

1 Experimental and theoretical study of nanofiltration of weak 2 electrolytes: $\text{SO}_4^{2-}/\text{HSO}_4^-/\text{H}^+$ system

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12 Abstract

13 Over recent years, nanofiltration (NF) has been considered as an effective way to improve
14 processing steps in metallurgical and hydrometallurgy applications dealing with mixtures of
15 metal ions in sulphuric-acid-dominated solutions. The principal advantage of NF membranes
16 over reverse osmosis (RO) membranes is their ability to allow for a practically free passage of
17 acid, while metallic species, especially multi-charged species, are efficiently rejected. In
18 general, these sulphuric solutions cover a range from strongly acidic solutions with pH below 1
19 up to moderately acidic solutions of pH 3. Over this range, changes in the feed acidity influence
20 both the aqueous electrolyte solution speciation ($\text{SO}_4^{2-}/\text{HSO}_4^-/\text{H}^+$) and the membrane acid–
21 base properties (protonation of carboxylic and amine groups). However, few studies have been
22 published on the trans-membrane transport of inorganic species coupled to changes in their
23 speciation as well as to the properties of the membrane phase.

24 In this study, experimental data on the sulphuric acid rejection for pH values from 1 to 3 have
25 been obtained with an aromatic poly(piperazine)amide membrane (NF270) at various trans-
26 membrane pressures. The results were modelled by a novel version of the Solution-Electro-
27 Diffusion model taking into account equilibrium reactions, and a general (quasi)analytical
28 solution was obtained for the transport of weak electrolytes of arbitrary valence type. The
29 equilibrium weak acid reaction made the total sulphate ($\text{SO}_4^{2-}/\text{HSO}_4^-$) rejection decrease
30 strongly as the fraction of single-charged hydrogen sulphate (HSO_4^-) in the feed increased.
31 From the modelling procedure, permeances to H^+ , HSO_4^- and SO_4^{2-} over the studied pH range
32 were determined.

33 Key words

34 Reactive transport, modelling, ionic permeances, NF270, sulphuric acid

35 1. Introduction

36 Sulphuric acid is one of the most produced chemicals worldwide. In the past the scale of its
37 production was even used as an indicator of industrial strength. Nowadays, sulphuric acid is
38 still widely used in a large number of applications (e.g. mineral processing, hydrometallurgy
39 and for pH control). In the mining industry, it is used as a leaching agent to dissolve minerals

40 for metal extraction [1]; sulphuric acid is also used in electroplating and acid pickling in the
41 metal-finishing industry [2]. In both cases, a large amount of effluent is generated. However,
42 new environmental legislation promotes sustainability and sulphuric acid recovering
43 technologies. Therefore, as an alternative to neutralisation/precipitation, several techniques,
44 such as electrolytic deposition, ion exchange, diffusion dialysis, electrodialysis, solvent
45 extraction [2–5] and more recently pressure-driven membrane processes, have been applied
46 to treat these effluents. Among them, nanofiltration (NF) offers unique opportunities for acid
47 purification. Typically, NF membranes show low rejection of acids, but high rejection of
48 multivalent ions. As the acid rejection is low, its concentrations on both sides of the membrane
49 are close to each other. Accordingly, the contribution of acid to the differential osmotic
50 pressure is very limited [6]. Then, in comparison with the techniques listed above, it has
51 relatively low energy consumption.

52 The use of NF for removing metal ions from acidic solutions has been previously studied [7–
53 13]. Nystrom et al. [10] separated metal (iron, chromium and nickel) sulphates and nitrates
54 from acidic media achieving high rejections (around 99%) with an NF-45 membrane. González
55 et al. [12] tested several reverse osmosis (RO) and NF membranes in the treatment of
56 concentrated phosphoric streams (from 2 to 8 M). NF allowed to obtain higher flows and
57 higher acid permeations than did RO. However, the metal rejection in RO was a little higher
58 than in NF. Skidmore and Hutter [13] patented a process for purification of phosphoric acid by
59 NF. High rejections of multivalent ions (Al(III), Fe(III), Mg(II)) were observed. Thus, the typically
60 high ion concentration of strongly acidic solutions, which may restrict the use of other
61 membrane techniques, can be overcome with nanofiltration.

62 Modelling of ion rejection in NF is useful for the process optimisation and scale-up. Ion
63 transport through NF membranes has been widely described by either non-equilibrium
64 thermodynamic models [14–17] or extended Nernst–Planck equations [18–21]. Among various
65 approaches, Yaroshchuk et al. [22] demonstrated that, for single salts, the Solution-Diffusion-
66 Film (SDF) model is applicable, and later it was extended to electrolyte mixtures by including
67 the coupling between the electro-diffusion fluxes of various ions via the electric field of the
68 membrane potential [23–25]. Taking this into account, a good description of ion rejection
69 dependence on the trans-membrane volume flow was achieved for a number of electrolyte
70 mixtures consisting of one dominant salt and trace ions [24,25]. This approach also accounts
71 for the existence of a concentration-polarisation layer where the ion transfer occurs via
72 electro-diffusion and convection. This description of trans-membrane mass transfer by the so-
73 called Solution-Electro-Diffusion-Film (SEDF) model allows for efficient determination of
74 membrane permeances to single ions from experimental data. However, to date only limited
75 attempts have been reported to describe membrane transport of solutes being in chemical
76 equilibrium with other solutes (e.g. anions with acid/base properties, or ions with complexing
77 properties) [7,26]. This is the case with the sulphuric acid solutions where, at pH values close
78 to the pK, HSO_4^- and SO_4^{2-} species are present in commensurable concentrations. In order to
79 model these results, the equilibrium reaction must be accounted for in the model. By using the
80 solution–diffusion model, Nir. et al. [27,28] considered proton and hydroxyl ions as traces in
81 RO, taking into account the chemical equilibrium between them. Niewersch et al. [29,30]
82 modelled numerically the trans-membrane ion fluxes under high-acidity conditions in NF. In
83 this paper we present a novel quasi-analytical approach to the modelling of NF transport in a

84 system of three components, coupled by an equilibrium chemical reaction. For simplicity, in
85 this study we neglected the concentration polarisation phenomena in the first approximation.

86 Another objective of this work is to extending the validation of the SEDF model towards the
87 transport of reacting species via comparison with experiment. A wide pH range was used in
88 order to explore the effect of changes in the acid–base equilibrium of the membrane
89 functional groups, and to evaluate the effect of incomplete acid dissociation. The experiments
90 were carried out using an NF270 membrane in a cross-flow experimental set-up. The results of
91 these measurements have been interpreted in terms of ionic permeances of the membrane to
92 HSO_4^- , SO_4^{2-} and H^+ .

93 **2. Reactive transport of ions in nanofiltration membranes:** 94 **model formulation**

95 The ion fluxes are described according to the Solution-Electro-Diffusion model. One of the
96 assumptions of this model is that there is no coupling between solute and solvent flows inside
97 the membrane or, in other words, reflection coefficients are assumed to be equal to one for all
98 species. The model uses ‘virtual’ concentrations, which are defined as those in a solution that
99 could be in thermodynamic equilibrium with an infinitely small volume inside the membrane.
100 ‘Virtual’ concentrations satisfy the chemical-equilibrium condition with the bulk association
101 constant. The partitioning coefficients, which quantify the ratios between the real and virtual
102 species concentrations in the membrane (and possible changes of association constant there)
103 are included in the ion permeances [24]. The ion transport is described by Eq 1.

$$j_i = -P_i \cdot c_i \cdot \left(\frac{d \ln c_i}{dx} + z_i \cdot \frac{d\varphi}{dx} \right) \quad (1)$$

104

105 where j_i (mol/m²s) is the trans-membrane volume flow of component i ; P_i is the membrane
106 permeability to ion i ; c_i and z_i stand for its virtual concentration and its charge, respectively; φ
107 is the dimensionless virtual electrostatic potential and x is the position across the membrane.

108 The transport of ions is:

109 – Subjected to the zero-current condition defined by Eq. 2.

$$\sum_i z_i \cdot j_i = 0 \quad (2)$$

110 – Subjected to the electro-neutrality condition in the virtual solution defined by Eq. 3:

$$\sum_i z_i \cdot c_i = 0 \quad (3)$$

111 – Virtual ion concentrations are not independent but related by a chemical equilibrium
112 condition at a given constant temperature and ionic strength ($\alpha = 10^{pK_a}$), where pK_a
113 is the acidity constant, defined by Eq. 4:

$$c_3 = \alpha \cdot c_1 \cdot c_2 \quad (4)$$

114

115 It should be stressed that the chemical reaction is subjected to charge conservation
 116 ($z_3 = z_1 + z_2$)

117 By using Eqs. 1 and 2 for a system of three ions (e.g. H^+ , HSO_4^- , SO_4^{2-}) an expression for the
 118 gradient of electrostatic potential in virtual solution can be derived (Eq. 5):

$$\frac{d\varphi}{dx} = - \sum_i \left(\frac{t_i}{z_i} \right) \cdot \frac{d \ln c_i}{dx} \quad (5)$$

119

120 where t_i is the transport number of ion 'i' defined by Eq. 6:

$$t_i = \frac{z_i^2 \cdot P_i \cdot c_i}{\sum_j z_j^2 \cdot P_j \cdot c_j} \quad (6)$$

121

122 From the chemical equilibrium condition, a relationship can be obtained between the
 123 derivatives of logarithms of concentrations of the species described by Eq. 7:

$$\frac{d \ln c_3}{dx} = \frac{d \ln c_1}{dx} + \frac{d \ln c_2}{dx} \quad (7)$$

124

125 By combining the chemical equilibrium and electro-neutrality conditions (in the virtual
 126 solution) we obtain this:

$$\frac{d \ln c_2}{dx} = \frac{1}{1 + \alpha \cdot \left(1 + \frac{z_1}{z_2} \right) \cdot c_1} \cdot \frac{d \ln c_1}{dx} \quad (8)$$

127

128 Due to the chemical reaction, the trans-membrane ion fluxes of all three components are not
 129 conservative (they change across the membrane). However, owing to the reaction
 130 stoichiometry, exactly the same amount of species '1' and '2' is consumed (produced) at a
 131 given point inside the membrane. Therefore, the difference in their fluxes is constant and
 132 equal to the product of trans-membrane volume fluxes and difference of their concentrations
 133 in the permeate. By using this, as well as Eqs. 5,7 and 8 it is possible to formulate just one
 134 transport equation in terms of concentration of only one of the species (for example, species
 135 '1'). In this equation, the variables (concentration of species '1' and space variable) can be
 136 separated so the equation can be solved in quadrature, and after some identical
 137 transformations, this could be obtained:

$$J_v L = \frac{\int_{c_{1p}}^{c_{1f}} F(c_1) dc_1}{c_{1p} \cdot \left(1 + \frac{z_1}{c_{1p} \cdot \alpha \cdot (z_1 + z_2) + z_2} \right)} \quad (9)$$

138

139 where c_{1f} , c_{1p} are the concentrations of species '1' in the feed and permeate, respectively, L is
 140 the active-layer thickness, and:

$$F(c_1) \equiv z_2 \cdot z_3 \cdot \frac{P_1 \cdot (z_2 + c_1 \cdot \alpha \cdot z_3) \cdot (P_2 + P_3 \cdot \alpha \cdot c_1) - P_2 \cdot P_3 \cdot z_1 \cdot \alpha \cdot c_1}{\frac{z_2 \cdot (P_2 \cdot z_2 - P_1 \cdot z_1) + c_1 \cdot \alpha \cdot z_3 \cdot (P_3 \cdot z_3 - P_1 \cdot z_1)}{z_2 + c_1 \cdot \alpha \cdot z_3 - z_1} \cdot \frac{1}{(z_2 + c_1 \cdot \alpha \cdot z_3)}} \quad (10)$$

141

142 **This general expression (Eq. 9) is valid for any weak electrolyte.** A detailed derivation of Eq. 9 is
 143 presented in Annex 1.

144 In principle, the integral in Eq. 9 can be taken analytically but the corresponding expression is
 145 too cumbersome to be shown here. Nonetheless, the availability of the relatively simple
 146 expression of Eq. 9 enables us to verify the solution by considering the limiting cases of no
 147 reaction ($\alpha = 0$) and dominant reaction ($\alpha \rightarrow \infty$). In both limiting cases, the integrand is
 148 independent of concentration, so the integral can be easily, taken to obtain:

$$\frac{c_{1f}}{c_{1p}} = 1 + \frac{J_v L}{P_{12}} \quad \text{at } \alpha = 0 \quad (11)$$

$$\frac{c_{1f}}{c_{1p}} = 1 + \frac{J_v L}{P_{13}} \quad \text{at } \alpha \rightarrow \infty \quad (12)$$

149

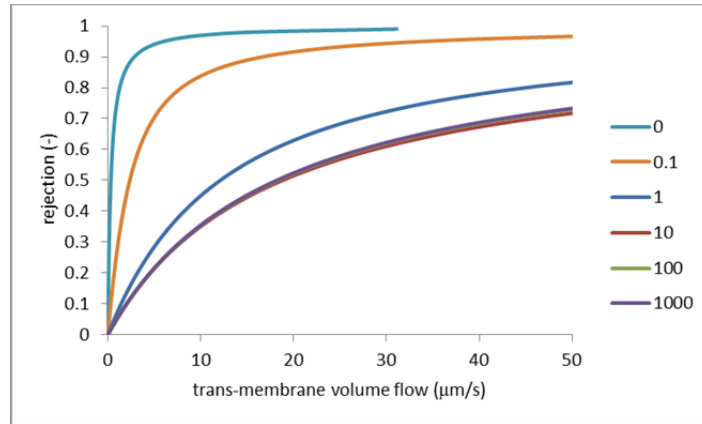
150 where it was denoted

$$P_{12} \equiv \frac{(z_1 - z_2) \cdot P_1 P_2}{z_1 P_1 - z_2 P_2} \quad P_{13} \equiv \frac{(z_1 - z_3) \cdot P_1 P_3}{z_1 P_1 - z_3 P_3} \quad (13)$$

151

152 It is easy to see that Eqs. 11 and 12 are just expressions (within the scope of the solution-
 153 diffusion model with negligible concentration polarisation) of the reciprocal transmission of
 154 either of two single salts consisting of ions '1' and '2' ($\alpha = 0$) or '1' and '3' ($\alpha \rightarrow \infty$), P_{12} and
 155 P_{13} being the corresponding salt permeabilities. Thus, the solution of Eq. 9 is in agreement
 156 with these two limiting cases. One can also see that though Eq. 9 is transcendental in c_{1p} it
 157 provides an explicit dependence of trans-membrane volume flow on the permeate
 158 concentration. This dependence can readily be used for the analysis of dependence of
 159 rejection on the trans-membrane volume flow.

160 If it is further assumed that $P_3 > P_2$, which is naturally due to the smaller charge magnitude of
 161 ion '3', one can show the integrand in the right-hand side of Eq. 9 to increase with the virtual
 162 concentration in a monotonic way. Interestingly, this does not mean that the rejection of weak
 163 electrolyte decreases monotonically with its feed concentration. Actually, due to the presence
 164 of the denominator in the right-hand side of Eq. 9, there is a slight minimum on the
 165 dependence of rejection (at a given trans-membrane volume flow) on the feed concentration
 166 (or association constant). This is illustrated by the results of some sample calculations shown in
 167 Figure 1.



168

169 **Figure 1. Dependence of rejection of ion ‘1’ on trans-membrane volume flow for various**
 170 **association constants**

171 One can see that initially the rejection decreases strongly with increasing product of feed
 172 concentration and chemical equilibrium dissociation constant. However, starting from
 173 $\alpha \cdot c_{1f} \approx 10$ there is practically no visible dependence. Below, we will see that this levelling
 174 out at higher concentrations is in agreement with our experimental data.

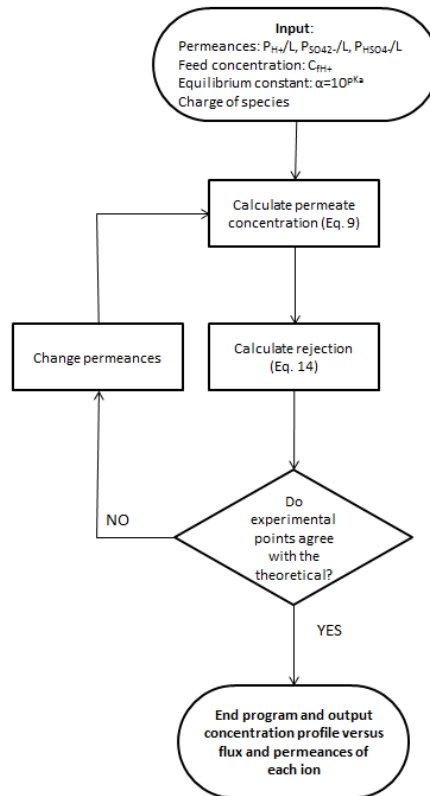
175 In this study, NF rejections of sulphuric acid at various feed acidities (concentrations) using a
 176 semi-aromatic polyamide NF membrane (NF270) were investigated experimentally. The
 177 observable rejections (R_{obs}) of the acid were calculated by using Eq. 14:

$$R_{obs} = 1 - \frac{C_p}{C_f} \quad (14)$$

178 where C_p and C_f are the concentrations of acid in the permeate and feed of the membrane
 179 experimental setup, respectively.

180 Eq. 9 was used to fit experimental data on the total sulphate rejection via determining a set of
 181 three ion permeances ($SO_4^{2-}/HSO_4^-/H^+$). The permeances are defined as the permeabilities
 182 divided by the barrier-layer thickness. From Eq. 9, one can see that just these quantities can be
 183 fitted to experimental data provided that the barrier-layer thickness is not specified.

184 The integral in the right-hand side of Eq.9 was calculated numerically. In Figure 1 a flow chart
 185 of the algorithm used is shown. The algorithm was implemented with Matlab® using the
 186 function *fminsearch* (based on Nelder–Mead algorithm) to reach converge criteria and the
 187 function *quad* to take the integral.



188

189 **Figure 2. Details on the Matlab® implemented algorithm**

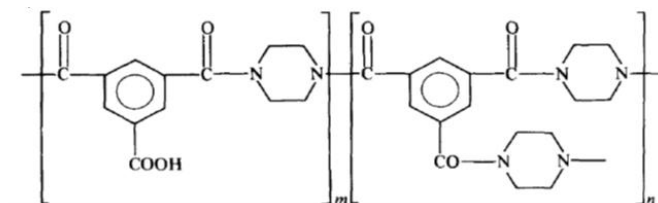
190 The objective function (OF) (Eq. 15) was used to minimise the relative error between the
191 measured and predicted permeate ion concentrations:

$$O.F. = \sum \frac{|C_{p_{model}} - C_{p_{experimental}}|}{C_{p_{experimental}}} \quad (15)$$

192 3. Experimental section

193 3.1. Membranes, reagents and solutions

194 We used an NF membrane with active layer of semi-aromatic poly(piperazine) amide (NF270,
195 Dow Chemical). Its chemical structure is shown in Figure 3.



196

197 **Figure 3. Chemical structure of the membrane active layer [31,32]**

198 Aromatic polyamide active layers prepared via interfacial polymerisation as in NF90 or ESNANF
199 membranes (similarly to NF270) have been characterised in terms of concentration of
200 ionisable functional groups (carboxylic (RCOOH/R-COO⁻) and amine (R-NH₃⁺/R-NH₂)) related

201 to the degree of polymer cross-linking [33,34]. These membranes have isoelectric points (IEPs)
202 of 3.5 and 4.3, respectively, which are very close to the isoelectric point of NF270 [35].
203 Although no data on the acidity constants of the carboxylic groups have been published, most
204 of the characterisation studies on the acid–base properties concluded that at pH values lower
205 than 3, carboxylic groups are protonated (R–COOH) and at pH values above 5 they are
206 deprotonated (R–COO⁻). Thus, in the present study the experiments were performed at pH
207 values supposedly below the isoelectric point so the carboxylic groups were protonated [36],
208 even though, at lower pH than the IEP, membrane was positively charged due to the presence
209 of protonated amine groups (R–NH₃⁺).

210 The experiments were carried out using sulphuric acid supplied by Sigma-Aldrich (95–97 wt%).
211 The pH of the feed solutions in our experiments lay between 3 and 1. In this pH range the
212 mono-charged sulphate species (hydrogen sulphate, HSO₄⁻) was present in solution in
213 noticeable amounts (Figure 4, [37]).

214 **3.2. Membrane cross-flow experimental setup**

215 We used an experimental setup as previously described[38]. Experiments were performed
216 with NF270 membrane (0.014 m²) in a cross-flow set-up equipped with a test cell (GE SEPA™
217 CF II) with a spacer-filled feed channel. In this setup the cross-flow velocity (cfv) and trans-
218 membrane pressure (TMP) could be adjusted independently.

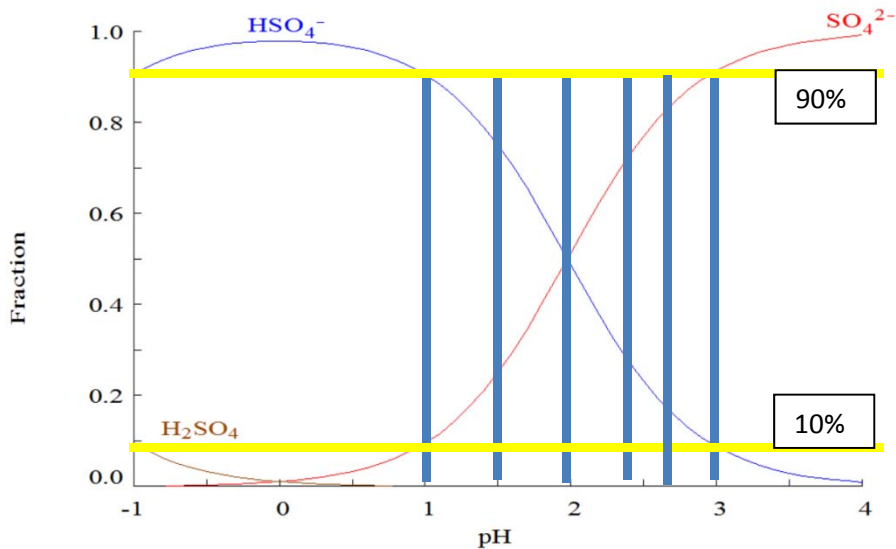
219 Feed working solutions were kept at constant temperature (21 ± 2 °C) in a thermostated 30 L
220 feed tank and pumped into the cross-flow filtration system with a high-pressure diaphragm
221 pump (Hydra-Cell, USA) at prefixed cross-flow velocities (cfv) and trans-membrane pressures.
222 The two output streams from the test cell, permeate and concentrate, were recirculated
223 through stainless steel pipes into the feed tank, thus providing a fairly constant concentration
224 in the feed solution. At a given cfv the TMP could be varied by a needle valve located in the
225 concentrate stream just at the exit from the test cell. The concentrate passed through a
226 manometer and a flow meter. A pre-filter cartridge was placed in the concentrate loop to
227 eliminate corrosion products and microorganisms. A major part of the permeate stream was
228 recycled back to the feed tank, and permeate samples for analysis were collected via a three-
229 way valve.

230 Prior to the measurements, the membranes were held in deionised water overnight to remove
231 conservation products. Then, the membrane was compacted with deionised water at 22 bar
232 and 1 m/s cfv. Before the experiments, the membrane was additionally compacted with the
233 feed solution at 22 bar and cfv of 1 m/s for 2 h. The experiment was performed at fixed cross-
234 flow velocity (0.7 m/s) and the TMP was varied.

235 The system is equipped with flow meters, pressure meters, a conductivity meter (Crison GLP
236 31 EC), a pH meter (Crison GLP 21) and a temperature sensor, to monitor the hydrodynamic
237 and chemical parameters. Furthermore, a data acquisition system programmed in LABVIEW®
238 version 8.6 (Laboratory Virtual Instrumentation Engineering Workbench) was developed.
239 Sensor calibrations were performed under the hydrodynamic conditions used in the
240 experimental work.

241 **3.3. Ion rejection experimental tests with H₂SO₄ solutions**

242 Aqueous solutions of H₂SO₄ were used as feed solutions. These selected model solutions
243 reproduced some common scenarios of acidity of mining and metallurgical process waters.
244 HYDRA-MEDUSA software was used to evaluate the dissociation of sulphuric acid [37]. The
245 experiments were performed with feed solutions having pH values between 1 and 3. The
246 speciation of H⁺, SO₄²⁻ and HSO₄⁻ species is shown in Figure 4. It can be seen that the
247 experiments covered the range from below the pKa of the SO₄²⁻/HSO₄⁻ equilibrium, where
248 HSO₄⁻ was the dominant species, up to values above the pKa, where SO₄²⁻ was dominant.



249

250 **Figure 4. Variation of the H₂SO₄/ HSO₄⁻/SO₄²⁻ molar fractions as a function of pH by using the**
251 **equilibrium code Medusa and using the thermodynamic data from the Hydra database**
252 **(Hydra) [37]. Vertical blue lines represent the pH values of the experiments carried out.**

253 As can be seen in Figure 4, in the studied pH range the presence of H₂SO₄ as species can be
254 disregarded as it is present only at pH values below -1 (e.g. C_{H₂SO₄} > 5 M).

255 Total concentration of sulphuric acid in the feed solutions varied between 0.5 and 50 mmol/L.
256 The details of experimental conditions are summarised in Table 1. Feed solution pH was
257 adjusted to the values shown in Table 1.

258

259

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266 **Table 1. Experimental conditions for the transport of multi-ion solutions of H₂SO₄ by the**
 267 **NF270 membrane at 21 ± 1 °C.**

Feed concentration		Cross-flow velocity	Trans-membrane pressure
C ^o (H ₂ SO ₄ (mol/L))	pH ^o	(m/s)	(bar)
5 x 10 ⁻²	1	0.7	4.0–14
1.6 x 10 ⁻²	1.5	0.7	4.0–14
5 x 10 ⁻³	2	0.7	4.0–14
2 x 10 ⁻³	2.3	0.7	4.0–14
9 x 10 ⁻⁴	2.7	0.7	4.0–14
5 x 10 ⁻⁴	3	0.7	4.0–14

268

269 The trans-membrane volume flow (J_v) was determined by monitoring the collected permeate
 270 volume. Ion concentrations in feed and permeate samples were measured by ion
 271 chromatography (Dionex ICS-1000). Total sulphate concentrations were analysed by using the
 272 IONPAC[®] AS23 anion-exchange column using a mixture of 45 mM Na₂CO₃ and 0.8 mM NaHCO₃
 273 as eluent solution. The pH of the feed and permeate solutions were measured with a pH
 274 electrode.

275 Measured total sulphate concentrations $[SO_4^{2-}/HSO_4^-]_t$ and proton activities:

$$pH = -\log(\alpha(H^+)) = -\log([H^+] \cdot \gamma'(H^+)) \quad (16)$$

276

277 were compared when necessary with those predicted by using HYDRA-MEDUSA code.
 278 Concentration values of ionic species in solution (H^+ , HSO_4^- , SO_4^{2-}) were calculated taking into
 279 account the weak-acid equilibrium (Eq. 17):



280

281 and the mass and electroneutrality balances (Eqs. 18-21):

$$[SO_4^{2-}/HSO_4^-]_t = [SO_4^{2-}] + [HSO_4^-] \quad (18)$$

$$[HSO_4^-] = \alpha \cdot [H^+] \cdot [SO_4^{2-}] \quad (19)$$

$$[SO_4^{2-}] = \frac{[SO_4^{2-}/HSO_4^-]_t}{[1 + H^+ \cdot \alpha]} \quad (20)$$

$$[H^+] = 2 \cdot [SO_4^{2-}] + [HSO_4^-] \quad (21)$$

282

283 From the general expression (Eq. 9), and taking the species 1, 2 and 3 as H^+ , SO_4^{2-} and HSO_4^- ,
 284 respectively, the following equation will describe the flux of H^+ through the membrane for the
 285 H_2SO_4 case (Eq. 22):

$$J_v L = \frac{\int_{c_{H^+p}}^{c_{H^+f}} F(c_{H^+}) dc_{H^+}}{c_{H^+p} \cdot \left(1 + \frac{1}{-c_{H^+p} \cdot \alpha - 2}\right)} \quad (22)$$

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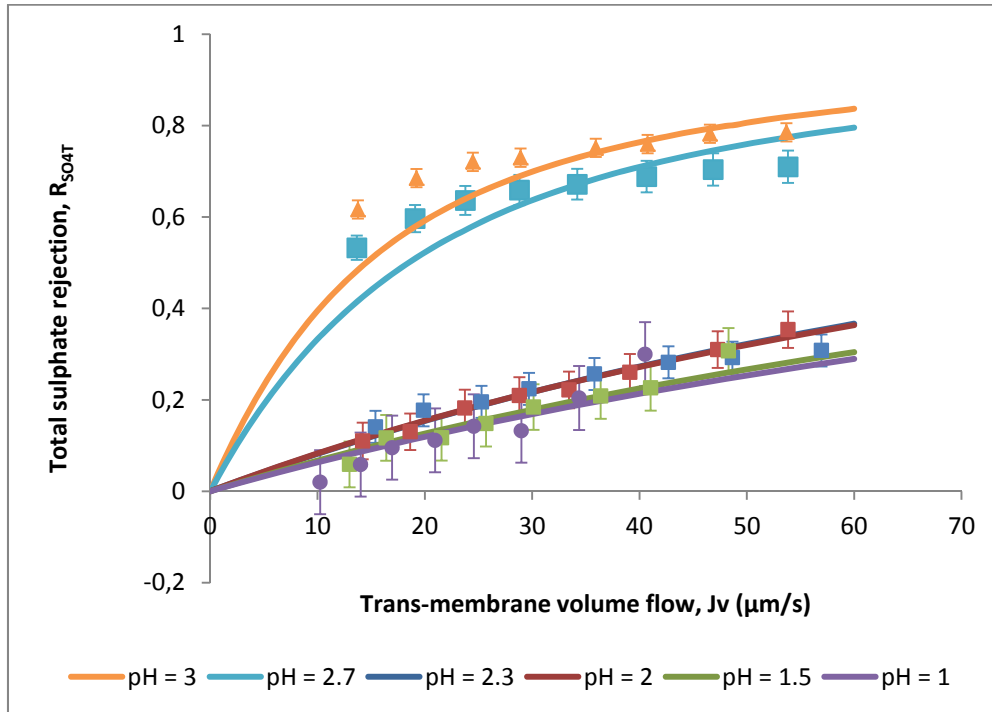
287 Where $\alpha = 10^{pK_a} = 10^{1.98}$ and

$$F(c_{H^+}) \equiv 2 \cdot \frac{P_{H^+} \cdot (-2 - c_{H^+} \cdot \alpha) \cdot (P_{SO_4^{2-}} + P_{HSO_4^-} \cdot \alpha \cdot c_{H^+}) - P_{SO_4^{2-}} \cdot P_{HSO_4^-} \cdot \alpha \cdot c_{H^+}}{-2 \cdot (-2 \cdot P_{SO_4^{2-}} - P_{H^+}) - c_{H^+} \cdot \alpha \cdot (-P_{HSO_4^-} - P_{H^+})} \cdot \frac{-2 - c_{H^+} \cdot \alpha - 1}{(-2 - c_{H^+} \cdot \alpha)} \quad (23)$$

288 4. Results and discussion

289 4.1. Sulphuric acid rejection by aromatic poly(piperazine) NF270 290 membrane: influence of pH on rejection

291 Figure 5 shows the observable rejection of total sulphate as a function of trans-membrane
 292 volume flow at various feed pH values. The symbols represent the experimental points and the
 293 lines were obtained by using the Solution-Diffusion-based model described in Section 2 (Eq. 9).



294

295 Figure 5. Total sulphate (SO_4^{2-}/HSO_4^-)_t rejection in solutions at different pH values as a
 296 function of trans-membrane volume flow. Solid lines were obtained by the Solution-Electro-
 297 Diffusion model.

298 Table 2 collects the experimental data from the experiments, including TMP, trans-membrane
 299 flow, sulphate concentration and pH.

300 **Table 2. Trans-membrane flow, total sulphate concentration and pH for feed and permeate**
 301 **in a range of TMP from 4 to 14 bar.**

ΔP (bar)	Feed	4	5.5	7	8.5	10	12	14
J_v ($\mu\text{m/s}$)		13.77	19.22	24.47	28.92	35.93	40.75	46.55
$[\text{SO}_4^{2-}/\text{HSO}_4^-]$ (ppm)	59.65	22.87	18.78	16.66	16.11	14.83	14.34	13.00
pH	2.95	3.32	3.41	3.48	3.53	3.57	3.55	3.55
J_v ($\mu\text{m/s}$)		13.66	19.09	23.77	28.81	34.20	40.67	46.86
$[\text{SO}_4^{2-}/\text{HSO}_4^-]$ (ppm)	86.84	41.51	35.84	32.31	30.28	29.15	27.68	26.29
pH	2.76	3.00	3.05	3.10	3.13	3.14	3.17	3.17
J_v ($\mu\text{m/s}$)		15.40	19.88	25.28	29.70	35.80	42.70	48.66
$[\text{SO}_4^{2-}/\text{HSO}_4^-]$ (ppm)	274.93	236.18	226.24	221.15	213.31	204.36	197.37	194.57
pH	2.36	2.39	2.41	2.43	2.43	2.45	2.46	2.46
J_v ($\mu\text{m/s}$)		14.21	18.63	23.72	28.80	33.43	39.10	47.29
$[\text{SO}_4^{2-}/\text{HSO}_4^-]$ (ppm)	603.33	536.92	524.79	493.36	476.96	469.59	446.29	416.31
pH	2.01	2.05	2.06	2.06	2.07	2.08	2.08	2.11
J_v ($\mu\text{m/s}$)		12.98	16.40	21.56	25.70	30.13	36.39	41.03
$[\text{SO}_4^{2-}/\text{HSO}_4^-]$ (ppm)	2462.87	2318.17	2175.28	2174.37	2098.26	2009.08	1949.15	1905.50
pH	1.52	1.54	1.55	1.57	1.57	1.58	1.60	1.60

J_v ($\mu\text{m/s}$)		10.24	14.02	16.95	20.95	24.56	28.99	34.38
$[\text{SO}_4^{2-}/\text{HSO}_4^-]$ (ppm)	6309.13	6182.82	5940.24	5707.34	5605.95	5411.45	5472.72	5021.21
pH	1.04	1.09	1.10	1.11	1.12	1.12	1.13	1.15

302

303 The rejections of total sulphate species ($\text{SO}_4^{2-}/\text{HSO}_4^-$) increased quasi-linearly for all the
304 experiments (apart from pH 3 and pH 2.7) up to values between 30 and 35% at the maximum
305 achieved trans-membrane volume flow (40–50 $\mu\text{m/s}$), without reaching the typical values
306 obtained with sulphate solutions at neutral to basic conditions (>99%) in the same range of
307 concentrations and trans-membrane volume flows [25,38,39]. At pH 3, the total sulphate
308 rejection reached values around 75%. It was still considerably reduced compared to the typical
309 sulphate rejections of >99%, due to the noticeable presence of monovalent specie HSO_4^- in the
310 feed. When pH was lowered, the amount of HSO_4^- species increased significantly, leading to
311 lower sulphate rejections (around 35%). Given the scattering, at pH < 2.3 the total sulphate
312 rejection was practically independent of pH, which is in agreement with the theoretical
313 calculations above (see Fig.1).

314 Ion rejections can be expected to be dependent on the membrane properties, especially
315 effective fixed charges, and on solution composition. The dependence on membrane
316 properties is related to the ion interactions with the polymer matrix (e.g. sorption or solution
317 of the ions into the polymer matrix) and to free functional groups (e.g. adsorption and
318 complexation of counter-ions to the fixed charge sites of the polymer membrane matrix, which
319 diminishes the effective fixed charge). NF270 has an isoelectric point around 3 [26,40] which
320 makes the membrane positively charged when pH is below the IEP. In all our experiments the
321 pH was lower than the IEP, making the membrane charge positive, and increasing when pH
322 was lowered. However, the low total sulphate rejection could also be primarily related to the
323 low degree of dissociation of the acid [7]. As pH decreases, the total amount of monovalent
324 sulphate species (hydrogen sulphate) increases and it becomes dominant in solution. This
325 change of feed composition results in a lower rejection of total sulphate.

326 Among various mechanisms invoked to describe the ion rejection by NF membranes, dielectric
327 exclusion should be taken into account. It is caused by the interactions of ions with bound
328 electric charges induced by ions at interfaces between media of different dielectric constants
329 (aqueous solution/polymer matrix). Initially, it was common to believe that the main rejection
330 mechanism in NF was the Donnan exclusion caused by fixed electric charges. That conclusion
331 was based, in fact, on the only observation that double-charge anions were rejected essentially
332 to a greater extent than were single-charge ones. However, that is characteristic of dielectric
333 exclusion, too, and is essentially even more pronounced because the ion-exclusion free energy
334 is proportional to the square of the ion charge (while the Donnan exclusion is linear to it). As a
335 result, for a 1:1 (e.g. $\text{HSO}_4^- \text{H}^+$) electrolyte, dielectric exclusion can be much weaker than for

336 electrolytes with doubly-charge ions 2:1 (e.g. Na₂SO₄) [41]. Table 3 summarises published
 337 results on the application of NF membranes in acidic solutions.

338 **Table 3. Comparison of total sulphate rejection data from dilute and concentrated aqueous**
 339 **solutions for various NF membranes.**

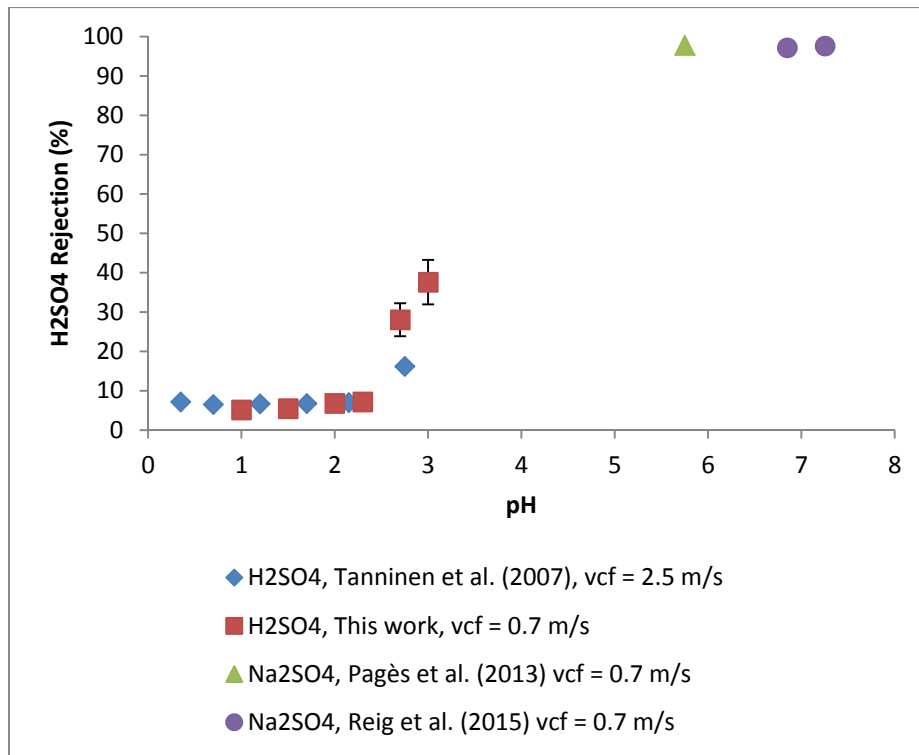
Name	IEP	Acid	Feed composition C _{H₂SO₄} (mol/L), pH	Trans- membrane flow, J _v	Rejection (%)	Reference
NF270	3.3 [42]	H ₂ SO ₄	8 wt%	n.a	27%	Tanninen et al. [42]
		H ₂ SO ₄	0.001-0.8 M	n.a.	-5%/ 20%	Tanninen et al. [9]
		H ₂ SO ₄	0.001-0.05	8 μm/s	7-15%	Tanninen et al. [43]
		H ₂ SO ₄	0.0005-0.05	13 μm/s	6-61%	This work
NF20	6.6 [43]	H ₂ SO ₄	0.001-0.05	8 μm/s	3-8%	Tanninen et al. [43]
NF45	7-8 [44]	H ₂ SO ₄	0.001-0.8 M	n.a.	-65%/10%	Tanninen et al. [9]
Desal DK	4 [43]	H ₂ SO ₄	0.001-0.8 M	n.a.	12%	Tanninen et al. [9]
		H ₂ SO ₄	0.001-0.8 M	n.a.	10%/15%	Tanninen et al. [9]
		H ₂ SO ₄	0.1-0.001	8 μm/s	2-40%	Tanninen et al. [43]
		H ₂ SO ₄	0.01-2 M	5-50 L/m ² ·h	25-5%	Soldenhoff et al. [45]
Desal KH	4.9 [43]	H ₂ SO ₄	8 wt%	n.a	22%	Tanninen et al. [42]
		H ₂ SO ₄	0.1-0.001	8 μm/s	10-40%	Tanninen et al. [43]
MPF34	-	H ₂ SO ₄	0.01-2	2-22 L/m ² ·h	20-10%	Soldenhoff et al. [45]
MPF	-	H ₃ PO ₄	28 wt%	0.06-0.1 μm/s	<1%	Diallo et al. [6]
		H ₂ SO ₄	1.6 – 2.0 wt%		6-1%	

340

341 Rejection values ranged from (slightly) negative up to ca. 40% for all the membranes.
 342 Generally, increase in the feed acid concentration resulted in a reduction of the rejection, as
 343 demonstrated by Diallo et al. [6]. Their experiments with concentrated mixtures of
 344 H₃PO₄/H₂SO₄ (28wt% and 1.8wt%, respectively) showed rejections below 6% for both
 345 compounds. In general, the increase of acidity (acid concentration) to values below pH 1
 346 resulted in rejection values below 2–5%, whereas rejection was above 30–40% for dilute
 347 solutions of pH above 3, where the dominant species in solution is SO₄²⁻.

348 In Figure 6 the total electrolyte rejections determined by Tanninen et al. [43], Pagès et al. [25]
 349 and Reig et al. [39] are compared with the results obtained in this work for the pH range from

350 0.3 to 3 (both studies used sulphuric acid solutions and NF270 membrane). Rejections of
 351 sodium sulphate at pH values from 5.75 to 8 are also included for comparison [25].



352

353 **Figure 6 - Comparison of sulphuric rejection as a function of pH for NF270 from this study**
 354 **and from Tanninen et al. [43], Pagès et al. [25] and Reig et al. [39] at trans-membrane**
 355 **volume flux of 8 $\mu\text{m/s}$.**

356 Both studies reveal similar behaviour, and the total sulphate/sulphuric rejection was
 357 approximately constant in the pH range from 0 (with HSO_4^- as predominant species) up to the
 358 vicinity of pKa, pH 2.5 (where the concentrations of HSO_4^- and SO_4^{2-} are comparable) with
 359 measured rejection values below 8%. Only when pH values were above pKa ($\text{pH} > \text{pKa} + 1$), i.e.
 360 when more than 90% of the sulphate was present as SO_4^{2-} , did rejection values start to
 361 increase from values above 30% at pH 3 to values of >90% for pH values above 5. It should be
 362 mentioned that, according to the IEP tests for NF270 in the literature, at pH higher than 3 the
 363 membrane becomes negatively charged and most of the carboxylic groups are present in
 364 deprotonated form (RCOO^-).

365 Finally, from the measured trans-membrane volume flow values (J_v), the membrane
 366 permeability toward water, $k_{\text{H}_2\text{O}}$, was determined by using Eq. 21:

367
$$J_v = k_{\text{H}_2\text{O}} \cdot (\Delta P - \Delta\pi) \quad (21)$$

368 where ΔP is the trans-membrane pressure and $\Delta\pi$ is the osmotic pressure difference across the
 369 membrane, assuming the reflection coefficient to be one. Table 4 shows the obtained values
 370 for the membrane permeability.

371

372 **Table 4. Hydraulic membrane permeability values (k_{H_2O}) for NF270 membrane with sulphuric**
 373 **and Na_2SO_4 solutions.**

pH	$C_{SO_4^{2-}}$ (ppm)	TMP (bar)	Jv ($\mu m/s$)	Total sulphuric rejection, $R_{SO_4^{2-}}$ (%)	K_{H_2O} ($\mu m/(bar \cdot s)$)	Reference
H_2SO_4 solution						
3	60	4 – 16	13.8 – 53.7	61.6 – 78.5	3.06	This work
2.7	90	4 – 16	13.3 – 53.8	53.3 – 71	3.25	This work
2.3	275	4 – 16	15.4 – 56.9	14 – 38	3.5	This work
2.0	600	4 – 16	14.2 – 53.8	11 – 35	3.3	This work
1.5	2465	4 – 16	12.9 – 48.3	5.8 – 31	3.0	This work
1.0	6030	4 – 16	10.2 – 40.5	5.8 – 30	2.5	This work
Na_2SO_4 solution						
5.8	9600	7 -20	6 – 43	97 – 99	3.1	Pagès et al. [25]
6.9	9600	7 -20	3.9 – 44.5	95 – 99	3.1	Reig et al. [39]
7.3	9600	7 -20	4.3 – 50	96 – 99	3.5	Reig et al. [39]

374

375 The membrane hydraulic permeability showed a decrease with the increase of sulphuric
 376 concentration (pH decrease). Permeability value is also shown for Na_2SO_4 feed solution; it
 377 approached the values for the most dilute sulphuric acid solutions.

378 **4.2. Determination of NF membrane permeances to ions:**
 379 **dependence on solution acidity**

380 The model parameters were membrane ion permeances: equal to the ion permeabilities
 381 divided by the active-layer thickness, $P_i^* = P_i/L$. The membrane ion permeances fitted to the
 382 experimental data shown in Fig. 5 are listed in Table 5. This presents the results obtained for
 383 each experiment (**calculated membrane permeances to ion from experimental data, P_i^***) as well
 384 as the relative error (given by Eq. 15) obtained when experiments were modelled. The highest
 385 deviations from model prediction to experimental data were observed when diluted sulphuric
 386 solutions (pH values 3.0 and 2.7) were filtered. At these two pH values, total sulphate (SO_4^{2-}
 387 $/HSO_4^-$)_t rejection started to increase with trans-membrane flow, until reaching a constant
 388 value. This behaviour was related to concentration polarisation phenomena, and it was
 389 responsible for the deviations between model and experimental data. In order to improve the
 390 mathematical model and to predict correctly the transport of a weak electrolyte through the
 391 membrane, a correction for concentration polarisation layer should be added.

392

393 **Table 5. Compilation of membrane permeance to ions ($\mu\text{m/s}$) of NF270 membrane in**
 394 **sulphuric solutions at 21 ± 1 °C using a cross-flow configuration and cfv of 0.7 m/s.**

pH	% HSO_4^-	Calculated membrane permeance to ions, P_i^* ($\mu\text{m/s}$)			% error
		H^+	SO_4^{2-}	HSO_4^-	
3.0 ± 0.1	10	191.4	0.25	67.1	13.2 %
2.7 ± 0.1	14	200.0	0.15	60.6	12.6 %
2.3 ± 0.1	31	220.0	0.39	149.7	2.5 %
2.0 ± 0.1	48	217.3	0.68	119.0	0.9 %
1.5 ± 0.1	75	220.9	0.32	117.2	1.7 %
1.0 ± 0.1	90	221.6	0.68	118.9	2.7 %

395

396 The ion permeance for HSO_4^- was much greater than for SO_4^{2-} , due probably to the dielectric
 397 exclusion mechanism as discussed in section 3.1. High values obtained for H^+ can be explained
 398 by the high mobility of this ion. Notably, the permeances do not show clear trends in their
 399 dependences on pH, which may point to a secondary role played by the membrane fixed
 400 charge in this context. However, the potential decrease in the permeances to cations due to
 401 the increasing positive fixed charge (with decreasing pH) could be approximately compensated
 402 by the increase in the feed ionic strength.

403 No published studies on the modelling of NF of sulphuric acid solutions at low pH values with
 404 the same membrane (NF270) could be found, so our results are compared with sulphate
 405 electrolytes in pH-neutral solutions. Permeances to sulphate for two types of strong sulphate
 406 electrolytes (Na_2SO_4 and MgSO_4) in mixtures of electrolytes as dominant or trace salts are
 407 summarised in Table 6. Permeances to SO_4^{2-} , determined by using the SEDF model, ranged
 408 from 0.03 to 0.90 $\mu\text{m/s}$ [24,25,39,46] and were similar to the values obtained in this study:
 409 0.15–0.68 $\mu\text{m/s}$.

410

411 **Table 6. Compilation of permeance values to SO_4^{2-} ($\mu\text{m/s}$) to the NF270 membrane in strong**
 412 **sulphate electrolytes as dominant and trace electrolyte at 21 ± 1 °C using cross flow**
 413 **configurations.**

Feed composition				cfv (m/s)	J_v ($\mu\text{m/s}$)	$R_{\text{SO}_4^{2-}}$ (%)	$P_{\text{SO}_4^{2-}}^*$ ($\mu\text{m/s}$)	Reference
Dominant salt	C_f (mol/L)	Ion trace	C_f (mol/L)					
Sulphate as dominant								
Na_2SO_4	0.1	NaCl	$5 \cdot 10^{-4}$	0.35; 0.7; 1	6 – 43	97 -99	0.05	[25]
Na_2SO_4	0.044	NaCl	$2.2 \cdot 10^{-4}$	1	7 – 93	97 – 99	0.07	[24]
MgSO_4	0.1	NaCl	$5 \cdot 10^{-4}$	0.35; 0.7; 1	4.5 – 52	98 – 99	0.05	[25]
MgSO_4	0.1	NaCl	$5 \cdot 10^{-4}$	0.7	6 – 48	98 – 99	0.04	[38]
MgSO_4	0.1	NH_4Cl	$5 \cdot 10^{-4}$	0.7	4.5 – 47	91 – 98	0.1	[38]
MgSO_4	0.1	NaNO_3	$5 \cdot 10^{-4}$	0.7	5 – 50	94 – 98	0.03	[38]
MgSO_4	0.1	NaI	$5 \cdot 10^{-4}$	0.7	4 – 47	98 -99	0.1	[38]
Sulphate as trace								
NaCl	0.1	MgSO_4	0.002	0.35; 0.7; 1	2.5 – 60	97 – 99	0.16	[25]
NaCl	0.088	Na_2SO_4	$2.7 \cdot 10^{-4}$	1	13 – 128	98 – 99	0.13	[24]
NaCl	0.1	MgSO_4	$2 \cdot 10^{-3}$	0.7	2 – 60	> 98	0.09	[38]
MgCl_2	0.1	Na_2SO_4	$5 \cdot 10^{-4}$	0.7	6 -40	97 -99	0.9	[38]

414

415 In addition, just a few studies to determine ion permeances in NF membranes considering
 416 reactive transport have been carried out. Niewersch et al. [29,30] studied the application of NF
 417 for the recovery of phosphoric acid from the processing of mono-incineration of municipal
 418 sewage-sludge bottom ash after leaching with sulphuric acid. Phosphoric/sulphuric solutions,
 419 also rich in potassium were treated with an aromatic polyamide membrane (Desal-DL), and the
 420 results were modelled by SEDM and accounting for chemical equilibria between species. Desal-
 421 DL has an IEP value of 3.0 [47]. High membrane permeances to hydrogen (H^+) were obtained
 422 (ranging from 4 to 400 $\mu\text{m/s}$), while relatively low permeances were obtained for both
 423 sulphate species (ranging from 3 to 20 $\mu\text{m/s}$) [29,30]. Table 7 collects the fitted values of
 424 membrane ion permeances of the Desal-DL membrane.

425

426 **Table 7. Compilation of Desal-DL membrane permeances ($\mu\text{m/s}$) to H^+ , HSO_4^- and SO_4^{2-} in**
 427 **sulphuric/phosphoric mixtures at 20 ± 1 °C using cross flow configurations and cfv of 0.5**
 428 **L/min, from Niewersch et al. [29,30]**

pH	$[\text{SO}_4^{2-}/\text{HSO}_4^-]_{\text{TOT}}$ (M)	$[\text{K}^+]$ (M)	$P_{\text{H}^+}^*$ ($\mu\text{m/s}$)	$P_{\text{SO}_4^{2-}}^*$ ($\mu\text{m/s}$)	$P_{\text{HSO}_4^-}^*$ ($\mu\text{m/s}$)
1.5	0.150	0.214	4	20	15
2	0.149	0.255	55	4	4
2	0.182	0.318	70	5	3
3	0.153	0.302	400	6	20

429

430 According to the reported values, membrane permeances to ion showed a trend in the values
 431 of membrane permeances for H^+ , in which its value decreased from 400 to 4 $\mu\text{m/s}$ when pH
 432 was lowered from 3 to 1.5. However, permeances for HSO_4^- and SO_4^{2-} showed values between
 433 3 to 20 $\mu\text{m/s}$.

434 5. Conclusions

435 Our experimental study of transport of sulphuric solutions within a pH range covering the
 436 ratios between SO_4^{2-} and HSO_4^- species from 1:10 to 10:1 revealed strong influence of the
 437 speciation on electrolyte rejection. The principal reason for this is the strong dependence of
 438 ion permeance of NF membranes on the magnitude of ion charge (probably due to dielectric
 439 exclusion).

440 The experimental data could be quantitatively interpreted by means of the Solution-Electro-
 441 Diffusion model extended to account for an equilibrium chemical reaction between the
 442 species. An analytical solution of this problem was obtained for the first time.

443 Although information on the membrane permeances to ions has remained empirical in this
 444 study, it can, in principle, be used further for the verification of self-consistency of various
 445 mechanistic models. The availability of three 'measurable' quantities: the membrane
 446 permeances to these two species ($\text{SO}_4^{2-}/\text{HSO}_4^-$) in equilibrium and to H^+ , in contrast to just
 447 one permeance to the salt available from measurements with strong electrolytes, can make
 448 model self-consistency checks much more conclusive. The results indicated that the fitted
 449 membrane ion permeances were approximately constant within the studied (acidic) pH range.
 450 In the case of sulphate ions, their values were comparable to those determined for the same
 451 membrane with strong sulphate electrolytes at neutral pH values.

452

453 **Nomenclature**

c_i	Concentration of component i inside the membrane
c_{ip}	Concentration of component i in permeate side
j_i	Ion flux
J_v	Trans-membrane volume flow
L	Membrane thickness
P_i	Membrane permeability to ion i
P_i^*	Membrane permeance to ion i
pK_a	Acidity constant
x	Dimensionless position in the membrane
z_i	Ion charge
α	Equilibrium dissociation constant
φ	Dimensionless electric potential in the membrane

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 460 computation stage.

461 **Appendix I - Derivation of transport equations for reacting**
 462 **species weak electrolytes**

463

464 Model development included the following steps:

465 Step 1) Formulation of the gradient potential from the zero-current flow condition
 466 ($\sum_i z_i \cdot j_i = 0$), according to Eq. A1-A2:

$$-\sum z_i \cdot P_i \cdot c_i \cdot \left(\frac{d \ln c_i}{dx} + z_i \cdot \frac{d\varphi}{dx} \right) \rightarrow -\sum z_i^2 \cdot P_i \cdot c_i \cdot \frac{d\varphi}{dx} \quad (A1)$$

$$= \sum z_i \cdot P_i \cdot c_i \cdot \frac{d \ln c_i}{dx}$$

$$\frac{d\varphi}{dx} = -\sum \frac{t_i}{z_i} \cdot \frac{d \ln c_i}{dx} \rightarrow \frac{d\varphi}{dx} = -\frac{t_1}{z_1} \cdot \frac{d \ln c_1}{dx} - \frac{t_2}{z_2} \cdot \frac{d \ln c_2}{dx} - \frac{t_3}{z_3} \cdot \frac{d \ln c_3}{dx} \quad (A2)$$

467

468 where t_i represents (Eq. A3):

$$t_i = \frac{z_i^2 \cdot P_i \cdot c_i}{\sum_j z_j^2 \cdot P_j \cdot c_j} \quad (A3)$$

469 Step 2) Combination of electro-neutrality condition ($\sum_i z_i \cdot c_i = 0$) and equilibrium
 470 condition ($c_3 = \alpha \cdot c_1 \cdot c_2$) to describe concentration of ion 2 (c_2) as a function of the
 471 concentration of the other two ions of the weak electrolyte equilibrium (Eq. A6):

$$z_1 \cdot c_1 + z_2 \cdot c_2 + (z_1 + z_2) \cdot \alpha \cdot c_1 \cdot c_2 = 0 \quad (\text{A4})$$

$$c_2 \cdot (\alpha \cdot c_1 \cdot (z_1 + z_2) + z_2) + z_1 \cdot c_1 = 0 \quad (\text{A5})$$

$$c_2 = -\frac{z_1 \cdot c_1}{\alpha \cdot c_1 \cdot (z_1 + z_2) + z_2} = -\frac{z_1}{\alpha \cdot (z_1 + z_2) + z_2/c_1} \quad (\text{A6})$$

472 Step 3) Definition of the logarithmic concentration gradient of each species in solution

473 (Eq. A7, A13)

474 Considering the equilibrium condition

$$\frac{d \ln c_3}{dx} = \frac{d \ln c_1}{dx} + \frac{d \ln c_2}{dx} \quad (\text{A7})$$

475

476 Considering the electro-neutrality condition:

$$z_1 \cdot c_1 \cdot \frac{d \ln c_1}{dx} + z_2 \cdot c_2 \cdot \frac{d \ln c_2}{dx} + (z_1 + z_2) \cdot \alpha \cdot c_1 \cdot c_2 \cdot \left(\frac{d \ln c_1}{dx} + \frac{d \ln c_2}{dx} \right) = 0 \quad (\text{A8})$$

$$\frac{z_1}{c_2} \cdot \frac{d \ln c_1}{dx} + \frac{z_2}{c_1} \cdot \frac{d \ln c_2}{dx} + (z_1 + z_2) \cdot \alpha \cdot \left(\frac{d \ln c_1}{dx} + \frac{d \ln c_2}{dx} \right) = 0 \quad (\text{A9})$$

$$\frac{d \ln c_1}{dx} \cdot \left(\frac{z_1}{c_2} + \alpha \cdot (z_1 + z_2) \right) + \frac{d \ln c_2}{dx} \cdot \left(\frac{z_2}{c_1} + \alpha \cdot (z_1 + z_2) \right) = 0 \quad (\text{A10})$$

$$\frac{d \ln c_2}{dx} = -\frac{\frac{z_1}{c_2} + \alpha \cdot (z_1 + z_2)}{\frac{z_2}{c_1} + \alpha \cdot (z_1 + z_2)} \cdot \frac{d \ln c_1}{dx} \quad (\text{A11})$$

$$c_2 = -\frac{z_1}{\alpha \cdot (z_1 + z_2) + \frac{z_2}{c_1}} \rightarrow \frac{z_1}{c_2} = -\left(\alpha \cdot (z_1 + z_2) + \frac{z_2}{c_1} \right) \quad (\text{A12})$$

$$\frac{d \ln c_2}{dx} = \frac{\frac{z_2}{c_1}}{\frac{z_2}{c_1} + \alpha \cdot (z_1 + z_2)} \cdot \frac{d \ln c_1}{dx} \quad (\text{A13})$$

477 Step 4) Definition of ion flux of component 1 and 2 (Eq.A14-A15):

$$-j_1 = P_1 \cdot c_1 \cdot \left(\frac{d \ln c_1}{dx} + z_1 \cdot \left(-\frac{t_1}{z_1} \cdot \frac{d \ln c_1}{dx} - \frac{t_2}{z_2} \cdot \frac{d \ln c_2}{dx} - \frac{t_3}{z_3} \cdot \frac{d \ln c_3}{dx} \right) \right) \quad (\text{A14})$$

$$-j_1 = P_1 \cdot c_1 \cdot \left[\frac{d \ln c_1}{dx} \cdot \left(1 - t_1 - \frac{z_1 \cdot t_3}{z_1 + z_2} \right) - \frac{d \ln c_2}{dx} \cdot \left(t_2 \cdot \frac{z_1}{z_2} + \frac{z_1 \cdot t_3}{z_1 + z_2} \right) \right] \quad (\text{A15})$$

$$-j_2 = P_2 \cdot c_2 \cdot \left[\frac{d \ln c_2}{dx} \cdot \left(1 - t_2 - \frac{z_2 \cdot t_3}{z_1 + z_2} \right) - \frac{d \ln c_1}{dx} \cdot \left(t_1 \cdot \frac{z_2}{z_1} + \frac{z_2 \cdot t_3}{z_1 + z_2} \right) \right] \quad (\text{A16})$$

478 Step 5) Taking into account that the difference of ion fluxes of 1 and 2 is conservative,

479 a new equation is obtained (Eq. A17).

$$-J_{\Delta} = P_1 \cdot c_1 \cdot \left[\frac{d \ln c_1}{dx} \cdot \left(1 - t_1 - \frac{z_1 \cdot t_3}{z_1 + z_2} \right) - \frac{d \ln c_2}{dx} \cdot \left(t_2 \cdot \frac{z_1}{z_2} + \frac{z_1 \cdot t_3}{z_1 + z_2} \right) \right] - P_2 \cdot c_2 \cdot \left[\frac{d \ln c_2}{dx} \cdot \left(1 - t_2 - \frac{z_2 \cdot t_3}{z_1 + z_2} \right) - \frac{d \ln c_1}{dx} \cdot \left(t_1 \cdot \frac{z_2}{z_1} + \frac{z_2 \cdot t_3}{z_1 + z_2} \right) \right] \quad (\text{A17})$$

480

481 By substituting equation A12 in the A17 and rearranging the terms, the equation presented in
482 the section 2 (Eq. 9) could be obtained (A18):

$$J_v L = \frac{\int_{c_{1p}}^{c_{1f}} F(c_1) dc_1}{c_{1p} \cdot \left(1 + \frac{z_1}{c_{1p} \cdot \alpha \cdot (z_1 + z_2) + z_2} \right)} \quad (\text{A18})$$

483 Where

$$F(c_1) = z_2 \cdot z_3 \cdot \frac{P_1 \cdot (z_2 + c_1 \cdot \alpha \cdot z_3) \cdot (P_2 + P_3 \cdot \alpha \cdot c_1) - P_2 \cdot P_3 \cdot z_1 \cdot \alpha \cdot c_1}{\frac{z_2 \cdot (P_2 \cdot z_2 - P_1 \cdot z_1) + c_1 \cdot \alpha \cdot z_3 \cdot (P_3 \cdot z_3 - P_1 \cdot z_1)}{z_2 + c_1 \cdot \alpha \cdot z_3 - z_1}} \cdot \frac{1}{(z_2 + c_1 \cdot \alpha \cdot z_3)} \quad (\text{A19})$$

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