Spacecraft Formations Reconfiguration using Finite Element Methodology with Adaptive Remeshing

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Abstract: We present a methodology to compute trajectories for formation flying reconfiguration problems using a methodology based on finite elements: FEFF (Finite Elements for Formation Flying). In this paper we center ourselves on the obtention of optimal meshes using adaptive remeshing. As an application we show how adaptive remeshing is applied to two kinds of reconfigurations: a class where is known that a bang-bang control is the optimal solution and another class where bang-bang would end up in collision. In the first case the methodology tends to the optimal bang-bang, while in the second one, tends to a low thrust control. Finally, we update the trajectories obtained with FEFF and adaptive remeshing to the JPL-ephemeris model to test the reliability of the methodology.

Keywords: astrodynamics, formation flying

1. INTRODUCTION

Formation flying of spacecraft is a concept that has an important role in technology applications and continues growing in importance, mainly for science and astrophysical missions. The reason is that formation flying enables a set of some small (and cheaper) spacecraft to act as a virtual larger satellite, obtaining a better information than a bigger one, with flexibility about the space observations that a formation can perform in the future.

Some formation flying missions are planed to be in orbits about the Earth, but also relevant science missions have big interest on the vicinity of libration points. The L_1 libration Sun-Earth+Moon point is nowadays the best place for observations of the Sun. The L_2 libration point is also an interesting place for deep space observations, where large telescopes or interferometry baselines could be located. This is the reason because projects like the Terrestrial Planet Finder (11) or Darwin (10) were planned to be in a libration point orbit about L_2.

The key technology that must be implemented in spacecraft formation flying is the control of a formation when doing observations. Mutual distances between spacecraft must be kept with high precision. A study of some of these control methodologies can be found in in Farrar, Thein and Folta (1) and references therein.

But also there are many other technologies that have to be taken into account for formation flying. The one we consider in this paper is the reconfiguration of the formation. This technique is important in the lapses between observations. It can be necessary to change the orientation, the target point of the formation or maybe also the shape or diameter of the cluster to give flexibility to the mission. Some representative techniques of reconfigurations are the proximity maneuvering using artificial potential functions studied by McInnes (8), or the technique used by Hadaegh, Beard, Wang and McLain (6).

In this paper we consider the reconfiguration of a formation by means of an optimization problem which uses the finite element methodology to obtain the controls that must be applied to each spacecraft. Additionally, our methodology can solve the problem of the transfer or deployment of the formation, which is another key problem in formation flying.

In this paper we mainly focus on the obtention of a suitable optimal mesh using an adaptive remeshing strategy, that assures us that the error produced by the finite element methodology is small enough. Once the methodology gives us the trajectory and an optimal mesh, we check it considering a vector field of full JPL-ephemeris.

2. THE FEFF METHODOLOGY

The main purpose of the FEFF methodology is to compute reconfigurations of a formation of spacecraft. In this paper we consider that the spacecraft are in the vicinity of a halo orbit of 120000 km of z-amplitude about L_2 in the Sun-Earth system, but the methodology can be applied to any other libration point orbit.

Here we present just a brief description of the basic FEFF methodology. More details about this point can be found in (2; 4).

We consider a formation of N spacecraft which must perform a reconfiguration in a fixed time T. The spacecraft are in a small formation (i.e., the distance between them is only of a few hundreds of meters, both in the initial and the final configurations). Our objective is to find a trajectory for each of the spacecraft which guides it to the goal position, with the minimum fuel consumption and avoiding collisions with other spacecraft.

As the formations are small with respect to the amplitude of the halo orbit, we consider the linearized equations of motion about the nonlinear orbit. We compute the trajectories using these equations and then we will deal with the non-linear part. For each of the spacecraft, we have the equation

$$\dot{\mathbf{X}}(t) = \mathbf{A}(t)\mathbf{X}(t), \tag{1}$$

where $\mathbf{A}(t)$ is a 6×6 matrix and $\mathbf{X}$ refers to the state of the spacecraft. The origin of the reference frame for the $\mathbf{X}$ coordinates is the nominal point on the base halo orbit at time t being the orientation of the coordinate axis parallel to the one of the RTBP.

The goal of the methodology is to find a set of optimal controls for each of the spacecraft. Including the initial and final states of the spacecraft in the reconfiguration problem and the controls, the equations that we deal with are

$$\begin{cases} \dot{\mathbf{X}}_i(t) = \mathbf{A}(t)\mathbf{X}_i(t) + \bar{\mathbf{U}}_i(t), \\ \mathbf{X}_i(0) = \mathbf{X}_i^0, \\ \mathbf{X}_i(T) = \mathbf{X}_i^T, \end{cases} \tag{2}$$

where $\mathbf{X}_i^0$ and $\mathbf{X}_i^T$ stand for the initial and final state of the i-th spacecraft of the formation, and $\bar{\mathbf{U}}_1, \ldots, \bar{\mathbf{U}}_N$, are the controls we are searching.

The key of procedure FEFF is that it uses the finite element methodology to obtain these controls (see (9) for references about the finite element method and (4) for a more detailed exposition about the FEFF methodology). Essentially, the time interval $[0, T]$ which we consider for the reconfiguration is split in M subintervals of the domain that we call elements. This mesh can have elements of different length and can be different for each of the spacecraft, depending on the nature of the trajectories of reconfiguration. We impose that controls are some maneuvers (in form of delta-v) that we apply in the points where two elements join (the nodes). The finite element methodology gives us a relation between the positions of the spacecraft in the nodes and the maneuvers, Δv, that we must apply.

Procedure FEFF reduces the reconfiguration problem to an optimization problem with constrains. The functional that must be minimized has to be related to fuel consumption, and since it is related to the sum of the norm of the delta-v, the functional we want to minimize is

$$J_1 = \sum_{i=1}^{N} \sum_{k=0}^{M_i} \rho_{i,k} ||\Delta \mathbf{v}_{i,k}||, \tag{3}$$

where $|| * ||$ denote the Euclidean norm and $\rho_{i,k}$ are weight parameters that can be used, for instance, to penalize the fuel consumption of selected spacecraft with the purpose of balancing fuel resources (here for clarity we consider that $\rho_{i,k}$ multiplies the modulus of the delta-v, but in a similar way we can impose a weight on each component).

The most important constraint in our problem is the avoidance of collision between spacecraft. This enters in the optimization problem as a constraint, imposing that each spacecraft is surrounded by a security sphere and the spheres of all the spacecraft cannot collide. We note that the partition of the time interval made by the finite element methodology gives us an efficient implementation to check this constraint.

3. ADAPTIVE REMESHING APPLIED TO RECONFIGURATIONS

When searching the optimal controls for the spacecraft, we must take into account two facts. One of them is related to ill conditioning problems and the other is related to the error due to the finite element approximation. Both of them lead us to think about a remeshing strategy.

When we consider the functional of equation (3), we see that it is ill conditioned to compute derivatives when delta-v values are small. But our objective is to find small delta-v! In order to avoid this problem, we have two strategies: the first one consists on considering an alternative functional to minimize,

$$J_2 = \sum_{i=1}^{N} \sum_{k=0}^{M_i} \rho_{i,k} ||\Delta \mathbf{v}_{i,k}||^2, \tag{4}$$

which is also related to fuel consumption and it is not ill conditioned. The second one is based on considering a remeshing strategy to suppress the nodes with a small delta-v.

On the other hand, the approximation of the solution via the finite element method gives us some errors associated to the approximation we make on each one of the elements of the mesh.

To solve these two facts, we will consider an adaptive remeshing strategy applied to our reconfiguration problem. The general idea of adaptive remeshing is that, given a threshold value e, to find a mesh that provides an approximate solution with error (understood as the difference between the solution of the problem and its approximation inside of an element) less than e in some norm.

In figure 1 we give a schema of the general idea of our procedure. It has two different phases. In the first one, it starts computing the trajectories for the spacecraft using the finite element method with a given mesh. In this first phase the objective is to find a rough approximation of the solution, so we start with a small number of elements (a maximum of 10). All of these elements have the same length and all the spacecraft start with the same mesh. Once we have obtained the trajectories, it computes an estimation of the error. This error is essentially obtained by comparison between the gradient obtained using the finite element model and the one obtained by integration of the equations of motion. The second step of the procedure is an iterative process which performs adaptive remeshing, recomputes the approximate solution with the new mesh and ends when the error is below a given tolerance. When

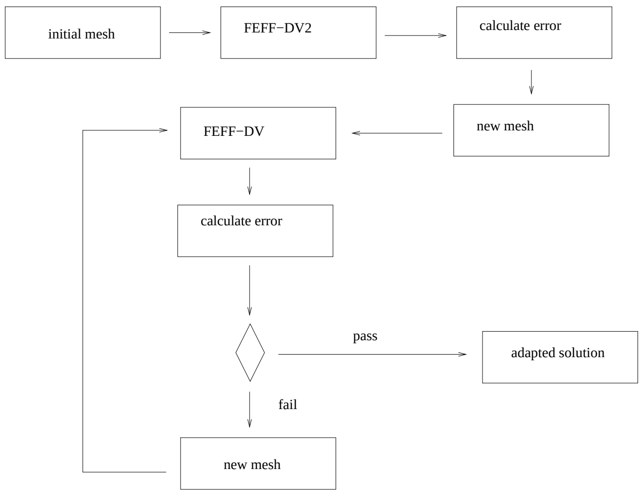

Fig. 1. Schema of the procedure of adaptive remeshing.

remeshing is necessary, the new mesh is adapted using the estimation of errors of the previous mesh.

Adaptive remeshing methods penalize the elements where the error is considered big, dividing them in smaller elements. On the other hand, if the estimation of the error is small in an element, then this element is made bigger in the next iteration. Since, essentially our estimation of the error is related to the value of the delta-v maneuvers to be implemented, this method tends to increase the length of the elements which have associated small delta-v and tends to decrease the length of the elements which have associated big delta-v's.

Essentially to decide whether the current mesh is good enough or not we base in a criterion which compares the modulus of the estimated error ($||e||$) with the total gradient of the solution,

$$||\bar{u}|| = \int_0^T \mathbf{v}_2 dt.$$

We accept the mesh when

$$||e|| \leq \nu ||\bar{u}||,$$

where ν is the acceptability criteria. We discuss about the value of ν taken on section 4.

In order to compute the new mesh, we use the Li and Bettess remeshing strategy (see (7)). This strategy is based on the idea that the error distribution on an optimal mesh is uniform,

$$||\hat{e}_k|| = \nu ||\bar{u}|| / \sqrt{\hat{M}},$$

where ν is again the acceptability criteria, e_k is the computed error on element k, M is the number of elements of the mesh and the hat distinguishes the parameters of the new mesh. The strategy consists on finding the new length of the elements using the number of elements of the new mesh, $\hat{M}$. Let us denote d the dimension of the problem and m the maximum degree of the polynomials used in the interpolation. Then, according to Li and Bettess, the number of elements needed by the new mesh is,

$$\hat{M} = (\nu ||\bar{u}||)^{-d/m} \left(\sum_{k=1}^{M} ||e_k||^{d/(m+d/2)} \right)^{(m+d/2)/m}.$$

Since we work with linear elements in dimension one, we have $m = 1$ and $d = 1$. The recommended number of elements of the new mesh is

$$\hat{M} = (\nu ||\bar{u}||)^{-1} \left(\sum_{k=1}^{M} ||e_k||^{2/3} \right)^{3/2}.$$

Once we have the estimation of the number of elements, we can find the length of the new elements:

$$\hat{h}_k = \left(\frac{\nu ||\bar{u}||}{\sqrt{\hat{M}} ||e_k||} \right)^{1/m+d/2} h_k,$$

that in our case, turns out to be

$$\hat{h}_k = \left(\frac{\nu ||\bar{u}||}{\sqrt{\hat{M}} ||e_k||} \right)^{3/2} h_k.$$

4. SIMULATIONS WITH ADAPTIVE REMESHING

When computing reconfigurations of spacecraft, we have two limiting cases: if there is no collision risk when the spacecraft follow a linear trajectory, we know that the optimal trajectory is a bang-bang control for each one of the spacecraft. This is the most critical case for our procedure, since the optimal maneuver consists in two delta-v: one at departure and another one at the arrival position. The remaining nodal delta-v must be zero, and so this is a case where the computation of derivatives for J_1 is very ill conditioned. On the other hand, for cases with collision risk, our methodology must tend to low thrust when the diameter of the mesh tend to zero. In this section we present an example of each of these two cases.

4.1 A bang-bang example

In this kind of problems, since there are no collision hazards, collision avoidance does not affect the trajectories of the spacecraft and the optimal trajectory for each spacecraft is independent from the others. For this reason, we can reduce the computations to obtain the optimal trajectory for a single spacecraft.

In order to exemplify the procedure, we consider a shift of a single spacecraft. We consider the reference frame for the equations (2) aligned with respect to the RTBP reference frame, but with origin on the nominal point of the base halo orbit (when $t = 0$ this point corresponds to the "upper" position of the Halo orbit, this is when it crosses the RTBP plane $Y = 0$ with $Z > 0$). The initial condition for this example is taken 100 meters far from the base nominal halo orbit in the X direction, and the goal is to transfer it to a symmetrical position with respect to the halo orbit in 8 hours. This is to 100 meters in the opposite X direction doing a shift of 200 meters for the spacecraft transfer maneuver.

For this particular case, we know that the optimal solution is a bang-bang control with maneuvers of 0.69 cm/s at departure and arrival.

Our procedure starts by minimizing the functional (4) and obtains a trajectory with the delta-v profile of figure 2. This is a typical profile result for the reconfigurations without collision risk.

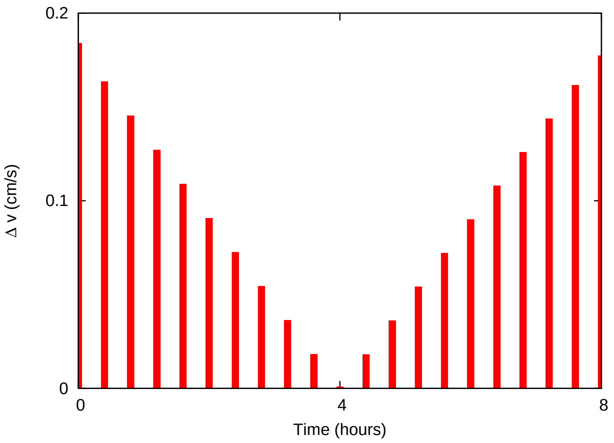

Fig. 2. Delta-v obtained with the minimization of functional (4) in the case of no collision risk. The methodology FEFF converges to a bang-bang solution with this initial seed.

We compute now the reconfiguration considering different values for the parameter ν. We note that this parameter does not only appear in the acceptability criteria: it is also used to obtain the new mesh. If we take a small value of ν, we can end up with a mesh with a big number of nodes, which results in an optimization problem with a very large number of variables, that could be unsolvable in practice. Note that a mesh with 100 elements and a single spacecraft ends up with an optimization problem with 594 variables. In the other way around, if we use a big ν, we could end up accepting some meshes with big errors. In table 1, we have a summary of the results obtained for different values of the parameter ν, the number of iterations needed to reach the bang-bang solution and the number of elements after the first iteration of the methodology.

We note that when ν is very small, there is no convergence. The case with ν equal to 0.0001 makes the optimal procedure awkward. When ν is 0.001, the final number of elements is greater than 1 (that we know is our final target number) although there are some very small delta-v. When ν is big, moreover there is no convergence: the final mesh contains more elements than expected, because it passes the acceptability criteria before converging to the bang-bang control. We can conclude that, in this bang-bang case, the best values for ν are inside the range $[0.04, 0.06]$. With values larger than 0.06, the algorithm does not converge.

4.2 A low-thrust example

In reconfigurations where the bang-bang trajectories end up with collisions with some of the spacecraft, FEFF obtains trajectories which of course are different from bang-bang. Our objective is to study whether these trajectories could tend to low thrust arcs.

For this case we consider a configuration based on the Terrestrial Planet Finder (TPF) model (see (11)). We assume that the satellites are initially contained in the local plane $Z = 0$, with the interferometry baseline aligned on the X axis. We simulate the swap between two pairs of satellites in the baseline: each inner satellite changes its location with the outer satellite which is closest to it in

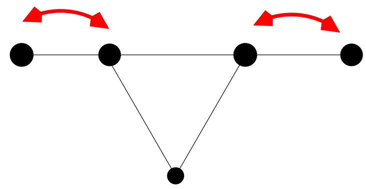

Fig. 3. Example of reconfiguration with collision risk: the switch of two pairs of spacecraft of the TPF formation.

position (this is inner satellites are maneuvered to attain outer positions and vice-versa as shown in figure 3). Again we consider 8 hours for the reconfiguration. The process of switching positions has a collision risk and simple bang-bang controls are no longer valid.

As in the bang-bang case, we consider different values for the parameter ν. A discussion like in the previous section is also valid here: using a small ν, we can end up with a mesh with many elements. For example, taking $\nu = 0.0005$, in the first iteration we have around 1000 elements. We do not only have the problem of having very small elements: the optimization problem that we end up with has 29970 variables and it is not desirable. Again, if we take a big ν, we can end up with a mesh with big errors, or with a mesh with only a few elements.

In table 2, we display a summary of the results obtained for different values of the parameter ν, the number of iterations until the methodology converges (Iter), the number of elements in the first iteration (N_1) and the number of elements in the last iterate (N_F).

As in the previous case, when ν is small, the number of elements is big, and the computation of the optimum is very expensive. Also, taking ν big, the number of elements may not be enough.

We note that now the best values are inside the range $[0.005, 0.05]$. With values larger than 0.05, the number of

Table 1. Number of iterations necessary to obtain the bang-bang solution depending on ν. We have indicated by "fail" the cases where the procedure does not converge.

ν	Elements it. 1	Iterations
0.0001	3008	Fail
0.001	301	Fail
0.002	149	25
0.005	61	16
0.01	31	14
0.02	15	10
0.03	11	6
0.04	7	4
0.05	6	4
0.06	4	2
0.07	4	Fail

elements is very small and with values smaller than 0.005 the number of elements makes the computation much more expensive.

Since the value $\nu = 0.05$ is appropriated for the two cases, we consider it for our computations.

4.3 Considerations about the value of ν

We have seen in the previous sections that a desirable value of ν must be in the range $[0.005, 0.05]$. This range gives us an idea of the value of ν we must choose.

We have applied the reconfiguration procedure to a test bench of 25 reconfigurations which include switches between spacecraft located at opposite vertices of polygons (6), switches in the TPF formation (9) and parallel shifts (10) of different size with a number of spacecraft from 3 to 10. 10 of the reconfigurations are converging to a bang-bang solution, the other 15 reconfigurations are converging to low-thrust. We have applied the methodology using different values of ν and we have computed the mean of the number of iterations of the the adaptive process necessary to converge. The results can be seen in table 3.

Again we conclude that the best value of ν is 0.05.

5. DEALING WITH NONLINEARITIES

We have pointed out that the size of the formations is very small (a few hundreds of meters) when comparing it to the size of the halo orbit. This fact gives us the possibility to work with linearized equations about the halo orbit. In this section we present the results of some simulations to show that the use of linearized equations is really a good model.

Our objective in this section is to give a way to measure how the truncated nonlinear terms, as well as other perturbations, affect the nominal reconfiguration trajectory, and the corrections that should be applied to the nominal maneuvers (corrective maneuvers) in order to reach the mission goal. For this purpose, we consider the trajectories given by the FEFF methodology as the nominal path for the spacecraft and compute the corrective maneuvers that guide the spacecraft through the nominal nodal states.

Table 2. Number of iterations and elements obtained with the swapping example of TPF depending on ν.

ν	N_1	Iter	N_F
0.0001	3504	Fail	
0.001	350	10	232
0.002	175	8	202
0.005	70	8	171
0.01	34	7	89
0.02	18	6	45
0.03	12	4	33
0.04	9	3	27
0.05	6	3	15
0.06	6	3	9
0.07	5	2	7

Table 3. Mean of the number of iterations as a function of ν for the 25 test bench reconfigurations.

ν	0.005	0.01	0.02	0.03	0.04	0.05	0.055	0.06
It.	10.2	8.4	7.1	4.2	3.7	3.2	4.3	5.2

The corrective maneuvers are computed using a similar strategy to (5). The main idea is that on each element we have the nodal states given by the FEFF methodology, and the difference between these states and the true states is corrected with some small maneuvers. For more details, see (3) or (4).

In the same line as in previous computations, we consider two different cases: the ones which end up in low thrust trajectories (there are many elements on the mesh, and these elements are small) and the ones that end up in bang-bang (the mesh is formed by a single element which is the same as the reconfiguration time).

The bang-bang case

Like we have pointed out in the previous section, in examples where there is no collision risk, we can focus in the results of a single spacecraft, because the results are independent.

In order to test the suitability of the linear approximation, we consider the shift of a spacecraft in the x direction (200 or 400 meters in 8 or 24 hours). In table 4 we give the value of the delta-v given by FEFF methodology (Δv_L), the number of corrections that are performed on each element (n), the maximum of corrective maneuvers ($\Delta \hat{v}_{LJmax}$), the total amount of corrective maneuvers ($\Delta \hat{v}_{LJ}$) and the percentage of the corrective maneuvers with respect to the maneuvers given by FEFF.

Table 4. Corrective maneuvers for some cases of bang-bang. Model equations considered correspond to JPL ephemeris and all delta-v are given in cm/s.

case	Δv_L	n	$\Delta \hat{v}_{LJmax}$	$\Delta \hat{v}_{LJ}$	%
200 m 8 h	0.69	3	3.3×10^{-3}	6.7×10^{-3}	0.96
	0.69	4	2.5×10^{-3}	6.1×10^{-3}	0.88
	0.69	5	2.2×10^{-3}	5.8×10^{-3}	0.84
	0.69	6	2.2×10^{-3}	5.6×10^{-3}	0.80
200 m 24 h	0.23	3	3.6×10^{-3}	7.5×10^{-3}	3.25
	0.23	4	2.7×10^{-3}	6.7×10^{-3}	2.89
	0.23	5	2.2×10^{-3}	6.4×10^{-3}	2.77
	0.23	6	2.2×10^{-3}	6.1×10^{-3}	2.65
400 m 8 h	2.8	3	5.3×10^{-3}	9.7×10^{-3}	0.35
	2.8	4	3.9×10^{-3}	9.2×10^{-3}	0.33
	2.8	5	3.1×10^{-3}	8.1×10^{-3}	0.29
	2.8	6	2.7×10^{-3}	7.5×10^{-3}	0.27

We note that all these corrective maneuvers are very small, both in absolute values and in percentage.

The low thrust case

When we compute the corrective maneuvers with cases of low thrust we must take into account that the elements are very small, and we will do a lot of corrective maneuvers during the reconfiguration time. So, each one of these maneuvers is expected to be smaller than the ones in the case of bang-bang.

We have considered three cases of low thrust examples, presented in table 5. The first one is the swap of two spacecraft which are at a distance of 100 meters. The reconfiguration time is 24 hours. The second example is the TPF swap presented in the previous section. And the third example deals with another case based on the TPF formation: to change the position of the collector towards a point symmetric to the departure position with respect to the interferometry baseline. The parameters of the table are the same ones of the previous example, except for the maximum of the delta-v, $(\Delta v/l)_{Jmax}$. In this case we give the maximum of delta-v divided by the length of the element instead, this is thrust acceleration as is usual for low thrust trajectories.

Table 5. Corrective maneuvers for some cases of low thrust. Model equations considered correspond to JPL ephemeris and all delta-v are given in cm/s.

case	Δv_L	n	$(\Delta v/l)_{Jmax}$	Δv_J	%
swap	0.63	3	9.1×10^{-3}	7.5×10^{-3}	1.19
2 sats	0.63	4	8.5×10^{-3}	6.9×10^{-3}	1.10
100 m	0.63	5	7.7×10^{-3}	6.7×10^{-3}	1.06
24 h	0.63	6	5.9×10^{-3}	6.4×10^{-3}	1.01
	2.34	3	9.9×10^{-3}	1.3×10^{-2}	0.51
TPF	2.34	4	8.2×10^{-3}	1.1×10^{-2}	0.44
swap	2.34	5	6.4×10^{-3}	1.0×10^{-2}	0.42
	2.34	6	2.6×10^{-3}	9.2×10^{-3}	0.37
TPF	1.26	3	8.3×10^{-3}	1.0×10^{-2}	0.77
symmetry	1.26	4	8.0×10^{-3}	9.4×10^{-3}	0.73
baseline	1.26	5	5.9×10^{-3}	8.9×10^{-3}	0.66
	1.26	6	5.5×10^{-3}	8.1×10^{-3}	0.60

We remark that delta-v's are again small, both in absolute value and in percentage. So, we can conclude that for small formations (of a few hundreds of meters) and small reconfiguration times (of a few hours), the corrective maneuvers are small, and the linear approximation about the nonlinear orbit is good enough.

6. CONCLUSIONS

This paper presents a methodology to find trajectories for reconfigurations of spacecraft, using the finite element method.

We have shown that the strategy of adaptive remeshing is suitable for the problem. It converges towards a bang-bang solution when there is no collision risk (that is known in this case to be the optimal control) and when there exists collision risk in the reconfiguration process, the procedure tends to low thrust arcs.

The computations have been done for formations of a few hundreds of meters, with reconfiguration times of few hours. We have shown, through some general examples, that the linearized model about the nonlinear orbit is suitable for the nominal computations.

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