

# Estimating fuel consumption from radar tracks

## A validation exercise using FDR data from descent trajectories

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**Abstract**—Fuel consumption is one of the major considerations for both the impact of aviation in the environment and the cost of operations. This paper assesses the accuracy of a method capable of producing aircraft fuel estimates based on their 4D trajectory and the weather forecast. Fuel consumption estimates generated for 2448 descents are compared with the flight data recorder (FDR) values provided by the airline. Fuel consumption is estimated by taking the 4D trajectory from two different sources: the FDR system itself and surveillance radar tracks. In both cases, the Base of Aircraft Data (BADA) and a model that fits manufacturers' performance data to polynomial functions are used to represent aircraft performance. Results obtained with the later show that fuel usage could be estimated with an accuracy of 16 kg (4.8%) by using the 4D trajectory as reported by the FDR system and 28 kg (7.8%) by using surveillance radar observations. It is also observed that the BADA 3.6 model underestimates the fuel consumption, illustrating the need for an improved performance model in the terminal manoeuvring area.

### I. INTRODUCTION

With the awareness of global warming and the rising of fuel prices, reducing fuel consumption (and therefore emissions) has become one of the main concerns of the different aviation stakeholders. One way of reducing fuel consumption is through the introduction of more optimal and environmentally friendly procedures in the Terminal Maneuvering Area (TMA), such as Continuous Descent Operations (CDO) [1], [2] and Continuous Climb Operations (CCO) [3]. Assessing the economic and environmental benefits of CDO and CCO requires computing the fuel savings for many flights performing these procedures. In this context, accurately estimating fuel consumption becomes a key factor to enable more informed policy and investment decisions for future airspace designs.

Traditionally, fuel consumption and emissions of aircraft have been estimated by using the International Civil Aviation Organization (ICAO) time-in-mode method [4]. This method assumes the reference Landing and Take-Off (LTO) cycle and takes the emission and fuel flow factors from the ICAO emissions database. Patterson et al. [5] compared actual fuel flow and time-in-mode as reported by several aircraft's Flight Data Recorder (FDR) systems with the ICAO standard, showing that this standard is not representative of current airline operations. In a later study [6], it was reported that fuel consumption in the TMA is generally overestimated by the ICAO method.

Aiming at facilitating environmental review activities, the Federal Aviation Administration (FAA) developed the Aviation

Environmental Design Tool (AEDT). AEDT models aircraft performance in space and time to estimate fuel consumption, emissions and noise, consolidating the modelling of these environmental impacts in a single tool. Even if being capable of accurately estimating fuel consumption in the TMA [7], its core algorithm takes the thrust profile directly from the FDR system [8], making fuel consumption a straightforward calculation not applicable when this data is not available. In such cases, AEDT relies on standard thrust and/or speed profiles to compute the environmental impacts. These standard profiles are not able to capture the effects of Air Traffic Management (ATM) procedures such as level flight segments at constant altitude or radar vectoring, which are common Air Traffic Control (ATC) separation practices in busy TMAs.

Previous work [9], [10] present a method to compute fuel flow without requiring neither FDR specific data nor an explicit model of the speed or thrust schedule of the aircraft operational procedure. Fuel is estimated using only the 4D trajectory data gathered from a sequence of surveillance observations, such as radar track data, along with a weather forecast/nowcast of the area of concern. The procedure consists on estimating aircraft and wind states, lift, drag and thrust using a point-mass representation of the dynamics of the aircraft. Both authors adopted the the EUROCONTROL's Base of Aircraft Data (BADA)[11] to model drag, thrust and the fuel flow as a function of these estimated states; and validated the procedure against FDR data using only a single example flight.

In this paper, the method presented in [9], [10] is implemented and validated against more than 2400 descent operations which took place in a certain European airport between 2012 and 2014. For all flights in this set, the accuracy of the algorithm is assessed by comparing the estimated fuel consumption using radar track data with the actual fuel consumption as reported by the FDR system. The sensitivity of the method to the quality of the data is also analysed by using FDR trajectory track data as input. Another contribution of this paper is to quantify the improvement in accuracy that could be achieved if instead of BADA performance data, accurate aircraft performance data from the manufacturer were used. In this context, performance data extracted from the Airbus Performance Engineering Programs (PEP) have been used.<sup>1</sup>

<sup>1</sup>Airbus PEP software provides high degree of precision in the certified aircraft performance data and uses specific Flight Management System (FMS) algorithms for the computations.

## II. AIRCRAFT DYNAMICS

The fuel flow is a function on many factors (mainly the aircraft thrust) and several modelling approaches could be adopted to capture these dependencies. Regardless of the final expression used to model the fuel flow, the method proposed in [9], [10] requires the definition of a mathematical model describing the aircraft dynamics, along with a model for certain atmospheric variables. These models will be used to estimate, from the track observations, certain aircraft states and atmospheric variables required to compute the fuel flow.

In Section II-A a model for the temperature, pressure and wind is presented. In Section II-B the equations describing the aircraft dynamics are presented and two different performance models are proposed: the BADA v3.x specification and a model specifically developed for this paper where Airbus performance data was fitted into continuous functions. However, it should be noted that the method presented in Section III is generic and valid for any other performance model.

### A. Weather model

Typical weather models for ATM studies assume the International Standard Atmosphere (ISA) [12] for both temperature ( $\tau$ ) and pressure ( $p$ ) profiles. In this paper, however, generic expressions are considered for  $\tau$  and  $p$ , which are supposed to be known as a function of the geometric altitude ( $h$ ), geodetic latitude ( $\lambda$ ) and longitude ( $\varphi$ ). Provided that  $\tau$  and  $p$  values are known, the density ( $\rho$ ) can be obtained by using the perfect gas law relationship:

$$p = \rho R \tau, \quad (1)$$

where  $R$  is the specific gas constant for dry air. The following normalised variables are also used in this paper:

$$\delta = \frac{p}{p_0}; \quad \theta = \frac{\tau}{\tau_0}; \quad \sigma = \frac{\rho}{\rho_0}; \quad (2)$$

where  $p_0$ ,  $\tau_0$  and  $\rho_0$  are, respectively the standard pressure, temperature and density values at sea level.

A similar rationale is adopted to model the wind field, where the horizontal wind is decomposed in North ( $w_n$ ) and East ( $w_e$ ) components, which are supposed to be known functions of  $h$ ,  $\lambda$  and  $\varphi$ . The vertical wind, which is usually two orders of magnitude smaller than the horizontal wind, is neglected.

### B. Aircraft point-mass model

In this paper, the state vector of the aircraft  $\mathbf{x} = [v_a \ \gamma_a \ \chi_a \ h \ n \ e \ m]^T$  is composed, respectively, by the true airspeed, the aerodynamic flight path angle, the aerodynamic heading, the altitude, the North and East positions and the mass of the aircraft. The dynamics of  $\mathbf{x}$  are expressed by the following set of non-linear differential equations, assuming a point-mass representation of the aircraft, neglecting the vertical component of the thrust vector (which is typically orders of magnitude below the aerodynamic lift), neglecting the sideslip angle and considering small angle of attack:

$$\dot{v}_a = \frac{T - D}{m} - g \sin \gamma_a \quad (3a)$$

$$\dot{\gamma}_a = \frac{1}{v_a} \left( \frac{L}{m} \cos \phi - g \cos \gamma_a \right) \quad (3b)$$

$$\dot{\chi}_a = \frac{1}{v_a \cos \gamma_a} \left( \frac{L}{m} \sin \phi \right) \quad (3c)$$

$$\dot{h} = v_a \sin \gamma_a \quad (3d)$$

$$\dot{n} = v_a \cos \gamma_a \cos \chi_a + w_n \quad (3e)$$

$$\dot{e} = v_a \cos \gamma_a \sin \chi_a + w_e \quad (3f)$$

$$\dot{m} = -f, \quad (3g)$$

where  $T$  is the thrust,  $g$  is the gravity acceleration (assumed to be constant) and  $\phi$  is the bank angle. The aerodynamic drag ( $D$ ) and lift ( $L$ ) forces are expressed as a function of the drag coefficient ( $C_D$ ) and lift coefficient ( $C_L$ ), respectively:

$$L = q S C_L, \quad D = q S C_D, \quad (4)$$

being  $S$  the wing surface area and  $q = \frac{1}{2} \rho v_a^2$  the dynamic pressure. In practice,  $C_D$  depends on  $C_L$ , the Mach number ( $M$ ) and the Reynolds number ( $Re$ ). Similarly, powerplant variables such as  $T$  and  $f$  are related through the throttle setting,  $M$ ,  $\tau$  and  $p$ . Several approaches have been proposed to model these dependences, from look-up tables coming directly from the manufacturer data, to more or less sophisticated function approximations that fit these data. Next, the widely used BADA v3.x performance model and an accurate model that uses data from the manufacturer are presented.

1) *BADA v3.x aircraft performance model*: Here, the well known drag polar is used to model  $C_D$ , which is composed by a parasite drag term plus a quadratic term function of  $C_L$ :

$$C_D = C_{D_0} + K_i C_L^2. \quad (5)$$

The coefficients appearing in Eq. (5) should depend on both  $M$  and  $Re$ . Nevertheless, BADA v3.x neglects the compressibility effects and models these coefficients as constants that depend only on the aircraft hyper-lift surfaces configuration.

Regarding the fuel flow, it is determined by the product of the thrust and the specific fuel consumption ( $\eta$ ), which in turn is modelled as a linear function of the true airspeed:

$$f = \eta T = C_1^f \left( 1 + \frac{v_a}{C_2^f} \right) T. \quad (6)$$

It should be noted that the thrust estimated from the radar tracks could be less than the minimum thrust generated by the engines due to errors in the drag model, in the aircraft weight and/or in the airspeed derivative estimation. BADA also provides an expression for the idle thrust ( $T_{idle}$ ), which is modelled as a fraction of the maximum climb thrust ( $T_{max}$ ):

$$T_{idle} = C_{des}^T T_{max}, \quad (7)$$

where  $T_{max}$  is calculated as a function the altitude and the temperature deviation from ISA ( $\Delta \tau_{ISA}$ ) as follows:

$$T_{max} = C_1^T \left( 1 - \frac{h}{C_2^T} + C_3^T h^2 \right) (1 - C_5^T \Delta\tau_{ISA_{EFF}}), \quad (8)$$

with  $\Delta\tau_{ISA_{EFF}} = \Delta\tau_{ISA} - C_4^T$ . Similarly, the fuel flow corresponding to idle conditions ( $f_{idle}$ ) is expressed as a linear function of the geometric altitude through:

$$f_{idle} = C_3^f \left( 1 - \frac{h}{C_4^f} \right). \quad (9)$$

The values for the coefficients appearing in Eqs. (5)-(9) are given in the Operations Performance File (OPF) of the BADA v3.x database, which contains these performance parameters for each specific aircraft model.

2) *Accurate aircraft performance model*: It is well known that BADA v3.x was optimised to model aircraft performance and fuel consumption in cruise phase, and several studies have already revealed that is not accurate enough to derive correct fuel consumption figures in the TMA [13], [7]. For instance, compressibility effects of aerodynamic drag cannot be neglected for typical cruising speeds of the majority of commercial aircraft. Moreover, it has been reported that BADA model tends to underestimate fuel flow in idle conditions. Aiming to obtain better estimations, a performance model that uses tabular data from the manufacturer is proposed herein.

Regarding the  $C_D$ , the following expression is used:

$$C_D = C_{D_0} + K_i (C_L - C_{L_0})^2. \quad (10)$$

where for  $M$  below the drag rise region (where compressibility effects start to be noticeable), the coefficients of Eq. (10) are assumed to be constant without compromising the accuracy of the model. Above certain  $M$ , a polynomial fitting similar to that proposed in [14] is used, giving a very accurate approximation of  $C_D$  when the compressibility cannot be neglected. In both cases, these coefficients are obtained after a fitting process with aerodynamic data obtained from PEP:

$$C_{D_0} = C_{D_{min}} + C_{D_1} M \quad (11a)$$

$$K_i = K_{i_{min}} + K_{i_1} M + K_{i_2} M^2 \quad (11b)$$

$$C_{L_0} = C_{L_{min}} + C_{L_1} M + C_{L_2} M^2. \quad (11c)$$

Relationships for the engine-related variables can be derived by using the Buckingham  $\Pi$  technique of dimensional analysis [15]. Applying simple mathematical analysis, it can be demonstrated that the maximum revolutions of the engine fan ( $N1_{max}$ ) and the residual revolutions when the throttle is zero ( $N1_{idle}$ ) are function of  $M$  and  $\theta$ . In this paper, these dependences are modelled with a third degree polynomial:

$$N1_k = \sum_{i=0}^3 \sum_{j=0}^3 C_{ij}^k M^i \theta^j \quad k \in \{max, idle\}. \quad (12)$$

Following the same methodology, the reduced revolutions of the engine fan ( $\frac{N1}{\sqrt{\theta}}$ ) is modelled as a function of  $M$  and the reduced thrust ( $\frac{T}{\delta}$ ):

$$\frac{N1}{\sqrt{\theta}} = \sum_{i=0}^2 \sum_{j=0}^5 C_{ij}^{N1} M^i \left( \frac{T}{n_e \delta} \right)^j \quad (13)$$

being  $n_e$  the number of engines of the aircraft. Finally, the reduced fuel flow ( $\frac{f}{\delta\sqrt{\theta}}$ ) is a function of  $M$  and  $\frac{N1}{\sqrt{\theta}}$  as:

$$\frac{f}{\delta\sqrt{\theta}} = n_e \sum_{i=0}^2 \sum_{j=0}^5 C_{ij}^f M^i \left( \frac{N1}{\sqrt{\theta}} \right)^j \quad (14)$$

The polynomial coefficients appearing in Eqs. (12)-(14) have been obtained after a least-squares fitting process using the engine performance tables of PEP's database. The relationships presented in these equations easily account for variations in temperature and pressure and are also of the form in which powerplant models are typically provided.

### III. FUEL FLOW ESTIMATION ALGORITHM

The performance models proposed in this paper require the aircraft position (either to derive the weather variables or the compute  $T_{idle}$  and  $f_{idle}$ ) along with the airspeed and thrust values to estimate the fuel flow, which can be computed by using either Eq. (6) if the BADA performance model is used or Eq. (14) if the accurate aircraft performance model is used.

For the remainder,  $(\hat{\cdot})$  will denote "estimated" while  $(\bar{\cdot})$  will be used to represent data directly taken from the FDR system.

#### A. Weather estimation from FDR data

Since the method proposed in this paper can be performed off-line, the required weather data could be obtained from observations stored in Gridded Information in Binary Form (GRIB) formatted files, such those generated by the Rapid Refresh (RAP) or the High-Resolution RAP (HRRR) models of the National Centers for Environmental Prediction (NCEP). These models are updated hourly, and provide weather data on a 13-km and 3-km resolution horizontal grid, respectively. Alternatively, geostatistical methods such that proposed in [16] could be used to estimate the weather from surveillance data. Finally, if access to such data were not available, the ISA model and/or basic wind models could be assumed instead.

As a first step towards validation, in this paper the weather data has been estimated from the FDR reports. The motivation behind this decision is to isolate the fuel estimation errors caused by inaccuracies in the aircraft performance models and flight data. In future works, a sensitivity study on the effects of inaccuracies in the weather data will be performed.

The temperature is estimated from the Total Air Temperature (TAT) and Mach number using the following expression:

$$\hat{\tau} = \overline{\text{TAT}} \left( 1 + \frac{\gamma - 1}{2} \bar{M}^2 \right)^{-1} \quad (15)$$

where  $\gamma = 1.4$  is the specific heat ratio of the air. The true airspeed (TAS) can be also estimated from the estimated temperature and the observed Mach number:

$$\hat{v}_a = \bar{M} \sqrt{\hat{\tau} \gamma R} \quad (16)$$

Operational altitudes in aviation are always given in terms of pressure altitude ( $h_p$ ), the altitude displayed by on-board barometric altimeters. This altitude only depends on the atmospheric pressure, and is computed assuming ISA and the principle of hydrostatic equilibrium. The inverse procedure can be performed to obtain the pressure from the pressure altitude:

$$\hat{p} = \begin{cases} p_0 \left( \frac{\tau_0 - \lambda_\tau \bar{h}_p}{\tau_0} \right)^{\frac{g}{\lambda_\tau R}} & \text{if } \bar{h}_p \leq h_{11} \\ p_{11} \exp \left( \frac{-g(\bar{h}_p - h_{11})}{R\tau_{11}} \right) & \text{if } \bar{h}_p > h_{11} \end{cases} \quad (17)$$

where  $\lambda_\tau$  is the ISA temperature lapse rate; and  $h_{11}$ ,  $\tau_{11}$  and  $p_{11}$  are the standard pressure altitude, temperature and pressure at the tropopause, respectively, as defined in the ISA model.

The calibrated airspeed (CAS) is the speed of an aircraft calibrated to reflect standard atmosphere adiabatic compressible flow at sea level. The following formula defines CAS:

$$v_{CAS} = v_a f(p, M) \sqrt{\frac{\rho}{\rho_0}} \quad (18)$$

where the function  $F$  is given by [17]:

$$F(p, M) = 1 + \frac{1}{8} \left( 1 - \frac{p}{p_0} \right) M^2 + \frac{3}{640} \left( 1 - 10 \frac{p}{p_0} + 9 \frac{p^2}{p_0^2} \right) M^4 \quad (19)$$

Therefore, the density of the air can be deduced by combining Eqs. (18) and (19) and solving for  $\rho$ :

$$\hat{\rho} = \rho_0 \left( \frac{\bar{v}_{CAS}}{F(\hat{p}, \bar{M}) \hat{v}_a} \right)^2 \quad (20)$$

Regarding the wind components, they can be also estimated provided that the FDR data contains the time history of  $\chi_a$ :

$$\hat{w}_n = \hat{s} \cos \bar{\chi}_g - \hat{v}_a \cos \hat{\gamma}_a \cos \bar{\chi}_a \quad (21a)$$

$$\hat{w}_e = \hat{s} \sin \bar{\chi}_g - \hat{v}_a \cos \hat{\gamma}_a \sin \bar{\chi}_a, \quad (21b)$$

where  $\hat{\gamma}_a$  is obtained by solving Eq. (3d) for the flight path angle, using the vertical speed as reported by the FDR system.

### B. Fuel flow estimation from radar tracks

When using radar tracks to estimate the fuel consumption, only information about the aircraft latitude, longitude and altitude time histories is available. In such circumstances, the following expression can be used to estimate the TAS.

$$\hat{v}_a = \sqrt{\left( \hat{n} - \hat{w}_n \right)^2 + \left( \hat{e} - \hat{w}_e \right)^2 + \hat{h}} \quad (22)$$

Unfortunately, North, East and vertical speeds are not provided by the surveillance radar. A first necessary step consists on computing  $\hat{n}$ ,  $\hat{e}$  and  $\hat{h}$  from the observed 4D trajectory.

In this paper, the Earth is assumed to be ellipsoidal and non-rotating as in the World Geodetic System 1984 (WGS-84), which specifies the standard coordinate system and reference ellipsoid used in civil aviation. The Earth's meridian ( $M_E$ ) and prime vertical radius of curvature ( $N_E$ ) are:

$$M_E = \frac{b(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{\frac{3}{2}}}, \quad N_E = \frac{b}{\sqrt{1 - e^2 \sin^2 \varphi}}, \quad (23)$$

being  $b$  the Earth's semi-major axis and  $e$  the first eccentricity. Taking the above consideration into account, the North and East speeds can be computed from the estimated latitude and longitude rates as follows:

$$\dot{n} = \dot{\lambda} (M_E + \hat{h}) \quad (24a)$$

$$\dot{e} = \dot{\varphi} (N_E + \hat{h}) \cos \hat{\lambda}, \quad (24b)$$

where the estimated altitude, latitude and longitude rates and accelerations can be obtained from the sequence of aircraft's position observations by using a state estimator such as a Kalman Filter [18], or a simplified form of observer such as a  $\alpha$ - $\beta$ - $\gamma$  tracker [19]. Alternatively, smooth noise-robust differentiators, smoothing splines [20] or methods to regularise the differentiation process itself [21] could be used. Different from other approaches, in [9] a Proportional-Integral-Derivative (PID) controller-based estimator was proposed. In this paper, the Savitzki-Golay [22] smoothing filter (also called least-squares polynomial smoothing filter) has been selected because of its simplicity and more than acceptable performance when filtering such kind of data.

Solving Eq. (3a) for  $T$  the following expression is obtained:

$$\hat{T} = \hat{D} + \hat{m} \left( \hat{v}_a + g \frac{\hat{h}}{\hat{v}_a} \right). \quad (25)$$

The true airspeed appearing in Eq. (25) is given by Eq. (22), which can be differentiated to obtain  $\dot{\hat{v}}_a$ :

$$\dot{\hat{v}}_a = \frac{\left( \dot{\hat{n}} - \dot{\hat{w}}_n \right) \hat{n} + \left( \dot{\hat{e}} - \dot{\hat{w}}_e \right) \hat{e} + \dot{\hat{h}}}{\hat{v}_a} \quad (26)$$

where the North and East accelerations are obtained by differentiating Eqs. (24a) and (24b), respectively, as follows:

$$\ddot{n} = \ddot{\lambda} (M_E + \hat{h}) + \dot{\lambda} \dot{\hat{h}} \quad (27a)$$

$$\ddot{e} = \ddot{\varphi} (N_E + \hat{h}) \cos \hat{\lambda} - \dot{\varphi} \dot{\lambda} (N_E + \hat{h}) \sin \hat{\lambda} + \cos \hat{\lambda} \dot{\varphi} \dot{\hat{h}} \quad (27b)$$

The aircraft drag depends on  $q$  and  $C_D$  (see Eq. (4)). The former can be computed provided that both  $\hat{v}_a$  and  $\hat{\rho}$  are known. The latter is directly related with  $C_L$  through Eq. (5).  $C_L$  is obtained with Eq. (4), where  $L$  can be estimated by either assuming that the  $L$  balances the weight (i.e.  $L = mg$ ) or by using the trajectory assumptions proposed in [9]. In this paper, the latter approach has been adopted to estimate  $L$ .

The aircraft mass appearing in Eq. (25) (which is also needed for the lift estimation) has a significant effect on total fuel consumption. The estimation of the aircraft mass is an active field of research and several methods have been

proposed recently. For instance, [10] estimates the weight at the initial point of the 4D trajectory based on the estimated take-off weight and how far the aircraft is from the departure airport. This method not only requires to know the aircraft route but also a previous estimation of the aircraft zero fuel weight; another option is to estimate the landing weight from the final approach speed (which values are strongly correlated) as proposed in [23]; using the least-squares or adaptive mass estimation methods compared in [24]; or machine learning techniques as proposed in [25]. It is out of the scope of this paper to assess these methods and, for the sake of the simplicity, the initial weight has been fixed to that reported by the FDR system at the Top Of Descent (TOD).

The estimated thrust is then used to compute the fuel flow using either Eq. (6) if the BADA performance model is used, or Eq. (14) if the accurate aircraft performance model is used. Finally, the fuel flow is numerically integrated (e.g. using a trapezoidal scheme) to obtain the total fuel consumption.

#### IV. RESULTS

In this section the performance of the algorithm presented in Section III is validated against a significant number of flights. The experimental setup is described in Section IV-A. Thereupon, an example of fuel consumption estimation applied to a single flight is shown. Finally, aggregated results are presented in Section IV-C.

##### A. Experimental Setup

The algorithm presented in Section III has been used to estimate the fuel consumption of 2448 flights descending at a certain European airport between 2012 and 2014. All these flights were performed by Airbus aircraft models.

At the lower part of the descent, and well before intercepting the Instrumental Landing System (ILS), aircraft start configuring with flaps according to a manufacturer-prescribed schedule that defines safe flap surface exposure as a function of aircraft speed. In practice, however, the exact moment at which flaps are extended and the landing gear is deployed depends on the particular circumstances and the pilot criteria. In absence of an algorithm capable of capturing changes of configuration, it was decided to consider only the aircraft trajectory from the TOD to 5,000 ft Above Ground Level (AGL).

For each flight, the 4D positions of the trajectory have been taken from two different sources: FDR data provided by airlines and the corresponding surveillance radar observations. In both cases, the BADA v3.x and the manufacturer performance models have been used to model the aircraft performance.

The coefficients of the generic BADA v3.x model have been particularised with those of v3.6, the available version at the moment of performing this study. However, the changes in the values of the coefficients between BADA v3.6 and the newer versions are significant. For example, the idle fuel flow of BADA v3.13 is higher, at all altitudes, than the one provided by BADA v3.6; as a reference, it increases by 8% at FL300. Results with newer versions of BADA are foreseen in future work. However, this paper already quantify how fuel estimations improve with more accurate performance data.

##### B. Example for a single flight

Figure 1 shows the lateral and vertical profiles obtained from the FDR system and the radar observations for a particular flight. The black line represents the aircraft's trajectory as observed by the radar, while the blue line is the trajectory as reported by the FDR system. The green dashed line represents the portion of the trajectory for which the fuel consumption has been estimated by using the algorithm presented herein.

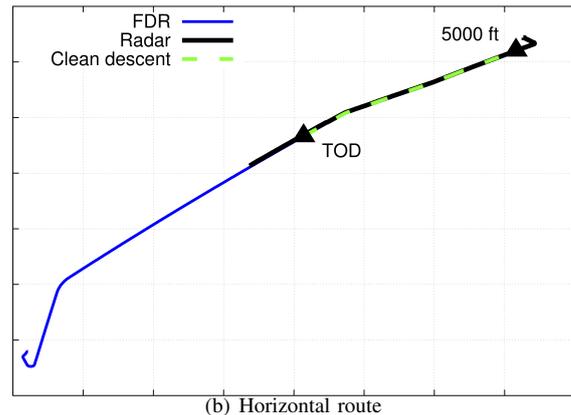
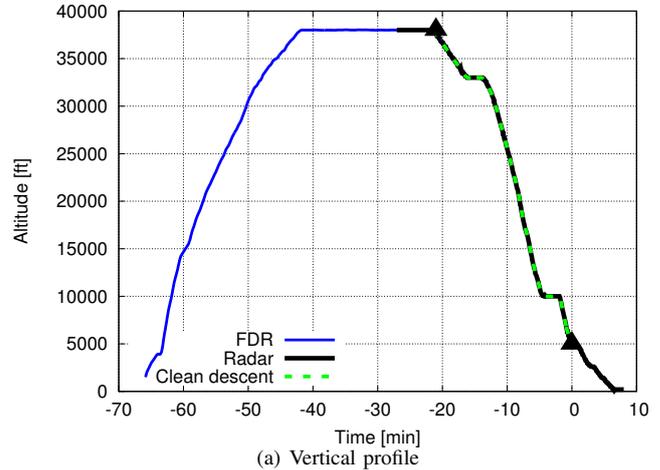
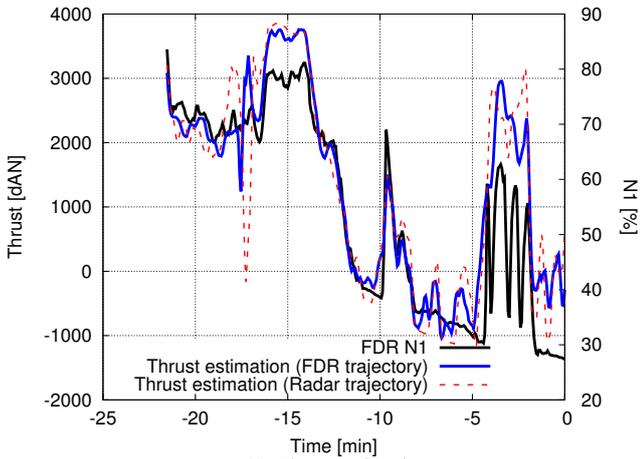


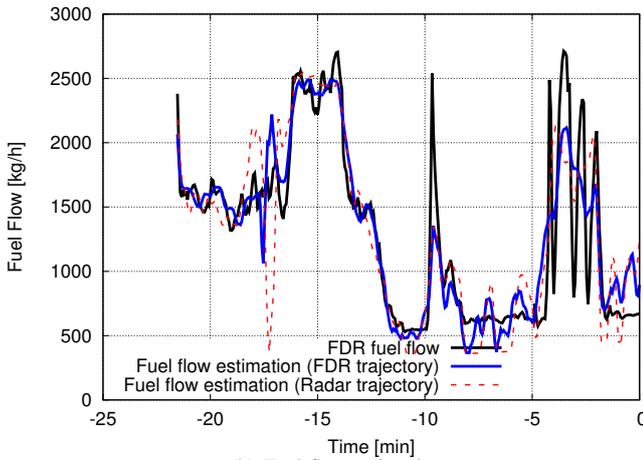
Fig. 1. Example of trajectory

Figure 2 shows the thrust and the fuel flow estimated by using the 4D trajectory as reported by the FDR system and by the radar. Additionally, Figs. 2(a) and 2(b) show the  $N1$  and the fuel flow directly taken from the FDR data, respectively.

According to Fig. 2, the estimated thrust (resp. fuel flow) follows the trend of the FDR and captures most of the  $N1$  (resp. fuel flow) changes during the course of the descent. As expected, thrust and fuel flow were found to be strongly correlated. The spikes appearing at approximately  $-17$  min are occasioned by a wrong and unrealistic estimation of the airspeed derivative induced by the noise in the data. A more appropriate state estimator that not only considers the observations but also the model of the aircraft dynamics (such as the Kalman Filter) would definitely help to fix this issue.



(a) Thrust estimation



(b) Fuel flow estimation

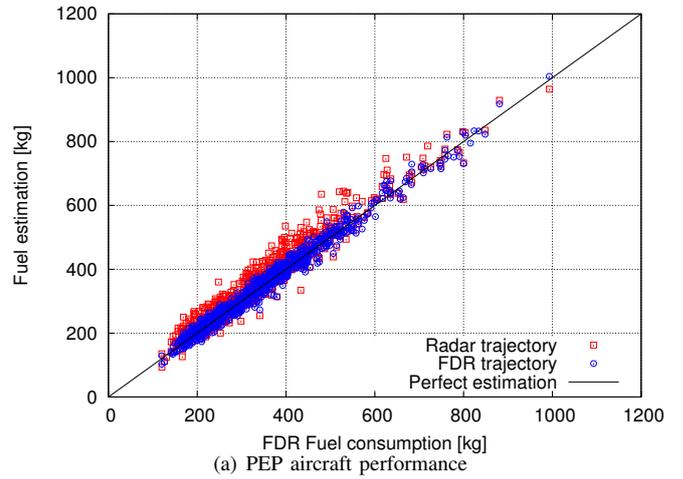
Fig. 2. Thrust and fuel flow estimation for the example flight

### C. Aggregated results

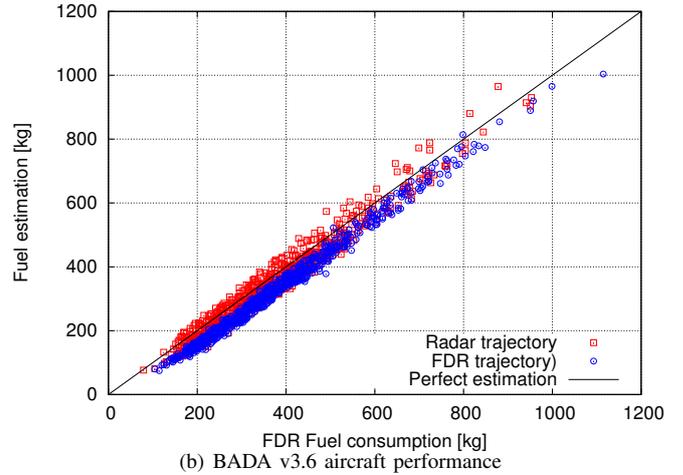
This section summarises the aggregated results obtained after applying the method presented in previous sections to the whole set of flights under study. In Fig. 3 each flight within this set is displayed in a scatter plot, in which the horizontal and vertical axis represent the fuel consumption as reported by the FDR and that estimated from radar tracks, respectively.

According to Fig. 3(a), when the manufacturer performance model is used, the estimation fits very well the FDR reported fuel consumption. The mean error is about 2 kg (0.4%) when using the FDR trajectory and 20 kg (5.1%) when using the radar observations. Results shown in Fig. 3(b) show that BADA v3.6 performance model underestimates the total fuel usage. These results were also reported in [13] and [9]. In this case, the mean error is about -48 kg (-18%) when using the FDR trajectory and -24 kg (-9%) when using the radar tracks.

Surprisingly, for the simulations in which the BADA v3.6 performance model was used, a better estimation was achieved by using the radar trajectory. It is worth noting that this fact does not imply that the fuel flow estimation algorithm performs better with poorer quality data (see Fig. 3(a) for



(a) PEP aircraft performance



(b) BADA v3.6 aircraft performance

Fig. 3. Fuel consumption estimation

instance). It is well known that the numerical differentiation process amplifies any noise in the data [21]. Under given circumstances, this amplified noise can cause wrong estimates for the airspeed derivative, affecting the calculation of the thrust and, consequently, the estimated fuel flow. When the thrust is overestimated, an unlimitedly higher fuel flow can be obtained. Conversely, when the thrust is underestimated, bounds for the minimum thrust and  $N1$  (see Eq. (9) and Eq. (12), respectively) prevent to estimate a fuel flow below that corresponding to idle thrust conditions. Accordingly, since the thrust errors are not balanced at all, the overall result is an overestimation of the total fuel usage. A more appropriate state estimator could help to improve the results.

In Fig. 3, it can be noticed that radar tracks lead to higher fuel consumption estimates compared to those obtained by using the 4D trajectory as reported by the FDR system (regardless of aircraft performance model considered), thus compensating the BADA v3.6 model underestimation.

Tables I and II show the most typical statistical indicators based on the absolute and relative errors, respectively.

According to Tables I and II, the Mean Absolute Error (MAE) ranges from 16 kg to 48 kg, which correspond to

TABLE I  
STATISTICS BASED ON ABSOLUTE ERRORS

Measurement	BADA v3.6 [kg]		PEP [kg]	
	FDR	ATC radar	FDR	ATC radar
MAE	48	32	16	29
MDAE	48	32	14	21
RMSE	50	37	20	38

TABLE II  
STATISTICS BASES ON RELATIVE ERRORS

Measurement	BADA v3.6 [%]		PEP [%]	
	FDR	ATC radar	FDR	ATC radar
MAPE	18.7	11.3	4.8	7.8
MDAPE	17.1	10.5	4.0	6.3
RMSPE	21.0	13.9	6.1	9.9

a Mean Absolute Percentage Errors (MAPE) of 4.8% and 18.7%, respectively. As expected, the lowest average differences between the estimated and the actual fuel consumption are achieved by using the performance model derived from the manufacturer data, regardless the source of the 4D trajectory.

Complementing the MAE and MAPE, other regularly employed indicators are the Root Mean Square Error (RMSE) and the Root Mean Square Percentage Error (RMSPE), which give a relatively high weight to large errors. Since the difference between MAE (resp. MAPE) and RMSE (resp. RMSPE) values shown in Tables I and II are rather small, it can be concluded that the variance in the individual errors is not significant and that the proposed method is quite robust for a given combination of data source and performance model.

Other interesting statistical measures are the Median Absolute Error (MDAE) and the Median Absolute Percentage Error (MDAPE), which represent the middle value of all the absolute errors ordered by magnitude. The advantage of MDAE and MDAPE is that they are not influenced by outliers, even if their meaning is less intuitive. According to Tables I and II, when both manufacturer performance model and FDR trajectory are used, the fuel consumption for half of the flights is estimated with an absolute error lower than 14 kg (4.0%). Using the radar trajectory this result worsens to 21.2 kg (6.3%). Conversely, when the BADA v3.6 is used, the MDAPE are 47.6 kg (17.1%) and 32.0 kg (10.5%) when taking the trajectory from the FDR system and from the radar, respectively.

Aiming at quantifying the statistical dispersion of the estimation errors, a Box-and-whiskers plot is shown in Fig. 4, allowing to easily perceive the medians and the Interquartile Range (IQR) of these errors, defined as the difference between the 3<sup>th</sup> and 1<sup>th</sup> quartiles. In this kind of plot, the bottom and top of the box represent the 1<sup>th</sup> and 3<sup>th</sup> quartiles, respectively. The lines extending vertically from the boxes (whiskers) indicate variability, while the ends of the whiskers represent the 2<sup>nd</sup> and the 98<sup>th</sup> percentiles. The band inside the box is the median and outliers are represented as black points.

According to Fig. 4, the best results are achieved when data from the manufacturer is used to model the aircraft performance. In such case, if the 4D trajectory is taken from the FDR system, the fuel consumption estimation error for

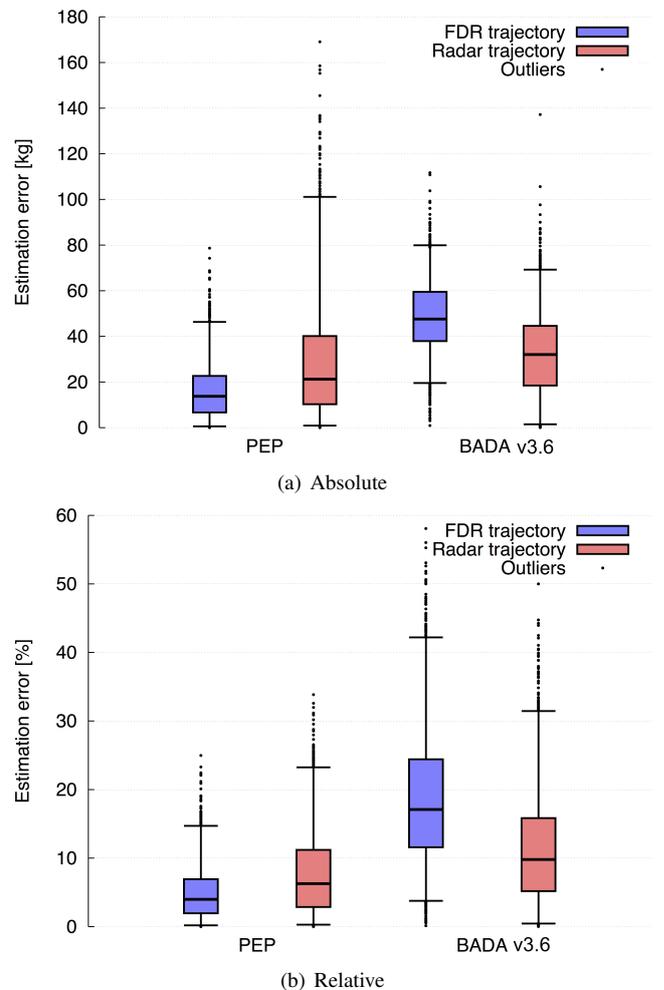


Fig. 4. Fuel consumption estimation errors

25% of the flights is lower than 6.4 kg (1.2 %) while for 75% of the flights this error is lower than 22.2 kg (6.9%). Using the manufacturer performance model and the radar trajectory the estimation error is higher but still acceptable: lower than 10.0 kg (2.9%) for 25% of the flights and lower than 40 kg (11.2%) for 75% of the flights. By using the BADA v3.6 performance model, fuel consumption estimation errors are more significant, especially when the 4D trajectory is taken from the FDR system. Finally, it should be noted that the IQR for the four combination is quite narrow, which is a good indicator of dispersion or variability.

## V. CONCLUSION

High fuel prices and global climate change have been a major concern for the aviation community for many years. Accordingly, fuel efficiency and aircraft emissions have nowadays a strong impact on major airspace designs. In this context, accurate estimation of aircraft fuel consumption is a key factor to enable more informed policy and investment decisions.

Flight Data Recorder (FDR) data is often extremely difficult to obtain. Event though airlines would be the first to feel the potential consequences of changes in the aviation system, they

do not make these data publicly available. In contrast, surveillance data (such as radar or automatic dependent surveillance) is typically more accessible but has some limitations. On one hand, the latency is lower and the quality of the data is worse if compared with those provided by the FDR system. On the other hand, surveillance radar only provides the observed aircraft's identification, latitude, longitude and altitude at a sequence of times (i.e. the 4D trajectory).

Aiming at dealing with the limitations of traditional environmental analysis tools, this paper assessed the accuracy of a method able to take the 4D trajectory of an aircraft along with the weather data and produce fuel estimates without specific knowledge of neither the thrust nor the calibrated airspeed/Mach profile. Yet, some important assumptions were made regarding the aircraft mass estimation.

Results obtained with accurate aircraft performance from the manufacturer showed that fuel consumption could be estimated with an accuracy of 16 kg (4.8%) by using the 4D trajectory as reported by FDR system and 29 kg (7.8%) by using radar observations. It was also proved that the EUROCONTROL's Base of Aircraft Data (BADA) v3.6 performance model underestimates the total fuel consumption, illustrating the need for an improved performance model in the terminal maneuvering area (TMA). In light of these results, the proposed method could be used to enhance the accuracy of TMA studies in which fuel is considered a major factor.

Future research will focus on improved state estimators and smoothers to capture with higher exactness the dynamics of the aircraft. The estimation of the mass of the aircraft is another key issue to be tackled. Moreover, a sensitivity study on the influence of the weather forecast errors (and in particular wind errors) on fuel consumption figures is also foreseen. Last but not least, the performance of the newly BADA v4 performance model for this particular application will be assessed and compared with the existing results; and the use of Automatic Dependent Surveillance - Broadcast (ADS-B) data (which includes information about other valuable flight parameters such as true airspeed and Mach number), combined with radar tracks, is also expected to be investigated.

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