

# Closed-Form Capacity Bounds for Downlink and Uplink Decoupling

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**Abstract**—Downlink (DL) and uplink (UL) decoupling (DUDe) is a new architectural paradigm where DL and UL are not constrained to be associated to the same base station (BS). Thus, a user having access to multiple BSs within a dense cellular network can receive the DL traffic from one BS and send its UL traffic through another. Building upon this architectural paradigm, the present paper provides tight analytical bounds in closed form for the UL ergodic capacity that depend solely on the density of the infrastructure. The devised bounds account for the backbone network congestion and the synchronization of the acknowledgments of the decoupled channels. The proposed bounds are compared against extensive numerical simulations demonstrating the tractability and accuracy of the expressions.

**Index Terms**—Downlink uplink decoupling; ergodic capacity; bound; network density; acknowledgment synchronization.

## I. INTRODUCTION

The advent of multimedia interactive services has induced a change in the entrenched perception of mobile networks, introducing a shift from asymmetric to symmetric traffic loads (i.e. symmetric with respect to the uplink (UL) and downlink (DL) traffic). Specifically, the rise of social media and online video gaming applications resulted in an unabated increase in the UL traffic, which in turn, mandated a dedicated optimization of the UL channel.

In this direction, a new architectural paradigm emerged, which allows for the standalone management of the UL and DL connectivity and, therefore, the dedicated optimization of the UL channel. In classical schemes both the DL and the UL connectivity were driven by the DL conditions, which significantly deteriorated the UL performance. As opposed to this, DL and UL decoupling (DUDe) [1] is a novel approach addressing UL and DL as separate network connections, where user equipment (UE) can be connected to a different serving node in the UL and the DL. Hence, DL and UL are no longer constrained to the same base station (BS) and the independent management and optimization of both channels can provide substantial capacity [1], [2] and power [3] gains.

The feasibility of this approach relies on the density of BSs in current heterogeneous networks (HetNets) and on the disparity between the transmit power of the network elements. The density and disparity of current HetNets is prompted by the need for a 1,000-fold capacity increase for the 5G networks within the next decade [4], which has made the need for ultra dense cellular networks more actual than ever. Building upon the documented success of spatial reuse and ultra-dense cellular topologies [5], disparate and low-power

heterogeneous elements (femto/pico BSs, distributed antennas, relays, etc.) are currently overlaid on top of the existing infrastructure of power-hungry macro BSs, offloading data traffic from those BSs to nearby low-power elements, reducing transmit distances and, hence, increasing spectral efficiency.

In this setting of dense deployments and power disparate network elements, the concept of DUDe came into existence. According to a DUDe policy, a UE residing within the coverage of a distant macro cell (MC) and a close small cell (SC) could receive a higher power from the distant MC in the DL due to the high power and gain of the MC BS. In this course, the UE would connect to the MC in the DL, maximizing the traffic rate. However, given the limited power of the UE in the UL, the connection to the nearby SC would be preferable in the UL over the connection to the distant MC. Thus, the independent management of UL and DL in DUDe provides flexibility which can engender substantial benefits for the network.

Multitude of research works hitherto have documented the gains arising from the employment of DUDe in dense cellular networks [1] and have provided expressions for the evaluation of the UL performance [2]. However, in those expressions the performance of the UL channel is assumed to be independent of the density of the infrastructure, for the sake of tractability, which is not the case for systems encountered in practice [6]. In fact, the performance of the channel depends heavily of the network density.

In this direction, the present paper provides tight analytical bounds in closed form for the UL ergodic capacity, which depend solely on the density of the network infrastructure (i.e. density of SCs and MCs). To the best of the authors' knowledge, all relevant attempts hitherto employed numerical approaches for the calculation of the capacity bounds. As opposed to these approaches, the present paper provides simple bounds for the UL capacity in closed form, building upon the results published by the authors in [7]. Enhancing the results of [7] (where the bounds depend on the distance of the UE to the MC BS), the proposed bounds depend solely on the density of the infrastructure. Hence, the devised bounds provide an insight into the minimum degree of densification, that guarantees meeting the Quality of Service (QoS) objectives. This sets out a densification road map for the network operator and designer of significant practical and commercial value. Moreover, the present analysis shifts from the binary analyses employed hitherto, which focus on the connection of

the UE either to the MC or to the SC in the UL, irrespective of the DL. In the present approach, the decoupled connection in the UL arises as a standalone case, allowing for addressing inherent drawbacks of DUDe. Thus, the present approach accommodates a holistic analysis of DUDe, while specifically accounting for the salient drawback of DUDe [8] which is the synchronization of the acknowledgments (ACK/NAK) of the decoupled channels through the backbone network, which leads to packet losses and reduction of the achievable rate.

The remainder of the paper is organized as follows. Section II presents the considered network architecture and the DUDe association policies. Section III introduces a novel methodology for the calculation of the UL capacity bounds in DUDe, focusing on a network design perspective and taking into account packet losses due to synchronization. Section IV derives the tight analytical bounds, associating the average user capacity in the UL with the density of the SCs. Section V presents the simulation results demonstrating the tight performance of the devised bounds, while Section VI concludes the paper and presents perspectives.

## II. THE WIRELESS CELLULAR NETWORK ARCHITECTURE

### A. The Wireless Network Scenario

A wireless cellular system is considered comprising a MC with a coverage area of radius  $R_0$ . On top of the coverage area of the MC a set of SCs are overlaid, whose positions follow a homogeneous Poisson point process (PPP) of density  $\lambda(SCs/m^2)$ . The MC BS is considered to transmit at a high power, whereas the SC BSs are considered to transmit at a low power. Moreover, for the sake of simplicity in the notation, all UEs and MC and SC BSs are considered to employ only one antenna, whereas the necessary methodology for the extension of the analysis to the multi-antenna case is provided in [7].

All intra-cell users are assumed to be sharing orthogonal resources, as is typically the case [2] and adjacent SCs are assumed to coordinate using different operating frequencies providing an interference free scenario. The soundness of the latter assumption relies on two pillars. On the one hand, since the analysis focuses on the UL channel, the limited transmit power of UEs minimizes inter-cell interference. On the other hand, spatial blockages in mmWave transmissions can facilitate the mitigation of interference within an arrangement of coordinated SCs.

To elaborate, since DUDe is tailored for ultra-dense 5G networks, which are expected to operate primarily in mmWave bands, the extension of the analysis to mmWave cellular networks is imperative and remains to be addressed in future work. In this course, the extension to the mmWave bands can give rise to a novel approach where interference free zones will emerge due to spatial blockages. The size of those isolated zones – which constitutes a frequency reuse radius – can be determined analytically, providing an interference free setup where the SCs within the aforementioned zones do not interfere due to coordination and SCs outside of these zones do not interfere due to the isolation provided by blocking.

### B. Association Policy

Having described the wireless cellular network architecture, the association policy of the assumed DUDe is described hereafter. The DUDe approach described in brevity in Section I gives birth to 3 distinct association cases depending on the distance of the UE to the surrounding BSs. These cases are:

- 1) DL-UL connected to a MC,
- 2) DL-UL connected to a SC,
- 3) DL connected to a MC and UL connected to a SC.

The selection criterion for each of the above association cases is based on the distance  $d_0$  of the UE to the MC BS and the distance  $d$  between the UE and the closest SC BS. Specifically, the UE connects to the closest SC in the DL if the following condition holds:

$$d \leq \mu d_0, \quad (1)$$

where  $\mu = \left(\frac{P_{SC}}{P_{MC}}\right)^{\frac{1}{\beta}} < 1$ ,  $P_{SC}$  is the transmit power of the SC,  $P_{MC}$  is the transmit power of the MC and  $\beta$  is the path-loss exponent [7]. That is, the connection criterion for the UE is the level of the received power from each BS<sup>1</sup>.

On the other hand, the UE connects to the closest SC in the UL if the SC BS is closer than the MC BS, namely if  $d \leq d_0$ . That is, since the UE transmit power is the same when transmitting to a MC or to a SC BS. However, to fully exploit the leeway provided by DUDe in selecting the optimum connectivity, a decision parameter  $\alpha$  is introduced in the notation and the criterion for connecting to the closest SC in the UL is redefined as follows:

$$d \leq \alpha d_0. \quad (2)$$

The decision parameter  $\alpha$  ( $\mu \leq \alpha \leq 1$ ) allows for the extension of the analysis in future work, toward optimizing the overall system connectivity. However, in conventional DUDe and in the present paper it is considered that  $\alpha = 1$ .

The combination of (1) and (2), leads to 3 association intervals for the respective association cases described above:

- 1) DL-UL connected to a MC:  $\mathcal{I}_1(d_0) = \{d : \alpha d_0 \leq d\}$ , (3)
- 2) DL-UL connected to a SC:  $\mathcal{I}_2(d_0) = \{d : d \leq \mu d_0\}$ , (4)
- 3) DL to MC, UL to SC:  $\mathcal{I}_3(d_0) = \{d : \mu d_0 < d < \alpha d_0\}$ . (5)

The probability of a random reference user to reside within the association interval defined in (3)-(5) can be calculated based on the probability density function (PDF) of the distance  $d$  to the closest SC, which for a homogeneous PPP deployment of SCs [7] is given by  $f_d(d) = 2\pi d \lambda \exp(-\lambda \pi d^2)$ . Thus, the probability  $P_1$  corresponding to the selection criterion (3) is:

$$P_1(d_0) = P(\alpha d_0 \leq d) = \int_{\alpha d_0}^{\infty} f_d(x) dx = \exp(-\lambda \pi \alpha^2 d_0^2), \quad (6)$$

<sup>1</sup>In the case of multi-antenna BSs the selection criterion is defined as in [7].

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$$\bar{\mathbf{R}}(R_0) \geq m_1 \log(1 + \gamma\rho \exp(-\beta\mathbb{E}_{\epsilon_1(d_0)}[\log(d_0)])) + m_2 \log(1 + \gamma\rho \exp(-\beta\mathbb{E}_{\epsilon_2(d,d_0)}[\log(d)])) + m_3 \log(1 + \gamma\rho \exp(-\beta\mathbb{E}_{\epsilon_3(d,d_0)}[\log(d)])) (1-p), \quad (7)$$

$$\mathbb{E}_{\epsilon_1(d_0)}[\log(d_0)] = \frac{\left(\text{Ei}(-(\alpha x)^2) - \psi - \frac{\log((\alpha x)^2)}{\exp((\alpha x)^2)}\right)}{2 - 2\exp(-(\alpha x)^2)} - \log(\alpha\sqrt{\lambda\pi}), \quad (8)$$

$$\mathbb{E}_{\epsilon_2(d,d_0)}[\log(d)] = \frac{1}{m_2} \left( -\frac{\psi}{2} + \frac{1}{2(\mu x)^2} \left( ((\mu x)^2 - 1) \text{Ei}(-(\mu x)^2) - 1 + \psi + \frac{1 + \log((\mu x)^2)}{\exp((\mu x)^2)} \right) - \log(\sqrt{\lambda\pi}) + \frac{\log(\sqrt{\lambda\pi})(1 - \exp(-(\mu x)^2))}{(\mu x)^2} \right), \quad (9)$$

$$\begin{aligned} \mathbb{E}_{\epsilon_3(d,d_0)}[\log(d)] &= \frac{1}{m_3} \left( \frac{1}{2(\alpha x)^2} \left( ((\alpha x)^2 - 1) \text{Ei}(-(\alpha x)^2) - 1 + \psi + \frac{1 + \log((\alpha x)^2)}{\exp((\alpha x)^2)} \right) \right) \\ &+ \frac{1}{m_3} \left( -\frac{1}{2(\mu x)^2} \left( ((\mu x)^2 - 1) \text{Ei}(-(\mu x)^2) - 1 + \psi + \frac{1 + \log((\mu x)^2)}{\exp((\mu x)^2)} \right) + \log(\sqrt{\lambda\pi}) \left( \frac{(1 - \exp(-(\alpha x)^2))}{(\alpha x)^2} - \frac{(1 - \exp(-(\mu x)^2))}{(\mu x)^2} \right) \right). \end{aligned} \quad (10)$$


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$P_2(d_0) = 1 - \exp(-\lambda\pi\mu^2 d_0^2)$  corresponding to (4) and  $P_3(d_0) = \exp(-\lambda\pi\mu^2 d_0^2) - \exp(-\lambda\pi\alpha^2 d_0^2)$  corresponding to (5).

### III. SYNCHRONIZATION AWARE ANALYSIS

Examining DUDe from a network design perspective, it becomes evident that the acknowledgments (ACK/NAK) of the decoupled links require strong synchronization and data connectivity (e.g. via fiber) between the BSs [8]. Therefore, the feasibility of DUDe relies heavily on the status of the backbone network and its capability to provide strong synchronization and data connectivity.

However, these network aspects have generally been disregarded in the literature. In this course, we introduce probability  $p$  of having packet losses in the backbone network, since the acknowledgments of the UL and the DL are routed through the backbone when the channels are decoupled. The introduction of  $p$  into the devised bounds allows for characterizing the performance of the UL channel more accurately while accounting for the network implementation aspects of DUDe. The latter constitutes one of the primal shortcomings of the scheme so far.

The association case subjected to the aforementioned synchronization issue is that corresponding to (5). Hence, in the ensuing analysis the UL capacity of the decoupled channel is weighted by the probability  $(1-p)$ , that is the probability of a successful acknowledgment synchronization of the decoupled channels via the network backbone.

### IV. THE UL CAPACITY BOUNDS

#### A. UL Ergodic Capacity vs Distance to MC BS

Having described the network architecture and the methodology for accounting for the synchronization of the decoupled acknowledgments, the analytical bounds for the UL capacity are derived hereafter. The average UL ergodic capacity is obtained by the sum of the conditioned UL ergodic capacities  $\mathbb{E}_h[R|1], \mathbb{E}_h[R|2], \mathbb{E}_h[R|3]$  in the case that each of the 3 association cases are selected, weighted by the probabilities  $P_1, P_2, P_3$  of each of these contingencies happening. The expectation of the ergodic capacities is with respect to the fading coefficient  $h$ , assuming a Rayleigh fading with  $\mathbb{E}[h^2] = 1$ .

Moreover, to obtain the average UL ergodic capacity, the expectation with respect to the distance  $d$  needs to be calculated. Hence, the instantaneous average UL ergodic capacity is given by:

$$\bar{\mathbf{R}}(d_0) = \mathbb{E}_h[R|1]P_1 + \mathbb{E}_{h,d|2}[R|2]P_2 + \mathbb{E}_{h,d|3}[R|3]P_3(1-p). \quad (11)$$

In the above expression the first term corresponds to the coupled connection to the MC and therefore, it is independent of the distance  $d$  to the closest SC. As opposed to that, the second and third terms are averaged over the distance  $d$ , conditioned to the fact that  $d$  falls within the association interval imposing the selection of the respective association.

Employing Jensen's inequality for the convex function  $\log(1 + \exp(x))$  the following lower, albeit tight bound holds:

$$\mathbb{E}[\log(1+g(x))] \stackrel{(\cdot)=\exp(\log(\cdot))}{\geq} \log(1+\exp(\mathbb{E}[\log(g(x))])), \quad (12)$$

where  $\log(\cdot)$  in all the expressions represents the natural logarithm and all bounds henceforth are given in (nats/s). Applying (12), to the first term of (11) which corresponds to the average over fast fading the following bound is obtained:

$$\mathbb{E}_h[R|1] = \mathbb{E}_h[\log(1 + d_0^{-\beta} h^2 \gamma)] \stackrel{(12)}{\geq} \log(1 + d_0^{-\beta} \gamma \rho). \quad (13)$$

where  $\gamma = \frac{P_{UE}}{\sigma^2 L_{ref}}$  is the SNR at the reference distance of 1 meter with  $P_{UE}$  being the transmission power of the UE,  $\sigma^2$  is the noise power, and  $L_{ref}$  is the equivalent path-loss at a reference distance, which includes also the effects of the transmit and receive antenna gains [7]. In this expression,  $\rho = \exp(\mathbb{E}_h[\log h^2])$  is calculated using the expectation of the logarithm of a Chi-square random variable, leading to [7]:

$$\rho = \exp(\mathbb{E}_h[\log h^2]) = \exp(-\psi), \quad (14)$$

where  $\psi \simeq 0.577$  is the Euler-Mascheroni constant [7]. In the case of a multi-antenna receiver, the preceding analysis is to be revised according to [7].

Since the the random variables  $h$  and  $d|i$  are independent, the following bound can be derived for  $\mathbb{E}_{h,d|i}[R|i]$ ,  $i = 2, 3$  (i.e. for the second and third terms of (11)):

$$\begin{aligned} \mathbb{E}_{h,d|i}[\log(1 + d^{-\beta} h^2 \gamma)] &\stackrel{(12)}{\geq} \\ \log(1 + \gamma \exp(-\beta\mathbb{E}_{d|i}[\log(d)] + \mathbb{E}_h[\log(h^2)])) &. \end{aligned} \quad (15)$$

In order to calculate  $\mathbb{E}_{d|i}[\log(d)]$ , the PDFs  $f_{d|i}(d|i)$ ,  $i = \{2, 3\}$  need to be employed, which are the truncated PDFs for the condition that the distance  $d$  falls within the association interval  $\mathcal{I}_2$  or  $\mathcal{I}_3$ . Hence, the two PDFs are defined as follows:

$$f_{d|i}(d|i) = \begin{cases} \frac{1}{k_i} 2\pi d \lambda \exp(-\lambda \pi d^2), & d \in \mathcal{I}_i(d_0), \\ 0, & \text{elsewhere,} \end{cases} \quad (16)$$

where  $k_i$  is a constant selected appropriately so that the area of  $f_{d|i}(d|i)$  is equal to 1. Accordingly,  $k_2 = P_2(d_0)$  and  $k_3 = P_3(d_0)$ , whereas the term  $\mathbb{E}_{d|i}[\log(d)]$  of (15) is given by:

$$\mathbb{E}_{d|i}[\log(d)] = \int_{\mathcal{I}_i(d_0)} \frac{\log(d) 2\pi d \lambda \exp(-\lambda \pi d^2)}{P_i(d_0)} dd. \quad (17)$$

The above bounds are provided in analytical expressions in [7] for a scenario disregarding the synchronization of the decoupled acknowledgments. However, the extension of the analysis to the current scenario is straightforward.

### B. UL Ergodic Capacity vs MC Radius

It is readily deduced that the above bounds still depend on  $d_0$ . Specifically the dependence is manifested in the probabilities  $P_i(d_0)$ , in (13), and in the integration limits of (17). In order to provide a comprehensive characterization of the UL channel over the whole MC coverage (which is defined by a disk of radius  $R_0$ ), (11) needs to be averaged over  $d_0$  and, thus, the UL ergodic capacity is given by:

$$\begin{aligned} \bar{\mathbf{R}}(R_0) &= \mathbb{E}_{d_0}[\bar{\mathbf{R}}(d_0)] \stackrel{(11),(13),(14),(15)}{\geq} \\ &\mathbb{E}_{d_0} \left[ \log(1+d_0^{-\beta} \gamma \rho) P_1(d_0) \right] + \mathbb{E}_{d_0} \left[ \mathbb{E}_{d|2} [\log(1+d^{-\beta} \gamma \rho)] P_2(d_0) \right] \\ &+ \mathbb{E}_{d_0} \left[ \mathbb{E}_{d|3} [\log(1+d^{-\beta} \gamma \rho)] P_3(d_0) \right] (1-p), \end{aligned} \quad (18)$$

where (14) has already been substituted in (15) before the latter is applied. Assuming that the users are uniformly distributed over the MC coverage, the PDF of the distance  $d_0$  is given by  $f_{d_0}(d_0) = \frac{2d_0}{R_0^2}$  ( $0 \leq d_0 \leq R_0$ ) and the 3 terms of (18) need to be calculated employing  $f_{d_0}(d_0)$ .

In order to apply the bound of (12) to (18), each individual term of (18) is expanded as follows:

$$\mathbb{E}_{d_0} \left[ \mathbb{E}_{d|i} [\log(1 + d^{-\beta} \gamma \rho)] P_i(d_0) \right] = \quad (19)$$

$$\begin{aligned} &\int_0^{R_0} \int_{\mathcal{I}_i(d_0)} \log(1+d^{-\beta} \gamma \rho) f_{d|i}(d|i) f_{d_0}(d_0) P_i(d_0) d d d d_0 = \\ &m_i \int_0^{R_0} \int_{\mathcal{I}_i(d_0)} \log(1 + d^{-\beta} \gamma \rho) \epsilon_i(d, d_0) d d d d_0 \stackrel{(12)}{\geq} \\ &m_i \log \left( 1 + \gamma \rho \exp(-\beta \mathbb{E}_{\epsilon_i(d, d_0)} [\log(d)]) \right). \end{aligned}$$

In (19),  $\epsilon_i(d, d_0)$  is a pseudo PDF, over which an auxiliary expectation  $\mathbb{E}_{\epsilon_i(d, d_0)}$  is applied to allow the employment of (12). Thus, for the three terms of (18) three pseudo PDFs emerge:  $\epsilon_i$ ,  $i = \{1, 2, 3\}$  which are valid for  $0 \leq d_0 \leq R_0$  and  $d \in \mathcal{I}_i(d_0)$ . For each of the pseudo PDFs  $\epsilon_i$  the constants  $m_i$ ,  $i = \{1, 2, 3\}$  are computed for the volume of each PDF to be

TABLE I  
LINK BUDGET PARAMETERS

Parameter	Value	Parameter	Value
Noise Spect. Dens.	-174 dBm/Hz	$P_{UE}$	33 dBm
Noise Power	-104 dBm	$P_{SC}$	33 dBm
Path Loss at $L_{ref}$	25.6 dB	$P_{MC}$	53 dBm
$\alpha$	1	BW	10 MHz
$\mu$	$(0.01)^{0.25} = 0.3$	Path Loss Exp.	4

equal to 1. Thus, after defining the constant  $x = R_0 \sqrt{\lambda \pi}$  – for brevity in the notation –  $\epsilon_i$  and  $m_i$  are given by:

$$\epsilon_1(d_0) = \frac{P_1(d_0) f_{d_0}(d_0)}{m_1}, \quad \epsilon_i(d, d_0) = \frac{f_d(d) f_{d_0}(d_0)}{m_i}, \quad i = \{2, 3\}, \quad (20)$$

$$m_1 = (1 - \exp(-(\alpha x)^2)) / (\alpha x)^2, \quad (21)$$

$$m_2 = 1 - \frac{1 - \exp(-(\mu x)^2)}{(\mu x)^2}, \quad (22)$$

$$m_3 = \frac{1}{x^2} \left( \frac{1 - \exp(-(\mu x)^2)}{\mu^2} - \frac{1 - \exp(-(\alpha x)^2)}{\alpha^2} \right). \quad (23)$$

Expanding (18) as in (19) and applying the bound of (12) we obtain in (7) the bound for the UL ergodic capacity<sup>2</sup>.

*Corollary 1:* The previous analysis and the employment of (12) and (14) can give rise to a remarkably simple bound for the case of a standalone MC, whereas all attempts in literature hitherto for the calculation of this bound involved cumbersome numerical approaches. The average MC ergodic UL capacity is given by:

$$\bar{\mathbf{R}}_{MC}(R_0) = \mathbb{E}_{h, d_0} [\log(1 + d_0^{-\beta} h^2 \gamma)] \geq \log \left( 1 + \gamma \rho R_0^{-\beta} \exp\left(\frac{\beta}{2}\right) \right), \quad (26)$$

whereas (26) can also be employed for the DL capacity, if  $\gamma$  is adjusted to account for the MC BS transmission power.

<sup>2</sup>After employing the following expressions obtained through integration by parts:

$$\int_0^\omega \phi \log(\phi) \exp(-\phi^2) d\phi = \frac{1}{4} \left( \text{Ei}(-\omega^2) - \psi - \frac{\log(\omega^2)}{\exp(\omega^2)} \right). \quad (24)$$

$$\begin{aligned} \int_0^\omega \phi \left( \text{Ei}(-\phi^2) - \frac{\log(\phi^2)}{\exp(\phi^2)} \right) d\omega &= \frac{(\omega^2 - 1) \text{Ei}(-\omega^2) - 1 + \psi}{2} \\ &+ \frac{1 + \log(\omega^2)}{2 \exp(\omega^2)}, \end{aligned} \quad (25)$$

where  $\text{Ei}(\phi) = \int_\phi^\infty \frac{e^{-t}}{t} dt$  is the exponential integral.,

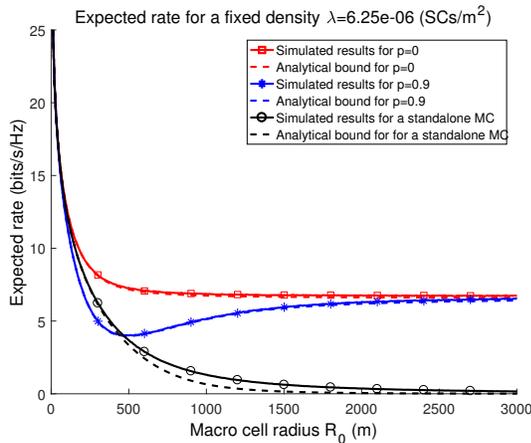


Fig. 1. Expected UL rate vs macro cell radius.

## V. SIMULATIONS

In order to demonstrate the accuracy of the devised bounds, the performance of (7) is compared against extensive Monte Carlo simulations for the link budget parameters tabulated in Table I. The comparison of the analytical bound against the simulation results is depicted in Fig.1, while the bound of (26) is plotted as well in Fig.1 for the sake of completeness.

For low values of  $R_0$  (which correspond to a small MC coverage) the probability of a coupled association to the MC is high. Conversely, for high values of  $R_0$  (which correspond to a large MC coverage) the probability of a coupled association to the SC is high. In those two cases the performance of the un-congested network ( $p = 0$ ) and of the congested network ( $p = 0.9$ ) is identical. However, for the intermediate values of  $R_0$  where the probability of a decoupled connection is high the effect of the packet losses due to the synchronization of the acknowledgments, becomes evident. Hence, the proposed bounds capture accurately the detrimental effect of synchronization in DUDe, while providing an extremely tight performance.

## VI. CONCLUSION

In the present paper we provided tight and closed form capacity bounds, which capture accurately both the performance of the system in terms of the instantaneous average UL ergodic capacity, as well as the detrimental effect of the ACK synchronization in DUDe. This allows for quantifying the effect of the network aspects of the system, to its performance and for addressing them a priori. Moreover, the devised bounds provide an insight into the minimum degree of densification, that guarantees meeting the Quality of Service (QoS) objectives. This, sets out a densification road map for the network operator and designer of significant practical and commercial value.

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