ACTIVE CONTROL OF HYSTERETIC BASE-ISOLATED STRUCTURES VIA DELAYED SIGNALS

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Abstract

This paper proposes an active control law of hysteretic base-isolated structures. The control law is obtained using a linearized model of the Bouc-Wen system and it uses delay signals. The numerical simulations show that the proposed active control system effectively reduces the structural response due to a major earthquake like "El Centro". For example, with the magnitude of control force of 19% of the total building's weight, the controller is capable of reducing the response up to about 89%. Moreover, it accomplishes a high performance from the beginning to the end of the earthquake.

1 Introduction

The control of structures subjected to seismic excitation represents a challenging task for the civil engineering profession. The traditional approach to seismic hazard mitigation is to design structures with sufficient strength capacity and the ability to deform in a ductile manner. Alternatively, newer concepts of structural control, as active control systems, have been growing in acceptance and may preclude the necessity of allowing for inelastic deformations in the structural system. Nowadays, active control has been developed over more than 25 years. Recently, there has been interest on active control of base-isolated hysteretic systems associated to a second-order structural plant, where the dynamic Bouc-Wen model is used to capture the hysteretic behavior. In this case, the main difficulty to the active control design is the presence of non-linear dynamics associated to the hysteretic behavior. However, as the base-displacement, under seismic perturbation, is small, a linearized model of the Bouc-Wen system is proposed in this work for control design. The linearization of the Bouc-Wen model produces a static system, which is desirable for control design, and a good estimation of the internal state variable of the Bouc-Wen system by only using the position measurement, which is important from the point of view of the observer design.

Time delays exist in many industrial processes. They are a source of instability and oscillations, (see ,for instance, Gu & Niculescu (2003),Gouaisbaut *et al.* (2002),Wu *et al.* (2006)). Recently, the problems of robust stability analysis and robust stabilization of time delay systems has been studied (see, for instance Xu *et al.* (2002), among many others). However, time delays can also be used to increase controller performance for vibration mitigation in mechanical structures as in Sipahi & Olgac (2003). Following this research line, we propose a control law that employs delay signals to improve its performance in mitigating seismic perturbation effects on a hysteretic base-isolation system. One of the contributions of this work is the proposal of an active control by means of delay signals (in the control design of structures the use of delay signals is still not very common, but an emerging technique). According with numerical experiments, where the hysteretic system is perturbed utilizing the "El Centro" earthquake, our controller can reduce 89% of the peak-base perturbed displacement with respect to the open-loop response by employing a peak-force of 19% of the weight of the base structure.

The paper is structured as follows. The problem statement is presented in Section 2. Next, the active control law is developed in Sections 3 and 4. To illustrate the efficiency of the proposed method, in Section 5 numerical simulations are analyzed for hysteretic structural systems in the presence of seismic excitations. The recorded earthquake "El Centro" is used as in Narasimhan *et al.* (2006). Finally, the conclusions are stated in Section 6.

2 Problem statement

Consider a second-order base-isolated structure with an active controller as illustrated in Figure 1. The passive component consists of a hysteretic base isolator (see Ismail *et al.* (2009)):

$$m\ddot{x}(t) + c\dot{x}(t) + \Phi(x,t) = f(t) + u(t) \tag{1}$$

where m and c are the mass and the damping coefficients, respectively; $\Phi(x,t)$ characterizes a nonlinear restoring force, x gives the position, $f(t)$ is an exciting unknown (but bounded) force given by the earthquake ground acceleration, and $u(t)$ is an active control force supplied by appropriate actuators. The nonlinear force $\Phi(x,t)$ presents a hysteresis phenomenon due to the use of inelastic rubber bearings and it can be described by the so-called Bouc–Wen model (see Ikhouane & Rodellar (2005)) in the following form:

$$\Phi(x,t) = \alpha_0 \kappa x(t) + (1-\alpha_0) D\kappa\omega(t), \tag{2}$$

$$\dot{\omega}(t) = D^{-1}(A\dot{x}(t) - \beta_0|\dot{x}(t)||\omega(t)|^{n-1}\omega(t) - \lambda\dot{x}(t)|\omega(t)|^n). \tag{3}$$

This model represents the restoring force $\Phi(x,t)$ by the superposition of an elastic component $\alpha_0\kappa x(t)$ and a hysteresis component $(1-\alpha_0)D\kappa\omega(t)$, in which $D > 0$ is the yield constant displacement and $\alpha_0 \in (0,1)$ is the post- to pre-yielding stiffness ratio (see Ikhouane & Rodellar (2007)). The hysteretic component involves a non-dimensional auxiliary variable $\omega(t)$, which is the solution of the nonlinear differential equation 3. In equation 1, A, β and λ are nondimensional parameters that control the shape and size of the transition from elastic to plastic response as can be seen in Ikhouane & Rodellar (2005); Smyth *et al.* (2002).

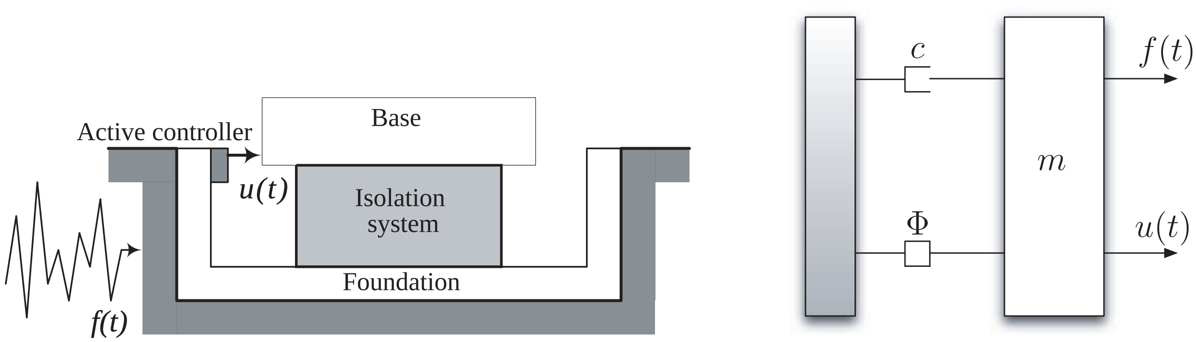

Figure 1: Base isolation system (left) and schematic model (right).

The control law assumes that the hysteretic isolated-base is represented by the Bouc–Wen model. This model has gained large consensus within the engineering community because it can capture a wide variety of different shapes of the hysteresis loops as stated in Ma *et al.* (2004)). Although the internal parameters of the Bouc–Wen model can be manipulated by applying a voltage in magnetorheological dampers, for simplicity, we assume that they are constant as in Ikhouane *et al.* (2005). Moreover, many base-isolated structures have hysteretic behavior with constant parameters (see Ikhouane & Rodellar (2007)). It is noteworthy that the internal variable $\omega(t)$ is uniformly bounded for any piecewise discontinuous signal $\dot{x}(t)$, for certain values of the parameters A, β and λ (see Ikhouane & Rodellar (2007)). Theorem 1 in Ikhouane *et al.* (2005) proves this statement and provides a way to compute the bound.

The main objective of this paper is to design an active controller using delay signals such that the seismic closed-loop perturbed system of the hysteretic structure 1-3 is locally stable.

3 Active controller design

In order to design the active controller, the linearization around the origin of equation 3 is taken,

$$\dot{\widehat{\omega}}(t) = D^{-1}A\dot{x}(t),$$

which implies that

$$\widehat{\omega}(t) = D^{-1}Ax(t). \tag{4}$$

It is noteworthy that equation 4 gives a simple tool for the posterior stability analysis and also, despite its simplicity, it will proof to be a useful observer of the internal dynamic of the Bouc–Wen model just using position measurements. Using equation 4 in 2 gives

$$\Phi(x,t) = \alpha_0\kappa x(t) + (1-\alpha_0)\kappa A x(t),$$

and substituting this last equation in 1 yields

$$m\ddot{x}(t) + c\dot{x}(t) + \kappa(\alpha_0 + (1-\alpha_0)A)x(t) = f(t) + u(t). \tag{5}$$

We propose the control law,

$$u(t) = \kappa(\alpha_0 + (1-\alpha_0)A)x(t) - k_1\dot{x}(t) - k_2 x(t), \quad k_1, k_2 > 0, \tag{6}$$

where k_1 and k_2 are design parameter. This control law yields the following closed-loop system 5-6,

$$m\ddot{x}(t) + (c + k_1)\dot{x}(t) + k_2 x(t) = f(t). \tag{7}$$

Note that equation 7 corresponds to an exponential stable linear system when $f(t) = 0$. When $f(t)$ is bounded the perturbed system is BIBO-stable.

4 Active controller design with delay signals

We propose the control law,

$$u_\tau(t) = \kappa(\alpha_0 + (1-\alpha_0)A)x(t) - k_1\dot{x}(t) - k_2 x(t) - k_3\dot{x}(t-\tau), \tag{8}$$

$$k_1 \geq 10c, \quad k_2 > 0, \quad 0 < k_3 \leq \sqrt{40ck_1 - 400c^2},$$

where τ represents a time delay, and k_1, k_2 and k_3 are design parameters. That is, the information of velocity with time delay has been incorporated to the control law proposed in equation 6. It should be noted that, as on classic linear control theory, zero initial conditions are assumed. The control law with time delay yields the following closed-loop system 5-8,

$$m\ddot{x}(t) + (c + k_1)\dot{x}(t) + k_2 x(t) + k_3\dot{x}(t-\tau) = f(t). \tag{9}$$

Theorem 1 *The control law 8 stabilizes the system 1-3 when $f(t) = 0$. When $f(t)$ is bounded the perturbed system is BIBO-stable.*

Proof 1 *Let's take the Lyapunov function*

$$V = \frac{k_2}{2}x^2(t) + \frac{m}{2}\dot{x}^2(t) + 10c\int_{t-\tau}^{t} \dot{x}^2(\xi)\, d\xi,$$

then the derivative along the system trajectories yields

$$\dot{V} = k_2 x(t)\dot{x}(t) + \dot{x}(t)\left[m\ddot{x}(t)\right] + 10c\dot{x}^2(t) - 10c\dot{x}^2(t-\tau).$$

Using equation 9

$$\dot{V} = k_2 x(t)\dot{x}(t) + \dot{x}(t)\left[-(c+k_1)\dot{x}(t) - k_2 x(t) - k_3\dot{x}(t-\tau) + f(t)\right] + 10c\dot{x}^2(t) - 10c\dot{x}^2(t-\tau),$$

which can be rewritten as

$$\begin{aligned}\dot{V} &= -(c+k_1)\dot{x}^2(t) - k_3\dot{x}(t)\dot{x}(t-\tau) + 10c\dot{x}^2(t) - 10c\dot{x}^2(t-\tau) + f(t)\dot{x}(t)\\ &= -(k_1 - 10c)\dot{x}^2(t) - k_3\dot{x}(t)\dot{x}(t-\tau) - 10c\dot{x}^2(t-\tau) - c\dot{x}^2(t) + f(t)\dot{x}(t).\end{aligned}$$

Finally, in matrix form,

$$\dot{V} = -\begin{pmatrix}\dot{x}(t) & \dot{x}(t-\tau)\end{pmatrix}\underbrace{\begin{pmatrix} k_1 - 10c & \frac{k_3}{2} \\ \frac{k_3}{2} & 10c \end{pmatrix}}_{A}\begin{pmatrix}\dot{x}(t)\\ \dot{x}(t-\tau)\end{pmatrix} - c\dot{x}^2(t) + f(t)\dot{x}(t). \tag{10}$$

Using Sylvester's criterion it can be seen that when

i) $k_1 - 10c \geq 0 \Longleftrightarrow k_1 \geq 10c$,

ii) $(k_1 - 10c)10c - \frac{k_3^2}{4} \geq 0 \Longleftrightarrow k_3 \leq \sqrt{40ck_1 - 400c^2}$,

the matrix A is positive semi-definite. Thus, taking parameters k_1 and k_3 satisfying conditions $i)$ and $ii)$ we can guarantee that when $f(t) = 0$ the system is stable, as $\dot{V}$ is negative semi-definite. When $f(t)$ is bounded, from equation 10, it can be written that

$$\dot{V} \leq -c\dot{x}^2(t) + f(t)\dot{x}(t) \leq -c\dot{x}^2(t) + |f(t)||\dot{x}(t)| = -|\dot{x}(t)|\left[c|\dot{x}(t)| - |f(t)|\right],$$

which is negative semi-definite when

$$|\dot{x}(t)| > \frac{|f(t)|}{c}. \tag{11}$$

In conclusion, taking parameters k_1 and k_3 satisfying conditions $i)$ and $ii)$, when $f(t)$ is bounded the system is BIBO-stable under the ultimate bound given by equation 11.

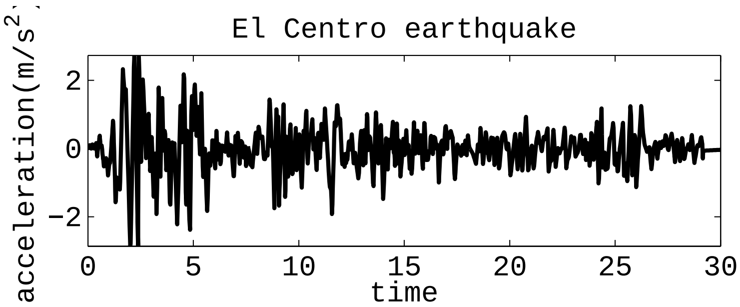

Figure 2: "El Centro" earthquake

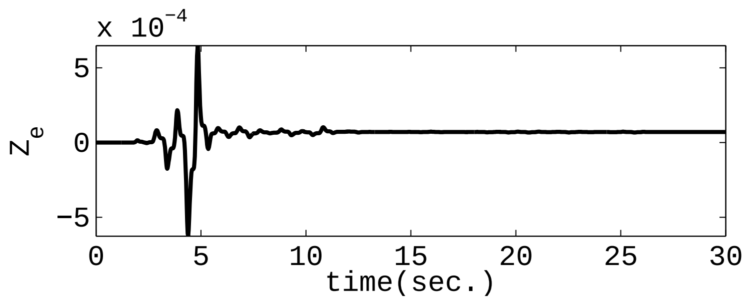

Figure 3: Comparison between the internal dynamic of the Bouc–Wen model, $\omega(t)$, and the proposed observer, $\widehat{\omega}(t)$.

5 Numerical simulations

In this section, responses of the proposed method subject to the accelerogram of "El Centro" 1940 earthquake are studied, see Figure 2. The parameters used in the simulations have the following nominal values: $m = 156 \times 10^3$ Kg, $c = 2 \times 10^4$ Ns/m, $\kappa = 6 \times 10^6$ N/m, $\alpha_0 = 0.6$, $D = 0.6$ m, $A = 1$, $\beta_0 = 0.1$, $\lambda = 0.5$, and $n = 3$ as in Ikhouane *et al.* (2005); Acho & Pozo (2009).

Firstly, the open-loop system is studied (equations 1-3 with $u(t) = 0$) to compare the internal dynamic of the Bouc–Wen model, $\omega(t)$, with respect to the observer defined in equation 4, $\widehat{\omega}(t)$. Figure 3 shows the difference between these two quantities, $Z_e(t) := \omega(t) - \widehat{\omega}(t)$. It can be seen that the proposed observer has a satisfactory behavior, as $Z_e(t) = \mathcal{O}(10^{-4})$, and thus it corroborates the usefulness of the proposed observer.

Secondly, the closed-loop system with active control without delay signals is studied (equations 1-3 and 6). Simulation results are shown in Figure 4. When $k2 = 100c$ the infinity norm of the applied active control without delay signals reaches a minimal value around $k_1 = 100c$ and the obtained infinity norm of the displacement base, for these values of k_1 and k_2, is also small. Thus, henceforth the value $k_1 = k_2 = 100c$ will be used for the numerical simulations. Figure 5 shows a time history of the base displacement comparing the open-loop and closed-loop responses. The proposed active control law without delay signals is effective in reducing the responses of the structure from the very best beginning of the earthquake.

Thirdly, the closed-loop system with active control with an incorporated delay signal is studied (equations 1-3 and 8). Figure 6 shows a comparison between the obtained displacement time history when using or not a delay term in the control law. When using the control law 8 we take the parameters $k_1 = k_2 = 100c, k_3 = 50c$ and $\tau = 0.1$. The control law with delay signal further reduces the displacement response of the structure with respect to the control law without delay. To illustrate this result the following index is

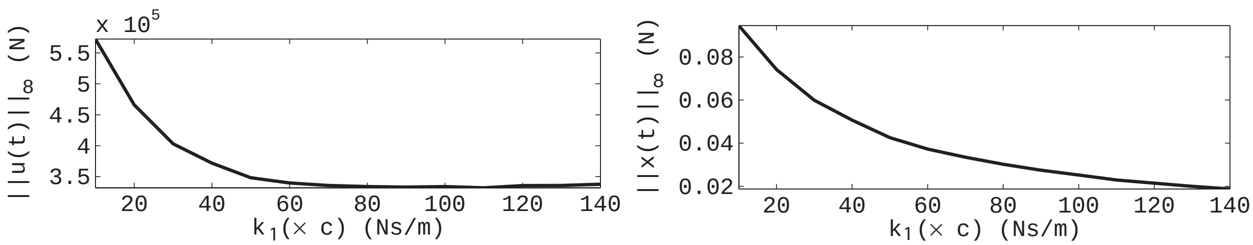

Figure 4: The infinity norm of the applied active control without delay signals (left) and the obtained infinity norm of the displacement base (right) with respect to the parameter k_1.

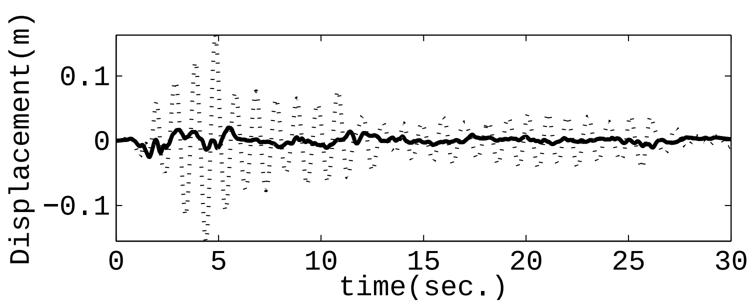

Figure 5: Displacement time history. Open loop (dashed) versus closed loop without delay signals (solid).

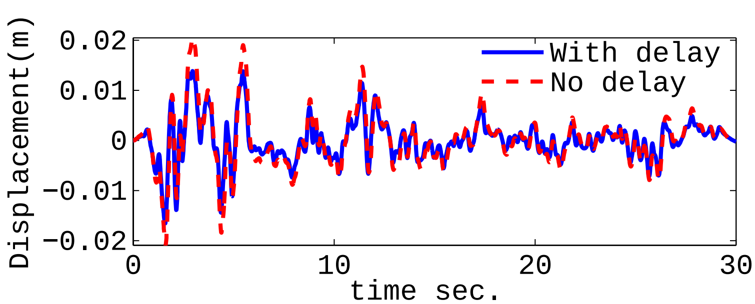

Figure 6: Displacement comparison between using or not a delay term in the control law.

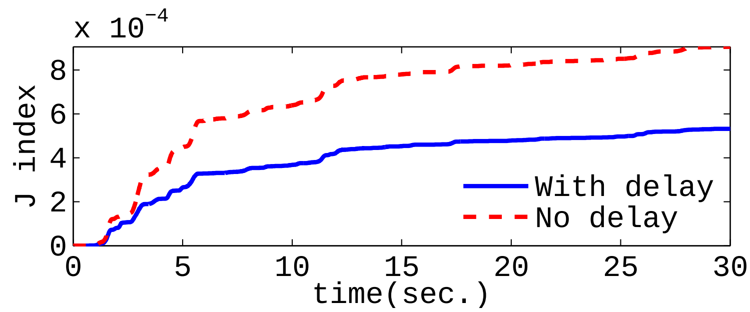

Figure 7: J index comparison between using or not a delay term in the control law.

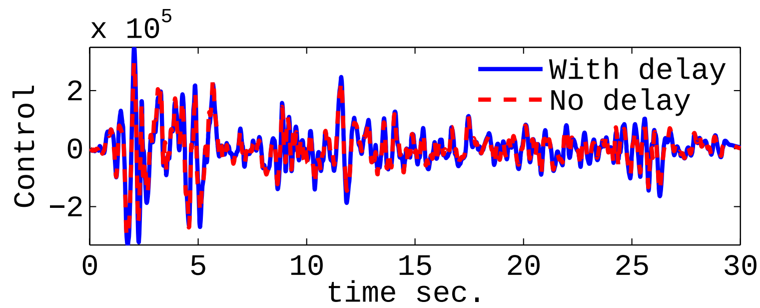

Figure 8: Control signal comparison between using or not a delay term in the control law.

computed

$$J = \int x^2 dt. \tag{12}$$

Figure 7 shows that the J index when using delay signals is halved versus the control law without delay terms. It is noteworthy that this improvement is accomplished without increasing the magnitude of the control signal, as can be seen in Figure 8. The proposed active control law with delay signal is effective in reducing the responses of the structure from the very best beginning of the earthquake. Moreover, as can be seen in Figure 8, the controller is capable of reducing the response up to about 89% with a magnitude of the control law that is below 19% of the structure weight. In order to decrease the magnitude of the control law a saturation function could be applied to the control signal. However, this is kept as future work.

Finally, it is important to note that a bad choice of the time delay, e.g. a value of τ too large, can lead to worse results than the controller without the time delay, as can be seen in Figure 9 where $\tau = 2$s.

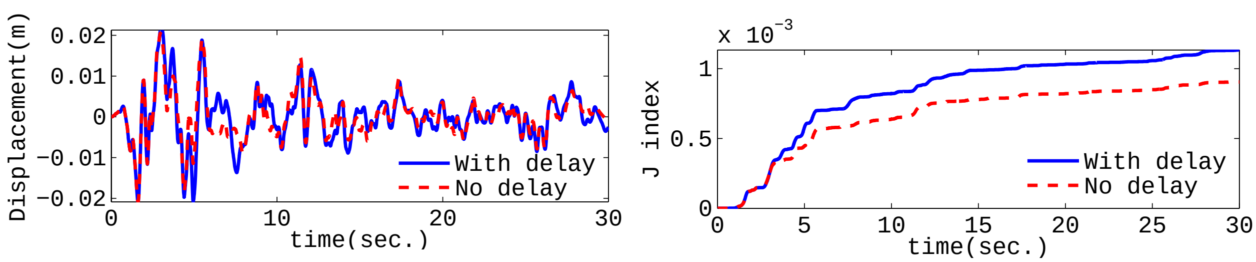

Figure 9: Displacement comparison (left), and J index comparison (right) between using or not a delay term in the control law when a too large value of the time delay has been chosen ($\tau = 2$s).

6 Conclusions

An observer of the internal dynamic of the Bouc?Wen model just using position measurements and an active control law using delay signals is contributed in this work. Numerical simulations, with data from a recorded earthquake, have demonstrated the highly effective performance and robustness of the proposed active control law in hysteretic base-isolated systems.

Acknowledgements

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