Recent works have proposed the combination of precoding and multiuser detection techniques [4], [5]. In both works, it is considered an overloaded scenario where at each beam two simultaneous frames are transmitted. In this context, it is shown that the simultaneous non-unique decoding receiver strategy [6] offers the largest attainable rates.

In contrast to the mentioned works, this paper explores the use of rate-splitting (RS) strategy [7]. In RS, along with each beam transmitted frame, a public frame containing information for all users is transmitted through all beams. This latter frame is coined as public message whereas each beam frame is coined as private messages. While private frames are only decodable by a set of users that are located in the same beam, the public frame is decodable by all users. In this context, every UT performs a successive interference cancellation (SIC) of the public frame and, posteriorly, it detects the frame devoted to the beam it is located at.

Guided by the sum-rate achievable rates in the max-min multigroup multicast optimization described in [8], we propose the use of RS in the sum-rate optimization of the multigroup multicast case. Note that multibeam satellite systems is a multigroup multicast transmission as each frame contains information from all intended UTs located in the same beam [3].

We first obtain an upper bound of the attainable rates. This is done considering the semidefinite program relaxation (SDR) of the original non-convex sum-rate optimization problem. With this, we point out that by using RS, certain gain can be obtained compared to the single stream (SS) case. Bearing this result in mind, we conceive a low complexity precoding design that is able to balance both the public and private messages transmit power. The design differs to the precoding method presented in [9] which does not consider the multigroup multicast transmission and the per-antenna power constraints as we propose in here.

The numerical results of the proposed precoding scheme show a certain gain compared to the benchmark SS design. The results are obtained considering a close-to-real scenario with a real multibeam satellite deployment.

The rest of the paper is organized as follows. Section II presents the system model. Section III describes the optimization problems to be solved for obtaining the achievable rates of RS in multibeam satellite systems. Section IV proposes a low-complexity scheme for precoding systems with RS. Section V contains the numerical results and Section VI concludes.

Notation: Throughout this paper, the following notations
are adopted. Boldface upper-case letters denote matrices and boldface lower-case letters refer to column vectors. \((\cdot)^H\), \((\cdot)^T\), \((\cdot)^\ast\) and \((\cdot)^+\) denote a Hermitian transpose, transpose, conjugate and diagonal (with positive diagonal elements) matrix, respectively. \(I_N\) builds \(N \times N\) identity matrix and \(0_{K \times N}\) refers to an all-zero matrix of size \(K \times N\). If \(A\) is a \(N \times N\) matrix, \([A]_{ij}\) represents the \((i,\, j)\)-th element of \(A\). \(\otimes\) and \(|\cdot|\) refer to the Kronecker product, the Hadamard product and the Frobenius norm, respectively. Vector \(1_N\) is a column vector with dimension \(N\) whose entries are equal to 1. vec \((\cdot)\) denotes the vectorization operator.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a multibeam satellite system where the satellite is equipped with an array fed reflector antenna with a total number of feeds equal to \(N\). These feed signals are combined to generate a beam radiation pattern forming a total number of \(K\) beams, which is considered fixed. For each frame, we assume that a total number of \(N_u\) users are simultaneously served belonging to the same user beam (i.e. the total number of served users by the satellite is \(KN_u\)). These users have been scheduled according to certain criteria based on the spatial signature and rate requirements.

A multibeam satellite system can be cast as a multi-group multicast multiple-input-single-output (MISO) transmission [3]. In this context, the satellite acts as a transmitter with \(N\) antennas, sharing information towards \(K\) groups where each group is composed by \(N_u\) users.

For the sake of the analytical tractability, it is common practice to gather users in disjoint sets. In the multibeam satellite context, we propose to create \(K\) groups of \(N_u\) users. The users of each group are always located in different beams. Following this approach, the objective is to mitigate inter-beam interference to the highest possible extent in all the \(K\) groups. It is worth mentioning that there are a lot of combinations to group the users. Ideally, the user selection and the interference mitigation strategy should be jointly designed. However, the complexity of the solution may not be affordable. To reduce the complexity, we will consider a two-step design, where the user selection is computed beforehand according to a given criteria. Then, the interference mitigation technique will be built upon this selection. User selection algorithms are not within the main scope of the paper and will be left for future work.

Considering that all beams radiate in the same frequency band, the received signal at the \(i\)-th user terminal of each beam in and arbitrary time instant can be modeled as

\[
y^{[i]} = H^{[i]}x + n^{[i]}, \quad i = 1, \ldots, N_u, \tag{1}
\]

being \(y^{[i]} \in \mathbb{C}^{K \times 1}\) the vector containing the received signals at the \(i\)-th UT (i.e. the value \([y^{[i]}]_{k}\) refers to the receive signal of the \(i\)-th UT from the \(k\)-th beam). Vector \(n^{[i]} \in \mathbb{C}^{K \times 1}\) contains the noise terms of each \(i\)-th UT. The entries of this vector are assumed to be Gaussian distributed with zero mean, unit variance and uncorrelated with both the desired signal and the rest of noise entries (i.e. \(E\left[n^{[i]}n^{[i]*}\right] = I_K\) \(i = 1, \ldots, N_u\)).

The \((k,\, n)\)-th entry of matrix \(H^{[i]} \in \mathbb{C}^{K \times N}\) can be described as follows

\[
H^{[i]}_{k, n} = \frac{G_R a^{[i]}_{k,n} e^{j\psi^{[i]}_{k,n}}}{4\pi d^{[i]}_{k,n} R B W}
\] \tag{2}

for \(k = 1, \ldots, K, \quad n = 1, \ldots, N, \quad i = 1, \ldots, N_u\). \(d^{[i]}_{k,n}\) is the distance between the \(i\)-th UT at the \(k\)-th beam and the satellite. \(\lambda\) is the carrier wavelength, \(K_B\) is the Boltzmann constant, \(B\) is the carrier bandwidth, \(G_R\) is the UT receive antenna gain, and \(T_R\) the receiver noise temperature. The term \(a^{[i]}_{k,n}\) refers to the gain from the \(n\)-th feed to the \(i\)-th user at the \(k\)-th beam. The time varying phase due to beam radiation pattern and the radiowave propagation is represented by \(\psi^{[i]}_{k,n}\). For the sake of notation clarity, we denote

\[
H^{[i]} = \begin{pmatrix} h^{[i]}_1, & \ldots, & h^{[i]}_K \end{pmatrix}^T.
\] \tag{3}

In here we consider a rate splitting (RS) approach which separates the transmit information into a public symbol \(s_0\) and \(K\) private symbols \(\{s_k\}_{k=1}^{K}\). The public symbol, \(s_0\), shall be decoded by all UTs (even though it may not be intended to all UTs) while the \(k\)-th private symbol is only required to be decoded by the UTs of the \(k\)-th beam. Figure 1 shows an example scenario with \(K = 2\) and \(N_u = 2\).

\[\text{Fig. 1. A 2-beam satellite system scheme with RS.}\]

The transmitted signal can be written as follows

\[
x = w_0 s_0 + \sum_{k=1}^{K} w_k s_0.
\] \tag{4}

It is assumed that all symbols are unit energy. Then, the attainable rates are ruled by the common message rate

\[
R_0 = \min_{k=1,\ldots,K} \min_{i=1,\ldots,N_u} R_{0,k}^{[i]},
\] \tag{5}

where

\[
R_{0,k}^{[i]} = \log_2 \left( 1 + \frac{|H^{[i]}_k w_0|^2}{\sum_{k=1}^{K} |H^{[i]}_k w_k|^2 + 1} \right)
\] \tag{6}

and each private message data rate

\[
R_k^{[i]} = \min_{i=1,\ldots,N_u} R_{k}^{[i]},
\] \tag{7}
for $k > 0$ and where

$$R_k[i] = \log_2 \left( 1 + \frac{|h_k[i].H w_k|^2}{\sum_{j\neq k,j>0} |h_k[i].H w_j|^2 + 1} \right). \quad (8)$$

In this context, under Gaussian signaling, the sum-rate becomes

$$ \mathcal{SR} = R_0 + \sum_{k=1}^K R_p^k. \quad (9)$$

It is important to remark that RS entails the transmission of $s_0$, which shall be decoded by all users. The optimization problem to be solved is the following

$$\begin{align*}
\text{maximize} & \quad \mathcal{SR} \\
\text{subject to} & \quad \left[ \sum_{k=0}^K w_k w_k^H \right]_{n,n} \leq P \quad n = 1, \ldots, N,
\end{align*} \quad (10)$$

where $P$ is the maximum available power per feed.

The optimization problem in (10) is a non-convex large scale problem. In order to observe the benefits of RS, we opt to compute the SDR of optimization problem in (10) The SDR provides an upper bound on the attainable rates of the system. Recent results in non-convex quadratically constraint quadratic programs (QCQP) techniques [10], [11] show that it is possible to obtain solutions with a performance very close to this upper bound. Yet a promising alternative would be the use of the weighted minimum mean square error technique (WMMSE) [12].

III. UPPER BOUND COMPUTATION

Prior to conceiving low complexity precoding techniques for the considered scenario, in this Section we show the prospective gains of employing RS. This is done by computing an upper bound of the optimization problem in (10).

We can formulate the SDP relaxation of the optimization problem in (10) so that

$$\begin{align*}
\text{maximize} & \quad \mathcal{SR} \\
\text{subject to} & \quad \left[ \sum_{k=0}^K w_k w_k^H \right]_{n,n} \leq P \quad n = 1, \ldots, N,
\end{align*} \quad (11)$$

where $G_k[i] = h_k[i].h_k[i]^H$. The optimization problem in (11) is a biconvex problem. This is, fixed $\{w_k\}_{k=0}^K$, the optimization problem becomes a linear optimization problem and when fixing $\{t_k\}_{k=0}^K$, the optimization problem becomes a SDP.

Biconvex problems can be solved via the alternating optimization method which is guaranteed to converge to a stationary point of the original problem [13]. Bearing in mind that the final solution depends on the initial point, we consider multiple initial random points and elect the solution with the largest sum-rate.

Note that when fixing $\{t_k\}_{k=0}^K$, the optimization problem in (11) becomes a feasibility problem as follows

$$\begin{align*}
\text{find} & \quad \{w_k\}_{k=0}^K \\
\text{subject to} & \quad (2^k - 1) \left[ \sum_{j=1}^K \text{Tr} \left( G_k[i] W_j \right) + 1 \right] - \text{Tr} \left( G_k[i] W_0 \right) \leq 0 \\
& \quad (2^k - 1) \left[ \sum_{j\neq k,j>0} \text{Tr} \left( G_k[i] W_j \right) + 1 \right] - \text{Tr} \left( G_k[i] W_k \right) \leq 0 \\
& \quad \sum_{k=0}^K W_k \leq P \quad n = 1, \ldots, N, \quad (12)
\end{align*}$$

The overall alternating optimization is described in Algorithm 1. It is important to remark that $\epsilon$ controls the stopping criteria of the optimization.

**Data:** $G_k[i]$ for $k = 1, \ldots, K, i = 1, \ldots, N_u$

**Result:** $\{W_k\}_{k=0}^K, \{t_k\}_{k=0}^K$

**Initialize** $\{t_k\}_{k=0}^K$;

**while** $\left| \sum_{k=0}^K t_k^{(n)} - \sum_{k=0}^K t_k^{(n-1)} \right| \geq \epsilon$ **do**

Compute $\{W_k\}_{k=0}^K, \{t_k\}_{k=0}^K$;

Set up $t_0^{(n)} = \min_{k=1,\ldots,K; i=1,\ldots,N_u} \log_2 \left( 1 + \frac{\text{tr}(G_k[i] w_0)}{\sum_{j=1}^K \text{tr}(G_k[i] W_j)} \right)$;

Set up $t_k^{(n)} = \min_{i=1,\ldots,N_u} \log_2 \left( 1 + \frac{\text{tr}(G_k[i] w_k)}{\sum_{j\neq k,j>0} \text{tr}(G_k[i] W_j)} \right)$;

$n \leftarrow n + 1$;

**end**

Output the final solution;

**Algorithm 1:** Alternating optimization for obtaining an efficient solution of (11).

For the case where RS is not employed (i.e. $W_0 = 0$), a similar alternating optimization method can be used as reported in [14].

Despite its efficiency, the mentioned optimization SDR approach presents a large computational complexity which limits its applicability in real systems. In this paper we propose a low-complexity alternative based on closed-form linear precoding techniques. We tackle the problem by first considering the precoding vectors associated to the private messages, $\{w_k\}_{k=1}^K$, and; posteriorly, the precoding vector devoted to the public message transmission, i.e. $w_0$. 
IV. PRECODING IN RS MULTIBEAM SATELLITE SYSTEMS

Based on previous studies, a variation of the minimum mean square error (MMSE) precoding under a simple scaling factor power allocation [3, 15] offers the best complexity-performance trade-off to design the precoders of the private messages. This design, \( \mathbf{W} = (\mathbf{w}_1, \ldots, \mathbf{w}_K) \), can be written as

\[
\mathbf{W} = \gamma \left( \hat{\mathbf{H}}^H \hat{\mathbf{H}} + \frac{1}{NF_{\text{private}}} \mathbf{I}_N \right)^{-1} \hat{\mathbf{H}}^H, \tag{13}
\]

where \( \gamma \) controls the transmit power. The channel matrices are mapped into a single metric, namely

\[
\hat{\mathbf{H}} = \frac{1}{N_u} \sum_{i=1}^{N_u} \mathbf{H}^{[i]}, \tag{14}
\]

and \( P_{\text{private}} \) is the maximum transmit power that the private transmission can have. In other words, \( \gamma \) is elected so that

\[
\gamma^2 = \frac{P_{\text{private}}}{\max_{n=1, \ldots, N} [\mathbf{W} \mathbf{W}^H]_{nn}}. \tag{15}
\]

Equation in (14) generates an equivalent channel matrix \( \hat{\mathbf{H}} \) based on the average channel matrix from all \( N_u \) users simultaneously served. Note that \( P_{\text{private}} \leq P \) and its value shall be elected a priori. For the sake of simplicity, we consider

\[
P_{\text{private}} = \alpha P, \tag{16}
\]

where \( 0 \leq \alpha \leq 1 \) controls the amount of power devoted to the private messages transmission.

It is noteworthy that in general only one feed out of \( N \) will transmit with \( P_{\text{private}} \). The rest will transmit with less power. In the following we will demonstrate how RS can take advantage of the unused power to multiplex one addition message and thus, increase the spectral efficiency. To the best of authors’ knowledge this strategy has not been previously adopted in the literature.

Once \( \mathbf{W} \) is computed, the optimization of \( \mathbf{w}_0 \) becomes

\[
\text{maximize}_{\mathbf{w}_0} \quad \min_{k=1, \ldots, K} \quad \frac{||\mathbf{h}_k^{[i]} H \mathbf{w}_0||^2}{\tau_k^{[i]}}, \tag{17}
\]

subject to

\[
[\mathbf{W} \mathbf{W}^H + \mathbf{w}_0 \mathbf{w}_0^H]_{nn} \leq P, \quad n = 1, \ldots, N, \]

where

\[
\tau_k^{[i]} = \sum_{l=1}^{K} ||\mathbf{h}_l^{[i]} H \mathbf{w}_l||^2 + 1. \tag{18}
\]

The optimization problem in (17) is non-convex but its concave-convex relaxation shows a performance very close to the optimal design [16]. It is important to remark that despite \( \alpha = 1 \), there might be the case where \( \mathbf{w}_0 \) takes a non-zero value as not all the feed will work on full power transmission.

V. NUMERICAL RESULTS

Prior to evaluate the potential of RS in multibeam satellite systems, we consider an example of Rayleigh distributed channel. This is depicted in Figure 2 where we have consider an scenario with \( K = 3, N = 4 \) and \( N_u = 3 \). The upper bound on the attainable rates has been obtained considering the alternating optimization method described previously. It can be observed that RS offers a substantial potential gain with respect to the benchmark case. For this case we consider the SDR upper bound of the sum-rate multigroup multicast optimization problem.

![Fig. 2. Achievable rates example with Rayleigh distributed channel scenario.](image)

We consider \( K = 3, N = 4 \) and \( N_u = 3 \) for different \( P \).

On the other hand, it is evident that the low complexity precoding scheme yields a poor performance. Alternatively, when combining this low complexity scheme with the optimized \( \mathbf{w}_0 \), certain gain can be obtained even with \( \alpha = 1 \). For this case, \( \alpha = 0 \) (i.e. pure public message transmission) attains the maximum sum-rate. The reason for this is the tentative strong channel co-linearity of users belonging to different groups which negatively impacts the data rates of the private message transmission. The results have been obtained in a Monte Carlo simulation with 500 runs.

We evaluate the considered technique in a multibeam satellite system with \( K = 8 \). For evaluating the aforementioned technique, a real coverage area provided by a geostationary satellite is considered. This data has been obtained in a study performed by the European space agency (ESA). We assume that at each time instant all bandwidth is shared by all beams. The simulation parameters are summarized in Table I. The considered figure of merit is the throughput defined as

\[
\mathcal{T} H = B \times SR. \tag{19}
\]

As a benchmark, we consider the design in (14) with \( P_{\text{private}} = P \).

In order to observe the variation of \( \mathcal{T} H \) over \( \alpha \), we consider a fixed maximum per feed available power of \( P = 55 \) Watts. This can be observed in Figure 3. The maximum throughput is attained via \( \alpha = 0.5, 0.09, 0.05, 0.04 \) for \( N_u = 2, 3, 4, 5 \) respectively.

In light of this simulation it is clear that in the considered multibeam satellite system the public message plays a central
TABLE I. USER LINK SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite height</td>
<td>35786 km (geostationary)</td>
</tr>
<tr>
<td>Satellite longitude, latitude</td>
<td>10° East, 0°</td>
</tr>
<tr>
<td>Earth radius</td>
<td>6378.137 Km</td>
</tr>
<tr>
<td>Feed radiation pattern</td>
<td>Provided by ESA</td>
</tr>
<tr>
<td>Number of feeds</td>
<td>8</td>
</tr>
<tr>
<td>Number of users $N_u$</td>
<td>2, 3, 4 and 5</td>
</tr>
<tr>
<td>Number of beams</td>
<td>8</td>
</tr>
<tr>
<td>User location distribution</td>
<td>Uniformly distributed</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>20 GHz (Ku band)</td>
</tr>
<tr>
<td>Total bandwidth $B$</td>
<td>500 MHz</td>
</tr>
<tr>
<td>Roll-off factor</td>
<td>0.25</td>
</tr>
<tr>
<td>User antenna gain</td>
<td>41.7 dBi</td>
</tr>
<tr>
<td>G/T in clear sky</td>
<td>17.68 dB/K</td>
</tr>
</tbody>
</table>


