Preprints of the 20th World Congress
The International Federation of Automatic Control
Toulouse, France, July 9-14, 2017

Reliability computation within an MPC health-aware framework

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Abstract: This paper presents a comparison between two different approaches for reliability consideration within a Health-Aware Control framework which takes into account system and component reliability by means of reliability importance measures. The two different approaches for reliability assessment consideration are the instantaneous reliability and the expected one. The system reliability performance under both approaches is compared in a control strategy applied to a drinking water network.

Keywords: reliability, control system, availability, health aware control, drinking water network

1. INTRODUCTION

Fault-tolerant control has been a relevant topic in control theory by decades, its objective is to allow system functioning after a fault occurrence, those faults can be in sensors or actuators (Zhang and Jiang, 2008). However, using Prognostic and Health Management (PHM) to prevent faults occurrence could be more interesting from and economical and safety point of view.

The prevention of faults occurrence in the control loop is also called Health-Aware Control (HAC). This technique uses proper on-line prognostic information of the system to modify the control actions or to change the mission objective in order to maintain a high level of system health.

Extending the operational time of the system and avoiding faults occurrence can be achieved by considering the level of system components reliability and their importance for the overall system reliability in the control algorithm (Salazar et al., 2015, 2016). Then, in over-actuated systems, it is possible to redistribute the control effort among the available actuators following the appropriate policy (Khelassi et al., 2011; Bicking et al., 2013).

In this paper, the redistribution policy is given by the reliability importance of the actuators to the overall system reliability in such way that the use of an actuator does not compromise the functioning of the system. This measure was proposed by Birnbaum (1969) and defines the amount of system reliability decay if the reliability of a component decreases to 0.

In this work, two ways of calculating and interpreting the component and system reliability are studied and discussed. The first one considers the most general definition of reliability as the probability that a component will perform its function under specified conditions and for a defined interval of time, in this case, reliability is computed at fixed intervals and is a decreasing function of time. The second one considers that a component remains fully reliable as long as it is not affected by a fault, but its expected reliability decays based on its usage (Chamseddine et al., 2014).

The proposed study is illustrated using a Drinking Water Network (DWN), where a redistribution of the control efforts among the actuators is proposed in order to improve the overall DWN reliability. DWNs are multivariable dynamic systems composed of several interconnected subsystems, such as tanks, pumps, valves, intersection nodes, water sources and consumer sectors.

Model Predictive Control (MPC) approach has proved to be an efficient technique that can predict the appropriate control actions to achieve optimal performance according to physical constraints and multiobjective cost functions (Maciejowski, 2002).

Recently, some studies dedicates to DWN control are focussed on MPC to perform an optimal management of the DWN system and supply the consumer demand while preserving the DWN reliability, e.g. by tuning the weights of the optimization problem according to a wear index (Grosso Pérez et al., 2012), according to reliability importance measures (Salazar et al., to appear), or by imposing constraints concerning actuator reliability (Robles et al., 2016).

★ This work was supported by Spanish Government (MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD) and FEDER under project DPI2014-58104-R (HARCRICS).

reliable as long as it is not affected by a fault. Moreover, its reliability at mission time t_f evaluated at current time τ is denoted as:

$$R_i^\tau(t_f) = e^{-\int_\tau^{t_f} \lambda_i^\tau(v)dv} \quad \forall\, i = 1, \ldots, m \tag{7}$$

where $\lambda_i^\tau(t)$ is the failure profile of the ith actuator determined at time τ and $R_i^\tau(t_f)$ denotes the reliability at the end of the mission time t_f computed at time instant τ (Fig. 3). This approach was introduced in Chamseddine et al. (2014).

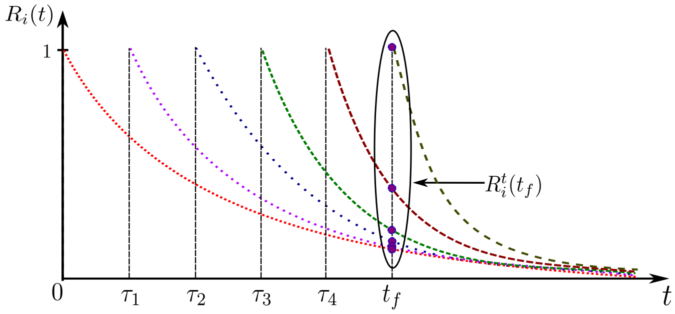

Fig. 3. Expected reliability of the actuator $R_i^t(t_f)$.

This second reliability interpretation will be called the *expected reliability of the actuator*. The actuator can be characterized at instant time τ by a constant failure rate in the time interval $[\tau, t_f]$. Hence, the expected reliability of the actuator becomes:

$$R_i^\tau(t_f) = e^{-\lambda_i(\tau)\times(t_f-\tau)} \quad \forall\, i = 1, \ldots, m \tag{8}$$

Regarding system reliability, under the first interpretation it will be called the *instantaneous system reliability,* denoted as $R_S(t)$. Under the second interpretation it will be called the *expected system reliability,* denoted as $R_S^\tau(tf)$.

2.4 Importance reliability measures

To measure and quantify the impact of actuator failures over the functioning of the system, several indicators concerning reliability importance have been proposed, each of them with a particular purpose (Kuo and Zhu, 2012).

In this work, the Birnbaum's importance measure $\mathrm{I_B}$ (Birnbaum, 1969) will be considered. This importance measure quantifies the maximum decrease of system reliability due to reliability changes of the ith actuator. It can be defined as:

$$\mathrm{I}_{\mathrm{B}i}(t) = \frac{\partial R_S(t)}{\partial R_i(t)} = R_S(1_i, t) - R_S(0_i, t) \tag{9}$$

The notation $R_S(1_i, t)$ denotes the reliability of the system in which the ith actuator is replaced by a fully reliable one, while $R_S(0_i, t)$ denotes the reliability of the system in which the ith actuator has failed.

According to both reliability interpretations, two Birnbaum's measures are proposed. On the one hand, under the instantaneous reliability interpretation, the actuator Birnbaum's importance measure will be computed as in (9), whereas under the expected reliability interpretation the following Birnbaum's importance measure will be considered.

$$\mathrm{I}_{\mathrm{B}i}^\tau(t_f) = \frac{\partial R_S^\tau(t_f)}{\partial R_i^\tau(t_f)} = R_S^\tau(1_i, t_f) - R_S^\tau(0_i, t_f), \tag{10}$$

which is the actuator Birnbaum's importance measure at mission time instant t_f computed at current time τ.

3. HEALTH-AWARE MPC

3.1 MPC formulation

Consider the following linear discrete-time model described in the state-space form of an over actuated system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + E\varepsilon(k) \\ y(k) &= Cx(k) \end{aligned} \tag{11}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^p$ is the control input vector with $u(k) \geq 0\ \forall\ k$, $y(k) \in \mathbb{R}^q$ is the measured output vector, $\varepsilon(k) \in \mathbb{R}^m$ is the disturbance vector, $A \in \mathbb{R}^{n\times n}$ is the state matrix, $B \in \mathbb{R}^{q\times p}$ input matrix, $E \in \mathbb{R}^{n\times m}$ is the disturbance matrix, and $C \in \mathbb{R}^{q\times n}$ is the output matrix.

The MPC algorithm uses a model of the system to predict the future output of the system and compute the optimal control actions aimed at optimizing a given cost function over a prediction horizon H_p. This cost function is minimized subject to a set of physical and operational constraints over a control horizon $H_c \leq H_p$. Once the minimization is performed, a vector of control actions is obtained and just the first component is applied to the system. The procedure is repeated for the next time instant following a receding horizon strategy and taking into account feedback system measurements and future set-points.

In this work, the multiobjective optimization problem is formulated as follows:

$$\begin{aligned} \underset{\substack{(\hat{u}(k|k),\ldots,\\ \hat{u}(k+Hc-1|k),\\ \Delta\hat{u}(k|k),\ldots,\\ \Delta\hat{u}(k+Hc-1|k))}}{\text{minimize}} \quad & J(k) = \sum_{j=0}^{H_c-1}\sum_{i=1}^{p} \Delta\hat{u}_i(k+j|k)^2 \\ & \qquad + \sum_{j=0}^{H_c-1}\sum_{i=1}^{p} \rho_i(k)\hat{u}_i(k+j|k)^2 \\ \text{subject to} \quad & \underline{u} \leq \hat{u}(k+j|k) \leq \overline{u} \quad j = 0, .., H_c - 1 \\ & \underline{x} \leq \hat{x}(k+l|k) \leq \overline{x} \quad l = 1, .., H_p \end{aligned} \tag{12}$$

where $\rho_i(k)$ is a weight, $\underline{u}$ and $\overline{u}$ denote the minimum and maximum actuator bounds, and $\underline{x}$ and $\overline{x}$ denote the minimum and maximum state bounds. The notation $k+j|k$ allows a future time instant $k+j$ to be referred at current time instant k, and $\Delta\hat{u}_i(k) \triangleq \hat{u}_i(k) - \hat{u}_i(k-1)$.

The first term of the objective function aims at guaranteeing a smooth actuator operation whereas the second term penalizes actuator operation according to their use cost $\rho_i(k)$. In this work, the control law dependence on reliability will be achieved through ρ_i weights.

Although a linear model has been taken into account in this work, a nonlinear model could also be considered. In such case, the model should be either linearized around an equilibrium point or Nonlinear Model Predictive Control (Grüne and Pannek, 2011) could be applied.

The Health-Aware Control scheme used in this paper was proposed in Salazar et al. (to appear) and facilitates the exploration of different redistribution policies without significant changes in the control algorithm.

3.2 Redistribution policy

One of the objectives is to extend the overall system reliability by performing the optimal control actions over the system given a redistribution policy which is achieved by tuning the weight ρ_i.

Three weights assignments for ρ_i are proposed in this paper under both reliability interpretations.

Under the instantaneous reliability interpretation, the first assignment focuses actuators reliability in discrete time:

$$\rho_i(k) = 1 - R_i(k) \tag{13}$$

The objective of this weight assignment is to preserve the individual reliability of each actuator as the optimization algorithm will further penalize those actuators with lower reliability (Salazar et al., 2015).

The second weight assignment focuses the overall system reliability using the Birnbaum importance measure in discrete time:

$$\rho_i(k) = \mathrm{I}_{\mathrm{B}i}(k) \tag{14}$$

The objective is further penalizing those actuators with a greater impact of their reliability change on the system reliability (Salazar et al., 2015).

Similarly, under the expected reliability interpretation, the first weight assignment corresponds to:

$$\rho_i(k) = 1 - R_i^k(k_f), \tag{15}$$

and the second one to:

$$\rho_i(k) = \mathrm{I}_{\mathrm{B}i}^k(k_f) \tag{16}$$

In the third weight assignment, which is common for both reliability interpretations, no reliability is taken into account, i.e., $\rho_i(k) = 1$.

In Salazar et al. (2015), weights were already assigned following the instantaneous reliability approach. The novelty of this work consists in assigning the weights under the expected reliability interpretation.

3.3 Reliability performance assessment

Different indexes are proposed to compare both reliability interpretations. Firstly, the cumulative control effort index (U_{cum}) which indicates the amount of energy spent controlling the system,

$$U_{\mathrm{cum}} = T_s \sum_{k=0}^{T_{sim}/T_s} \left[u(k)^T u(k)\right]. \tag{17}$$

Next, the cumulative system reliability, which indicates the aggregated system reliability over the simulation time. Under the instantaneous reliability interpretation, it is denoted in discrete time as:

$$R_{S\mathrm{cum}} = T_s \sum_{k=0}^{T_{sim}/Ts} R_s(k), \tag{18}$$

and under the expected reliability interpretation, it is denoted as:

$$R_{S\mathrm{cum}}^{k_f} = T_s \sum_{k=0}^{T_{sim}/Ts} R_s^k(k_f) \tag{19}$$

4. DRINKING WATER NETWORK APPLICATION

4.1 System description

In this work, an application over a Drinking Water Network (DWN) is presented. A DWN is a system composed of sources (water supplies), sinks (water demand sectors), reservoirs, pipelines that link sources to sinks through pumps and valves. The network consists of 5 sources and 1 demand sector (Fig. 4).

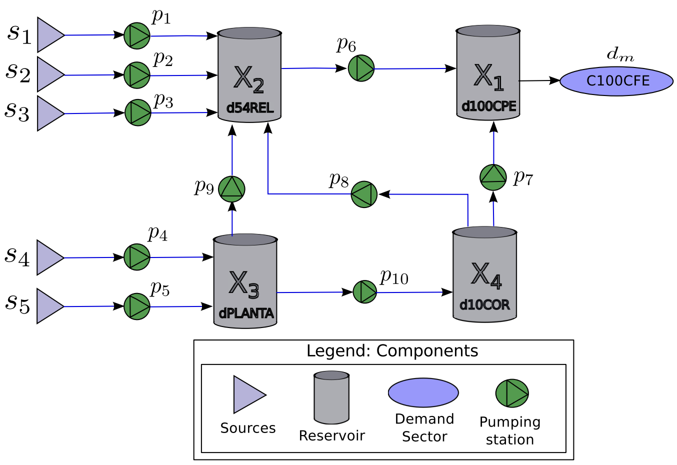

Fig. 4. Drinking Water Network system example

It is assumed that the demand forecast (d_m) at the sink is known and that any single source can satisfy this required water demand (Fig. 5).

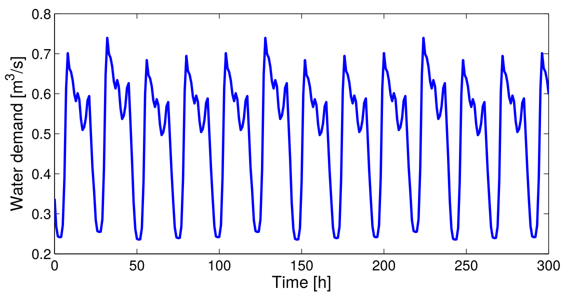

Fig. 5. Drinking water demand.

By applying mass balance at each tank, the following linear discrete-time model is obtained:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_d d_m(k) \\ y(k) &= Cx(k) \end{aligned} \tag{20}$$

where $x(k) \in \mathbb{R}^n$ are tanks volume, $u(k) \in \mathbb{R}^p$ are the control inputs (pump commands), $y(k) \in \mathbb{R}^q$ are the measured tanks volume, $d_m(k) \in \mathbb{R}^m$ is the water demand, $A \in \mathbb{R}^{n\times n}$ is the state matrix, $B \in \mathbb{R}^{q\times p}$ input matrix, $B_d \in \mathbb{R}^{n\times m}$ is the disturbance matrix, and $C \in \mathbb{R}^{q\times n}$ is the output matrix.

The objective of the MPC scheme (12) is to maintain the pumps and tanks under their bounds and extend the reliability of the system.

The simulation parameters are presented in Table 1.

Table 1. Simulation parameters

Parameter	Value				
H_p $[h]$	24				
H_c $[h]$	8				
T_s $[h]$	1				
β_i	10^{-2} $\forall\ i \in [1, 10]$				
$\overline{u}_i$ $[m^3/s]$	0.75	0.75	0.75	1.20	0.85
	1.60	1.70	0.85	1.70	1.60
$\underline{u}_i$ $[m^3/s]$	0 $\forall\ i \in [1, 10]$				
λ_i^0 $[h^{-1} \times 10^{-4}]$	9.85	10.70	10.50	1.40	0.85
	0.80	11.70	0.60	0.74	0.78
$\overline{x}_i$ $[m^3]$	65200	3100	14450	11745	
$\underline{x}_i$ $[m^3]$	25000	2200	5200	3500	
$x_i(0)$ $[m^3]$	45100	2650	9825	7622	

4.2 Reliability analysis

The five ρ_i assignments proposed at Section 3.2 have been considered in the MPC control of the DWN. All cases will be assessed under both reliability interpretations.

Figure 6 shows the instantaneous system reliability evolution and Fig. 7 provides the expected system reliability at the end of the mission time t_f. In Table 2, the reliability performance indexes are shown.

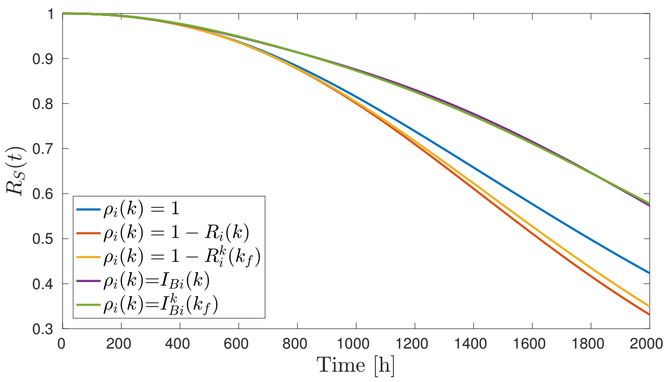

Fig. 6. Instantaneous system reliability.

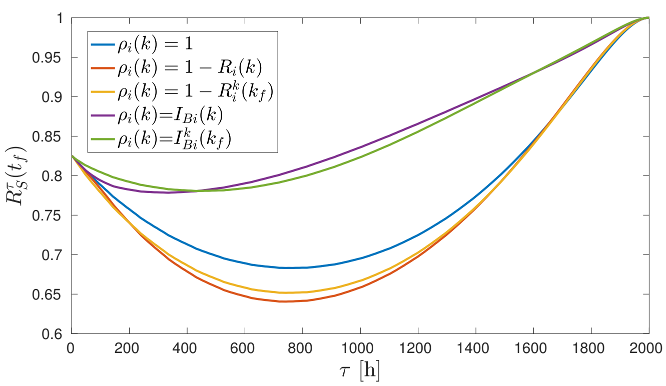

Fig. 7. Expected system reliability at mission time t_f.

Results show that the best reliability performance is attained when using a redistribution policy based on the Birnbaum's importance measure, i.e., (14) and (16), with a small improvement when the instantaneous reliability interpretation is followed, i.e., (14). Focusing on actuator reliability (i.e., (13) and (14)) does not optimize system reliability. However, targeting system reliability leads to a greater actuator energy expense.

Table 2. Reliability performance indexes.

$\rho_i(k)$	$R_{S\text{cum}}$ $[\times 10^6]$	$R_{S\text{cum}}^{k_f}$ $[\times 10^6]$	U_{cum} $[\times 10^6]$
1	5.6131	5.5583	1.5370
$1 - R_i(k)$	5.4046	5.4054	1.9687
$1 - R_i^k(k_f)$	5.4525	5.4340	1.9002
$\mathrm{I}_{\mathrm{B}i}(k)$	6.1006	6.1653	3.2158
$\mathrm{I}_{\mathrm{B}i}^k(k_f)$	6.0915	6.1447	3.5040

In general, remark that both reliability interpretations are almost equivalent since both approaches provide similar results.

Figures 8 and 9 provide the pump control actions and the tank volumes for the best redistribution policy, corresponding to (14). Note that, the DWN is able to supply the required water demand maintaining pump control efforts and tank volumes within the specified bounds.

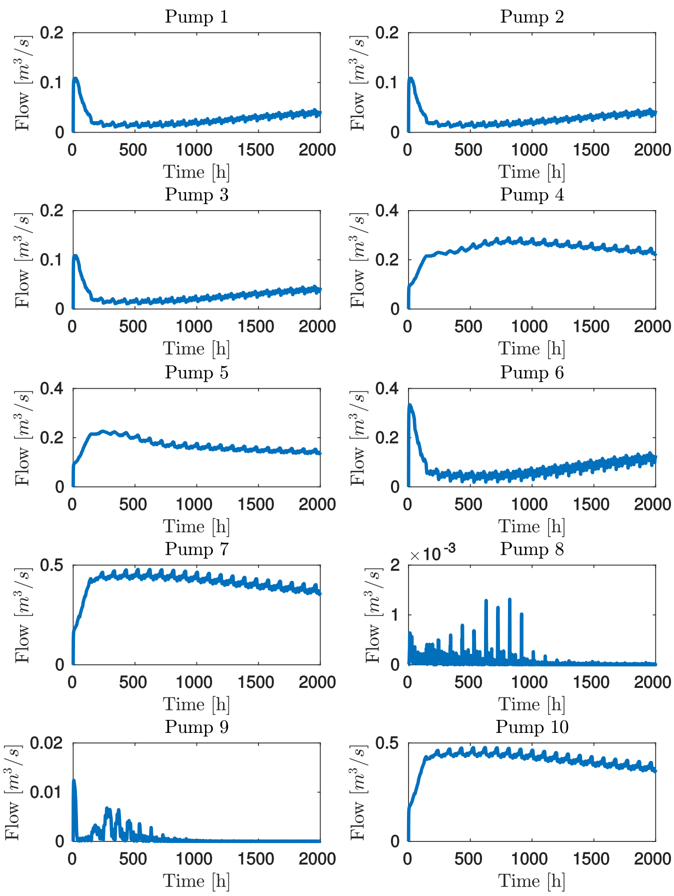

Fig. 8. Pump commands corresponding to $\rho_i(k) = \mathrm{I}_{\mathrm{B}i}(k)$.

5. CONCLUSIONS

In this paper, two reliability interpretations have been presented and illustrated using a DWN system. Both

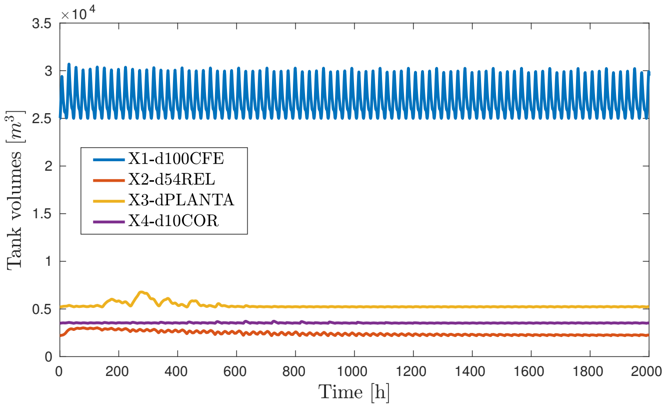

Fig. 9. Tank volumes corresponding to $\rho_i(k) = \mathrm{I}_{\mathrm{B}i}(k)$.

approaches have been applied to a Health-Aware Control scheme based on an MPC algorithm with the objective of improving system reliability.

The results of the redistribution policy provide similar results in terms of reliability enhancement independently of the reliability interpretation. Thus, both interpretations are virtually equivalent.

Additionally, a better-suited modeling of the failure rate considering the aggregated usage of the actuators has been proposed.

Future research will focus on the study of the robustness of both reliability approaches, i.e. determine how robust or sensitive are these approaches respect to the parameters selection.

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