LOAD FORECASTING USING SINGULAR SPECTRUM ANALYSIS

Bachelor Thesis
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1. Abstract

Nowadays, the energy market has become more relevant and the electricity demand is more active, this means that it is less predictable. This has caused an increase in the development of a prediction with greater certainty and accuracy. It has the aim of improving energy efficiency, power systems planning and operations. [1]

The majority of the load forecasting articles have been a timely forecasting, which is easier to obtain the expected values. There are different ways to predict the load consumption, primarily statistical techniques and artificial intelligent techniques. [2]

2. Introduction

As mentioned above, a good approximation is fundamental, in order to try to produce more accurately the energy that will be consumed and to make good use of it.

In this document a prediction of the power consumption is made, where trying to make it as accurate as possible. A method called Singular Spectrum Analysis (SSA) is developed, this method does not use Artificial Neural Networks (ANN), which is known as connectionist systems.

For the prediction of the load consumption, a database is taken from 2010 to 2013. The prediction of the year 2013 will be made and then compared with the real values of data. There will also be a comparison with different prediction methods, Holt-Winters and Artificial Neural Networks.

Finally, it will be demonstrated if the method studied is the appropriate to predict the load consumption, or it is better to consider a new method.
3. Singular Spectrum analysis

SSA is a nonparametric spectral estimation method, is a technique of time series analysis and forecasting, combination of classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing.

SSA tries to decompose the original series into a sum of series, which have interpretable components. And reconstructed of the original series, this reconstruction is used for forecasting the new load points.

It based in the value decomposition of a matrix, which constructed on the time series. [3][4][5][6].

SSA has two stages:

- The decomposition, that consists of embedding and singular value decomposition (SVD).
- The reconstruction, that consists of Eigen-trip grouping and diagonal average.

3.1. Embedding

First of all is mapping the original time series onto the trajectory matrix $T_x$. Considered $N$ the length of the time series ($Y^N = Y_1 \ldots Y_N$), where $2 < N$ and $Y$ is a non-zero series ($Y_i \neq 0$). The $L$, called the window length, is an integer number, which $1 < L < N$, so we know that $K = N - L + 1$. $L$ is the lagged copies of the time series $Y^N$.

Now can start the embedding, the initial time series is applied into a sequence of vectors. The size of lagged vectors is $L$, creating $K = N - L + 1$ lagged vectors of lagged $L$.

The trajectory matrix($X$) is a Hankel matrix, the components of the rows and columns are subseries of the original series.

$$
X = \begin{bmatrix}
Y_0 & \ldots & Y_K \\
\vdots & \ddots & \vdots \\
Y_L & \ldots & Y_N
\end{bmatrix}
$$
*The most typical selection of \( L \) for our case is a multiple of the seasonal cycles of our time series. \( L \) can’t be bigger than \( \frac{N}{2} \).

### 3.2. **Singular Value Decomposition (SVD)**

This step is applied to the trajectory matrix, the \( L \times L \) (\( XX^N \)) matrix is calculated with Eigen-triples. The matrix \( XX^N \), expressed by \((\lambda_i, i=1,\ldots,L)\) vector, the eigenvalues, of \( XX^N \) in decreasing order, \( \lambda_1 \) the biggest and \( \lambda_L \) the smallest but bigger than 0. The vector \( U_i \) is express the system of eigenvectors \( U_1,\ldots,U_L (i=1,2,\ldots,L) \).

With \( d=\text{rank} (X) \) of the matrix \( X \), can define with the factor vector \( V_i \frac{X^T U_i}{\sqrt{\lambda_i}} \) \((i=1,2,\ldots,d)\), and redefine the elements of \( X \). Defining the trajectory matrix:

Eigen-triple is the collection of the three vectors, \( V_i, U_i, \sqrt{\lambda_i} \).

### 3.3. **Eigen-triple Grouping**

The next step is divide the matrices \( X_i \) in groups and summing the matrices in each group. The objective of this groups is separate the several additive components of the time series.

\[
X=X_{l1}+\ldots+X_{lm}
\]  

(2)

Eigen-triple grouping is the product of the set \( I=l_1,\ldots,l_m \) is the Eigen-triple grouping. *If \( m=d \) and \( l_j={j} \), \( j=1,\ldots,d \) the grouping is called elementary.

### 3.4. **Diagonal Averaging**

The last step is the reconstruction, the matrixes are transforms in a new times series with length \( N \). Have \( Y \) the matrix \( L \times K \). The new matrix \( Y \) constructed using diagonal averaging.
4. Application of the method

4.1. PARAMETERS

In the first stage, the window length \( N \) must be determined. This parameter is used for the trajectory matrix \( T \). The value of \( L \) fixed the value \( K \) and the length of the vectors obtained by the decomposition of the initial time series. For small values of \( L \) the rank of the trajectory matrix is small too and lambda bigger, but it tend to be rough and irregular. However, for bigger values of \( L \) the rank of the trajectory matrix is bigger too and lambda is smaller, but the tend is smooth and regular. [7]

In the grouping step, the initial parameters include window length determined the noise of the forecasting and the trend. The correct value of window length allows the properly separation of the initial time series.

Other important parameter is the periodicity of the initial time series \( M \). \( M \) is used to establish a seasonality, it’s the number of the points that compose one of the decomposition of the initial time series.

4.2. ERROR

To verify that the value of \( L \) and \( M \) are the correct, the error is checked using the mean squared error (MSE). When the error is small means that the values of \( L \) and \( M \) are quite accurate. The equation used is:

\[
\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2
\]

Were \( Y_i \) is the \( i^{th} \) value of the variable to be predicted and \( \hat{Y} \) is the predicted value. [8]
5. Predictions


For the first case, the third Wednesday of the years 2010, 2011 and 2012 have been used as data to predict the third Wednesday of 2013 (16th January). The choice of this day is because it is a working day. So, our samples size for this scenario is 3 days (). The choice of the parameters in this case follow the next steps:

- M is the seasonality of the time series, for the prediction of 16th, as commented before it is one day, so 24 hours (24 points).
- N is the window length, this parameter delimits the length of initial time series, in this case is 72 points, of the 72 hours of 20/01/2010, 19/01/2011 and 18/01/2012.
- L is fixed by us, the common value of L is the half of N (36 in this case), this value is a good initial reference to choose the best L, fine the best L is the most important and difficult parameter, the best approximation is a smooth and regular trend, so L it will be bigger than 36, with the help of MSE error and by trial and error the value of N with smaller error is 54.

With N=54 is MSE=0,0103, and the error with L=36 is MSE=1,3921.

As shown in the graphs, in the first with a bigger L (L=54), the prediction is more regular, however for a smaller L (L=36), the trend of the prediction is more irregular and rough, with more pikes and abrupt changes.
Graphic.1.- Prediction of the load consumption of 1st of January \((L=54)\). (Source:MATLAB).

Graphic.2.- Prediction of the load consumption of 1st of January \((L=36)\). (Source:MATLAB).
5.2. Prediction of 7th January to 3rd February 2013

In this scenario, the 4 first weeks of the year of 2010, 2011 and 2012 are considered, it has been taken into account, that the weeks are taken from Monday to Sunday. the parameters are:

- **M** for the prediction of 4 weeks of January 2013. The value of M is 672 points, because in 28 days it is 672 hours.

- **N** is the window length, this parameter delimits the length of initial time series, it is 2016 points, of the hours between 4 and 31 of January 2010, 3 to 30 of January 2011 and 2 to 29 of January 2012.

- **L** is fixed by us, the common value of L is the half of N (1008 in this case), this value is a good initial reference to choose the best L, fine the best L is the most important and difficult parameter, the best approximation is a smooth and regular trend, so L it will be bigger than 1116, with the help of MSE error and by trial and error the value of L with smaller error is 1500. In this case L is very close to the value of N.

![Graphs showing load consumption for different periods](image_url)

*Graphic 3.* Prediction of the load consumption of January. (Source: MATLAB).
5.3. Prediction of 2013

For the last prediction:

- M for the prediction of year 2013, the daily average of load consumption considered is of the years 2010 to 2012. The value of M is 365 points, for 365 days of whole year.
- N is the window length, this parameter delimits the length of initial time series, in the last case is 1095 points, the days in the years 2010, 2011 and 2012.
- L is fixed by us, the common value of L is the half of N (548 in this case), this value is a good initial reference to choose the best L, find the best L is the most important and difficult parameter, the best approximation is a smooth and regular tend, so L it will be bigger than 548, with the help of MSE error and by trial and error the value of L with smaller error is 875. In this case L is less close to the value of N than the last one.

In this case the hourly load data consumption isn’t considered, due to the heaviness of the algorithm, Matlab can’t run that much data. Therefore, we only use the daily average.

Graphic.4.-Prediction of the load consumption of 2013. (Source:MATLAB).
Graphic 5.- Extension of January from graphic 4. (Source: MATLAB).
6. Comparing methods

This thesis studies a statistical algorithms capable of predicting values following the trends of a database, Singular Spectrum Analysis (SSA). The results obtained by the method presented in this thesis have been compared with those obtained by the method discussed in the thesis. [10][11]

In this section, both methods are compared with a third, which uses Artificial Neural Networks (ANN). Since most algorithms for data prediction use this type of model, it has been decided not to delve into it. The data have been entered in a free code to obtain results comparable to our study.

To compare the methods and see the accuracy of the results, three scenarios are presented. In each scenario, the real consumption values of 2013 are compared with those obtained in the predictions of each method.

The MSE (mean squared error) of the prediction is calculated as an indicator of the accuracy of the algorithm, and it will also be used to compare the methods.

The graphs are also used to visually compare the predictions with the actual load consumption values.

6.1. Prediction a single day in January 2013

For this prediction, three days of data are considered: 13th January 2010, 12th January 2011 and 11th January 2012 to predict 16th January 2013. These days are the second Wednesday of each year, chosen because in the first week of 2011 an unusual behaviour is seen and because Wednesday it is the middle day of the work week.
The errors found in this prediction for Holt-Winters, SSA and ANN methods are similar. In this case, the least number of timesteps are predicted, only 24, except ANN, that predicts 22 timesteps due to the training parameter. The errors are of the same order ($e^{-2}$) which indicates that is a good prediction, since the error is considerably low.

In the SSA method the error is smaller, that means that it is more accurate than the Holt-Winters and ANN.
<table>
<thead>
<tr>
<th></th>
<th>ERROR MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holt-Winters</td>
<td>0,0178</td>
</tr>
<tr>
<td>SSA</td>
<td>0,0103</td>
</tr>
<tr>
<td>ANN</td>
<td>0,0141</td>
</tr>
</tbody>
</table>

Table 1.- MSE error in the prediction of 16th January 2013.

6.2. Prediction the 4th first weeks of 2013

For this prediction, the hourly load data consumption considered is the 4 first weeks of the years 2010, 2011 and 2012, as described before.

In this case the error is quite similar to the scenario seen before (for the Holt-Winters and SSA), the order of magnitude of the error is the same. The error in ANN is much bigger than in the other methods. This method is more precise in the sections with less abrupt variations, but when peaks in the consumption are found, it is less accurate.

It is necessary to value that the number of predicted points is much bigger, 672 instead of 24.

In the SSA algorithm, error’s order of magnitude is almost 1,5 times bigger than in the Holt-Winters. But it’s still a good prediction, with an error considerably small.

[Graphic 8.--Prediction of the load consumption 7th January to 3rd February 2013 (Source: MATLAB).]
6.3. **Prediction year 2013**

For this forecast, the hourly load data consumption considered is the entire year, from 2010 to 2012.

In the SSA method and ANN, the prediction of a whole year is made with the daily average values of power consumption, because of the complexity of the algorithm, so the prediction vector has 365 peaks.

On the other hand, Holt-Winters prediction is made with 24 hours per day, so the prediction vector has 8,760 peaks.

But still this difference, both errors are comparable since the MSE is used. The SSA error is still smaller than the Holt-Winters. The error with ANN method is such bigger than the others.
Since there are so many points in the graph, a zoom has been made to get a better comparison between both methods.
There are now two more scenarios. The first zoom focuses on a part of the sample in which SSA produces a good prediction, from the 26\textsuperscript{th} day of the year until the 46\textsuperscript{th}, this is from 26\textsuperscript{th} January (Saturday) till the 15\textsuperscript{th} February (Friday).

Same days are taken by Holt-Winters. It can be seen that the approximation is also good, considering that here are very low peaks in data, difficult to predict.

If we look at SSA, we will appreciate that there is trend every 7 days, being the day of the greatest consumption on January 31\textsuperscript{st} 2010, which was Thursday.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{SSA_prediction}
\caption{Zoom of Prediction of the load consumption of 2013 with SSA. (Source:MATLAB).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Holt_Winters_prediction}
\caption{Zoom of Prediction of the load consumption of 2013 with Holt-Winters. (Source:MATLAB).}
\end{figure}
The second zoom focuses on a part of the sample in which SSA produces a worst prediction than before, from the 185th day of the year until the 205th, this is from Thursday the 4th July till Wednesday de 24th July.

**Graphic.14.-Second Zoom of Prediction of the load consumption of 2013 with SSA.**
*(Source:MATLAB).*

**Graphic.15.-Second Zoom of Prediction of the load consumption of 2013 with Holt-Winters.**
*(Source:MATLAB).*
7. Conclusion

Reading some load forecasting papers, we saw that many technicians use Artificial Neural Networks. We realized that deepening this powerful method with our knowledge would take more time than we had. We decided to choose two known statistics methods, which still gave good results, to be compared later with a simple Neural Networks tool from Matlab. This tool has proven to be more difficult than we thought, and the results have not been as good as expected.

The simulation results indicate that the three algorithms have more accuracy with short-term forecasting (like most predictions seen in papers). As we have checked in the different predictions, SSA method is more accurate than Holt-Winters and ANN. The three methods, are more exactly in the first scenario, with less points to predict. So, we can say that the three methods are good for short-term forecasting.

One of the important points for having a good prediction, is the similarity of the data and the prediction, it means that is important to match the weeks (if the database starts in the 23rd week of the year, the prediction must start the 23rd as well). For this, both the data and the prediction must start on the same day of the week.

Both methods (Holt-Winters and SSA) are based on the seasonality of the data, so they can only be applied with data that follow a periodicity. This is why these scenarios have been chosen as they are presented.

So, we have worked with three methods on the same data, this has allowed us to have a deep vision of forecasting.

A possible extension of this project would be to deepen more with ANN and to introduce more data, such as meteorological, holidays...
8. References


ANNEX I

Matlab code SSA algorithm (ssaformod.m)

[12]

function [F] = ssaformod(Y, L, N, M)

% Inputs:
% Y - initial time series
% L - number of reconstructed components input data series to be decomposed N in this doc
% N - number delimits de seasonality M in this doc
% M - number of points to be forecasted
% Outputs:
% F - sum of the first N reconstructed components a M points forecasted

T = length(Y);
K = T - L + 1;

% trajectory matrix
X = zeros(L,K);
for i = 1:K
    X(:,i) = Y((1:L)+i-1).';
end
x=X*X;

% Determination of the eigentriples
[V,LANDA] = eig(x);
V=flip(V);
% diagonal elements
LANDA=-diag(LANDA);
[LANDA,ind]=sort(LANDA);
d=-LANDA;

% principal components
PC=X'*V;

% Grouping
Vt=PC';
rca=V(:,N)*Vt(N,:);

% Reconstruction
for i = 1:L
    VV(:,i) = X.'*V(:,i);
end
% Eigentriple grouping

Z = cell(L,1);
Q = zeros(K,L);
for i = 1:L
    Z{i} = V(:,i)*V(:,i).';
    Z{i} = fliplr(Z{i});
    for j = (-L + 1):(K-1)
        Q(L+j,i) = sum(diag(Z{i},j))/length(diag(Z{i},j));
    end
end
Q = flipud(Q);

A = zeros(L-1,1);
for i = 1:N
    A = A + V(L,i)*V(1:L-1,i);
end
v = norm(V(L,1:N));
A = A/(1-v^2);
A = flipud(A);
G = sum(Q(:,1:N),2);
F = [G.', zeros(1,M)].';
for i = T+1:T+M
    for j = 1:L-1
        F(i) = F(i) + A(j)*F(i-j);
    end
end
F = F(T+1:T+M);
F = [G.', F.'].';
Matlab code initial time series (SSAload.m)

```matlab
function [H, D, Jan, pJan] = SSAload()
%Initial data of load consumition

A = importdata('excel bueno.xls');
i = 1;
j = 1;

for m=1:24:1153
    for d = 5:35
        for h= m:(23+m)
            if h >1152
                break
            end
            if A.data(h,d) > 0.00
                Mat(i,j)=A.data(h,d);
                i = i+1;
            end
        end
        if h >1152
            break
        end
        i=1;
        j = j+1;
        if A.data(h,d) == 0.00
            j=j-1;
        end
    end
end
%construction a vector of the load
H=[Mat(:)]';

%construction vector average load day, week and month
n = 1;
for i = 1:24:length(H)
    D(n) = sum(H(i:i+23))/24;
    n = n+1;
end
n = 1;

n=1;
h=1;
for d=8:35;
    for h=1:24;
        Jan(n)=A.data(h,d);
        n=n+1;
    end
end
for d=7:34;
    for h=289:312;
        Jan(n)=A.data(h,d);
        n=n+1;
    end
end
for d=6:33;
    for h=577:600;
        Jan(n)=A.data(h,d);
    end
end
```

iii
n = n+1;
end
end
for d=11:35;
    for h=865:888;
        Jan(n) = A.data(h, d);
        n = n+1;
    end
end
for d=5:7;
    for h=889:912;
        Jan(n) = A.data(h, d);
        n = n+1;
    end
end
n = 1;
for d=24;
    for h=1:24;
        pJan(n) = A.data(h, d);
        n = n+1;
    end
end
for d=23;
    for h=289:312;
        pJan(n) = A.data(h, d);
        n = n+1;
    end
end
for d=22;
    for h=577:600;
        pJan(n) = A.data(h, d);
        n = n+1;
    end
end
for d=20;
    for h=865:888;
        pJan(n) = A.data(h, d);
        n = n+1;
    end
end
end
Prediction and plots of 16th of January 2013 (codeSSA1Jan.m)

% SSA algorithm
p = path;
path(p, '../..');

% Initial data

[H, D, Jan, pJan] = SSAload();

A = pJan(1:72);
F = ssaformod(A, 49, 24, 24);

% Mean squared error
error = 0;
F = F';
for i = 72:length(F)
    error = error + ((pJan(i) - F(i))^2);
end
error = error/24
pJan = pJan*1000;
F = F*1000;

figure(1);

subplot(4, 1, 1)
plot (pJan(1:24), 'r');
title('January 2010');
axis([0 25 70 1500]);

subplot(4, 1, 2)
plot(pJan(25:48), 'r');
title('January 2011');
axis([0 25 70 1500]);

subplot(4, 1, 3)
plot(pJan(49:72), 'r');
title('January 2012');
axis([0 25 70 1500]);

subplot(4, 1, 4)
hold on
plot(pJan(73:96), 'r', 'DisplayName', 'DATA');
plot(F(73:96), 'b', 'DisplayName', 'PREDICTION');
title('1st January 2013');
axis([0 25 80 1500]);
grid
xlabel('Time [day]')
ylabel('Load consumption [KW]')
legend('show')
hold off
% SSA algorithm

% Initial data

[H, D, Jan, pJan] = SSAload();

p = path;
path(p, '../..');

A = Jan(1:2016);
F = ssaformod(A, 1500, 672, 672);

% Mean squared error
error=0;
F=F';
for i=2233:length(F)
    error=error+((Jan(i)-F(i))^2);
end
error=error/length(F)

Jan=Jan*1000;
F=F*1000;

figure(1);

subplot(4,1,1)
plot (Jan(1:672), 'r');
title('4th to 31st January 2010')

subplot(4,1,2)
plot(Jan(673:1344), 'r');
title('3rd to 30th January 2011')

subplot(4,1,3)
plot(Jan(1345:2016), 'r');
title('2nd to 9th January 2012')

subplot(4,1,4)
hold on
plot(Jan(2017:2688), 'r', 'DisplayName', 'DATA');
plot(F(2017:2688), 'b', 'DisplayName', 'PREDICTION');
title('7th January to 3rd February 2013')
axis([0 768 0 inf])
grid
xlabel('Time [h]')
ylabel('Load consumption [KW]')
legend('show')
hold off
Prediction and plots of 2013 (codeSSA1.m)

% SSA algorithm

% Initial data
[H, D] = SSAload();

p = path;
path(p, './.');

A = D(1:1096);

F = ssaformod(A, 875, 365, 364); F=F';

% Mean squared error
error=0;
for i=1096:1460
    error=error+((D(i)-F(i))^2);
end
error=error/364

D=D*1000;
F=F*1000;

figure(1);

subplot(4,1,1)
plot (D(1:365), 'r');
title ('2010')
axis([0 370 0 1200]);

subplot(4,1,2)
plot(D(366:730), 'r');
title ('2011')
axis([0 370 0 1200]);

subplot(4,1,3)
plot(D(732:1095), 'r');
title ('2012')
axis([0 370 0 1200]);

subplot(4,1,4)
hold on
plot(D(1096:1460), 'r', 'DisplayName', 'DATA');
plot(F(1096:1460), 'b', 'DisplayName', 'PREDICTION');
title ('2013')
xticks([185:205]);
axis([185 205 0 1200]);
grid

xlabel('Time [day]')
ylabel('Load consumption [kW]')
legend('show')
hold off