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A simple spatiotemporal evolution model of a transmission power grid

M. Rosas-Casals, S. Valverde and R. Solé

***Abstract* — In this paper we present a model for the spatial and temporal evolution of a particularly large human made network: the 400 kV French transmission power grid. This is based on (a) an attachment procedure that diminishes the connection probability between two nodes as the network grows and (b) a coupled cost function characterizing the available budget at every time step. Two differentiated and consecutive processes can be distinguished: a first global space filling process and a secondary local meshing process that increases connectivity at a local level. Results show that even without power system engineering design constraints (i.e., population and energy demand), the evolution of a transmission network can be remarkably explained by means of a simple attachment procedure. Given a distribution of resources and a time span, the model can also be used to generate the probability distribution of cable lengths at every time step, thus facilitating network planning. Implications for network's fragility are suggested as a starting point for new design perspectives in this kind of infrastructures**

***Index Terms* — Networks, evolution, transmission power grid.**

I. Introduction

Human made networks built with, both, the generic objective to establish means of communication and information exchange or to distribute resources and commodities of any kind, are known as technological networks [1]. Internet [2] and electronic circuits [3] would belong to the former, while networks such as the oil, gas [4] and water pipelines, those formed by air flights and airports [5]–[7], roads and streets [8] or railway and subway [9] would belong to the latter. Technological networks are normally characterized by a huge number of heterogeneous and spatially distributed components, usually connected in a non-trivial way. They tend to display functional patterns not deducible from the analysis of their individual components. It is therefore mandatory to understand the structure and dynamic behavior of these networks if we want to favor and improve our ability to maintain and guarantee (a) their structural integrity, (b) the security of supply and (c) the transport efficiency [10], [11].

Within the networks that distribute resources and commodities of any kind, there exists a particular important subset: those technological networks that deal with energy sources and vectors. Water, oil and gas networks would be the historically first energy networks [12]. Soon afterwards, the advent of electricity made it possible to develop the most essential technological characteristic of our times: the power grid [13], [14]. This network was built mostly during the last century and it is actually formed by all the generators, transformers and substations that connect electricity consumers with producers by means of wires, cables and different voltage levels.

Most technological networks evolve in time and space following unknown driving forces. They have been continuously going through changes, spanning and crossing urban and natural systems from their early stages, adapting and being adapted by human societies, landscapes and territories. This paper analyzes the topological patterns aroused in the growing process of one major technological energy network (Figure 1): the French transport power grid (http://www.rte-france.com). Evolved from 1962 until nowadays, the French electricity transport system is an example of technological network where societal, economic, political and lastly, environmental shaping processes are at play. This system defines a planar graph [15] which changes in time as the network grows. Approaches found in the literature apply, for example, optimization methodologies to planning one [16] or multivoltage-level distribution networks [17]. In contrast, here we present a characterization of the topological change displayed by a transmission network using a very simple attachment procedure which explains its evolution without optimization processes, or population and energy constraints.

The paper is organized as follows: in section II we present the dataset to be analyzed (a time series of biennial network snapshots covering a period of 43 years). In section III, basic topological patterns are measured following the graph evolution. In sections IV and V we present the spatiotemporal evolution model and a coupled cost function as a tentative dynamical explanation of its growth process. Finally, in

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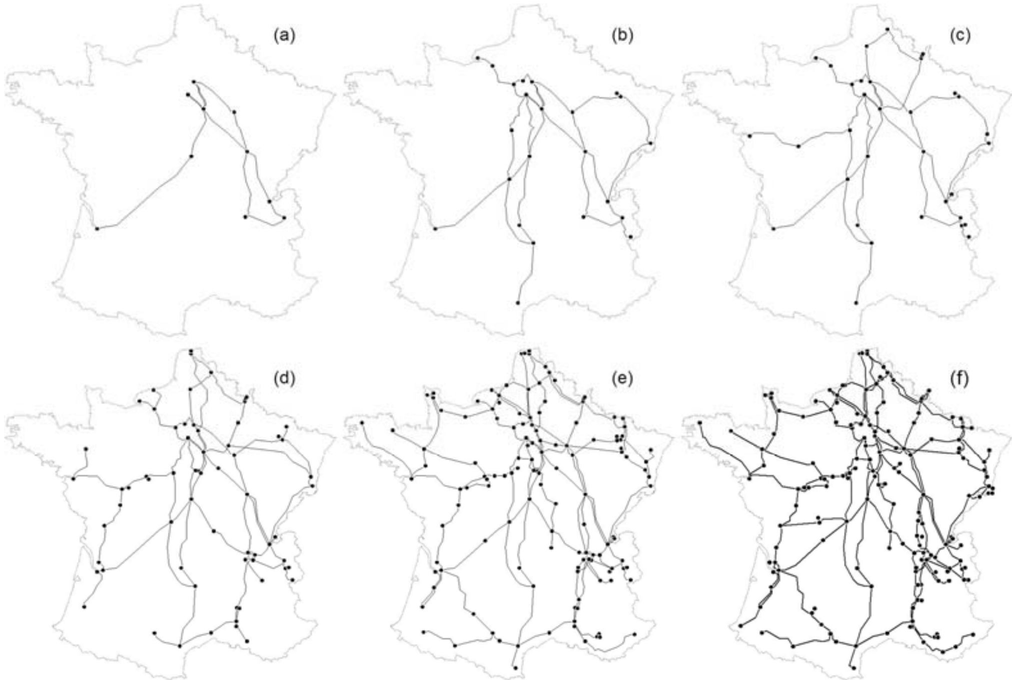


Fig. 1. Snapshots of the evolution of the French transmission power grid at different years. (a) 1962, (b) 1972, (c) 1976, (d) 1982, (e) 1992 and (f) 2005.

section VI we summarize our results and discuss further implications.

II. Power grid dataset

Nowadays, power grids define a continuum network of electric cables transporting alternating current from generators to consumers at different voltage levels. Generally speaking, the higher the voltage level, the more distance the electricity can travel and more power can be delivered. In Europe, three main voltage levels can be found. They define, at the same time, three different subnetworks, interconnected by transformers. These are: (1) the transport network at high (between 30 and 110 kV) and very high (between 110 and 400 kV) voltage levels; (2) the distribution network at medium voltage levels (between 1 and 30 kV); and (3), the consumer network at low voltage levels (below 1 kV).

The evolution of the French transport power grid (i.e., 400 kV) was downloadable from the RTE (Gestionnaire du Réseau du Transport d'Electricité) for some time. Nowadays this information is not accessible anymore, but it can be downloaded as supplementary information to this paper (see Appendix). Though power grid data can be obtained from year 1946 until year 2007, from 1946 to 1960 the French transport grid relays on 220 kV technology and it is mainly formed by disconnected lines and substations. It is not until 1962 that a main 400 kV connected core of 10 substations and some 1850 km of electric lines is detected. From 1976 to 1980, the 400 kV power grid begin to noticeably and effectively grow, due to the increase in both, electricity consumption and nuclear power generation equipment.

Although the topology of this network has been already characterized in the literature [18], and some aspects of the topological characterization of the network are obviously coincident, our contribution in this work is mainly focused on the definition of a very simple spatial and temporal model that explains and reproduces significantly well its topological features.

Here we study historical data from year 1964 until year 2000, due to cost and model considerations (see text). Last historical data (year 2000, upwards) can be found in the various actualizations of the ENTSO-E (European Network of Transmission System Operators for Electricity) map (http://www.entsoe.eu).

III. Topological patterns in power grid evolution

The power grid can be described formally in terms of a graph G [19], [20]. It is defined as a pair $G = (V, E)$, where $V = \{v_i\}$, $i = 1, \dots, V$, is the set of V vertices (generators, transformers and substations in a power grid) and $E = \{v_i, v_j\}$ is the set of edges or connections between nodes (cables and electric lines in a power grid). Here $e_{i,j} = (v_i, v_j)$ indicates that there is an edge (and thus a link) between nodes v_i and v_j. The degree k_i of a node $v_i \in E$ is the number of edges that connect it with other nodes, and its average for the whole graph is called the mean degree $\langle k \rangle$.

Figure 2 shows the size (i.e., cumulative number of nodes $|V(t)|$ and mean degree $\langle k \rangle$ of the very high voltage level French transport power grid (i.e., 400 kV) through the years. The slightly S-shaped network size function follows the usual sigmoid growing process found in most technological networks [1]. The growth begins rather slowly and then there

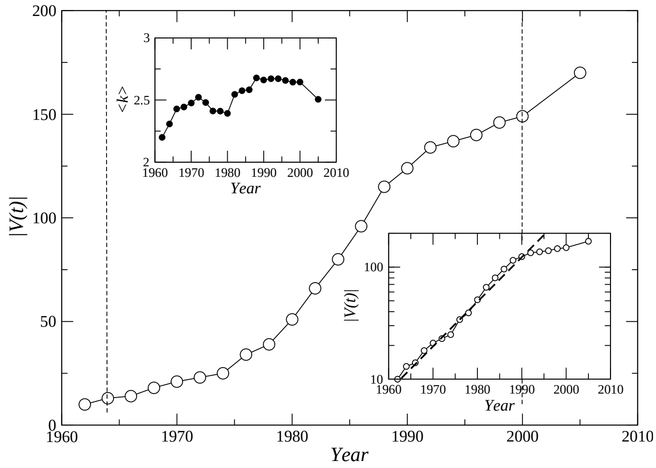


Fig. 2. Size (white circles) and mean degree (upper inset, black circles) evolution for the French transmission power grid. (Dashed vertical lines delimit the studied time span, i.e., 1964 - 2000; see text)

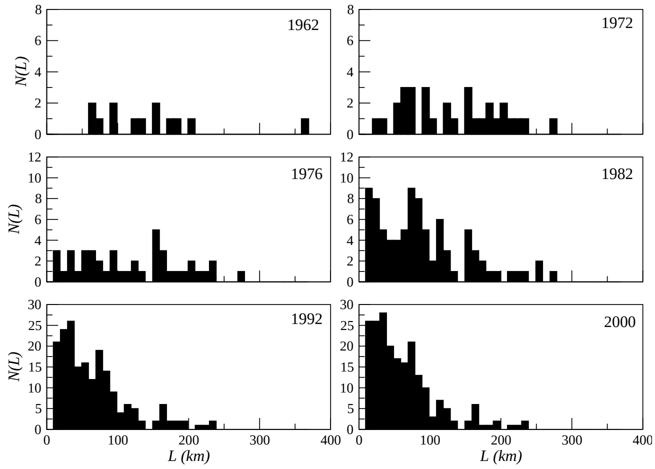


Fig. 3. Histograms for the lengths of edges L (in km) of the evolution of the French transmission power grid at different years. The bias towards shorter edges appears once long transmission lines have been built, by mid-1970's, and the initially longest line (right extreme value in 1962) is split into two in 1966.

is a noticeable steep that finally, in an ideal situation, saturates and decays. In the lower inset, $|V(t)|$ is displayed in linear-log scale. We notice an exponential growing phase (until approximately 1988) of the form $|V(t)| \sim Ke^{\lambda t}$, where for $t = 0,1,2,\ldots$ years, we empirically find $\lambda \approx 0.09$ and $K \approx 9.7$.

Table I shows the quantitative evolution of $\langle k \rangle$. As we can see, the mean degree is kept almost constant with a slight decrease at the point where the grid begins to increase (from 1975 to 1980) due to the fast addition of new lines from nodes with $k = 1$ with, clearly, one objective: to reach as much territory with as less time and cost as possible. From 1980 onwards, $\langle k \rangle$ increases slightly due to the meshing process of the grid, in order to attain a reliable $(N - X)$ criteria [21]. Table I shows as well the evolution of two other characteristic topological measures: the clustering coefficient C and the topological characteristic path length ℓ. These measures have been traditionally used to characterize complex networks [14], [22], [23] and will be used in the next section to validate the assumptions of the model presented. We recall that the clustering coefficient is here defined as an average [24]:

$$C = \frac{1}{N}\sum_{i} C_i \tag{1}$$

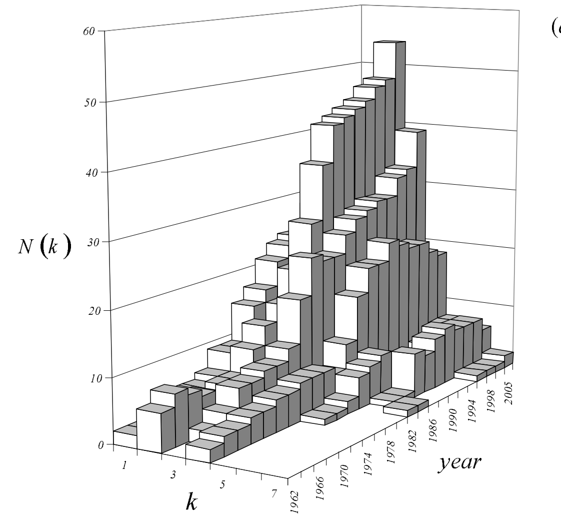
where $C_i = 2l_i/(k_i^2 - k_i)$ is the clustering coefficient of a given vertex i, if k_i is the number of neighbors of vertex i and l_i is the number of edges between the neighbors of i. On the other hand, the characteristic path length ℓ is defined as the mean value of the geodesic distances $d_{i,j}$ between nodes i and j:

$$\ell = \frac{1}{N(N-1)}\sum_{i>j} d_{i,j} \tag{2}$$

As we can see in Table I, while C remains more or less constant with a slight increase in time, ℓ increases with the number of nodes, as expected [22], [23].

We have chosen another simple way to describe the evolution of these networks which is the plot of the distribution of the lengths of the existing transmission lines at every time period considered. In Figure 3 we show the evolution of the histogram of the lengths of edges (in km) of the French transport power grid at six characteristic years. From 1962 until 1976, the amount of edges increases notably, but no characteristic mean is observed. In fact, almost every length is being used, even with the appearance of the longest line (354 km) before year 1972. This fact would suggest that neither economic factors nor technical ones would matter too much at the beginning of this growing process, other than a maximum spatial coverage objective. From 1976 onwards, a clear tendency towards shorter lines appear, as it seems to happen in other types of spatial networks [25]. After the covering process has been finished, there begins the meshing process, in order to reassure the connectivity and (hopefully enough) the reliability of the grid.

We finish the description of the evolution of the structure of the grid with Figure 4. It shows both degree and edge length distributions for the growing process of the French power grid. Likewise reported for other power grids [19], [20], [26], the degree distribution follows an exponential of the form $P(k) \sim e^{-k/\gamma}$ where k is the degree of a node and γ is a characteristic parameter. Figure 4 (b) shows the simultaneous increase in number of edges and decrease in existing individual lengths with time: $N_{\geq}(L)$ increases as L diminishes, showing a process that moves from one objective, i.e. expanding the network as far and quick as possible at the beginning, to another one, i.e. assuring a more stable grid, reducing the longitude of the edges and increasing their short-range connections.



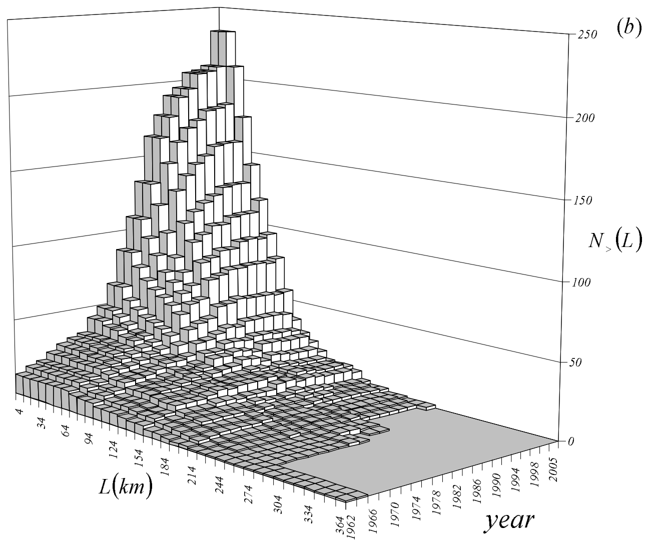


Fig. 4. Degree distribution (a) and edge longitude cumulative distribution (b) as function of time, where k is the node degree, $N(k)$ is the number of nodes with degree k at each specified year, L is the edge longitude (in km) and $N_{\geq}(L)$ is the cumulated number of edges with longitude equal or higher than L.

IV. Spatial evolution model

In order to characterize the spatial and temporal evolution of the French transport power grid, a model of spatial growth has been coupled with a cost function. Here we define the spatial growth procedure by means of the multipoint problem [27]. In its static version, the multipoint problem assumes that a function $cost: R \rightarrow \mathfrak{R}^{+}$ is defined for a network N where R is the set of all possible routes on N. The static problem is formalized as follows. Given a network N and a connection request c, find a route R_c such that $cost(R_c)$ has a minimum value among all possible routes for c. We consider this cost function associated with Euclidean distance d. The spatial model follows [28] essentially. It starts with a node randomly placed in a squared two dimensional space. At every time step t, a new node position is randomly chosen with coordinates in the interval [0, 1]. In this work though, and unlike [28], the connections of the new node n_t with each existing node at a distance d is established with probability

$$p(d, n_t) = \beta e^{-\alpha d(n_t/N)} \tag{3}$$

where N is the maximum size of the network (in number of nodes) at the end of the growing process, and α and β are the model parameters. The ratio (n_t/N) allows the modeling of the evolution of the length of individual edges as shown in Figure 3. At the beginning of the process (i.e., for lower values of n_t) the probability is almost one for any distance d. As the network increases its number of nodes with time (i.e., higher values of n_t) the probability of establishing links between nodes at higher distances decreases exponentially.

Figure 5 (right) shows several time steps in the evolution of one realization of this spatial model. The time evolution has been normalized with the final size N of the network in year 2000, so every time step is now characterized by the percentage of nodes already introduced in the network. These fractions follow the pace of the percentage of nodes introduced in the real network showed in the snapshots of Figure 1. The model has been implemented on NetLogo [29] and it can be downloaded as supplementary information to this paper (see Appendix). For the particular realization shown in Figure 5 and Table I, (3) has been used with α = 150, β = 1 and N = 149. It allows us to write

$$p(d, n_t) \approx e^{-d(n_t)} \tag{4}$$

Link lengths have been normalized as well, so the maximum length a link can attain is 1. As we can see, the distribution of link lengths over time greatly resembles the real observed one, except in the last two final stages, where the exponential function used in the model arises more clearly (Figure 5, left). This deviation is due to a phenomenon that this model cannot reflect and that is the split of long lines into many shorter ones. The capacity of this simple model to reflect the reality is nonetheless remarkable. In order to fully characterize the evolution of both real and modeled networks, mean degree, clustering coefficient and topological characteristic path length have been monitored at the same time steps. Table I shows significant agreement in the results. The errors shown are standard deviations averaged over 1000 realizations.

We could not find evolution data for the time span 2000 – 2015, a fact that would allow proving the accuracy of the model. Nonetheless, topological data for recent evolution phases of the French transmission network can be found in the literature [30] and can be used to validate, to a small extent, its long term accuracy. The French transmission grid has increased its number of transmission buses from 149 nodes in 2000, to 1400 nodes and 1819 branches in 2015. But at this point in time, it had similar topological properties than those

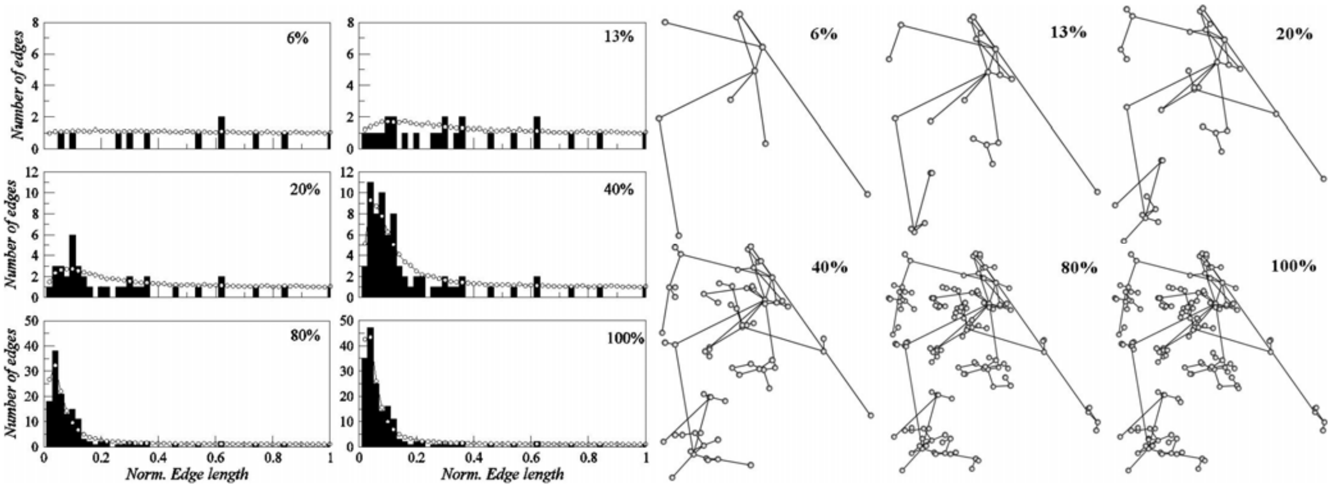


Fig. 5. Histograms of the lengths of edges (left) and snapshots (right) of the evolution of the spatial model, at different stages of completion (shown as percentage of nodes introduced with respect to final size). As for Table I, this realization uses $\alpha = 150$, $\beta = 1$ and $N = 149$ in (3). Circles stand for the normalized edge length mean value after 1000 realizations.

TABLE I
NETWORK PARAMETERS EVOLUTION

			RTE			Model		
Year	Completion (%)	Accum. Nodes	$\langle k \rangle$	C	ℓ	$\langle k \rangle$	C	ℓ
1962	6	10	2.2	0.00	2.38	2.4 ± 0.4	0.22 ± 0.17	2.15 ± 0.27
1972	13	23	2.5	0.03	3.94	2.6 ± 0.4	0.15 ± 0.09	3.02 ± 0.33
1976	20	34	2.4	0.02	4.67	2.7 ± 0.3	0.14 ± 0.07	3.52 ± 0.35
1982	40	66	2.5	0.06	5.41	2.6 ± 0.2	0.12 ± 0.05	4.56 ± 0.41
1992	80	134	2.6	0.10	7.01	2.5 ± 0.1	0.11 ± 0.03	6.67 ± 0.43
2000	100	149	2.6	0.06	7.76	2.5 ± 0.1	0.10 ± 0.03	7.04 ± 0.44

For every year considered, the table shows the percentage of completion (number of nodes introduced with respect to final size), accumulated number of nodes for the RTE network, mean degree $\langle k \rangle$, clustering coefficient C and topological characteristic path length ℓ. The model results and standard errors have been averaged over 1000 realizations using, in this particular case, $\alpha = 150$, $\beta = 1$ and $N = 149$.

in year 2000, except for the path length: $\langle k \rangle = 2.6$, $C = 0.06$ and $\ell = 11.3$. With 1400 nodes as final size and $\alpha = 1400$, our model gives $\langle k \rangle = 1.9 \pm 0.5$, $C = 0.0005 \pm 0.0002$ and $\ell = 8.8 \pm 1.6$. Although it underestimates clustering coefficient, the model is robust and remains accurate for degree and characteristic path length to the same order of magintude.

V. COUPLED COST FUNCTION

The characterization of the topological change displayed by this network cannot be completed without a tentative explanation of the characteristic S-shape observed in Figure 2 for the network size evolution. In fact, many natural processes and complex system display a history-dependent progression from small beginnings that accelerates and approaches a climax over time [31]–[33]. The simplest description of this process is the sigmoid curve, which often refers to the special case of the logistic function defined for each time step t like:

$$f(t) = \frac{1}{e^{-\epsilon t}} \qquad (5)$$

where ϵ is a characteristic parameter.

Long term engineering projects are usually characterized by uneven distribution of resources, with budgets normally distributed over time [34], [35]. For the French power grid, this distribution of costs is shown in Figure 6, with biennial data. We have considered a constant cost for nodes and a linear length dependent cost for links, as it has been historically, and nowadays, the case for electric transmission networks construction [21], [36]. Links (squares) and nodes (circles) share 80% and 20% of the total cost (stars) per two-year period respectively, regardless of any particular stage of the project completion. As the evolution of the expenditure over time is not constant, the accumulated cost follows a characteristic sigmoid function, similar (almost qualitatively exact) to that observed in Figure 2 for the evolution of the size of the network. This can also be observed in Figure 6, inset, where accumulated normalized costs scale linearly with network size.

In order to consider this cost constraint, the spatial model has been coupled with a sigmoid like accumulated cost function. It defines the maximum budget available for constructing new nodes and establishing new links during each two-year period. It takes the form

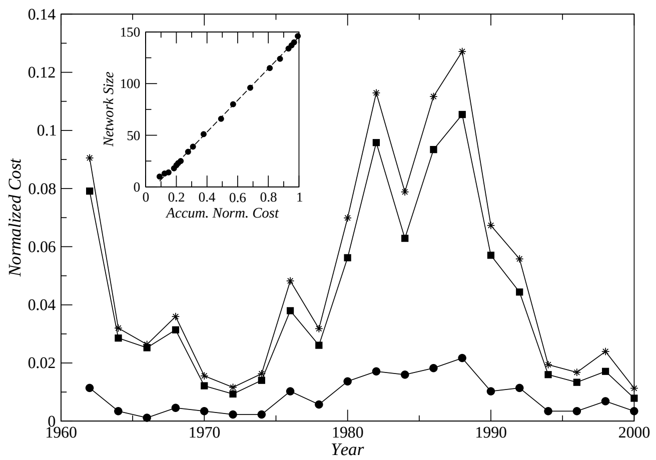


Fig. 6. Normalized costs distribution for the French power grid construction. Total normalized cost (stars) accounts for a constant cost for substations (circles) and linearly dependent one with distance for cables (squares). Inset: accumulated normalized cost and network size scale linearly over time. Normalization have been done over the total cost expenditure of the analyzed time span (i.e., 1964-2000).

$$C_A(b) = \frac{1}{1 + e^{-(b-\mu)/\sigma}} \qquad (6)$$

where b is the accumulated biennial period, and μ and σ are the function parameters. Equation 6 have been adjusted with $\mu = 10.57$ and $\sigma = 2.47$, for a time span of eighteen biennial periods (i.e., 1964 - 2000), in order to avoid the initial offset (ten nodes and some 1.850 km of lines, already in place) observed in year 1962. This offset comes from the last stages of the previous grid upgrade (that of 220 kV).

The cost function is coupled with the spatial model in the following way. At every time step t, the model accumulates the total cost C_T of nodes and links introduced in the network and compares it with C_A. If $C_T \geq C_A$, we consider that two years have passed. Each biennial period includes nodes and links introduced in several time steps. If C_n is the cost of a node and C_L is the cost of a line per length unit, C_T can be written as:

$$C_T = C_n \sum n_t + C_L \sum L_{n_t} \qquad (7)$$

where L_{n_t} is the total length of electric cable (i.e., length of all links) associated with the introduction of the node n_t at time step t.

Equations 6 and 7 are coupled in time. No a priori analytic methodology exists to obtain for each $b = [0, 1, \ldots, 18]$ (eighteen biennial periods, from 1964 to 2000) an optimal pair C_n and C_L, other than to sweep the costs parameter space. Figure 7, inset, shows the relation between node and link costs that give rise to eighteen biennial periods. This optimal relation follows a linear fitting ($R^2 = 0.9977$) of the form:

$$C_n = 0.05 \cdot C_L + 0.0013 \qquad (8)$$

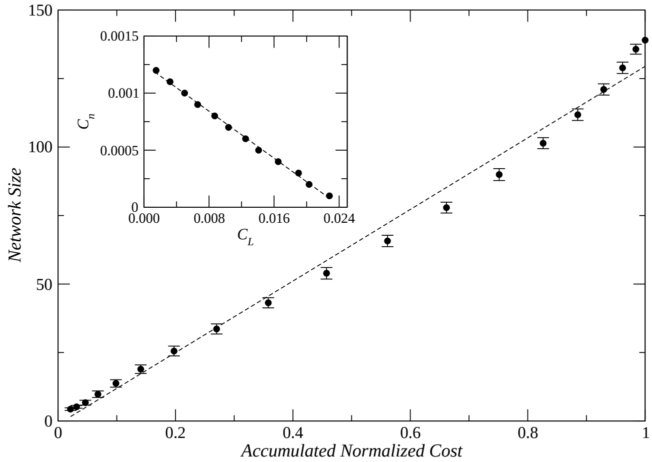


Fig. 7. Modeled cost and network size. Inset: linear relation between costs of nodes and links that give rise to optimal solutions for the model.

Figure 7, principal, shows the mean and standard deviations (error bars) of 1000 realizations of one cost combination: $C_n = 0.001$ and $C_L = 0.005$. The space model have been now used with $\alpha = 150$, $\beta = 1$ and $N = 139$ (149 less 10 node offset). Similarly to Figure 6, inset, both modeled accumulated normalized cost and network size, scale almost linearly over time. Although a slight deviation can be observed, the approximation given by this simple model is conspicuous.

VI. Conclusions

This paper presents a simple model for the spatial and temporal evolution of a transmission power grid. Power grids are examples of spatial networks, where nodes can be precisely located on Euclidean space and edges do have characteristic and measurable lengths. These facts highly coerce their topology and constrain their evolution. And as a consequence they also have a sensible impact on the overall performance of these technological systems [37]. Although historical and technological events of many kinds can participate in the molding and forming of a network such as the power grid, its growth can be simply modeled as a tradeoff between resources and a spatial filling process that minimizes the global connectivity cost. In this particular case, labor and cost requirements have been characterized by a sigmoid function which follows the three stages that characterize most engineering projects: a first acceleration stage, when civil work predominates and the number of man-hours usually builds up; the steady state stage, when all crafts are able to work on the project and the number of man-hours reaches its peak; and finally the deceleration stage, when the work of major craft is essentially complete, and the number of man-hour is reduced accordingly.

This example of network evolution cannot really be wholly compared with that of an innovation but rather with that of a partial upgrade of a lower voltage level existing grid, at least in its early stages of development. Nonetheless, it has evolved independently after this initial upgrading process and follows two clearly differentiated and consecutive evolving processes: a first global space filling process and a secondary local meshing process that increases connectivity at a local level. We assume that these processes are followed by transmission power grids in general. Since meshing processes seem to increase fragility [38], [39], it seems plausible to suggest that global fragility in spatial networks of this kind arises when local efficiency and reliability increases. This fact has been also suggested in relation with capacity-load optimization models and network traffic fluctuations [40].

In summary, this model challenges a particular vision of the evolution of technology and design of engineered systems which affirms that the expansion of the power grid is essentially energy demand driven, with underlying factors being mostly human population or industrial manufacturing. Our spatial model, which ignores power system engineering constraints, shows that the evolution of a transmission network can be remarkably explained by means of one simple equation which tells us it will growth following fundamental systemic trends, also observed in other networked systems coming from biology, life sciences or sociology. Given a distribution of resources and a time span, the model can also be used to generate the probability distribution of cable lengths at every time step and to facilitate infrastructure planning.

Complex networks can only be understood when simultaneously considering the three dimensions that characterize its meaning. Namely: (1) structure, (2) dynamics and (3) evolution. Most effort have been done in the characterization of the first two, being the latter usually less studied, mostly because of a lack of available or significant data. It is our hope that this work will help in finding new ways to tackle the optimal, organic (rather than hierarchical) design and modeling of this type of networks.

APPENDIX

RTE evolution data:
http://tinyurl.com/jukktyw

NetLogo [29] implementation of the model:
http://tinyurl.com/zoa6zet

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