ to denote varibles of flow desciption such as depth, velocity, physical coefficients, sluicegate position etc. Then, it can be written

$$[M] \begin{pmatrix} \frac{\partial x_R}{\partial \phi} \\ \frac{\partial y_R}{\partial \phi} \\ \frac{\partial v_R}{\partial \phi} \\ \frac{\partial t_R}{\partial \phi} \end{pmatrix} = -[N] \begin{pmatrix} \frac{\partial y_P}{\partial \phi} \\ \frac{\partial v_P}{\partial \phi} \\ \frac{\partial y_Q}{\partial \phi} \\ \frac{\partial v_Q}{\partial \phi} \end{pmatrix} \tag{1.10}$$

where:

$$[M] = \begin{bmatrix} \frac{\partial f_1}{\partial x_R} & \frac{\partial f_1}{\partial y_R} & \frac{\partial f_1}{\partial v_R} & \frac{\partial f_1}{\partial t_R} \\ \frac{\partial f_2}{\partial x_R} & \frac{\partial f_2}{\partial y_R} & \frac{\partial f_2}{\partial v_R} & \frac{\partial f_2}{\partial t_R} \\ \frac{\partial f_3}{\partial x_R} & \frac{\partial f_3}{\partial y_R} & \frac{\partial f_3}{\partial v_R} & \frac{\partial f_3}{\partial t_R} \\ \frac{\partial f_4}{\partial x_R} & \frac{\partial f_4}{\partial y_R} & \frac{\partial f_4}{\partial v_R} & \frac{\partial f_4}{\partial t_R} \end{bmatrix}$$

$$[N] = \begin{bmatrix} \frac{\partial f_1}{\partial y_P} & \frac{\partial f_1}{\partial v_P} & 0 & 0 \\ \frac{\partial f_2}{\partial y_P} & \frac{\partial f_2}{\partial v_P} & 0 & 0 \\ 0 & 0 & \frac{\partial f_3}{\partial y_Q} & \frac{\partial f_3}{\partial v_Q} \\ 0 & 0 & \frac{\partial f_4}{\partial y_Q} & \frac{\partial f_4}{\partial v_Q} \end{bmatrix}$$

It is neccessary to note that the set (1.8) is equal to the last when $\phi = y_P$ and thus has a more general character. From the physical point of view the set (1.10) "moves" or "modifies" the influence of a parameter ϕ on the points P and Q at point R.

The way of calculating the influences shown in this section are closely linked to the plan of characteristic curves. Usually however, this plan is not exactly used because it gives the solution of the unknown co-ordinates for a point R beforehand. These co-ordinates are also the solution of the same set of equations and normally it is more important to know the solution of the flow conditions at specific points of the channel and at regular points of time. To solve this problem there are two possibilities: first solve and then interpolate, or first interpolate and then solve.The second option will be the one used in this dissertation and is presented in the next section.

1.2.3 Applying to a structured grid

All what has been said so far has problems for finding the solution in desired points because x_R and t_R are the solution to the system. In figure 1.3 you can see how by placing the characteristic curves net (figure 1.3 a)) on top of a structured net (figure 1.3 b)) a plan where the variables for points P and Q can be obtained (figure 1.3 c)). In this way we can obtain the flow conditions for the fixed point R. Obviously, the same set of equations (1.7) is solved, but now with the new unknowns x_P, y_R, v_R and x_Q. A structured grid like this one creates a new nomenclature. Thus every variable will have a double index, where k refers to time and i to space. Therefore, y_i^k and v_i^k represent the values for water level and average velocity at the co-ordinates $x_i = i\Delta x$ and $t^k = k\Delta t$ where Δx and Δt take predetermined values. As interpolation has to be also of second order (in order to be coherent with the numerical plan used) I have used the Lagrange factors (a way of representing quadratic splines). For a dummy variable z the result is (see representation in figure 1.4):

$$
\begin{gathered}
s^k\left(x, z_{i-1}^k, z_i^k, z_{i+1}^k\right) = \\
= \left(\frac{x-x_i}{\Delta x}\right)\left(\frac{x-x_{i-1}}{2\Delta x}\right) z_{i+1}^k + \left(\frac{x-x_{i-1}}{\Delta x}\right)\left(\frac{x-x_{i+1}}{-\Delta x}\right) z_i^k + \left(\frac{x-x_i}{-\Delta x}\right)\left(\frac{x-x_{i+1}}{-2\Delta x}\right) z_{i-1}^k
\end{gathered}
$$

In this way the variables y_P, v_P, y_Q and v_Q become functions of x_P and x_Q, as shown here

$$
\begin{aligned}
y_P\left(x_P\right) &= s\left(x_P, y_{i-1}^k, y_i^k, y_{i+1}^k\right) \\
v_P\left(x_P\right) &= s\left(x_P, v_{i-1}^k, v_i^k, v_{i+1}^k\right) \\
y_Q\left(x_Q\right) &= s\left(x_Q, y_{i-1}^k, y_i^k, y_{i+1}^k\right) \\
v_Q\left(x_Q\right) &= s\left(x_Q, v_{i-1}^k, v_i^k, v_{i+1}^k\right)
\end{aligned}
\tag{1.11}
$$

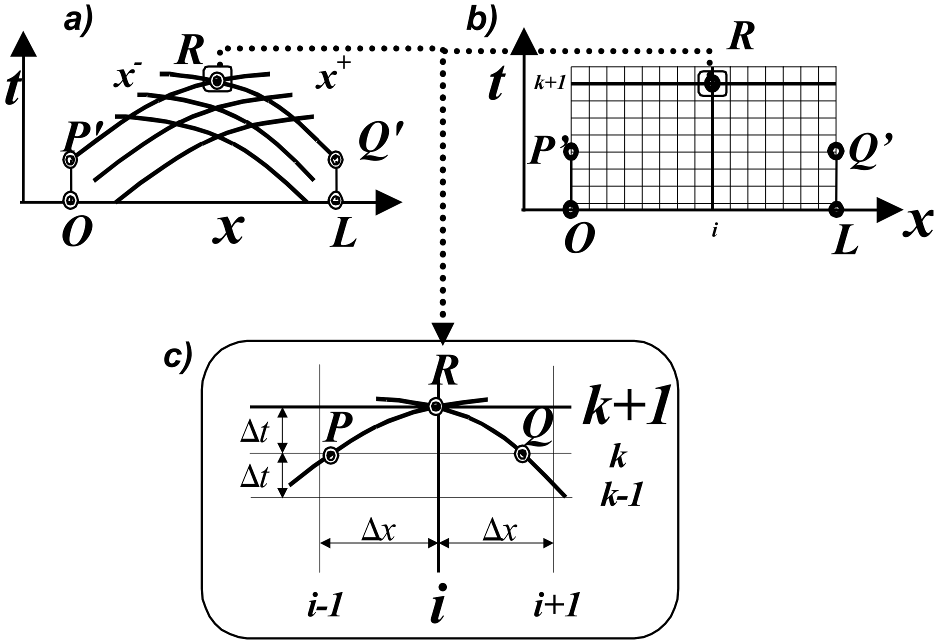

Figura 1.3: *The steps for the interpolation onto a structured grid.*

From now on it is necessary to determine the set to be solved in order to find the evolution of the influence of a parameter through a structured grid as has been done for finding (1.10) from the characteristic curves. Contrary to what might be expected, finding the "transfer" of the influences (or evolution of the influence in time or space) in an interpolation grid simplifies the problem. The reason for this is that the influence of the general parameter on the position and moment in time —$\frac{\partial x_R}{\partial \phi} = \frac{\partial x_i}{\partial \phi}, \frac{\partial t_R}{\partial \phi} = \frac{\partial t^{k+1}}{\partial \phi}$— loses all meaning, because we want to find the solution to specific x and y axes (see figure 1.5). Therefore, if a disturbance is introduced to any hydraulic variable at the moment in time k, then by solving the four equations (1.7) with x_P, y_i^{k+1}, $v_i^{k+1}$and x_Q as unknowns, two sets of characteristics are obtained —x^+, x'^+ and x^-, x'^-—, two solutions for point R —$\left(y_i^{k+1}, v_i^{k+1}\right)$ and $\left(y_i^{k+1}, v_i^{k+1}\right)'$— and two sets of interpolated x axes —x_P, x_Q i $x_{P'}, x_{Q'}$ are obtained, whereas the position of point R remains unaltered.

Applying once more the theory of implicit function to the same equations (1.7) with the supposition that y_{i-1}^k, v_{i-1}^k, y_i^k, v_i^k, y_{i+1}^k, v_{i+1}^k, y_i^{k+1} and v_i^{k+1}

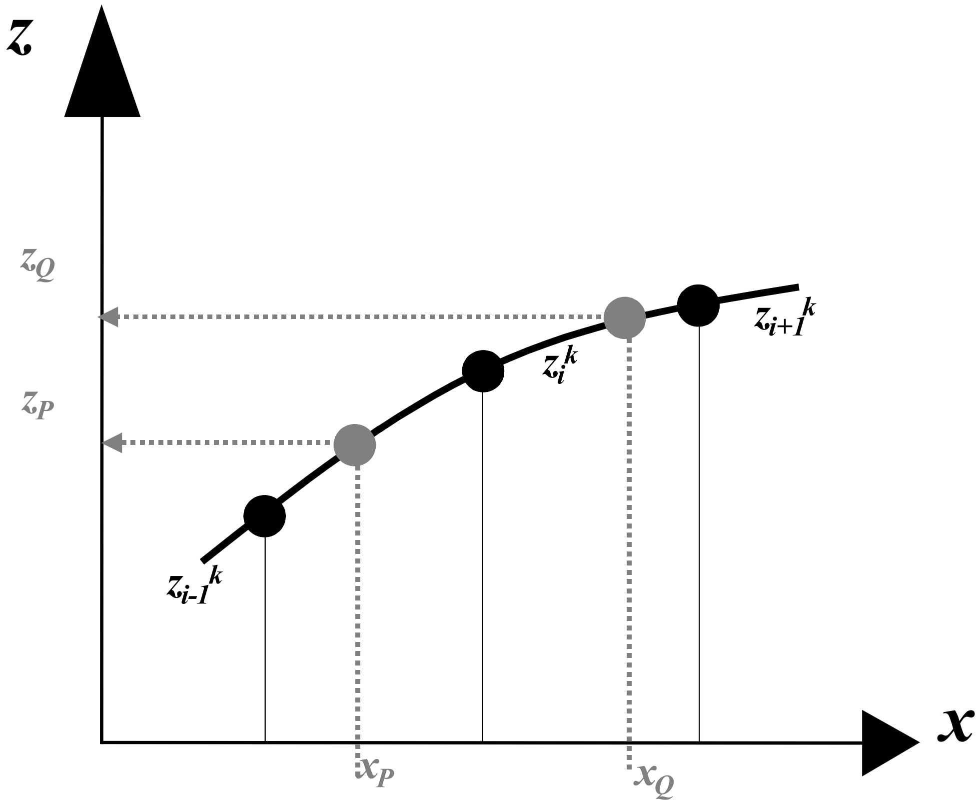

Figura 1.4: *Functions of interpolation.*

now depend on a general parameter ϕ, shows a set similar to (1.10):

$$[M]\begin{pmatrix} \frac{\partial x_P}{\partial \phi} \\ \frac{\partial y_i^{k+1}}{\partial \phi} \\ \frac{\partial v_i^{k+1}}{\partial \phi} \\ \frac{\partial x_Q}{\partial \phi} \end{pmatrix} = -[N]\,[S]\begin{pmatrix} \frac{\partial y_{i-1}^k}{\partial \phi} \\ \frac{\partial v_{i-1}^k}{\partial \phi} \\ \frac{\partial y_i^k}{\partial \phi} \\ \frac{\partial v_i^k}{\partial \phi} \\ \frac{\partial y_{i+1}^k}{\partial \phi} \\ \frac{\partial v_{i+1}^k}{\partial \phi} \end{pmatrix} \tag{1.12}$$

where:

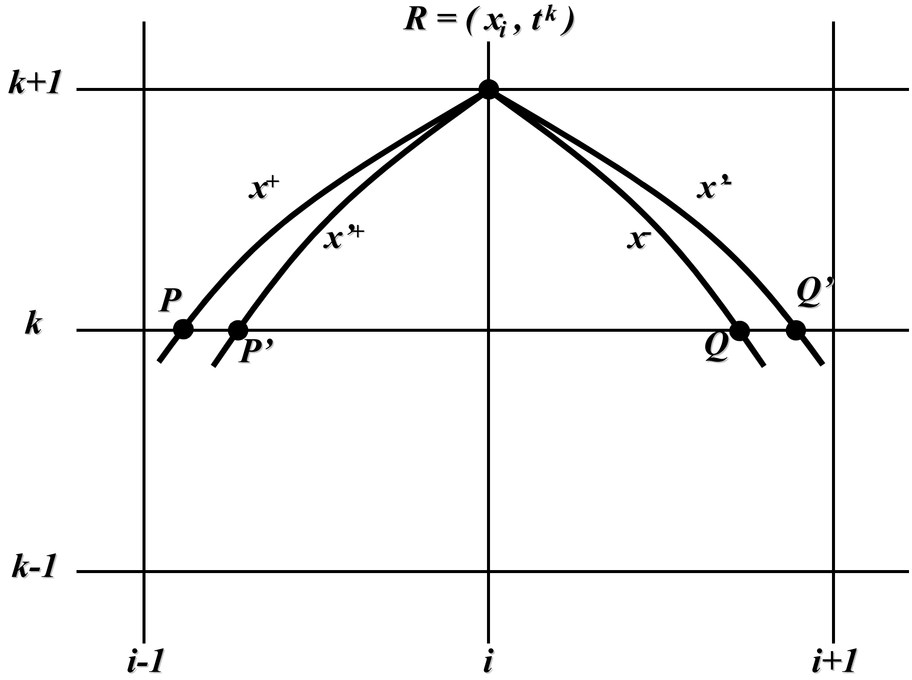

Figura 1.5: *A pair of sets of characteristic curves passing through point R.*

$$[M] = \begin{bmatrix} \frac{\partial f_1}{\partial x_P} & \frac{\partial f_1}{\partial y_i^{k+1}} & \frac{\partial f_1}{\partial v_i^{k+1}} & 0 \\ \frac{\partial f_2}{\partial x_P} & \frac{\partial f_2}{\partial y_i^{k+1}} & \frac{\partial f_2}{\partial v_i^{k+1}} & 0 \\ 0 & \frac{\partial f_3}{\partial y_i^{k+1}} & \frac{\partial f_3}{\partial v_i^{k+1}} & \frac{\partial f_3}{\partial x_Q} \\ 0 & \frac{\partial f_4}{\partial y_i^{k+1}} & \frac{\partial f_4}{\partial v_i^{k+1}} & \frac{\partial f_4}{\partial x_Q} \end{bmatrix}$$

$$[N] = \begin{bmatrix} \frac{\partial f_1}{\partial y_P} & \frac{\partial f_1}{\partial v_P} & 0 & 0 \\ \frac{\partial f_2}{\partial y_P} & \frac{\partial f_2}{\partial v_P} & 0 & 0 \\ 0 & 0 & \frac{\partial f_3}{\partial y_Q} & \frac{\partial f_3}{\partial v_Q} \\ 0 & 0 & \frac{\partial f_4}{\partial y_Q} & \frac{\partial f_4}{\partial v_Q} \end{bmatrix}$$

$$[S] = \begin{bmatrix} \frac{\partial y_P}{\partial y_{i-1}^k} & 0 & \frac{\partial y_P}{\partial y_i^k} & 0 & \frac{\partial y_P}{\partial y_{i+1}^k} & 0 \\ 0 & \frac{\partial v_P}{\partial v_{i-1}^k} & 0 & \frac{\partial v_P}{\partial v_i^k} & 0 & \frac{\partial v_P}{\partial v_{i+1}^k} \\ \frac{\partial y_Q}{\partial y_{i-1}^k} & 0 & \frac{\partial y_Q}{\partial y_i^k} & 0 & \frac{\partial y_Q}{\partial y_{i+1}^k} & 0 \\ 0 & \frac{\partial v_Q}{\partial v_{i-1}^k} & 0 & \frac{\partial v_Q}{\partial v_i^k} & 0 & \frac{\partial v_Q}{\partial v_{i+1}^k} \end{bmatrix}$$

It should be said that of the four values obtained ($\frac{\partial x_P}{\partial \phi}$, $\frac{\partial y_i^{k+1}}{\partial \phi}$, $\frac{\partial v_i^{k+1}}{\partial \phi}$ and $\frac{\partial x_Q}{\partial \phi}$) we are only interested in keeping ($\frac{\partial y_i^{k+1}}{\partial \phi}$ and $\frac{\partial v_i^{k+1}}{\partial \phi}$) to find the solution to $k+2$, as the two other values never intervene in the right part of set (1.12).

Of the differences from (1.10) we should underline the presence of the matrix $[S]_i^k$, the values of which can be obtained from (1.11). It should also be said that from the known values $\frac{\partial y_{i-1}^k}{\partial \phi}$, $\frac{\partial v_{i-1}^k}{\partial \phi}$, $\frac{\partial y_i^k}{\partial \phi}$, $\frac{\partial v_i^k}{\partial \phi}$, $\frac{\partial y_{i+1}^k}{\partial \phi}$ and $\frac{\partial v_{i+1}^k}{\partial \phi}$ at the moment in time k, $\frac{\partial y_i^{k+1}}{\partial \phi}$ and $\frac{\partial v_i^{k+1}}{\partial \phi}$ at the moment in time $k+1$ are obtained which shows the concept of influence and dependence domain.

For each point of the structured grid, a set of equations of the type (1.12) can be solved except for the interior points with sluicegate and the boundary conditions. The study of these will be analysed in the two following sections.

1.2.4 Sluicegate equations

There are many flow control structures in channels which allow flow modelling according to the desire of the lock-keeper. Almost all the structures present in channels, except the actual walls and floor, can be considered control structures. The individual study of each of these structures is impossible in this thesis and does not fall within its aims. However, we will present as an example a commonly found structure. It is a control point where you can find a vertically sliding sluicegate, an overflow point and a pumping point, as seen in figure 1.6. The way this control structure has of interacting with the flow can be described according to the principles of mass and energy conservation. These principles establish two mathematical relations between the flow conditions immediately upstream and downstream of the point,

$$\left.\begin{array}{c} S\left(y_e\right)\frac{dy_e}{dt} = A\left(y_e\right)v_e - q_b - q_s\left(y_e\right) - A\left(y_s\right)v_s \\ A\left(y_s\right)v_s = k_c u\left(y_e - y_s + d\right)^{\frac{1}{2}} \end{array}\right\} \tag{1.13}$$

where:

- $S\left(y_e\right)$ is the real horizontal area of the reception area upstream of the sluicegate,
- $A\left(y_e\right)v_e$ is the incoming flow, defined in terms of depth and speed of entry,
- $A\left(y_s\right)v_s$ is the outgoing flow which continues along the channel, described in terms of depth and speed of exit,
- q_b is the outgoing flow through pumping which is predetermined,

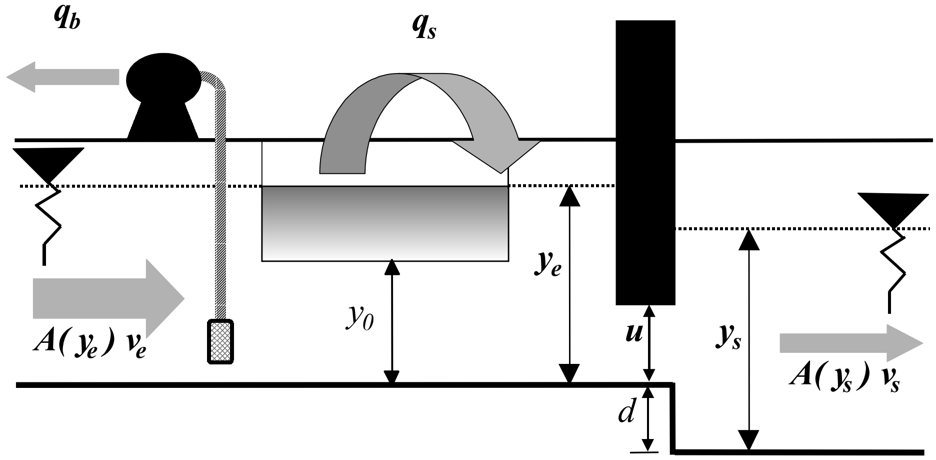

Figura 1.6: *Diagram of a sluice gate with lateral overflow and pumping point.*

- $q_s(y_e) = C_s a_s (y_e - y_0)^{\frac{3}{2}}$ is the outgoing lateral flow via the overflow point where C_s is the coefficient of the drainage system at the overflow point, a_s is the width of the lip and y_0 is the height of the lip measured from the bottom of the channel,

- $k_c = \sqrt{2g} C_c a_c$ where C_c is the coefficient of the drainage of the sluicegate and a_c is the width of the sluicegate which is normally rectangular,

- d is the height of the step, and

- u is the opening of the sluicegate.

This thesis considers that the deposit effect resulting from the presence of the sluice reception area is completely irrelevant from the control point of view, and therefore will be ignored from now on.

From the control point of view it is necessary to note the important difference between both types of lateral outflow: the first, represented by pumping, is predetermined by the lock-keeper, and the second, represented by the overflow, depends on the existing depth upstream from the control point and therefore is controlled by this. The difference lies in the fact that by pumping the desire flow can be produced with the only condition being having enough water, whereas with the overflow the water obtained will depend on the depth maintained. Therefore, in this second case the control will be much more difficult. Apart from the cases in which pumping is compulsory (due level limitations) the overflow system is preferred because it has lower energy costs even though it is more difficult to control.

1.2.5 The discretization of the control point equations

The presence of control points in the middle of a channel leads towards the sub-division of this channel into sections, in a way that there is a section between two control points, and there is a control point between two sections. Therefore y_n^{k+1} represents the existing water depth at node n in the section upstream of the control point at the moment in time $k+1$, that is to say, incoming depth y_e. In the same way y_1^k is defined as the existing water depth at the first node of the section downstream from the control point at the same moment in time $k+1$, and y_s the outgoing depth at the control point (see figure 1.6). The same can be said for the velocities $v_n^{k+1}$and v_1^{k+1}.

If discretization is carried out with time and we rewrite the control point equations (1.13), join them with the characteristics of (1.7) and then change the nomenclature, we arrive at the following set of six equations:

$$
\begin{aligned}
&f_1 \equiv x_n - x_P - \tfrac{1}{2}\Delta t\left[v_n^{k+1} + c_n^{k+1} + v_P + c_P\right] = 0\\
&f_2 \equiv \left(v_n^{k+1} - v_P\right) + \tfrac{g}{2}\tfrac{c_n^{k+1}+c_P}{c_n^{k+1}c_P}\left(y_n^{k+1} - y_P\right) + g\Delta t\left(\tfrac{{S_f}_n^{k+1}+S_{f_P}}{2} - S_0\right) = 0\\
&f_3 \equiv \left(v_1^{k+1} - v_Q\right) - \tfrac{g}{2}\tfrac{c_1^{k+1}+c_Q}{c_1^{k+1}c_Q}\left(y_1^{k+1} - y_Q\right) + g\Delta t\left(\tfrac{{S_f}_1^{k+1}+S_{f_Q}}{2} - S_0\right) = 0\\
&f_4 \equiv x_n - x_Q - \tfrac{1}{2}\Delta t\left[v_1^{k+1} - c_1^{k+1} + v_Q - c_Q\right] = 0\\
&f_5 \equiv A\left(y_n^{k+1}\right)v_n^{k+1} - q_b - q_s\left(y_n^{k+1}\right) - A\left(y_1^{k+1}\right)v_1^{k+1} = 0\\
&f_6 \equiv A\left(y_1^{k+1}\right)v_1^{k+1} - k_c u\left(y_n^{k+1} - y_1^{k+1} + d\right)^{\frac{1}{2}} = 0
\end{aligned}
\tag{1.14}
$$

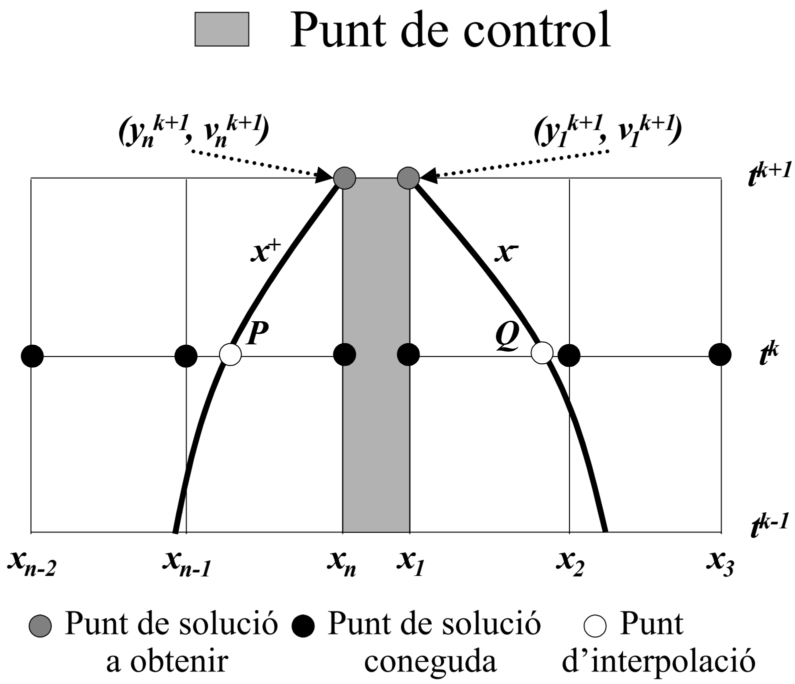

Figura 1.7: *Graph with discretization of the control point equations.*

where

$$\begin{array}{cc} \Delta t = t^{k+1} - t_P & \Delta t = t^{k+1} - t_Q \\ y_P\left(x_P\right) = s\left(x_P, y_{n-2}^k, y_{n-1}^k, y_n^k\right) & y_Q\left(x_Q\right) = s\left(x_Q, y_1^{k+1}, y_2^{k+1}, y_3^{k+1}\right) \\ v_P\left(x_P\right) = s\left(x_P, v_{n-2}^k, v_{n-1}^k, v_n^k\right) & v_Q\left(x_Q\right) = s\left(x_Q, v_1^{k+1}, v_2^{k+1}, v_3^{k+1}\right) \\ c_n^{k+1} = c\left(y_n^{k+1}\right) & c_1^{k+1} = c\left(y_1^{k+1}\right) \\ {S_f}_n^{k+1} = S_f\left(y_n^{k+1}, v_n^{k+1}\right) & {S_f}_1^{k+1} = S_f\left(y_1^{k+1}, v_1^{k+1}\right) \end{array}$$

and where the unknowns of the set are x_P, $y_n^{k+1}, v_n^{k+1}, y_1^{k+1}, v_1^{k+1}$ and x_Q.

In order to continue with the calculation of the influences of a general parameter ϕ, it is necessary to first assume that this parameter defines the opening of the sluicegate, this is $u\left(\phi\right)$, and then assume that the unknowns x_P, $y_n^{k+1}, v_n^{k+1}, y_1^{k+1}, v_1^{k+1}$ and x_Q also depend on ϕ. Therefore, applying once more the theory of the implied function to the set (1.14) we obtain

$$
[M]\begin{pmatrix} \frac{\partial x_P}{\partial \phi} \\ \frac{\partial y_n^{k+1}}{\partial \phi} \\ \frac{\partial v_n^{k+1}}{\partial \phi} \\ \frac{\partial y_1^{k+1}}{\partial \phi} \\ \frac{\partial v_1^{k+1}}{\partial \phi} \\ \frac{\partial x_Q}{\partial \phi} \end{pmatrix} = [N]\,[S]\begin{pmatrix} \frac{\partial y_{n-2}^{k}}{\partial \phi} \\ \frac{\partial v_{n-2}^{k}}{\partial \phi} \\ \frac{\partial y_{n-1}^{k}}{\partial \phi} \\ \frac{\partial v_{n-1}^{k}}{\partial \phi} \\ \frac{\partial y_n^{k}}{\partial \phi} \\ \frac{\partial v_n^{k}}{\partial \phi} \\ \frac{\partial y_1^{k}}{\partial \phi} \\ \frac{\partial v_1^{k}}{\partial \phi} \\ \frac{\partial y_2^{k}}{\partial \phi} \\ \frac{\partial v_2^{k}}{\partial \phi} \\ \frac{\partial y_3^{k}}{\partial \phi} \\ \frac{\partial v_3^{k}}{\partial \phi} \end{pmatrix} + (L)\,\frac{\partial u}{\partial \phi} \tag{1.15}
$$

where:

$$[M] = \frac{\partial(f_1,f_2,f_3,f_4,f_5,f_6)}{\partial\left(x_P,y_n^{k+1},v_n^{k+1},y_1^{k+1},v_1^{k+1},x_Q\right)}$$

$$[N] = -\frac{\partial(f_1,f_2,f_3,f_4,f_5,f_6)}{\partial\left(x_P,y_P,v_P,y_Q,v_Q,x_Q\right)}$$

$$(L) = -\left(0,0,0,0,0,\frac{\partial f_6}{\partial u}\right)^T$$

$$[S] = \frac{\partial\left(x_P,y_P,v_P,y_Q,v_Q,x_Q\right)}{\partial\left(y_{n-2}^k,v_{n-2}^k,y_{n-1}^k,v_{n-1}^k,y_n^k,v_n^k,y_1^k,v_1^k,y_2^k,v_2^k,y_3^k,v_3^k\right)}$$

So far we have spoken about the influence a general parameter (ϕ) has on the solution without entering into too many details about what it could represent, and now, for the first time, it appears explicitly in the description of the sluicegate opening $u(\phi)$. Despite the fact that the specific form of this function is still unknown, the set of equations (1.15) shows that the influence of the parameter ϕ on flow conditions at moment in time $k+1$ is the sum of the indirect influence of the conditions at moment in time k and the direct influence of the opening at the moment in time $k+1$ through the term $\left.\frac{\partial u}{\partial \phi}\right|_{n,1}^{k+1}$, which represents the variation in the sluicegate opening when the parameter ϕ changes.

1.2.6 Sluicegate trajectories

Normally the variable observed in the set is considered as a descriptive parameter of a physical phenomenum. The calculation process of the aforementioned parameter usually consists, first of all, of the taking of measurements of the dependent varibles which make the phenomenum evident (in our case these will be discharge, velocity and/or depth). After this it is necessary to adjust the values of these variables in a way that once introduced in the equations of the model, these equations reflect as close as possible the measured values. This process is known as the inverse problem resolution, and in some cases when solving it, it is necessary to have the "hydraulic influence" of the parameter (usually known in this case as the "sensitivity matrix").

One of the new things that this thesis wants to present is to consider that the unknowns of the inverse problem are the parameters which describe the trajectories of the sluicegate position. Therefore the aim is to try to find the sluicegate trajectories (and more specifically, the parameters that describe these trajectories) that foresee a behaviour more similar to the desired one rather than to the measured one.

Therefore it is very important to have a tool (like the hydraulic influence matrix) that permits us to quantify the influence on flow behaviour caused by a variation in a sluicegate parameter. To continue putting examples, we have taken the trajectory of a sluicegate as a presentation model but any other type of control can be used. In irrigation channel control, the duration

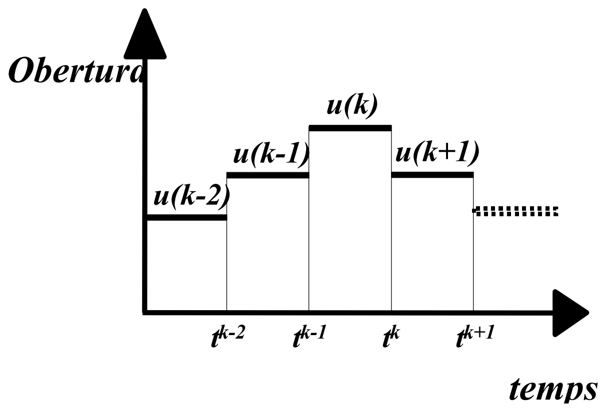

Figura 1.8: *Mathematical representation of a sluicegate trajectory.*

time of a sluicegate operation (i.e. a change of position) can be considered negligible if compared to the rest of the time that the sluicegate remains without moving. The reason for this fact is that any movement of sluicegate has a cost[1] and there is the dilemma between achieving the best control and the least number of operations.

This is the reason why the best form of mathematically representing the sluicegate trajectory is with a discontinuous temporal function, divided into sections as seen in figure 1.8. This mathematical representation allows parameterization of the problem, which is the process of identifying the variables of the problem. By simply identifying the parameter ϕ as the position of the sluicegate $u(K)$ at interval K between the points in times t^{K-1} and t^K, the process is finished. However, there are other representations of sluicegate trajectory which can be parameterised but this is not the time nor place to introduce them.

A sluicegate trajectory is defined as the following vector of parameters

$$u = \left[\begin{array}{ccccc} u(1) & \cdots & u(K) & \cdots & u(\lambda) \end{array} \right] \tag{1.16}$$

where λ is the last interval in which the function of the sluicegate trajectory is defined, and in the context of control, this is called prediction horizon.

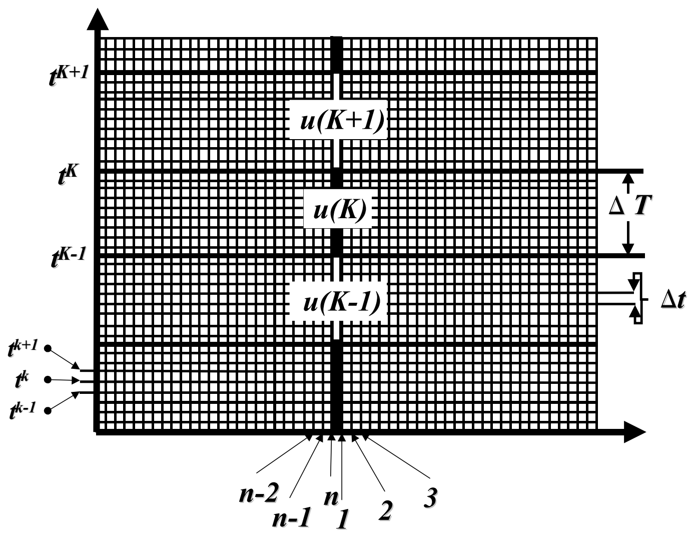

Figura 1.9: *Graph of the two time discretizations used.*

[1] Whether the sluice gate is repositioned by an operator (usually the lock keeper) or by an electric motor (which has a limited period of life), every adjustment has a cost which can be assessed. Therefore it is necessary to reduce the total number and magnitude of operations.

A description of a sluicegate trajectory such as this implies some very important consequences which are described here:

- In order to solve the problem it will be necessary to use a set of Saint-Venant equations, for example those described in the set of equations (1.7). It will be necessary to limit the increase in integrated time Δt for reasons of stability, as described in the stability conditions of Courant-Friedrichs-Levi (CFL). This condition says that the maximum increase allowed is the time it takes a wave to travel the integration area Δx. If, for each instant a sluicegate parameter is defined, the total number of parameters will depend on the CFL, that is to say, on the existing flow conditions for each instant. Given that, it would be very difficult, or even impossible, to solve the problem. Therefore it is necessary to establish a time discretization of the sluicegate trajectory independent of that of the simulation. To show this difference the superscript K (as a capital letter) will be used for the time discretization of the sluicegate trajectories and k (as a small letter) will be used for the discretization of the simulation, as shown in figure 1.9.

- The form of function by sections represented in figure 1.8 implies $\frac{\partial u}{\partial \phi} = 1$ in the set (1.15). Therefore, from now on, we will talk about the opening of any sluicegate $u\left(K\right)$ during the K^n interval of time instead of a general parameter ϕ as used so far (figure 1.9).

- With (1.16) the parameter are given a characteristic of time which allows us to state:

$$\frac{\partial y_i^k}{\partial u\left(K\right)} = 0 \quad \text{if} \quad t^k < T^K$$

1.2.7 The boundary conditions

To conclude the study of the influence a sluicegate movement has on the flow conditions at a point in the channel at a specific moment in time, it is necessary to describe the boundary conditions as well as to find out how the influences evolve when they arrive at the edge of the channel and "bounce". Crandall (1951) demonstrates a second theory of uniqueness for second order sets of equations like the one studied in this thesis. With the help of the graph in figure 1.10, the second theory states the following: *If the flow conditions at a characteristic intersection S are known and if only one variable is known from the two non-characteristic curves $\widetilde{SP}$ and $\widetilde{SQ}$, then the solution and uniqueness is guaranteed in the shaded area $\widetilde{SPRQ}$.*

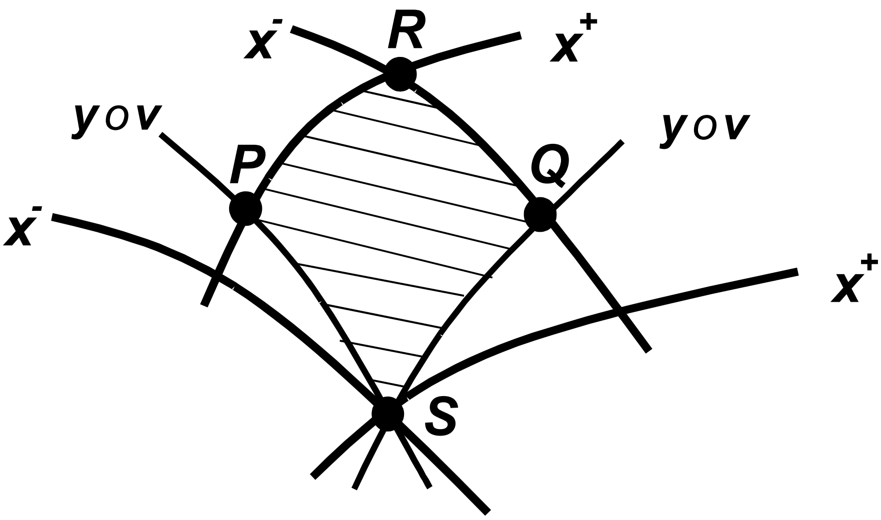

Figura 1.10: *Graph showing the formulation of the second theory of uniqueness for second order sets of hyperbolic equations.*

The joint application of this theory, and the one mentioned previously, (see figure 1.1) entitles us to state that by establishing two conditions, one at each extreme of the studied solution interval, uniqueness can be guaranteed in the shaded areas of figure 1.11. That is, if flow conditions at points P and Q on two characteristic curves and any condition of the y axis at both extremes are known, then the solution for points R and R' can be obtained.

It should be said that in a subcritical regime (the usual working regime in irrigation channels, and the only considered in this thesis) a condition for each extreme of the channel must be established (as stated in the previous paragraph) because the gradients of the characteristic curves x^+ and x^- have opposite signs since the velocity of the wave is higher than the velocity of the environment it is travelling through. From (1.3) we obtain

$$\frac{dx^+}{dt} \equiv v + c(y) > 0$$
$$\frac{dx^-}{dt} \equiv v - c(y) < 0$$

There are a large number of conditions that can be set at the extremes of a channel. While the previous explanation was simple and attractive using some specific examples of equations, now it is exactly the opposite.

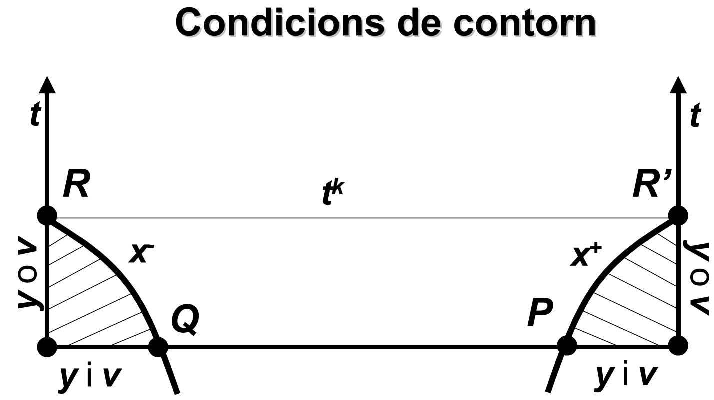

Figura 1.11: *Graph representing the application of the two uniqueness theories in flow problem context.*

Continuing with the nomenclature of the structured grid, the surrounding conditions are established as two expressions of the following type

$$\begin{array}{r} f_7\left(y_1^{k+1}, v_1^{k+1}\right) \equiv 0 \text{ as an upstream condition} \\ f_8\left(y_n^{k+1}, v_n^{k+1}\right) \equiv 0 \text{ as a downstream condition} \end{array} \tag{1.17}$$

Taking the third and fourth equations of the set (1.14) and the first from (1.17) gives a new set that needs to be solved in order to find the flow conditions at the upstream limit:

$$\left.\begin{array}{l} f_3 \equiv \left(v_1^{k+1} - v_Q\right) - \frac{g}{2}\frac{c_1^{k+1}+c_Q}{c_1^{k+1}c_Q}\left(y_1^{k+1} - y_Q\right) + g\Delta t\left(\frac{{S_f}_1^{k+1}+S_{f_Q}}{2} - S_0\right) = 0 \\ \\ f_4 \equiv x_n - x_Q - \frac{1}{2}\Delta t\left[v_1^{k+1} - c_1^{k+1} + v_Q - c_Q\right] = 0 \\ \\ f_7\left(y_1^{k+1}, v_1^{k+1}\right) \equiv 0 \end{array}\right\} \tag{1.18}$$

By doing the same we can find the set of equations corresponding to the downstream limits if we take the first and the second equations of (1.14) and the second from (1.17), which gives:

$$\left.\begin{array}{l} f_1 \equiv x_n - x_P - \frac{1}{2}\Delta t\left[v_n^{k+1} + c_n^{k+1} + v_P + c_P\right] = 0 \\ f_2 \equiv \left(v_n^{k+1} - v_P\right) + \frac{g}{2}\frac{c_n^{k+1}+c_P}{c_n^{k+1}c_P}\left(y_n^{k+1} - y_P\right) + g\Delta t\left(\frac{{S_f}_n^{k+1}+S_{fP}}{2} - S_0\right) = 0 \\ f_8\left(y_n^{k+1}, v_n^{k+1}\right) \equiv 0 \end{array}\right\} \tag{1.19}$$

Applying once more the hypothesis that all variables are dependent on a sluicegate opening $u(K)$ as we did in order to find (1.12) and (1.15), for the upstream point we can write:

$$\boxed{[M]\begin{pmatrix} \frac{\partial y_1^{k+1}}{\partial u(K)} \\ \frac{\partial v_1^{k+1}}{\partial u(K)} \\ \frac{\partial x_Q}{\partial u(K)} \end{pmatrix} = [N]\,[S]\begin{pmatrix} \frac{\partial y_1^k}{\partial u(K)} \\ \frac{\partial v_1^k}{\partial u(K)} \\ \frac{\partial y_2^k}{\partial u(K)} \\ \frac{\partial v_2^k}{\partial u(K)} \\ \frac{\partial y_3^k}{\partial u(K)} \\ \frac{\partial v_3^k}{\partial u(K)} \end{pmatrix} + (L)} \tag{1.20}$$

where:

$$[M] = \frac{\partial(f_3, f_4, f_7)}{\partial\left(y_1^{k+1}, v_1^{k+1}, x_Q\right)}$$

$$[N] = -\frac{\partial(f_3, f_4, f_7)}{\partial\left(, y_Q, v_Q, x_Q\right)}$$

$$(L) = -\left(0, 0, \frac{\partial f_7}{\partial u(K)}\right)^T$$

$$[S] = \frac{\partial\left(y_Q, v_Q, x_Q\right)}{\partial\left(y_1^k, v_1^k, y_2^k, v_2^k, y_3^k, v_3^k\right)}$$

and for the downstream point:

$$\boxed{[M]\begin{pmatrix} \frac{\partial y_n^{k+1}}{\partial u(K)} \\ \frac{\partial v_n^{k+1}}{\partial u(K)} \\ \frac{\partial x_P}{\partial u(K)} \end{pmatrix} = [N]\,[S]\begin{pmatrix} \frac{\partial y_{n-2}^k}{\partial u(K)} \\ \frac{\partial v_{n-2}^k}{\partial u(K)} \\ \frac{\partial y_{n-1}^k}{\partial u(K)} \\ \frac{\partial v_{n-1}^k}{\partial u(K)} \\ \frac{\partial y_n^k}{\partial u(K)} \\ \frac{\partial v_n^k}{\partial u(K)} \end{pmatrix} + (L)} \tag{1.21}$$

where:

$$[M] = \frac{\partial(f_1,f_2,f_8)}{\partial\left(y_n^{k+1},v_n^{k+1},x_P\right)}$$

$$[N] = -\frac{\partial(f_1,f_2,f_8)}{\partial(,y_P,v_P,x_P)}$$

$$(L) = -\left(0,0,\frac{\partial f_8}{\partial u(K)}\right)^T = (0,0,0)^T$$

$$[S] = \frac{\partial(y_P,v_P,x_P)}{\partial\left(y_{n-2}^k,v_{n-2}^k,y_{n-1}^k,v_{n-1}^k,y_n^k,v_n^k\right)}$$

1.3 Hydraulic influence of a sluicegate trajectory parameter on the status vector

In the structured grid of figure 1.12 you can see a time-space discretization of a channel made up of two sections, *Section I* subdivided into $n_I - 1$ cells and *Section II* subdivided into $n_{II} - 1$ cells. In the grid you can also see how from the solutions for the moment in time k (some of which are represented by black dots) the solution for all the $n_S = n_I + n_{II}$ sections (represented by grey dots) can be found at the moment in time $k+1$. Programming logic allows us to find sequencially the solution at $k+1$ for all the computational nodes, assuming that you identify the type of each of these nodes and you solve the corresponding set of equations. There is a summary of this in the following table:

	Section	**Solution at $k+1$**	**Set**
Upstream node	*I*	$i = 1$	(1.18)
Interior computational node	*I*	$i = 2,\ldots,n_I - 1$	(1.7)
Control point	*I and II*	$n_I, 1$	(1.14)
Interior computational node	*II*	$i = 2,\ldots,n_{II} - 1$	(1.7)
Downstream node	*II*	$i = n_{II}$	(1.19)

So far in this summary we have only talked about the solution of the equations for the simulation. However what is really important here is the study of the time-space evolution of the influence of any parameter in the trajectory (1.16).

For each moment of time, the new value of the influence of any sluicegate parameter $u(K)$ is calculated at each computational node using some of the previously mentioned sets of equations: (1.12), (1.15), (1.20) or (1.21). The matrices that make up these sets for each computational node can be

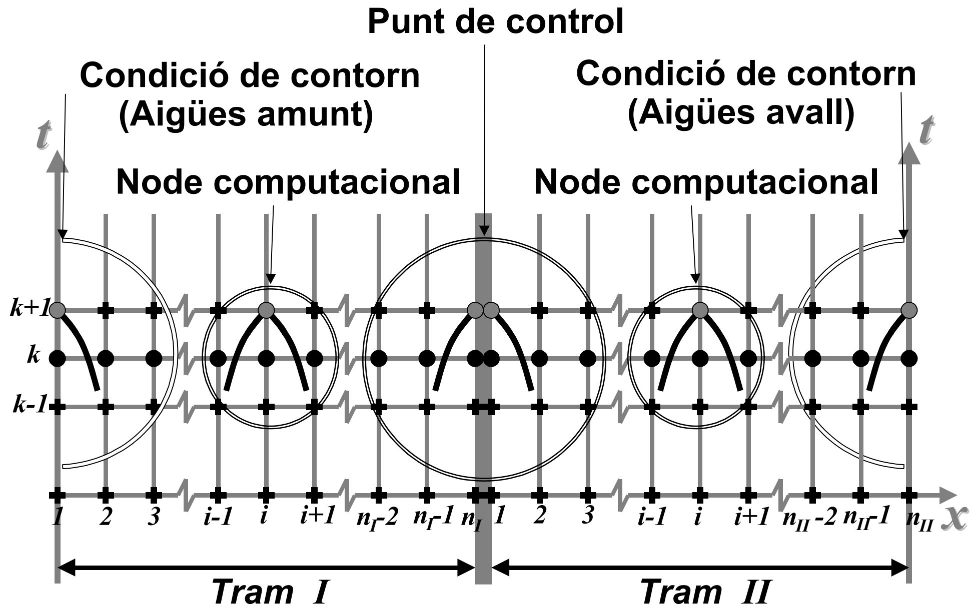

Figura 1.12: *Diagram to show the kind of computational node according to the position of each set of equations.*

adequately compiled into a single set that can be represented in the following manner:

$$\frac{\partial x^{k+1}}{\partial u(K)} = [A] \frac{\partial x^k}{\partial u(K)} + b\left[x^{k+1}, u(K)\right] \tag{1.22}$$

where $[A] = A\left[x^{k+1}, x^k, u(K)\right]$ is a square matrix of the order $(2 \times n_S) \times (2 \times n_S)$ where n_S is the total number of sections of the simulation calculation, $b\left[x^{k+1}, u(K)\right]$ is the direct influence vector of $2 \times n_S$ components and x is the status vector of dimension $2 \times n_S$ and, at the moments in time k and $k+1$, has the value:

$$\begin{aligned} x^{k+1} &= \left(\begin{array}{cccccccccc} y_1^{k+1} & v_1^{k+1} & \cdots & \cdots & y_i^{k+1} & v_i^{k+1} & \cdots & \cdots & y_{n_S}^{k+1} & v_{n_S}^{k+1} \end{array} \right)^T \\ x^k &= \left(\begin{array}{cccccccccc} y_1^k & v_1^k & \cdots & \cdots & y_i^k & v_i^k & \cdots & \cdots & y_{n_S}^k & v_{n_S}^k \end{array} \right)^T \end{aligned} \tag{1.23}$$

The set of equations (1.22) shows that the influence of the sluicegate parameter $u(K)$ on the status vector at the moment in time $k+1$ is the total of the new values of the influence at the moment in time k plus the direct influence of $u(K)$ on the status vector at $k+1$. With this, if instead of implementing sluicegate position $u(K)$ in the channel you implemented a slightly modified one $(u(K)+\Delta u(K))$ then the status vector at the moment in time $k+1$ would be $x^{k+1} + \frac{\partial x^{k+1}}{\partial u(K)} \Delta u(K)$. If we look at the diagram in

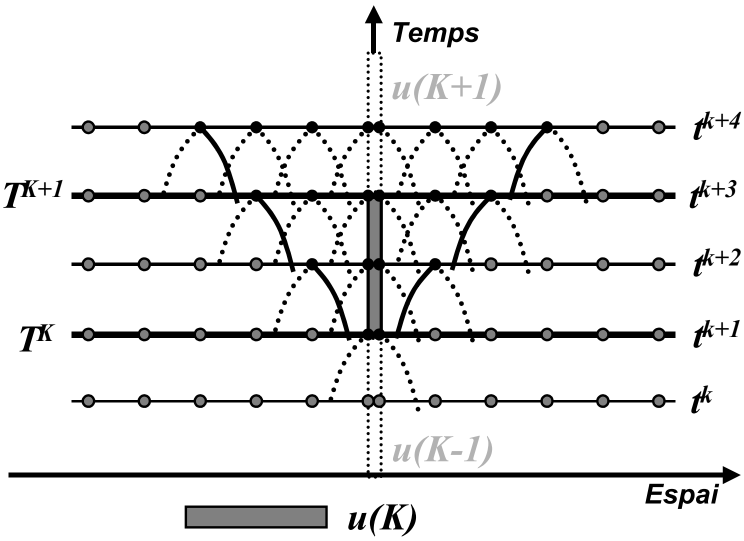

Figura 1.13: *The evolution of the influence of the sluicegate parameter $u(K)$ through time and space. The grey dots correspond to the points where there is no influence: $\frac{\partial x_i^k}{\partial u(K)} = 0$ and the black dots where there is influence: $\frac{\partial x_i^k}{\partial u(K)} \neq 0$.*

figure 1.13, we need to make the following considerations:

- When $t^{k+1} < T^K$, then $\frac{\partial x^{k+1}}{\partial u(K)} = 0$ and there is no need to give new values because the influence domain of a parameter backwards in time is nil.

- When $t^{k+1} = T^K$, the set of equations (1.22) becomes

$$\frac{\partial x^{k+1}}{\partial u(K)} = b\left[x^{k+1}, u(K)\right] \quad \text{if} \quad t^k = T^K$$

  since $\frac{\partial x^k}{\partial u(K)} = 0$. This set is useful to find the initial value of the influence of the parameter $u(K)$ on the flow conditions x^{k+1}, that is to say, the direct influence.

- The fulfilled set of equations (1.22) is the one used when $T^K < t^{k+1} \leq T^{K+1}$:
$$\frac{\partial x^{k+1}}{\partial u(K)} = [A] \frac{\partial x^k}{\partial u(K)} + b\left[x^{k+1}, u(K)\right]$$

- Finally, when $t^{k+1} > T^{K+1}$ the set (1.22) loses the direct influence of the parameter and becomes
$$\frac{\partial x^{k+1}}{\partial u(K)} = [A] \frac{\partial x^k}{\partial u(K)}$$

After determining the values that the status vector takes by applying the corresponding sets of equations, the influence domain of the parameter $u(K)$ can be considered described and quantified. This is what the black dots in figure 1.13 represent. In the same figure 1.13 you can also see how the influence on the characteristic curves evolves and how the influence domain of the parameter through the characteristic curves widens through time. We should highlight the indices in capital letters which refer to the time discretization adopted by the definition of the sluicegate trajectory and the indices in small letters of the discretization of the simulation. Both measurements of time are related to the travel time of a disturbance, this is to say, to the gradient of the characteristic curves with Courant's condition, as previously explained and represented in figure 1.9.

We can illustrate still further the concept of the influence of a parameter of sluicegate trajectory on the status vector, this is to say on all the sections of the channel and on all the moments in time. Look at a numerical example based on a simulation carried out on a $28Km$ long channel with eight control points or sluicegates. In the example there is a control point $18000m$ from the start of the channel which has a sluicegate that has a trajectory like the one defined in (1.16). The value of the influence of the parameter $u(22)$ (which corresponds to the existing opening between the moments in time $T^{22} = 22 \times \triangle T = 22 \times 300s = 6600s$ and $T^{23} = 23 \times \triangle T = 23 \times 300s = 6900s$ from the definition of the sluicegate trajectory divided into sections of $\Delta T = 300s$) was calculated with the status vector corresponding to the moments in time $6900s$, $7200s$ and $7500s$, this is, x^{6900}, x^{7200} and x^{7500}. The results can be seen in figure 1.14 where you can see how the influence of the parameter $u(22)$ evolves some moments after. After considering this graph, we need to point out that :

- A small movement of the sluicegate influences the flow conditions, both upstream and downstream, as would be expected in a subcritical regime.

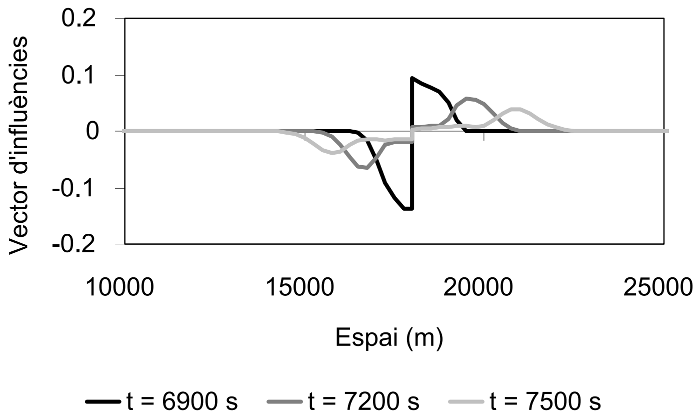

Figura 1.14: *An example of the evolution of the influence on the depths of sluice gate position u (22) (corresponding to the moments in time between 6600s and6900s) on the status vector at the moments in time 6900s, 7200s and 7500s.*

- The further the influence travels in time and space the smaller it becomes. This can be seen as a gentler curve of influence.
- If, for example, a change in the opening occurs (that is, a $\Delta u(22) > 0$) there will be a drop in depth that will travel upstream and an increase in depth downstream. The opposite will be true if the opening is reduced (this is, $\Delta u(22) < 0$).

The graph in figure 1.14 is a profile taken at three determined moments in time for the influence of the parameter $u\,(22)$, that is to say, the influence of the sluicegate opening between the moments in time $6600s$ and $6900s$. If all the profiles of the influence of this parameter on the status vector at all moments in time, this is to say on the whole simulation domain (that goes from $0s$ to $15000s$ and from $0m$ to $28000m$) are drawn, and the part that goes from the moment in time $6300s$ to $15000s$ is considered, the result can be seen in the image in figure 1.15.

If we consider the image in figure 1.15, there are a number of things that we need to comment on:

- We notice "influence leftover" that remains in the whole influence domain and makes it impossible to return to the initial state (grey colour) until a considerable time after the moment $6900s$, where the direct influence disappears.

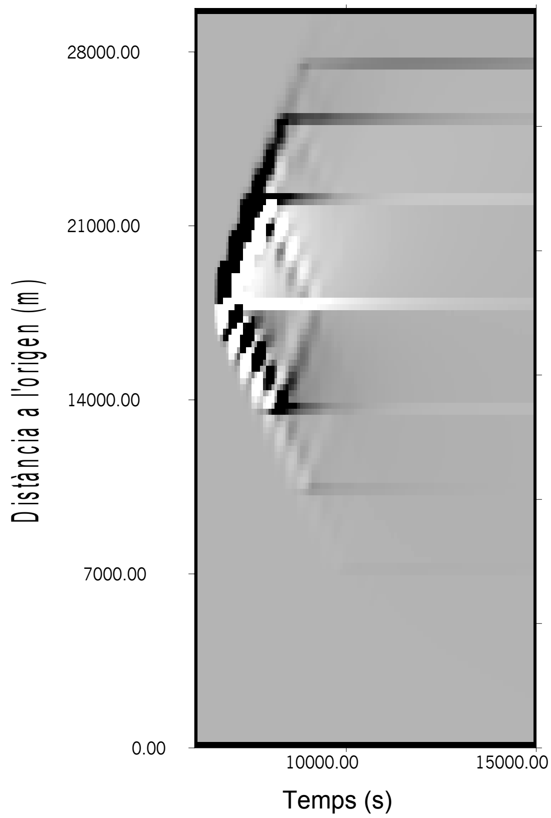

Figura 1.15: *Influence of the depths of the sluicegate position* $u(22) = u([6600s, 6900s])$ *on all points of the simulation domain. The grey area represents the absence of influence, the lighter tones represent the negative influences (drops in level) and the darkest tones the positive influences (increases in level).*

- We also notice the influence arrives at the following control point, a part "bounces" (changes direction) and another part continues in the same direction.

As in the definition of the simulation vector $X_{k_I+1}^{k_F}$[2], all the values of the status vector x^k were compiled from the initial moment $k = k_I$ to the final state $k = k_F$,

$$X_{k_I+1}^{k_F}(u) = \left[\begin{array}{ccccc} x^{k_I}(u)^T & \cdots & x^k(u)^T & \cdots & x^{k_F}(u)^T \end{array} \right]^T \quad (1.24)$$

we can now compile into only one vector, in a similar way to before, all the values of the influence of a parameter of any sluicegate trajectory $u(K)$ on

[2]See chapter 3.

the status vector. This new vector we will call the vector of influence of a parameter on the simulation vector

$$\frac{\partial X_{k_I+1}^{k_F}(u)}{\partial u(K)} = \left[\begin{array}{cccccc} \frac{\partial x^{k_I}(u)}{\partial u(K)}^T & \frac{\partial x^{k_I+1}(u)}{\partial u(K)}^T & \cdots & \frac{\partial x^{k}(u)}{\partial u(K)}^T & \cdots & \frac{\partial x^{k_F-1}(u)}{\partial u(K)}^T & \frac{\partial x^{k_F}(u)}{\partial u(K)}^T \end{array} \right]^T \quad (1.25)$$

1.4 The hydraulic influence matrix

1.4.1 Definition

Having arrived at this point, it is possible to define compilation that leads us to what we will call "the hydraulic influence matrix". If all the vectors of influence meet on the simulation vector (1.25) of all the defining parameters of the trajectory j of any sluicegate (1.16), the following hydraulic influence matrix denoted by $[I_M]_j$ is obtained

$$\begin{aligned} [I_M(u)]_j &= \left[\begin{array}{ccccccc} \frac{\partial X_{k_I+1}^{k_F}}{\partial u_j(1)} & \frac{\partial X_{k_I+1}^{k_F}}{\partial u_j(2)} & \cdots & \frac{\partial X_{k_I+1}^{k_F}}{\partial u_j(K)} & \cdots & \frac{\partial X_{k_I+1}^{k_F}}{\partial u_j(\lambda-1)} & \frac{\partial X_{k_I+1}^{k_F}}{\partial u_j(\lambda)} \end{array} \right] = \\ &= \left[\begin{array}{ccccccc} \frac{\partial x^{k_I}}{\partial u_j(1)} & \frac{\partial x^{k_I}}{\partial u_j(2)} & \cdots & \frac{\partial x^{k_I}}{\partial u_j(K)} & \cdots & \frac{\partial x^{k_I}}{\partial u_j(\lambda-1)} & \frac{\partial x^{k_I}}{\partial u_j(\lambda)} \\ \frac{\partial x^{k_I+1}}{\partial u_j(1)} & \frac{\partial x^{k_I+1}}{\partial u_j(2)} & \cdots & \frac{\partial x^{k_I+1}}{\partial u_j(K)} & \cdots & \frac{\partial x^{k_I+1}}{\partial u_j(\lambda-1)} & \frac{\partial x^{k_I+1}}{\partial u_j(\lambda)} \\ \vdots & & \ddots & & & \vdots & \vdots \\ \frac{\partial x^{k}}{\partial u_j(1)} & \frac{\partial x^{k}}{\partial u_j(2)} & & \frac{\partial x^{k}}{\partial u_j(K)} & & \frac{\partial x^{k}}{\partial u_j(\lambda-1)} & \frac{\partial x^{k}}{\partial u_j(\lambda)} \\ \vdots & \vdots & & & \ddots & \vdots & \vdots \\ \frac{\partial x^{k_F-1}}{\partial u_j(1)} & \frac{\partial x^{k_F-1}}{\partial u_j(2)} & \cdots & \frac{\partial x^{k_F-1}}{\partial u_j(K)} & \cdots & \frac{\partial x^{k_F-1}}{\partial u_j(\lambda-1)} & \frac{\partial x^{k_F-1}}{\partial u_j(\lambda)} \\ \frac{\partial x^{k_F}}{\partial u_j(1)} & \frac{\partial x^{k_F}}{\partial u_j(2)} & \cdots & \frac{\partial x^{k_F}}{\partial u_j(K)} & \cdots & \frac{\partial x^{k_F}}{\partial u_j(\lambda-1)} & \frac{\partial x^{k_F}}{\partial u_j(\lambda)} \end{array} \right] \end{aligned} \quad (1.26)$$

The following points should be noted for this matrix:

- The number of rows is much higher than the number of columns. This is because:
  - Each term $\frac{\partial x^k}{\partial u_j(K)}$ is a vector which contains $2 \times n_S$ rows.
  - The temporal discretization of the simulation $\triangle t^k$ is much smaller than that of the sluicegate trajectory $\triangle T^K$.
- Not all the terms have a value different from zero. As previously commented, all the terms $\frac{\partial x^k}{\partial u_j(K)}$ which fulfil $t^k < T^K$ are null. The arrangement of (1.26) shows that the "not null" terms of the matrix

are found in blocks in the sub-diagonal triangular part as $\triangle t^k << \triangle T^K$, this is to say $k_F - k_I >> \lambda$.

- When there are many sluicegates that control a channel, the hydraulic influence matrix for each needs to be compiled into one. In this case the hydraulic influence matrix becomes:

$$[I_M(U)]_X = \left[\left[\frac{\partial X_{k_I+1}^{k_F}(U)}{\partial u_1(1)} \cdots \frac{\partial X_{k_I+1}^{k_F}(U)}{\partial u_{n_C}(1)} \right] \cdots \left[\frac{\partial X_{k_I+1}^{k_F}(U)}{\partial u_1(\lambda)} \cdots \frac{\partial X_{k_I+1}^{k_F}(U)}{\partial u_{n_C}(\lambda)} \right] \right] \quad (1.27)$$

where the subindex X denotes the hydraulic influence matrix on the simulation vector; n_C is the total number of sluicegates and $U = \left(u_1^T, \ldots, u_{n_C}^T\right)^T$ is the vector of the compilation of all the sluicegate trajectories.

1.4.2 The discrete observer

In most cases we only need to know the values of the simulation and the influence of specific points of the channel and instants in time. In the representation shown in figure 1.16 it is possible to see an example of two channel sections with two control sluicegates. For the discretization of the simulation, the graph shows that there are a number of sections of $n_S = n_I + n_{II}$ and that the result of the simulation has given (remember that this depends on the Courant condition) a total number time segments $n_T = k_F - k_I$. As for the discretization of the sluicegate trajectories it should be said that it has a number of sluicegates $n_C = 2$, each one with a defined trajectory through four intervals $\lambda = 4$. As a result the total number of parameters becomes $n_U = \lambda \times n_C = 8$. It can also be seen in the graph that there is a line of n_E points —points coloured in grey— for which information is required.

Therefore the dimensions of the vectors and matrices shown in this chapter are the following:

- The dimension of the simulation vector $X_{k_I+1}^{k_F}$: $n_X = 2 \times n_T \times n_S$
- The dimension of the trajectories vector U: $n_U = \lambda \times n_C = 8$
- The dimension of the hydraulic influence matrix on the simulation vector $[I_M(U)]_X$: $n_X \times n_U \quad (= 2 \times n_T \times n_S \times n_U)$
- The result of the simulation at certain study points $Y_{k_I+1}^{k_F}(U)$: n_Y

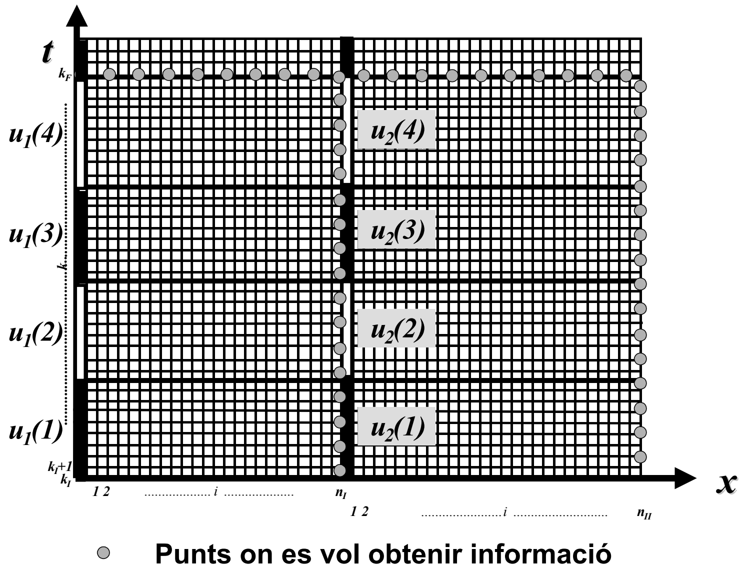

Figura 1.16: *An example to show the difference between the time discretization of the sluicegate trajectories and that of the simulation.*

- The dimension of the hydraulic influence matrix on the discrete observer set $[I_M(U)]_Y$: $n_Y \times n_U$

In order to express this information mathematically, a new matrix needs to be introduced called "discrete observer matrix", denoted as $[C]$ in the control literature. This matrix is made up exclusively of "zeros" and "ones", so they represent, respectively, the simulated behaviour at certain points of the channel and at certain moments in time, and the influence matrix of the trajectories at these points.

$$Y_{k_I+1}^{k_F}(U) = [C]\, X_{k_I+1}^{k_F}(U) \tag{1.28}$$

$$[I_M(U)]_Y = [C]\,[I_M(U)]_X \tag{1.29}$$

1.4.3 Verification of the Hydraulic influence matrix $[I_M(U)]_X$: a numerical point of view

In order to demonstrate the fact that the influence of the sluicegate parameters depend also on the actual flow conditions in the channel, it is necessary to explain a test that was carried out when checking the part of the programme where the hydraulic influence matrix was calculated. The description of this test also gives another point of view, a numerical one, on the influence concept, which is why it is introduced in this chapter.

If a simulation vector is calculated ($X_{k_I+1}^{k_F}(U)$) using specific sluicegate trajectories U and then it is simulated again after changing some term of U ($\Delta U = \begin{bmatrix} 0 & \cdots & \Delta u_j(K) & \cdots & 0 \end{bmatrix}^T$) obtaining the new result $X_{k_I+1}^{k_F}(U+\Delta U)$, then the difference between the simulation vectors ($X_{k_I+1}^{k_F}(U+\Delta U) - X_{k_I+1}^{k_F}(U)$) can be attributed to the absolute influence of the changed parameter ($u_j(K)$) at each point of the simulation domain. Furthermore, it needs to be verified that

$$\frac{\partial X_{k_I+1}^{k_F}(U)}{\partial u_j(K)} \approx \frac{X_{k_I+1}^{k_F}(U+\Delta U) - X_{k_I+1}^{k_F}(U)}{\|\Delta U\|} \tag{1.30}$$

where the terms on the left-hand side are the columns of the hydraulic influence matrix $[I_M(U)]_X$ found analytically and those on the right-hand side, numerically.

All the results obtained in the numerous verification tests ($\forall K = 1, \ldots, \lambda$ and $\forall j = 1, \ldots, n_C$) applied to specific examples had a margin of error lower than 10^{-6}, which provided a high level of exactitude in the calculation of the terms of the hydraulic influence matrix.

It is clear, therefore, that the concept of hydraulic influence of some sluicegate trajectories on the simulation vector is absolutely related to the state of the channel. This is to say it is a non-lineal characteristic as such..

1.4.4 Requisits of the hydraulic influence matrix

The most important characteristic that the hydraulic influence matrix must have in the form $[I_M(U)]_Y$ is that it is not a singular matrix. This requisit is justified by the fact that in the optimisation process, the matrix has to be inverted through its pseudo-inversion (see algorithms from sections 5.3.3 and 5.4.3). In other words, if we want to know that the modification ΔU that has to be introduced into the opening of a sluicegate trajectory U that is necessary to rectify an undesired change of the behaviour ΔY, it is necessary to solve the pseudo-inversion

$$\Delta U = \left\{ [I_M(U)]_Y^T [I_M(U)]_Y \right\}^{-1} [I_M(U)]_Y^T \Delta Y \tag{1.31}$$

where $\left\{ [I_M(U)]_Y^T [I_M(U)]_Y \right\}^{-1}$ is the so-called pseudo-inverse matrix, $\Delta Y = Y_{k_I+1}^{k_F}(U+\Delta U) - Y_{k_I+1}^{k_F}(U)$ is the undesired change in behaviour. Therefore, if the hydraulic influence matrix is not singular, then the pseudo-inversion will be able to be inverted and the set of equations (1.31) can be solved.

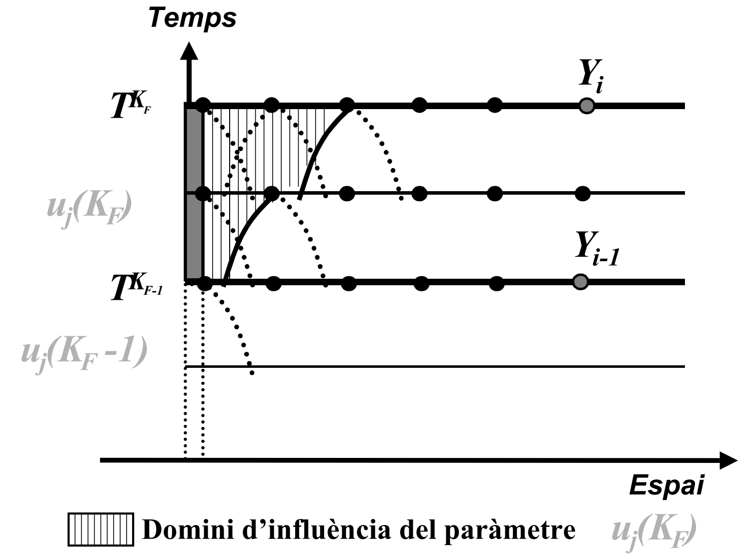

Figura 1.17: *An example of a sluicegate parameter which has no influence on any point.*

Apart from being a bad definition of the discrete observer matrix, the hydraulic influence matrix can become singular when a position $u_j(K)$ of the sluicegate j has no influence on any of the n_Y points of $Y_{k_I+1}^{k_F}(U)$, which is expressed as

$$\frac{\partial Y_{k_I+1}^{k_F}(U)}{\partial u_j(K)} = 0 \tag{1.32}$$

This is to say, a whole column of $[I_M(U)]_Y$ is full of "zeros". This is demonstrated in figure 1.17. In this example it can be seen how the parameter $u_j(K_F)$ has no influence on the levels Y_i and Y_{i-1}. This is the result of a faulty setting of the discrete observer matrix: It would be necessary for at least one value of Y_i to be in the influence domain of $u_j(K_F)$ to make the hydraulic influence matrix singular.

As a final conclusion, it can be said that the study of the hydraulic influence matrix could allow the establishment of the control parameters, such as the determination of the testing period. With this matrix, it is possible to find the stability condition of the algorithms that use the predictive control strategy, and which depends on the "inversion charateristic" of the influence matrix (and more specifically, on its condition). It is also possible to study the time period from the action on the system (the repositioning of the sluicegate) to the response. To summarise, it would be possible to carry out a study on the sensitivity of the system to modifications of the sluicegate position, etcetera. However, the study of this is not included in this thesis, but should be mentioned.

1.4.5 Example

In order to help clarify ideas, I would like to give a simple example of how, from a specific discretization problem, we will arrive to the hydraulic influence matrix compilation (see figure 1.18). The discretization in the example in the figure has two channel sections, each of them with a control sluicegate upstream $n_C = 2$. The space discretization is done through eight cells, which become $n_s = 10$ sections (from section "A" to "J"). The time discretization $\triangle t$, which depends on the condition CFL, will be $n_T = 16$, where $k_I = 0$ and $k_F = 16$. Therefore the simulation vector will have $n_X = 2 \times n_T \times n_S = 320$ components and the value will be

$$X_{k_I+1}^{k_F}(U) = X_1^{16}(U) = \begin{pmatrix} y_A^1 & v_A^1 & \cdots & v_i^k & v_i^k & \cdots & y_J^{16} & v_J^{16} \end{pmatrix}^T \tag{1.33}$$

A definition exists for the sluicegate trajectories with $\triangle T = 4 \times \triangle t$ formed by 4 sections ($\lambda = 4$) for each sluicegate. Therefore, it will have $n_U = \lambda \times n_C = 8$ unknowns which will define the following trajectories vector

$$U = \begin{pmatrix} u_1(1) & u_2(1) & u_1(2) & u_2(2) & u_1(3) & u_2(3) & u_1(4) & u_2(4) \end{pmatrix}^T \tag{1.34}$$

With this, the Jacobian matrix is expressed as follows:

$$\left[\nabla_U X_1^{16}(U)\right]_{(16\times 320)} = \begin{bmatrix} \frac{\partial y_A^1}{\partial u_1(1)} \frac{\partial v_A^1}{\partial u_1(1)} & \cdots & \frac{\partial y_i^k}{\partial u_1(1)} \frac{\partial v_i^k}{\partial u_1(1)} & \cdots & \frac{\partial y_J^{16}}{\partial u_1(1)} \frac{\partial v_J^{16}}{\partial u_1(1)} \\ \frac{\partial y_A^1}{\partial u_2(1)} \frac{\partial v_A^1}{\partial u_2(1)} & & \frac{\partial y_i^k}{\partial u_2(1)} \frac{\partial v_i^k}{\partial u_2(1)} & & \frac{\partial y_J^{16}}{\partial u_2(1)} \frac{\partial v_J^{16}}{\partial u_2(1)} \\ \vdots & \ddots & \vdots & & \vdots \\ \frac{\partial y_A^1}{\partial u_j(K)} \frac{\partial v_A^1}{\partial u_j(K)} & \cdots & \frac{\partial y_i^k}{\partial u_j(K)} \frac{\partial v_i^k}{\partial u_j(K)} & \cdots & \frac{\partial y_J^{16}}{\partial u_j(K)} \frac{\partial v_J^{16}}{\partial u_j(K)} \\ \vdots & & \vdots & \ddots & \vdots \\ \frac{\partial y_A^1}{\partial u_1(4)} \frac{\partial v_A^1}{\partial u_1(4)} & & \frac{\partial y_i^k}{\partial u_1(4)} \frac{\partial v_i^k}{\partial u_1(4)} & & \frac{\partial y_J^{16}}{\partial u_1(4)} \frac{\partial v_J^{16}}{\partial u_1(4)} \\ \frac{\partial y_A^1}{\partial u_2(4)} \frac{\partial v_A^1}{\partial u_2(4)} & \cdots & \frac{\partial y_i^k}{\partial u_2(4)} \frac{\partial v_i^k}{\partial u_2(4)} & \cdots & \frac{\partial y_J^{16}}{\partial u_2(4)} \frac{\partial v_J^{16}}{\partial u_2(4)} \end{bmatrix} \tag{1.35}$$

We must say that this matrix is full of "zeros" because, for example, the terms $\frac{\partial y_A^1}{\partial u_1(4)}$, $\frac{\partial v_A^1}{\partial u_1(4)}$, $\frac{\partial y_A^1}{\partial u_2(4)}$ and $\frac{\partial v_A^1}{\partial u_2(4)}$ are exactly 0, which means that the sluicegate positions $u_1(4)$ and $u_2(4)$ have no influence on the depths and velocities at the moment of time $k = 1$. Whatsmore, the terms that are not

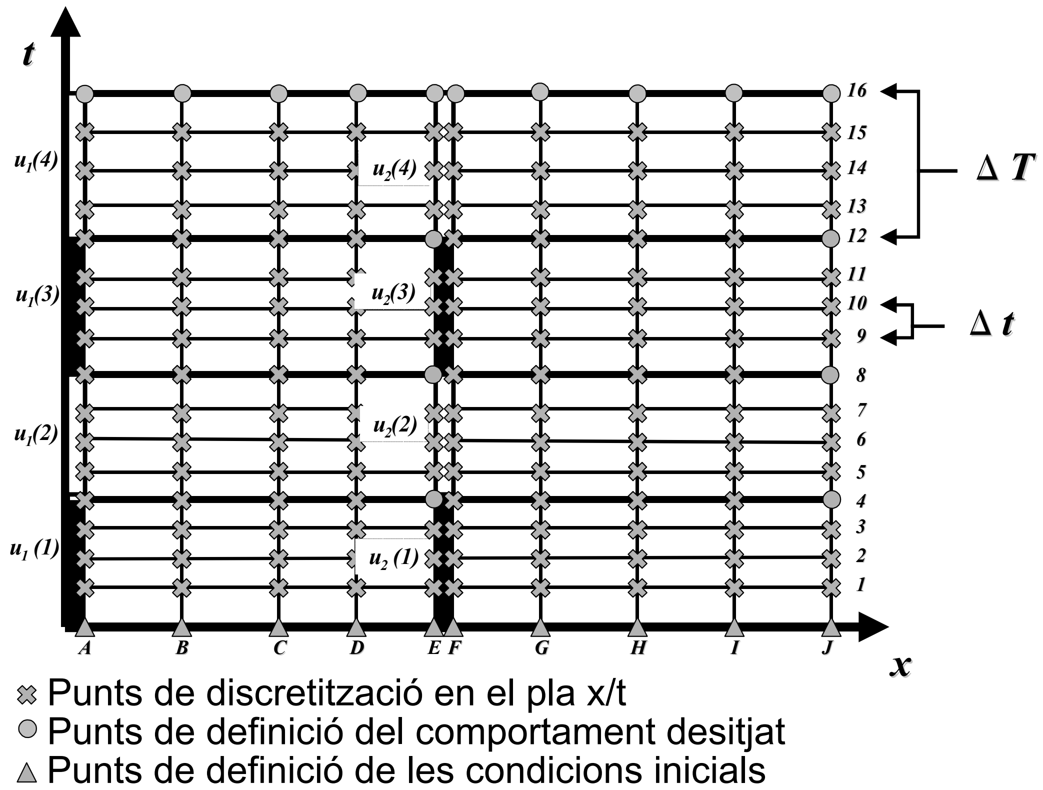

Figura 1.18: *Representation of a numerical example of the hydraulic influence matrix compilation.*

null discover the influence domain of each sluicegate position along the whole channel. Therefore, the first expression of the hydraulic influence matrix will be:

$$[I_M(U)]_X = \left[\nabla_U X_1^{16}(U)\right]^T \tag{1.36}$$

On the other hand, we want to obtain a behaviour reference which is most similar to a desired behaviour defined in terms of depth, for example, in the $n_Y = 16$ points of the axes x/t in the graph in figure 1.18. This desired behaviour can be expressed by specifying Y^* from (**??**):

$$Y^* = \left(\overline{y}_E^4\,\overline{y}_J^4\,\overline{y}_E^8\,\overline{y}_J^8\,\overline{y}_E^{12}\,\overline{y}_J^{12}\,\overline{y}_A^{16}\,\overline{y}_B^{16}\,\overline{y}_C^{16}\,\overline{y}_D^{16}\,\overline{y}_E^{16}\,\overline{y}_F^{16}\,\overline{y}_G^{16}\,\overline{y}_H^{16}\,\overline{y}_I^{16}\,\overline{y}_J^{16}\right)^T \tag{1.37}$$

It is necessary to note that the last 10 components of the vector Y^* (which define the state of the depths at $k = 16$) represent the final state at which we want to arrive, and the rest of the components represent the trajectory that we want to follow for passing from the initial state to the final one.

Similarly the vector Y_1^{16} is full of the values predicted by the model;

$$Y_1^{16}(U) = \left(y_E^4 \, y_J^4 \, y_E^8 \, y_J^8 \, y_E^{12} \, y_J^{12} \, y_A^{16} \, y_B^{16} \, y_C^{16} \, y_D^{16} \, y_E^{16} \, y_F^{16} \, y_G^{16} \, y_H^{16} \, y_I^{16} \, y_J^{16} \right)^T \tag{1.38}$$

and can be found by defining the discrete observer matrix of (**??**) specified in the following statement;

$$Y_1^{16} = [C] \, X_1^{16}(U) \tag{1.39}$$

where the composition of the matrix $[C]_{(16\times 320)}$ is left to the reader.

Finally, the second expression of the hydraulic influence matrix $[I_M(U)]_Y$, which is a (16×8) matrix, can be written

$$[I_M(U)]_Y = [C] \, [I_M(U)]_X \tag{1.40}$$

or

$$[I_M(U)]_Y = \\
= \begin{bmatrix}
\frac{\partial y_E^4}{\partial u_1(1)} & \frac{\partial y_E^4}{\partial u_2(1)} & 0 & \frac{\partial y_E^4}{\partial u_2(2)} & 0 & 0 & 0 & 0 \\
\frac{\partial y_J^4}{\partial u_1(1)} & \frac{\partial y_J^4}{\partial u_2(1)} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial y_E^8}{\partial u_1(1)} & \frac{\partial y_E^8}{\partial u_2(1)} & \frac{\partial y_E^8}{\partial u_1(2)} & \frac{\partial y_E^8}{\partial u_2(2)} & 0 & \frac{\partial y_E^8}{\partial u_2(3)} & 0 & 0 \\
\frac{\partial y_J^8}{\partial u_1(1)} & \frac{\partial y_J^8}{\partial u_2(1)} & \frac{\partial y_J^8}{\partial u_1(2)} & \frac{\partial y_J^8}{\partial u_2(2)} & 0 & 0 & 0 & 0 \\
\frac{\partial y_E^{12}}{\partial u_1(1)} & \frac{\partial y_E^{12}}{\partial u_2(1)} & \frac{\partial y_E^{12}}{\partial u_1(2)} & \frac{\partial y_E^{12}}{\partial u_2(2)} & \frac{\partial y_E^{12}}{\partial u_1(3)} & \frac{\partial y_E^{12}}{\partial u_2(3)} & 0 & \frac{\partial y_E^{12}}{\partial u_2(4)} \\
\frac{\partial y_J^{12}}{\partial u_1(1)} & \frac{\partial y_J^{12}}{\partial u_2(1)} & \frac{\partial y_J^{12}}{\partial u_1(2)} & \frac{\partial y_J^{12}}{\partial u_2(2)} & \frac{\partial y_J^{12}}{\partial u_1(3)} & \frac{\partial y_J^{12}}{\partial u_2(3)} & 0 & 0 \\
\frac{\partial y_A^{16}}{\partial u_1(1)} & \frac{\partial y_A^{16}}{\partial u_2(1)} & \frac{\partial y_A^{16}}{\partial u_1(2)} & \frac{\partial y_A^{16}}{\partial u_2(2)} & \frac{\partial y_A^{16}}{\partial u_1(3)} & \frac{\partial y_A^{16}}{\partial u_2(3)} & \frac{\partial y_A^{16}}{\partial u_1(4)} & \frac{\partial y_A^{16}}{\partial u_2(4)} \\
\frac{\partial y_B^{16}}{\partial u_1(1)} & \frac{\partial y_B^{16}}{\partial u_2(1)} & \frac{\partial y_B^{16}}{\partial u_1(2)} & \frac{\partial y_B^{16}}{\partial u_2(2)} & \frac{\partial y_B^{16}}{\partial u_1(3)} & \frac{\partial y_B^{16}}{\partial u_2(3)} & \frac{\partial y_B^{16}}{\partial u_1(4)} & \frac{\partial y_B^{16}}{\partial u_2(4)} \\
\frac{\partial y_C^{16}}{\partial u_1(1)} & \frac{\partial y_C^{16}}{\partial u_2(1)} & \frac{\partial y_C^{16}}{\partial u_1(2)} & \frac{\partial y_C^{16}}{\partial u_2(2)} & \frac{\partial y_C^{16}}{\partial u_1(3)} & \frac{\partial y_C^{16}}{\partial u_2(3)} & \frac{\partial y_C^{16}}{\partial u_1(4)} & \frac{\partial y_C^{16}}{\partial u_2(4)} \\
\frac{\partial y_D^{16}}{\partial u_1(1)} & \frac{\partial y_D^{16}}{\partial u_2(1)} & \frac{\partial y_D^{16}}{\partial u_1(2)} & \frac{\partial y_D^{16}}{\partial u_2(2)} & \frac{\partial y_D^{16}}{\partial u_1(3)} & \frac{\partial y_D^{16}}{\partial u_2(3)} & \frac{\partial y_D^{16}}{\partial u_1(4)} & \frac{\partial y_D^{16}}{\partial u_2(4)} \\
\frac{\partial y_E^{16}}{\partial u_1(1)} & \frac{\partial y_E^{16}}{\partial u_2(1)} & \frac{\partial y_E^{16}}{\partial u_1(2)} & \frac{\partial y_E^{16}}{\partial u_2(2)} & \frac{\partial y_E^{16}}{\partial u_1(3)} & \frac{\partial y_E^{16}}{\partial u_2(3)} & \frac{\partial y_E^{16}}{\partial u_1(4)} & \frac{\partial y_E^{16}}{\partial u_2(4)} \\
\frac{\partial y_F^{16}}{\partial u_1(1)} & \frac{\partial y_F^{16}}{\partial u_2(1)} & \frac{\partial y_F^{16}}{\partial u_1(2)} & \frac{\partial y_F^{16}}{\partial u_2(2)} & \frac{\partial y_F^{16}}{\partial u_1(3)} & \frac{\partial y_F^{16}}{\partial u_2(3)} & \frac{\partial y_F^{16}}{\partial u_1(4)} & \frac{\partial y_F^{16}}{\partial u_2(4)} \\
\frac{\partial y_G^{16}}{\partial u_1(1)} & \frac{\partial y_G^{16}}{\partial u_2(1)} & \frac{\partial y_G^{16}}{\partial u_1(2)} & \frac{\partial y_G^{16}}{\partial u_2(2)} & \frac{\partial y_G^{16}}{\partial u_1(3)} & \frac{\partial y_G^{16}}{\partial u_2(3)} & \frac{\partial y_G^{16}}{\partial u_1(4)} & \frac{\partial y_G^{16}}{\partial u_2(4)} \\
\frac{\partial y_H^{16}}{\partial u_1(1)} & \frac{\partial y_H^{16}}{\partial u_2(1)} & \frac{\partial y_H^{16}}{\partial u_1(2)} & \frac{\partial y_H^{16}}{\partial u_2(2)} & \frac{\partial y_H^{16}}{\partial u_1(3)} & \frac{\partial y_H^{16}}{\partial u_2(3)} & \frac{\partial y_H^{16}}{\partial u_1(4)} & \frac{\partial y_H^{16}}{\partial u_2(4)} \\
\frac{\partial y_I^{16}}{\partial u_1(1)} & \frac{\partial y_I^{16}}{\partial u_2(1)} & \frac{\partial y_I^{16}}{\partial u_1(2)} & \frac{\partial y_I^{16}}{\partial u_2(2)} & \frac{\partial y_I^{16}}{\partial u_1(3)} & \frac{\partial y_I^{16}}{\partial u_2(3)} & \frac{\partial y_I^{16}}{\partial u_1(4)} & \frac{\partial y_I^{16}}{\partial u_2(4)} \\
\frac{\partial y_J^{16}}{\partial u_1(1)} & \frac{\partial y_J^{16}}{\partial u_2(1)} & \frac{\partial y_J^{16}}{\partial u_1(2)} & \frac{\partial y_J^{16}}{\partial u_2(2)} & \frac{\partial y_J^{16}}{\partial u_1(3)} & \frac{\partial y_J^{16}}{\partial u_2(3)} & \frac{\partial y_J^{16}}{\partial u_1(4)} & \frac{\partial y_J^{16}}{\partial u_2(4)}
\end{bmatrix} \tag{1.41}$$

After seeing this hydraulic influence matrix, it should be said that the null elements in (1.41) are identical. The rest can also be null by numerical dependence but not by definition.

If the matrix is defined as such, it is evident that the discrete observer matrix has been chosen adequately for the hydraulic influence matrix to have a complete range, this is to say, range 8. There is nothing wrong if a line of (1.41) is all null, but if a column is null then the range of the matrix cannot be 8. Therfore, it is not possible for a pseudo-inverse matrix to exist, and as a consequence the set of equations de-stabilised. To conclude, the choice of matrix $[C]$ is of vital importance for the stability of the algorithms presented previously.

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