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A Passivity-based Controller without Velocity Measurements for the Leaderless Consensus of Euler-Lagrange Systems

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Abstract: This paper deals with the problem of achieving consensus of multiple interconnected Euler-Lagrange (EL) systems using the energy shaping plus damping injection principles of passivity–based control. It proposes a novel decentralized controller that is capable of solving the leaderless consensus problem in networks of fully-actuated EL-systems with interconnecting time-varying delays and without employing velocity measurements. The paper also presents experimental results that provide evidence of the performance of the novel controller.

Keywords: Consensus; Euler-Lagrange Systems; Passivity-based Control

1. INTRODUCTION

It is widely known that the Euler-Lagrange (EL) equations of motion describe the behavior of a wide number of physical systems—including mechanical, electrical and electromechanical systems (Ortega et al., 1998). The first results on consensus (synchronization) of a particular class of EL-agents were reported in (Rodriguez-Angeles and Nijmeijer, 2004; Chopra and Spong, 2005) and, the case of general, nonidentical, EL-systems with delays was first reported in (Nuño et al., 2011). Since then, a plethora of different controllers have been proposed to solve consensus problems, from simple Proportional plus damping (P+d) schemes (Ren, 2009; Nuño et al., 2013b,a) to more elaborated adaptive (Chung and Slotine, 2009; Nuño et al., 2011; Meng et al., 2014; Abdessameud et al., 2015; Chen et al., 2015) and sliding-mode controllers (Klotz et al., 2015).

In this paper we consider a network of N, *fully-actuated* n–DoF, EL-systems of the form

$$\frac{d}{dt}\left(\nabla_{\dot{\mathbf{q}}_i}\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\right)-\nabla_{\mathbf{q}_i}\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)=\boldsymbol{\tau}_i,$$

where $\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ is the Lagrangian defined as

$$\mathfrak{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)={}^s\mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)-{}^s\mathcal{U}_i(\mathbf{q}_i),$$

with

$${}^s\mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i):=\frac{1}{2}\dot{\mathbf{q}}_i^\top\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i,$$

the kinetic energy and ${}^s\mathcal{U}_i(\mathbf{q}_i)$ the potential energy. $\mathbf{q}_i,\dot{\mathbf{q}}_i\in\mathbb{R}^n$ are the generalized position and velocity, respectively, $\mathbf{M}_i(\mathbf{q}_i)\in\mathbb{R}^{n\times n}$ is the generalized inertia matrix, which is positive definite and bounded, $\boldsymbol{\tau}_i\in\mathbb{R}^n$ is the vector of external forces and $i\in\bar{N}:=[1,N]$. For these systems we design *decentralized* controllers, i.e., one controller for each agent, to solve the following problem:

(LC) Leaderless Consensus Problem. The EL-systems have to asymptotically reach a consensus position. That is, *there exists* a constant $\mathbf{q}_c\in\mathbb{R}^n$ such that, for all $i\in\bar{N}$, $\lim_{t\to\infty}|\dot{\mathbf{q}}_i(t)|=0$, and $\lim_{t\to\infty}\mathbf{q}_i(t)=\mathbf{q}_c$.

Most of the previous reported control schemes require velocity measurements for their implementation. Among the few controllers that do not rely on velocities are the following: in (Aldana et al., 2014), using a velocity filter, and in (Ren, 2009), with a bounded controller, the leaderless consensus is solved for undelayed networks of EL-systems; Abdessameud et al. (2012) solves the consensus problem for the attitude of rigid bodies by using a *virtual* system for each agent, and Abdessameud and Tayebi (2013), the consensus problem is solved for linear second-order systems. Zheng and Wang (2012) solves the leaderless consensus problem for linear heterogeneous—first and second order systems—but without interconnecting delays.

Recently, in (Nuño, 2015, 2016), a solution to the **LC** problem with time-varying interconnection delays is proposed. The solution incorporates the Immersion and Invariance velocity observer reported in (Astolfi et al., 2010). The main drawback of this scheme is that the implementation of the observer requires the *exact knowledge* of the complete EL-dynamics, which in several practical scenarios is unrealistic.

The proposed control scheme follows the energy shaping plus damping injection methodology where the energies of the system and the controller are added to make the resulting total energy a suitable Lyapunov function, and damping is added to achieve asymptotic stability (Ortega et al., 1998). In (Ortega and Spong, 1989) it was proved that passivity is the key property underlying the stabi-

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lization mechanism and the, now widely popular, term passivity-based control (PBC) was coined. The key feature of PBC that we exploit in this work is that the damping needed to ensure asymptotic stability—that for EL-systems is usually achieved feeding-back the velocity, *i.e.*, the d term in P+d controllers—can be *injected through the controller* without velocity measurements. The history of this important observation—in the context of robotics—may be found in (Ortega et al., 1998, 2016). Adopting the previous procedure in this paper leads to a novel *decentralized* controller that solves the **LC** problem in networks of fully-actuated EL-systems with interconnecting time-varying delays and without employing velocity measurements. To the best of the authors' knowledge, this is the first work that provides a globally asymptotically stable (GAS) solution to this challenging problem without requiring the knowledge of the complete dynamics of the agents.

The following *notation* is used throughout the paper. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. $\mathbf{I}_k$ represents the identity matrix of size $k \times k$. $\mathbf{1}_k$ is a column vector of size k with all entries equal to one. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \sup_{t \geq 0} |\mathbf{f}(t)|$, $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 := (\int_0^t |\mathbf{f}(\sigma)|^2 d\sigma)^{1/2}$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively. The argument of all time dependent signals is omitted, *e.g.*, $\mathbf{x} \equiv \mathbf{x}(t)$, except for those which are time-delayed, *e.g.*, $\mathbf{x}(t - T(t))$. The subscript $i \in \bar{N} := \{1, ..., N\}$, where N is the number of nodes of the network.

2. DYNAMIC MODEL OF THE EL–NETWORK

Each agent's EL-equations of motion can be written as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \nabla_{\mathbf{q}_i} {}^s\mathcal{U}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i \tag{1}$$

where $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n \times n}$ is the Coriolis and centrifugal forces matrix, defined via the Christoffel symbols of the first kind. Piling up the vectors $\mathbf{q}_i$ and $\boldsymbol{\tau}_i$ as

$$\mathbf{q} := \mathrm{col}(\mathbf{q}_i), \quad \boldsymbol{\tau} := \mathrm{col}(\boldsymbol{\tau}_i), \quad \forall i \in \bar{N},$$

the Hamiltonian (total energy) of the complete N EL-systems is

$${}^s\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}}) = {}^s\mathcal{K}(\mathbf{q}, \dot{\mathbf{q}}) + {}^s\mathcal{U}(\mathbf{q}),$$

where

$${}^s\mathcal{K}(\mathbf{q}, \dot{\mathbf{q}}) := \sum_{i \in \bar{N}} {}^s\mathcal{K}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i), \quad {}^s\mathcal{U}(\mathbf{q}) := \sum_{i \in \bar{N}} {}^s\mathcal{U}_i(\mathbf{q}_i),$$

are the total kinetic and potential energies, respectively. All the agents dynamics can be compactly written as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \nabla_{\mathbf{q}} {}^s\mathcal{U}(\mathbf{q}) = \boldsymbol{\tau}. \tag{2}$$

where we defined the overall inertia and Coriolis matrices as

$$\mathbf{M}(\mathbf{q}) := \mathrm{blockdiag}\{\mathbf{M}_i(\mathbf{q}_i)\},$$
$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) := \mathrm{blockdiag}\{\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\}.$$

The following well-known property of EL-systems is instrumental for the sequel (Duindam et al., 2009; Hatanaka et al., 2015; Ortega et al., 1998).

Fact 1. The system (2) defines a cyclo–passive[1] operator $\Sigma_s : \boldsymbol{\tau} \to \dot{\mathbf{q}}$ with storage function ${}^s\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}})$. More precisely,

$${}^s\dot{\mathcal{T}}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}^\top \dot{\mathbf{q}}.$$

◁

It is assumed that the EL-agents exchange information according to some prespecified invariant pattern. This is characterised by N sets $\mathcal{N}_i \subset \bar{N}$, where $\mathcal{N}_i$ contains the index of agents transmitting information to the ith agent. This interconnection of the agents is modeled via the Laplacian matrix $\mathbf{L} := \{L_{ij}\} \in \mathbb{R}^{N \times N}$, whose elements are defined as

$$L_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i} a_{ij} & i = j \\ -a_{ij} & i \neq j \end{cases} \tag{3}$$

where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise (Cao and Ren, 2011). The following assumption on the interconnection topology is imposed throughout the paper.

A1. The EL-agents interconnection graph is *undirected and connected.*

By construction, $\mathbf{L}$ has zero row sum. Moreover, Assumption **A1**, ensures that $\mathbf{L}$ is symmetric, has a single zero-eigenvalue and the rest of its spectrum is strictly positive. Thus, $\mathrm{rank}(\mathbf{L}) = N - 1$. Therefore, exists $\alpha \in \mathbb{R}$ such that $\ker(\mathbf{L}) = \alpha \mathbf{1}_N$.

In the paper we also consider delays in the information exchange between agents, for which we assume that:

A2. The communications, for every pair of i, j agents, is subject to a variable time-delay $T_{ji}(t)$ with a known upper-bound ${}^*T_{ji}$. Hence, it holds that

$$0 \leq T_{ji}(t) \leq {}^*T_{ji} < \infty. \tag{4}$$

Furthermore, $|\dot{T}_{ji}(t)|$ is bounded.

The following lemma serves as instrumental in the proof and has been borrowed from (Nuño et al., 2009).

Lemma 1. For any vector signals $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\alpha > 0$, the following inequality holds

$$-\int_0^t \mathbf{x}^\top(\sigma) \int_{-T(\sigma)}^0 \mathbf{y}(\sigma + \theta) d\theta d\sigma \leq \frac{\alpha}{2}\|\mathbf{x}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{y}\|_2^2.$$

◇

3. PASSIVITY-BASED CONTROLLER DESIGN

In the PBC methodology[2] the controller is another EL-system with its own generalized coordinates and Lagrangian function, that we interconnect with the plant to be controlled via a power–preserving interconnection. In this way, the plant and controller energies and dampings are *added* up in the overall system, being able then to shape the energy and add the required damping.

[1] The difference between cyclo–passive and passive operators is that the storage function of the former is not necessarily bounded from below.

[2] In the terminology of Ortega et al. (1998) this kind of PBC is called "Standard", to distinguish it from other PBC techniques, like Interconnection and Damping Assignment or Control by Interconnection, developed for port–Hamiltonian systems (Duindam et al., 2009).

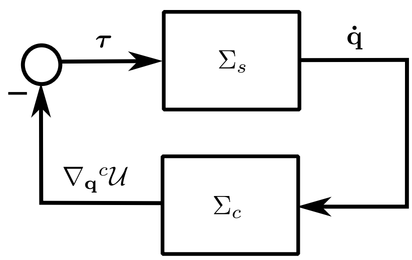

Fig. 1. Interconnection of the EL-system (2) and the EL-controller (6).

Let us denote the generalized coordinates of the controller as $\boldsymbol{\theta} \in \mathbb{R}^{Nn}$. Then its total energy function can be written as

$$^c\mathcal{T}(\mathbf{q}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) := {}^c\mathcal{K}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + {}^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta}) \tag{5}$$

where

$$^c\mathcal{K}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \frac{1}{2}\dot{\boldsymbol{\theta}}^\top \mathbf{M}_c \dot{\boldsymbol{\theta}}$$

is the controller's kinetic energy with $\mathbf{M}_c \in \mathbb{R}^{Nn\times Nn}$ its *constant* positive semi-definite inertia matrix and ${}^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta})$ the potential energy. Applying the EL-equations of motion the controllers dynamics will be

$$\mathbf{M}_c\ddot{\boldsymbol{\theta}} + \mathbf{D}\dot{\boldsymbol{\theta}} + \nabla_{\boldsymbol{\theta}}{}^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta}) = \mathbf{0}_{Nn}. \tag{6}$$

where $\mathbf{D} := \text{blockdiag}\{d_i\mathbf{I}_n\} > 0$ is an $Nn \times Nn$ *damping* matrix.

The controller dynamics (6) verifies the following obvious input–output property.

Fact 2. The controller (6) defines a cyclo–passive operator $\Sigma_c : \dot{\mathbf{q}} \to \nabla_{\mathbf{q}}{}^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta})$ with storage function ${}^c\mathcal{T}(\mathbf{q}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$, i.e.,

$$^c\dot{\mathcal{T}}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \dot{\mathbf{q}}^\top \nabla_{\mathbf{q}}{}^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta}) - \dot{\boldsymbol{\theta}}^\top \mathbf{D}\dot{\boldsymbol{\theta}}.$$

◇

The next step in the PBC design is to interconnect the plant with the controller via

$$\boldsymbol{\tau} = -\nabla_{\mathbf{q}}{}^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta}), \tag{7}$$

as shown in Fig. 1. It is clear from the figure and Facts 1 and 2 that the resulting system is the negative feedback interconnection of two passive subsystems. Consequently, the total (desired) energy function of the closed–loop system is the sum of energy of the system plus the energy of the controller, that is,

$$^d\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) := {}^s\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}}) + {}^c\mathcal{T}(\mathbf{q}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}), \tag{8}$$

and it, clearly, verifies

$$^d\dot{\mathcal{T}}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = -\dot{\boldsymbol{\theta}}^\top \mathbf{D}\dot{\boldsymbol{\theta}} \leq 0. \tag{9}$$

The controller dynamics (6) is now selected to, first, ensure that there exists an equilibrium point of the overall system where the control objective is achieved, say $(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = (\mathbf{q}_\star, \mathbf{0}_{Nn}, \boldsymbol{\theta}_\star, \mathbf{0}_{Nn})$ and, second, to render this equilibrium point stable (in the sense of Lyapunov). Towards this end, we postulate the total energy ${}^d\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ as a Lyapunov function. From (9) it can be concluded that it is a *nonincreasing* function, therefore it only remains to make this function positive definite, which is tantamount to proving that it has a unique and isolated minimum at the equilibrium point. The PBC design is completed establishing *asymptotic* stability of the equilibrium. Since, almost inevitably, the Lyapunov function is not strict—as seen in (9)—this is done by invoking LaSalle's invariance principle. In particular, it is necessary to prove that $\dot{\boldsymbol{\theta}}$ is a *detectable* output for the interconnected system. Namely, that $\dot{\boldsymbol{\theta}}(t) \equiv \mathbf{0}_{Nn}$ implies that

$$\lim_{t\to\infty}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t)) = (\mathbf{q}_\star, \mathbf{0}_{Nn}, \boldsymbol{\theta}_\star, \mathbf{0}_{Nn}), \tag{10}$$

holds true.

4. SOLVING THE CONSENSUS PROBLEM

A simple, natural choice for the controller energy (5) is to take $\mathbf{M}_c = \mathbf{I}_{Nn}$ and

$$^c\mathcal{U}(\mathbf{q}, \boldsymbol{\theta}) = -{}^s\mathcal{U}(\mathbf{q}) + \frac{1}{2}(\mathbf{q}-\boldsymbol{\theta})^\top \mathbf{K}(\mathbf{q}-\boldsymbol{\theta}) + \frac{1}{2}\boldsymbol{\theta}^\top(\mathbf{PL}\otimes\mathbf{I}_n)\boldsymbol{\theta},$$

where $\mathbf{K} := \text{blockdiag}\{k_i\mathbf{I}_n\} > 0$ is the $Nn \times Nn$ matrix of the springs stiffness coefficients, $\mathbf{P} := \text{diag}\{p_i\} > 0$ is a $N \times N$ gain matrix and $\otimes$ is the standard Kronecker product. This choice cancels the potential energy of the agents and interconnects them through linear springs. The desired energy (8) has a global minimum at

$$(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = ((\mathbf{1}_N \otimes \mathbf{q}_c), \mathbf{0}_{Nn}, (\mathbf{1}_N \otimes \mathbf{q}_c), \mathbf{0}_{Nn}), \tag{11}$$

where $\mathbf{q}_c \in \mathbb{R}^n$ that, as is well known (Nuño et al., 2011), coincides (in the undelayed case) with the average of the initial conditions of the agents positions. Consequently, ${}^d\mathcal{T}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is a Lyapunov function and the equilibrium is stable. Once it is proved that $\dot{\boldsymbol{\theta}}$ is detectable, then (10) holds. Hence, (11) is a GAS equilibrium.

The control signal (7) and the controller dynamics (6) of the ith-EL-system are given by

$$\boldsymbol{\tau}_i = \nabla_{\mathbf{q}_i}{}^s\mathcal{U}_i(\mathbf{q}_i) - k_i(\mathbf{q}_i - \boldsymbol{\theta}_i) \tag{12}$$

and

$$\ddot{\boldsymbol{\theta}}_i = -d_i\dot{\boldsymbol{\theta}}_i - k_i(\boldsymbol{\theta}_i - \mathbf{q}_i) - p_i\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j),$$

respectively. Clearly, this controller is decentralized and its implementation does not require velocity measurements. When communication delays are present (12) remains unaltered. However, the controller dynamics changes to

$$\ddot{\boldsymbol{\theta}}_i = -d_i\dot{\boldsymbol{\theta}}_i - k_i(\boldsymbol{\theta}_i - \mathbf{q}_i) - p_i\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t))). \tag{13}$$

In the delayed case, LaSalle's invariance principle cannot longer be invoked and a different proof approach has to be followed.

At this point we state our main result.

Proposition 1. Consider the network of EL-agents (2) with the interconnection graph verifying Assumptions **A1** and **A2**. The controller (12), (13) solves the **LC** problem provided the gains satisfy

$$2d_i > p_i\sum_{j\in\mathcal{N}_i} a_{ij}\left(\alpha_i + \frac{{}^*T_{ij}^2}{\alpha_j}\right), \quad \forall i \in \bar{N} \tag{14}$$

for any $0 < \alpha_i < \infty$, $\forall i \in \bar{N}$. ◇

Proof. Using the properties of the Laplacian matrix, as in (Nuño et al., 2013b), it is easy to show that the time derivative of the desired energy function (8)—evaluated along (2), (12) and (13)—is given by

$$
\begin{aligned}
{}^{d}\dot{\mathcal{T}} = &- \dot{\boldsymbol{\theta}}^\top \mathbf{D}\dot{\boldsymbol{\theta}} + \dot{\boldsymbol{\theta}}^\top (\mathbf{PL} \otimes \mathbf{I}_n)\boldsymbol{\theta} \\
&- \sum_{i\in\bar{N}} p_i \dot{\boldsymbol{\theta}}_i^\top \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t))).
\end{aligned}
$$

Since

$$
\dot{\boldsymbol{\theta}}^\top (\mathbf{PL} \otimes \mathbf{I}_n)\boldsymbol{\theta} = \sum_{i\in\bar{N}} p_i \dot{\boldsymbol{\theta}}_i^\top \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j),
$$

then ${}^{d}\dot{\mathcal{T}}$ can be written as

$$
{}^{d}\dot{\mathcal{T}} = -\sum_{i\in\bar{N}} \left(d_i|\dot{\boldsymbol{\theta}}_i|^2 + p_i \dot{\boldsymbol{\theta}}_i^\top \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_j - \boldsymbol{\theta}_j(t - T_{ji}(t))) \right).
$$

From

$$
\boldsymbol{\theta}_j - \boldsymbol{\theta}_j(t - T_{ji}(t)) = \int_{t-T_{ji}(t)}^{t} \dot{\boldsymbol{\theta}}_j(\theta)d\theta,
$$

we get

$$
{}^{d}\dot{\mathcal{T}} = -\sum_{i\in\bar{N}} \left(d_i|\dot{\boldsymbol{\theta}}_i|^2 + p_i \sum_{j\in\mathcal{N}_i} a_{ij} \dot{\boldsymbol{\theta}}_i^\top \int_{t-T_{ji}(t)}^{t} \dot{\boldsymbol{\theta}}_j(\theta)d\theta \right).
$$

Integrating ${}^{d}\dot{\mathcal{T}}$, from 0 to t, yields

$$
\begin{aligned}
{}^{d}\mathcal{T}(t) - {}^{d}\mathcal{T}(0) = &- \sum_{i\in\bar{N}} d_i \int_0^t |\dot{\boldsymbol{\theta}}_i(\sigma)|^2 d\sigma \\
&- \sum_{i\in\bar{N}} p_i \sum_{j\in\mathcal{N}_i} a_{ij} \int_0^t \dot{\boldsymbol{\theta}}_i^\top(\sigma) \int_{\sigma-T_{ji}(\sigma)}^{\sigma} \dot{\boldsymbol{\theta}}_j(\theta)d\theta d\sigma.
\end{aligned}
$$

invoking Lemma 1 on the double integral term and following the same steps as in (Nuño et al., 2013b), it can be shown, that setting the controller's gains such that (14) is satisfied, then there exists $\lambda_i > 0$ such that

$$
{}^{d}\mathcal{T}(0) \geq {}^{d}\mathcal{T}(t) + \sum_{i\in\bar{N}} \lambda_i \|\dot{\boldsymbol{\theta}}_i\|_2^2.
$$

This last, and the fact that ${}^{d}\mathcal{T}(t) \geq 0$, for all $t \geq 0$, ensures that $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2$ and ${}^{d}\mathcal{T} \in \mathcal{L}_\infty$.

Since ${}^{d}\mathcal{T}$ is positive definite and radially unbounded with respect to to $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i, |\mathbf{q}_i - \boldsymbol{\theta}_i|, |\boldsymbol{\theta}_i - \boldsymbol{\theta}_j|$ then all these signals are bounded.

$\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ ensures that $|\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t))| \in \mathcal{L}_\infty$. With all these bounded signals it follows from (13) that $\ddot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$. Barbalăt's Lemma allows to conclude that $\lim_{t\to\infty} \dot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$.

Now, differentiating (13) yields

$$
\begin{aligned}
\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i = &- d_i \ddot{\boldsymbol{\theta}}_i - k_i(\dot{\boldsymbol{\theta}}_i - \dot{\mathbf{q}}_i) \\
&- p_i \sum_{j\in\mathcal{N}_i} a_{ij} \left(\dot{\boldsymbol{\theta}}_i - (1 - \dot{T}_{ji})\dot{\boldsymbol{\theta}}_j(t - T_{ji}(t)) \right).
\end{aligned} \tag{15}
$$

The fact that $\ddot{\boldsymbol{\theta}}_i, \dot{\boldsymbol{\theta}}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and boundedness of $\dot{T}_{ji}$, ensure that $\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$. Therefore, $\ddot{\boldsymbol{\theta}}_i$ is uniformly continuous and, since

$$
\lim_{t\to\infty} \int_0^t \ddot{\boldsymbol{\theta}}_i(\sigma)d\sigma = \lim_{t\to\infty} \dot{\boldsymbol{\theta}}_i(t) - \dot{\boldsymbol{\theta}}_i(0) = -\dot{\boldsymbol{\theta}}_i(0),
$$

we have that $\lim_{t\to\infty} \ddot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$. Invoking the same arguments, it can be established that $\lim_{t\to\infty} \frac{d}{dt}\ddot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$. Consequently, from (15), $\lim_{t\to\infty} \dot{\mathbf{q}}_i(t) = \mathbf{0}_n$.

The proof is completed, first, showing that the controllers generalized coordinates $\boldsymbol{\theta}$ converge to a consensus point; second, proving that the systems generalized coordinates $\mathbf{q}$ converge to $\boldsymbol{\theta}$. For the first step we use the fact that

$$
\boldsymbol{\theta}_i - \boldsymbol{\theta}_j(t - T_{ji}(t)) = \boldsymbol{\theta}_i - \boldsymbol{\theta}_j + \int_{t-T_{ji}(t)}^{t} \dot{\boldsymbol{\theta}}_j(\theta)d\theta,
$$

and since $\lim_{t\to\infty} \dot{\boldsymbol{\theta}}_i(t) = \mathbf{0}_n$, from (13), it holds that

$$
\lim_{t\to\infty} \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i(t) - \boldsymbol{\theta}_j(t)) = \mathbf{0}_n.
$$

In matrix form and making use of the Laplacian, this last expression can be written as

$$
\lim_{t\to\infty} (\mathbf{L} \otimes \mathbf{I}_n)\boldsymbol{\theta}(t) = \mathbf{0}_{Nn}.
$$

The proof of the first claim is completed invoking the properties of the Laplacian.

For the second claim, notice that the closed-loop system (2) and (12) is given by

$$
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q} - \boldsymbol{\theta}) = \mathbf{0}_{Nn}.
$$

The fact that $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \dot{\mathbf{q}}_i(t) = \mathbf{0}_n$ implies, by Barbalăt's Lemma, that $\lim_{t\to\infty} \mathbf{q}_i(t) - \boldsymbol{\theta}_i(t) = \mathbf{0}_n$ as required. □

Remark 1. Although α_i are free to choose, for all $i \in \bar{N}$ and as long as they are positive, to improve performance they should be chosen as small as possible.

Remark 2. The PBC reported here generalizes different control schemes that rely on velocity measurements, as the P+d controller, or those which do not require velocity measurements. Interestingly, the conditions on the gains (14) of the new PBCs—that do not require velocity measurements—are the same as those for the P+d controller reported in (Nuño et al., 2013b).

Remark 3. The bound (14) has a very clear physical interpretation. On one hand, it captures the obvious fact that if the proportional gain of the controller, corresponding to the springs stiffness coefficients p_i, is increased to obtain a faster response, the dissipation gain d_i must also be increased. On the other hand, it reveals the more subtle pernicious effect of communication delays. In the absence of delays, the interconnection of EL-systems (via virtual springs) is power preserving. This is no longer the case in the presence of delays, which induce "energy losses" that must be compensated by the controller. This additional energy is a function of the delays bound ${}^{*}T_{ji}$, see (4), and the interconnection *strength* a_{ij}. See also (Schiffer et al., 2015; Pasumarthy and Kao, 2009) for some results on the effect of delays on interconnections of passive systems.

Remark 4. As stated in the Introduction, up to the authors' knowledge, the P+d controller with the *I&I* velocity observer of (Nuño, 2015, 2016) is the only scheme capable of solving the consensus problem in the presence of interconnecting delays and without using velocity measurements. The stability condition of this controller is given by

$$
d_i > p_i \sum_{j\in\mathcal{N}_i} a_{ij} \left(\alpha_i + \frac{{}^{*}T_{ij}^2}{\alpha_j} \right), \quad \forall i \in \bar{N},\ j \in \mathcal{N}_i.
$$

Compared to (14) it is clear that the observer–based controllers require twice the amount of *damping*, making more

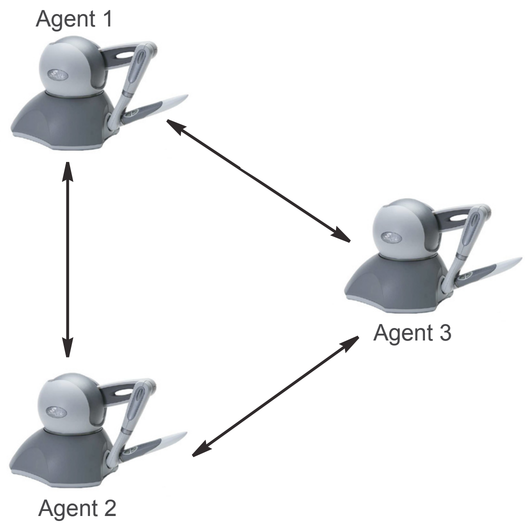

Fig. 2. Experimental testbed, composed of three Geomagic® Touch haptic devices.

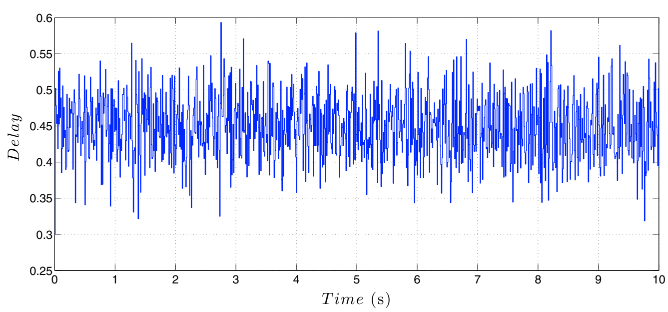

Fig. 3. Emulated UDP/IP Internet delay.

sluggish its response. Furthermore, in order to implement the $I\&I$ observer, the complete dynamics has to be exactly known, and it has a complicated expression that requires long computation times.

5. EXPERIMENTS

This section shows experimental evidence that the novel PBC (12) with (13) solves the **LC** problem. The experimental setup is composed of three fully-actuated 3-DOF mechanical systems. These devices are the Geomagic® Touch haptic devices (*www.geomagic.com*), as shown in Fig. 2. Two devices run in the same computer and the other is connected through a local WiFi network. The controller and all software is implemented in Matlab Simulink®. To handle the computer–device communication, we have used the PhanTorque libraries (Aldana et al., 2014) and the blocks *UDP send* and *UDP receive*, from the *Instrument Control Toolbox*, for the communications over the Internet.

Since the communication delays are negligible, an artificial delay has been included to show the robustness of the proposed scheme. For simplicity, the variable time-delays for all agents are the same and they emulate an ordinary UDP/IP Internet delay with a normal Gaussian distribution with mean, variance and seed equal to 0.45, 0.005 and 0.45, respectively (Salvo-Rossi et al., 2006). Along the duration of the experiments, these delays have been bounded by $^*T_{ij} = 0.65$s, as it is shown in Fig. 3. It should be underscored that compared to the real Internet delays in (Nuño et al., 2009), these delays are larger.

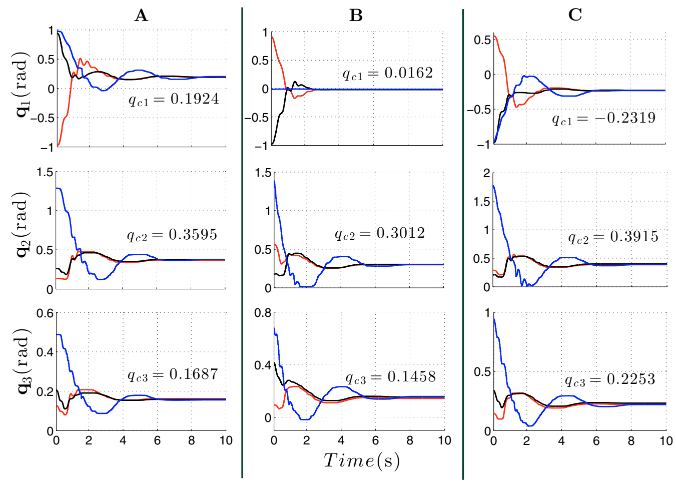

Fig. 4. Leaderless consensus experimental results for a network of three 3-DOF Geomagic® Touch haptic devices controlled by (12), (13).

The corresponding gravity vector, used in (12), is given by

$$\nabla_{\mathbf{q}_i}{}^s\mathcal{U}_i(\mathbf{q}_i) = \begin{bmatrix} 0 \\ \delta_{1_i}\sin(q_{2_i}+q_{3_i}) + \delta_{2_i}\cos(q_{2_i}) \\ \delta_{1_i}\sin(q_{2_i}+q_{3_i}) \end{bmatrix},$$

where $\delta_{1_i} = gm_{3_i}l_{2_i}$ and $\delta_{2_i} = gm_{3_i}l_{2_i} + gm_{2_i}l_{1_i}$. These physical values have been experimentally estimated yielding $\delta_{1_i} = 0.0095$Nm and $\delta_{2_i} = 0.0127$Nm.

The interconnection topology has the following Laplacian matrix

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

The controller gains have been set as: $d_i = 1.4$, $p_i = 1$, $k_i = 2$. Clearly, these gains satisfy (14) with $\alpha_i = 0.5$.

Fig. 4 depicts the leaderless consensus results where each column (A, B and C) has different initial positions for the three joints. Clearly, a consensus agreement point is found and therefore controller (12), (13) solves the **LC**, as expected.

6. CONCLUSIONS

This paper proposes a novel PBC that solves the leaderless consensus problem in networks of multiple EL-agents. Its main contribution is the proof that the resulting controller is robust to interconnecting variable time-delays and, more importantly, that it does not require velocity measurements. In contrast with the P+d controllers, the proposed PBC injects the dissipation required for asymptotic stability through the controller dynamics, which then propagates to the system. In the presence of delays, the dissipation has to be increased to compensate for the "losses" induced by the information exchange. The paper also presents experimental results that depict the performance of the novel controller.

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