1 Introduction

The main function of weather radars is to provide a description of the precipitation field, typically in four dimensions: temporal evolution of a 3D field, the determination of precipitation, rainfall intensity and the path of storms. Some radars, as the one used in this study, are designed to observe only vertical profiles.

The radar used in this work a Frequency Modulated Continuous Wave Radar (FMCW) (Strauch et al., 1976) radar. The main difference between a pulsed and a FMCW radar is that the latter transmits continuously and can change its frequency modulation to change the vertical resolution while the pulsed radar emits a great power in a short time at regular intervals. Pulsed transmitters are typically used in weather radars systems working at mid and long ranges while FMCW systems are used in low power transmitters often for short ranges applications. The next section describes some characteristics of the FMCW radar used in this study, the Micro Rain Radar (MRR).

The MRR manufacturer offers the equipment with its own software of analysis, but some researchers (for example (Maahn and Kollias, 2012), (Adirosi et al., 2016) among others) have proposed other approaches. The premise of the software is to consider that the raindrop is falling at terminal velocity, which is usually valid for stratiform precipitation, but not for convective storms. Another hypothesis is that the hydrometeors always are in liquid phase, but in numerous occasions this conditions is not met. The shape of snow, crystals or flakes, is completely different from water drops and also the terminal velocity and the complex refractive index.

2 Equipment and Measuring principles

2.1 Equipment

The main equipment is the meteorological radar from Metek, model MRR-2 (Micro Rain Radar). The characteristics of the MRR are shown in Table 1.
<table>
<thead>
<tr>
<th>Table 1: Main characteristics of Micro Rain Radar.</th>
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</thead>
<tbody>
<tr>
<td>Manufacturer</td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
</tr>
<tr>
<td>Radar Type</td>
</tr>
<tr>
<td>Transmit Power (mW)</td>
</tr>
<tr>
<td>Receiver</td>
</tr>
<tr>
<td>Radar Power consumption (W)</td>
</tr>
<tr>
<td>System power consumption (incl. antenna heating) (W)</td>
</tr>
<tr>
<td>Number of range gates</td>
</tr>
<tr>
<td>Range resolution (m)</td>
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<tr>
<td>Resulting measuring range (km)</td>
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<tr>
<td>Antenna diameter (m)</td>
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<td>Beam width (2-way, 6 dB)</td>
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<tr>
<td>Nyquist velocity range (m s$^{-1}$)</td>
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<tr>
<td>Number of spectral bins</td>
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<tr>
<td>Spectral resolution (m s$^{-1}$)</td>
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<tr>
<td>Averaged Spectra (Hz)</td>
</tr>
</tbody>
</table>

The equipment can be divided in 2 hardware element: the antenna dish with the transmitter, and the processing unit data. A heating system is available in the dish to avoid the accumulation of snow.

In the characteristics from MRR radar the power emission is low (50 mW) and its dish diameter is around 0.6 meters. These values make this radar safe, so it can be located in a city, or town, without a big protecting building as in the case of pulsed radars.

Another important factor is the number of range gates, or number of range bins, that the radar is able to process. In this case, the effective range bins number is 31 because the first gate corresponding to 0 m is removed in the processing. The locations of this height can be changed with the frequency modulation; in this case the vertical resolution can be changed from 10 to 200 meters. The choice in this work is a resolution of 100 meters. The choice of vertical resolution marks the maximum height of measurements, in our choice 3.1 km.

The frequency of the equipment is 24.23 GHz thus the wavelength emission from the equipment is around 12 mm. This point is very important because the wavelength is comparable with the size of the drop, so Rayleigh scattering approximation is not valid and the Mie scattering is used.

The number of spectral bins per range gate is another important parameter. Each spectral bin corresponds to a velocity. The resolution of the spectral bin is given from frequency, speed light, number of bins and number of gates. Table 1 shows the speed resolution is around 0.19 m·s$^{-1}$. This limitation implies that the velocity range is from 0 to 12 m·s$^{-1}$.

2.2  Measurement principles

The radar emits vertically and receives the backscattering signal. This signal is function of the velocity of the hydrometeor that are falling, and also of their dielectric constant (for example liquid water, ice and wet snow). In this work is assumed the hydrometeors are in liquid phase similarly as ordinary weather radars.

From this point there are two physical approaches for the treatment of signal. The first one is to consider that the diameter of the raindrops is similar to the radar wavelength, and the other way is to consider that the raindrops are much smaller than the wavelength. The consequences of this decision transform all necessary calculations to obtain $Z$ (dBZ) and the falling velocity $W$(m·s$^{-1}$). In fact, this paper analyses and makes comparisons between these two approaches, where the first one is supported by manufacturer (Messtechnik, 2015) because it assumes the wavelength of radar is similar to the diameter of the raindrop. Thus it is necessary to calculate the backscattering cross section with the hypothesis that the raindrops are falling at
terminal velocities using the Mie theory. The second way adopted by (Maahn and Kollias, 2012) and use the Rayleigh approximation because assumes that the diameter of hydrometeors is smaller than the wavelength, and can be used the usual equation for meteorological radar without hypothesis about the terminal velocities.

The paper explains in detail the two ways and the choice chosen.

3 Calculation process

In this section are described all steps necessary to calculate the moments as reflectivity and the falling velocities from the two methods, terminal and non-terminal velocities:

3.1 Format from file

The starting point for both methods is the radar raw data. The values in raw data files are in ASCII and contain several variables: the transfer function (TF(i), specific by each range bin), and the constant of calibration (specific for each radar), and the values of f(n,i) expressed in engineer units for all Doppler bin n and range bin i.

In both methods the conversion of f(n,i) to η(n,i) is necessary. Figure 2 shows the first lines of a raw data file.

![Figure 2: Example of data from a MRR raw data file](image)

It is important to notice that the Digital Signal Processing (DSP) of the MRR generates about 10 power spectra per second, and with the standard operation parameters, the DSP transfers averaged spectra every 10 seconds to the data acquisition unit. The data transfer via the serial interface takes about 4 seconds. During this time no measurement is possible. Hence the raw data file is created in 10 seconds where the first 6 seconds are acquisition of data and the other 4 seconds are used to make the raw data file. Thus the stochastic standard deviation of spectral powers is reduced to 13% or 0.53 dB, because the system average 60 spectra instead of 100 spectra.

3.2 Signal determination

An evaluation criterion of variance, to detect per each height if there is relevant signal, is used to improve the speed of processing program. The criteria is detailed in (3.1):

\[ \frac{V}{E} \geq 0.6 \sqrt{\Delta t} \]  \hspace{1cm} (3.1)

where E is the average spectrum, V is the variance of the spectrum and Δt is the time of the spectrum, in this case 10 s.

If the value of the ratio between E and V is higher than the other terms the spectrum pass the filter and will be evaluated. If the spectrum does not pass the criteria is considered as “no data”. The value of 0.6 is quiet conservative, but it is used as starting point.

The second filter is based in (Hildebrand and Sekhon, 1974) and the filter condition is shown in (3.2). If the spectrum does not pass the filter then it is considered as “no data”

\[ \frac{E^2}{V} \geq n \]  \hspace{1cm} (3.2)

where E is the average spectrum and V is the variance of the spectrum. The value of n= 58 is acceptable for a spectrum recorded every 10 s.
3.3 Noise determination

In this point resides the main contribution of this work to improve the post-processing of MRR raw data. In the case of signal detection, the noise evaluation from MRR radar data is to consider the noise as “white noise”, or independent of the frequency. \((3.2)\) is used to determine this noise.

In the manufacturer’s processing (Messtechnik, 2015) exists a loop where the maximum signal is eliminated and then it checks again the algorithm details in \((3.2)\). This loop stops when the condition is not accomplished. When it occurs, the remained signal value is estimated as noise and subtracted from signal. (See Figure 3.). Following this approach the process of noise determination does not use all bins. Figure 5 shows the minimum bin (blue) until the maximum bin (red) used, hence the manufacturer does not use whole interval of velocities, only a part of available spectral bins.

![Figure 3: Process to calculate the noise level in (Messtechnik, 2015) way](image)

In case of (Maahn and Kollias, 2012) the process is dynamic and is calculated for each range bin. The dynamic detection of the noise level at each range allows the detection of weak echoes and the elimination of artifacts caused by radar receiver instabilities. Similar as the (Messtechnik, 2015) method with \((3.2)\), but it only gets the maximum and adjacent bins. Another process to detect false signals has been applied and detailed at (Maahn and Kollias, 2012). In the code of IMPRoToo from (Maahn and Kollias, 2012) various fields are interpolated for this purpose, specially the gates number 0, 1 and 30, corresponding to heights, and the bins 1, 2 and 64 corresponding to velocities.

![Figure 4: Relation of the bin used by (Messtechnik, 2015)](image)
In this work the noise determination is realized using an new algorithm to detect the peak value. The advantage of this method is to consider all bins in all heights. Which allows to detect weak echoes.

In Figure 5, the left panel) the detection of a weak echo and the correction of noise level is shown. The middle panel shows the use of bins that the other methodologies have not used and the right panel the algorithm marks as “no data” an input signal.

![Figure 5](image)

**Figure 5:** Example of our own algorithm of noise determination. Left: at 100 m, the first height, of a weak echo from raw data file of real values. (Source: 0116.raw). Middle: at 1300 m. (Source: 0325.raw) Right: at 1600 m (Source: 0325.raw)

3.4 Spectrum average

In section 2.1 it was detailed the processing of the spectrums for all heights created every 10 seconds, but it is interesting to determinate the value for other period. The most usual is 60 seconds. For this reason, it is necessary to average the input signal. This seems easy, but here, the processing task is very important. In this work the program finds the null values and applies a configurable filter. This filter consists in determining the number of null on the number of samples at each height. The number of null number is configurable and is shown in Figure 6 wherein the limit acceptance is set to 50 %. Thus in the best case in a minute the system generates 6 spectrums, with this filter only will be good if the detection ise during 30 seconds.

![Figure 6](image)

**Figure 6:** Filter function in spectrum average

If the numbers of nulls is less than 50% of the size sample the sample is considered good and the averaged is done with the non nulls values. If the number is over 50%, the size sample result is a null sample. This filter allows increasing the threshold of detection.

![Figure 7](image)

**Figure 7:** Example noise detection signal from raw data file of real values. (Source: 0325.raw)
3.5 Dealing with folded velocities (Velocity folding)

Dealing with folded velocities is proposed by (Maahn and Kollias, 2012). This phenomenon occurs when the observed Doppler velocity exceeds the Nyquist velocity boundaries (±6 m s$^{-1}$) of MRR. The MRR Doppler spectra records have a velocity range from 0 to +12 m s$^{-1}$. Thus, by default the MRR assumes the absence of updrafts (negative velocity) considering that all negative velocities are from hydrometeors with a terminal velocity that exceed +6 m s$^{-1}$.

This hypothesis of terminal velocity is good enough for rain but does not apply to snow. For this reason to apply the dealing methodology is recommended. Figure 8 shows the contribution of the dealing process in the analysis of data and Figure 9 shows the detection from this work.

The treatment of dealing process consists in the evaluation of the spectrum at one height and the immediately upper and lower range bins at the same time. First it is necessary to calculate the reflectivity at this height, and then to calculate the velocity estimated, using the equation from (Atlas et al., 1973) and doing the average between the velocity estimated by snow and rain.

The dealing process consists in the evaluation of three range bins: current, lower and upper bins, where the velocities are from 0 m s$^{-1}$ to 12 m s$^{-1}$, from -12 m s$^{-1}$ to 0 m s$^{-1}$ and from 12 m s$^{-1}$ to 24 m s$^{-1}$ respectively. The maximum of each height closest to the velocity estimated will be considered to be the true value, and indicates the existence of updrafts if closest maximum is the lower range bin, downdrafts if the closest maximum is the upper range bin or none if the maximum is the current height.

The dealing is also implemented in the program done, but note that it is only interesting when there is a snowstorm because then the vertical velocities are very influenced by updrafts and its fall velocities can be less than 2 m s$^{-1}$.

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**Figure 8**: Right: Example "Waterfall" diagram from (Maahn and Kollias, 2012), left: Example correction by dealing from the work (Adirosi et al., 2016)

**Figure 9**: Example of detection of the three velocities from our application
3.6 Drop size distribution \( N(D,i) \)

Assuming that hydrometeors fall with their terminal fall speed is the MRR manufacturers choice (Messtechnik, 2015). This choice allows to relate some parameters as the diameter of the raindrops, with their velocity and then to calculate the backscattering cross sections following the relation of (Atlas et al., 1973) and (Gunn and Kinzer, 1949).

If the diameter is known, the particle backscattering cross section, \( \sigma(D) \) can be calculated using the Mie Theory. However this theory has not an analytical equation. The result is a matrix of values in function of height \( i \) and Doppler spectrum \( n \) where the diameters from raindrop must be calculated detailed in (3.3):

\[
D_{mn} = -\frac{1}{0.6} \cdot \ln \left( \frac{1}{10.3} \left( 9.65 - \left( \frac{\nu}{\delta \nu} \right) \right) \right) \quad (3.3)
\]

where :
\( \nu \) is the terminal velocity from (3.4)

\[
\nu(n) = n \cdot \Delta f \cdot \frac{\lambda}{2} \quad (3.4)
\]

And \( \delta \nu \) is the correction for height \( \delta \nu \) and \( h \) is in meters (3.5).

\[
\delta \nu = 1 + 3.68 \cdot 10^{-5} \cdot h + 1.71 \cdot 10^{-9} \cdot h^2 \quad (3.5)
\]

Once the diameters from drops, in every range bin, have been calculated, it is important to remember that the \( \sigma(D) \) is also a function of the complex refractive index \( m \). Basically it is necessary to calculate the value for \( |K|^2 \) following (3.6):

\[
|K|^2 = \left| \frac{m^2 - 1}{m^2 + 1} \right|^2 \quad (3.6)
\]

where the value of \( |K|^2 \) for liquid water is 0.92 and for ice spheres of 0.18.

In this work, the matrix of the backscattering cross section is implemented using a Matlab functions from (Mätzler, 2002), and the results are saved in a txt file. The txt file will be used in our python code. The reason to use Matlab is because the function of Mie theory in python has not been checked and in Matlab it is implemented.

Once the backscattering cross section has been calculated the next parameter is the drop size distribution \( N(D) \) detailed in (3.7), that will be necessary for the moments calculations

\[
N(D) = \frac{\eta(D,i)}{\sigma(D)} \quad (3.7)
\]

where:
\( \sigma(D) \) is the backscatter cross section
\( \eta(D,i) \) is calculate follows (3.8)

\[
\eta(D,i) = \eta(v,i) \cdot 6.18 \cdot \delta \nu(i \cdot \Delta h) \cdot \exp(-0.6 \cdot D) \quad (3.8)
\]

where \( D \) is the diameter from raindrop in mm.

For the case of (Maahn and Kollias, 2012) these parameters are not necessary because follow the Rayleigh approximation.

The use of Mie Theory or Rayleigh approximation affect at the backscattering cross section as shown in Figure 10,
3.7 N(D) with attenuation correction

The intensity of radar electromagnetic waves is attenuated on the propagation path by different processes. The absorption by water vapour at 24 GHz, around 0.2 dB/km, is neglected because the path considered is around 3 km. But the rain intensity can attenuate significantly at moderate and higher rain rates, when the altitude is considered. To deal this problem, the attenuation correction is implemented follows the method discussed in detail by (Peters et al., 2010). Thus a loop for all height and all Doppler spectra is necessary to calculate the Path Integrated rain Attenuation denoted as PIA. In this loop parameter used is the single particle extinction coefficient, denoted by $\sigma_e$, and described in the Mie theory.

The calculation starts in the first range gate $i=1$ ,

$$PIA(r_i) = 1$$

$$N_p(D_{nn}, r_i) = N_a(D_{nn}, r_i) \cdot PIA(r_{i-1})$$

$$\kappa_p(r_i) = \sum_{nn_{\text{max}}(h)}^{nn_{\text{min}}(h)} \sigma_e(D_{nn}) \cdot N_p(D_{nn}, r_i) \cdot \Delta D_{nn}$$

$$N(D_{nn}, r_i) = -N_p(D_{nn}, r_i) \cdot \frac{\ln(1 - 2 \cdot \kappa_p(r_i) \cdot \Delta r)}{2 \cdot \kappa_p(r_i) \cdot \Delta r}$$

$$\kappa(r_i) = \sum_{nn_{\text{min}}(h)}^{nn_{\text{max}}(h)} \sigma_e(D_{nn}) \cdot N(D_{nn}, r_i) \cdot \Delta D_{nn}$$

$$PIA(r_i) = PIA(r_{i-1}) \cdot \exp(2 \cdot \kappa_p(r_i) \cdot \Delta r)$$

If $PIA(r_{i,j}) > 10$ then the loop finishes, but if the condition is false then the counter is incremented $i=i+1$ and starts again in equation (3.9). In these equations the subindex a denoted the parameter without attenuation correction, $\kappa$ is the specific rain attenuation, $\Delta r$ is the range resolution from the range bins and $\Delta D_{nn}$

$$\Delta D_{nn} = (D_{nn+1} - D_{nn-1}) / 2$$

for all $nn$ except for the minimum value of $nn$ where $\Delta D_{nn}$

$$\Delta D_{nn} = D_{nn+1} - D_{nn}$$

and except for the maximum value of $nn$ where $\Delta D_{nn}$

$$\Delta D_{nn} = D_{nn} - D_{nn-1}$$

This loop is only applied for $PIA \leq 10$ because the algorithm becomes unstable for large values

3.8 Reflectivity calculation

Reflectivity is a basic target magnitude for a weather radar. It is obtained from noise detection, filtering and data connection with PIA. Different calculation approaches are used by (METEK, 2016) and (Maahn and Kollias, 2012).

Figure 10: Single particle backscatter cross section of water droplets at 24.1 GHz normalized with the Rayleigh backscatter cross section. (Extracted from (Messtechnik, 2015))
In the case of (METEK, 2016) the equation (3.18) used:

\[
Z = 10 \cdot \log \left[ 10^{-3} \sum_{n=n_{\text{min}}}^{n=n_{\text{max}}} \frac{N_n \cdot D_n^6 + N_{n-1} \cdot D_{n-1}^6}{2} \cdot (D_n - D_{n-1}) \right] \tag{3.18}
\]

while the option of (Maahn and Kollias, 2012) follows (3.19):

\[
Z = 10^{18} \cdot \frac{A^4}{\pi^2} \cdot |K|^2 \cdot \int \eta(v) dv \tag{3.19}
\]

this work uses the same equation of (Messtechnik, 2015).

Another important parameter is the fall velocity \(W\), which is calculated (METEK, 2016) follows (3.20):

\[
W = \frac{\sum_{n=n_{\text{min}}}^{n=n_{\text{max}}} n \cdot F_n}{\sum_{n=n_{\text{min}}}^{n=n_{\text{max}}} F_n} \tag{3.20}
\]

where \(n\) is the number of spectral bin and the \(F_n\) is the spectral reflectivity.

In the approach of Maahn and Kollias (2012) The fall velocity follows (3.21)

\[
W = \frac{\int v \cdot \eta(v) dv}{\int \eta(v) dv} \tag{3.21}
\]

In our application it uses (3.22) during the dealiasing process runs.

\[
W = \frac{\sum v \cdot \eta(v)}{\sum \eta(v)} \tag{3.22}
\]

It is interesting to remark that in the processing by Maahn and Kollias (2012) it absence of the terminal velocity assumption allows to work with \(\eta(v)\), but when assuming the terminal velocities works on \(\eta(D)\). Other authors as Adirosi et al (2016) work with both approaches. The relation to transform \(\eta(n,i)\) to \(\eta(v,i)\) follows (3.23)

\[
\eta(v, i) = \eta(n, i) \cdot \frac{2}{\Delta f \cdot \lambda} \tag{3.23}
\]

where:

\(\lambda\) is the radar wavelength and \(\Delta f\) is the frequency resolution in power spectra, in Hz.

4 Product comparison and conclusions

This section shows a comparison of the three methods described above (Maahn and Kollias, 2012; METEK, 2016) and the one propose here) of as precipitation data recorded at the Cerdanya (NE Spain) on the 24 of March of 2017.
Figure 11: Reflectivity (dBZ, left column) and hydrometeor velocity (in m/s, right column) calculated using the methods by (METEK, 2016) upper panel, the proposal method in middle panel, and lower panel of (Maahn and Kollias, 2012)

Figure 11 shows various differences between the methodologies. The differences between the manufacturer results and our results are commented because the two methodologies work under the same hypothesis of the terminal velocities. The result of (Maahn and Kollias, 2012) is interesting to see the trend and the noise evaluation of the signal. The first difference is the value of reflectivity (dBZ) higher in our application than the manufacturer. The reason resides in 2 concepts. One is that in our application there are not limit in spectral bins (remember that the manufacturer only uses some spectral bins in function of height, Figure 4) but in our application all bins are used. The other reason is the determination of noise: while the manufacturer may obtain a negative value, in our application this is not possible. Thus the values from the manufacturer always is lower than in our application.

An application for processing data from raw data files has been created trying to improve the noise. The results of the application seem good quantitatively but it is necessary to contrast with other equipment to determinate the possible improvement.

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References


