Development of Reduced Order Models for Wind Farm Control

MEMÒRIA

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Convocatoria: Abril 2017
Realizado en: TUM
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Abstract

Background: Wind energy is one of the leading renewable energies that take part in the current transition from the usage of traditional fossil-fuel energies to green ones. The potential and development of wind power is increasing constantly. Therefore, continuous studies are being carried out in order to improve the power extraction of wind farms.

Study: This Master’s Thesis aims to obtain multiple-Input-multiple-Output Reduced-Order Models (IOROMs) that are able to capture the main dynamics present in wind turbine wake flows within wind farms during transients and during operation. This dynamics can be excited via different inputs, such as wind turbine’s yaw angle, generator torque, pitch angle, among others. In this work, the study will be conducted by analyzing the response of the system to yaw angle variations, although the same procedure explained is valid for other inputs. The models developed are also mainly intended for capturing the relation between output magnitudes, such as wind turbine power output, bending moments, lateral forces, etc., and the given inputs to the system. Beside, order reduction is key to this work, since the models are expected to reproduce with acceptable fidelity high computationally costly simulations. The study has been conducted based on CFD (Computational Fluid Dynamics) simulation data that are considered as the starting point of the work.

Results & Conclusions: The models developed in this work present considerable agreement with respect to the results obtained via CFD simulations. Owing to the symmetry that the yawing motion presents for the system, the operational range has been divided into two sides, namely the positive yaw angles side and the negative yaw angles side. The results for both cases are successful in terms of flow and power output reconstruction. Therefore, the models can predict within seconds the behavior of wind farms whose performance takes days to be known through CFD simulations. This work also presents several improvements as future upgrade for reduced-order models obtained from CFD data. Some of these improvements deal with operational ranges, sample time, and data collection, inter alia.
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Chapter 1

Introduction

This introductory chapter is divided into several sections. In Section 1.1, the interest of the topic tackled in this work is explained. Then, in Section 1.2, the objectives of this work are presented. Lastly, in Section 1.3, the different parts in which the document is divided are announced.

1.1 Interest of object of study

Wind power evolution over time

Wind energy, as well as other types of renewable energy sources, are in continuous increase and expansion. In fact, wind energy is one of the leading renewable energies that take part in the current transition from usage of traditional fossil fuels energies to green ones. This ongoing transition is being brought about by the environmental problems that traditional fossil fuel-based energy sources are causing, combined with a higher sensitivity of modern societies to environmental problems. Indeed, the Directive 2009/28/EC of the European Parliament and of the Council of 23 April 2009 on the promotion of the use of energy from renewable sources [14] aims to reach in overall Community’s energy consumption a mandatory target of a 20% share of energy from renewable sources by 2020. Under this context, wind energy is clearly one strong actor to achieve this goal.

The evolution of wind energy has been tremendous over the last one or two decades. The world’s cumulative installed capacity (Figure 1.1) as well as the annual installed capacity (Figure 1.2) have been growing along all these last years significantly. This shows the clear worldwide pronounced tendency of wind energy and the global commitment to wind energy’s economic and social investment.

Figure 1.1: Global cumulative installed capacity of wind energy (2000-2015) [1].
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Figure 1.2: Global annual installed capacity of wind energy (2000-2015) [1].

Figure 1.3: Annual installed capacity of wind energy by region (2007-2015) [1].

Observing the trends by region (see Figure 1.3), one can observe that Europe has been having almost regularly the same tendency during the last decade despite the Financial crisis of 2007-2008. Regarding Asia, in spite of its being below Europe by the beginning of last decade in terms of installed power capacity, nowadays is clearly the dominant continent in terms of wind energy production thanks to the great development of world giants China and India.

The future forecast developed by the Global Wind Energy Council (GWEC) for the wind energy sector (see Figure 1.4) predicts that the world’s wind power capacity will continue being increased, with future annual installed capacities from 64 GW to 79.5 GW, respectively, for the years 2016-2020.

Lastly, it is also interesting to have a look, in relation to this general overview of the current situation of wind energy worldwide, to the wind energy share with respect to other energy production technologies and its evolution over the last years (see Figure 1.5). From playing a marginal role in the beginnings of century, wind energy has now a penetration in the European’s energetic system of around 15%, close to classic energy sources such as Nuclear Power, or Coal Power.

After this general description of the situation of the wind energy worldwide and its penetration in the European countries, the next concept is much more related to the details of this project.
1.1. Interest of object of study

Figure 1.4: Wind energy installed capacity forecast for the period 2016-2020 [1].

Figure 1.5: European evolution of wind energy share in comparison with other energy production technologies. Adapted from: [2].
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Wind power operation

To integrate wind turbines into the energy grid, wind turbines and wind farms must have efficient control strategies to make them competitive with respect to other technologies. Control strategies gain even more importance if we look at the variability over time of the wind resource with inevitable even strong fluctuations. Many studies like, for example, the one conducted in [3] show that there may exist operating conditions where global power extraction from a wind farm as a whole may be smaller if each of the wind turbines of the wind farm is optimized separately, instead of a global optimization. Therefore, optimal global power extraction scenarios may imply that wind turbines are not operating at their maximum individual power production, but operating at a specific operating condition that allows the whole wind farm to extract the maximum power possible. An example of this phenomenon is extracted from [3] and illustrated in Figure 1.6.

![Figure 1.6: Power extracted from a two-turbine wind farm array versus axial induction factor. The two-turbine array is aligned with the prevailing wind flow direction. The two-turbine wind farm produces its maximum power at an upstream wind turbine induction factor \( a_1 = 0.2 \), different to that of maximum energy production for the upstream wind turbine \( a_1 = 1/3 \) [3].](image)

Another example related to trying to maximize the global power output from a wind farm without maximizing individually each wind turbine consist in taking advantage of the phenomenon of wake steering. This phenomenon consists in the deviation or steering of wind turbine wakes by yawing upstream wind turbines. This deviation allows downstream wind turbines to avoid part of wake generated by upstream wind turbines and have wind speeds of higher magnitude. Examples of this effect are complied in [15] where a field-test was carried out in a scaled wind plant test field, or in [16] where yaw variations were introduced in Computational Fluid Dynamics (CFD) simulations. In this regard, see Figure 1.7 from the TUM Wind Energy Institute.

Therefore, different and more modern control strategies based on, for example, optimal control [17] gain relevance for the future. These control strategies could lead to so far unknown operational conditions for wind farms, and these new operational conditions may imply a situation similar to the one described in the previous examples or, for instance, operational conditions where other magnitudes, such as critic loads of the structure or any fatigue indicator, are considered in the optimal control algorithm. All in all, they have the potential to improve global wind farm power and reduce structural loads by properly individually coordinating the wind turbines within the wind farm.
1.2 Objective

There are several multidisciplinary objectives in this work. Firstly, the main objective of this work is to develop Reduced-Order Models (ROMs) that can capture the main dynamics of wind farm wake flows, and their relation for each turbine with power outputs and other relevant physical magnitudes, for varying yaw angles of upstream turbines. The models should forecast, with considerably low computational effort, the behavior of both wind farms flows...
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and wind turbine physical magnitudes. The models are expected to be used for wind farm control, so as to develop optimal control strategies looking for, e.g., wind farm optimal power production or load/fatigue minimization. Therefore, they must have a tractable structure for that goal. Secondly, the CFD simulations performed will show the effect that yaw misalignment of upstream wind turbines have on downstream wind turbines. This is studied for a particular wind farm configuration, and particularly in order to look at the power production of each turbine (among other physical magnitudes) and the power production of the whole wind farm.

1.3 Structure of the document

To display the work done in this Master’s Thesis, the following document structure has been adopted: In Chapter 2 different wake models are introduced and commented. Then, Computational Fluid Dynamics (typically known as CFD) are explored in detail and the specific software used to perform the simulations of this work is explained. Finally, the setup and characteristics of the simulation are related. Then, in Chapter 3, a brief overview is given to the topic of System Identification, which plays an important role in order to achieve a useful excitation of the system from which the models are obtained. Subsequently, in Chapter 4, some methods and techniques available in the literature to obtain ROMs (Reduced-Order Models) are presented and compared according a series of criteria detailed in Section 4.1. The results of this work are explained in Chapter 5, and the conclusions, future upgrades, and outlook will end this document in Chapter 6.
Wind Farm Wake Models & CFD Simulations

This chapter explains some of the different wake models existing in the literature in order to arrive to the one used to perform the simulations of this work: CFD (Computational Fluid Dynamics). Firstly, in Section 2.1, a short summary of different wake models present in the literature is presented. Since this work focuses on wind turbine wake propagation and evolution, an overall understanding of the state-of-the-art work so far is necessary. Then, in Section 2.2, one of these models, the simulation software tool SOWFA (Simulation fOr Wind Farm Applications), is described in a generic way and according to the literature and software’s manuals. Afterwards, Section 2.3 explains the specific features and configuration of the simulations carried out during this work.

2.1 Wake Models - State of the art

2.1.1 Static Park Model

The simplest wake model widely used in the literature is the so-called Park model [19]. Developed back at the beginning of the 80’s, this simple method considers steady uniform flow, linear wake expansion, and uniform wind speed at each cross-section downstream. For this model, wind turbines are modeled as actuator disks whose inductor factor $a$ is a parameter that is allowed to change, but uniformly for the entire actuator disk.

The Park model is modeled in the following terms. The coordinate system used is a two-dimensional coordinate system where $(x, y)$ denote a position streamwise and spanwise, respectively, being the origin of the coordinate system at the wind turbine’s hub. Let $U_\infty \,[\text{m/s}]$ denote the freestream velocity, let $D \,[\text{m}]$ denote the rotor diameter of a wind turbine, and $a \,[-]$ denote a uniform induction factor of the wind turbine along the whole rotor disk, which in turn corresponds to the ratio between the velocity reduction with respect to the freestream velocity, i.e.

$$a = \frac{U_i}{U_\infty} \quad \text{(2.1)}$$

where $U_i$ denotes a virtual velocity or deficit in velocity corresponding to the subtraction of speed due to the interaction of the flow with the turbine.

The streamwise velocity $U$ depends on the position $(x, y)$ and on the axial induction factor $a$. Its variation downstream is calculated by subtracting a $\delta U$, which is the factor that indeed depends on $x, y, a$. That is:

$$U = U(x, y, a) = U_\infty(1 - \delta U(x, y, a)) \quad \text{(2.2)}$$
Chapter 2. Wind Farm Wake Models & CFD Simulations

The velocity deficit, $\delta U$, due to the wake generation and propagation is

\[
\delta U = \begin{cases} 
2a \left( \frac{D}{D + 2\kappa r} \right)^2, & \text{if } r \leq \frac{D + 2\kappa r}{2} \\
0, & \text{else}
\end{cases}
\tag{2.3}
\]

Therefore, and according to the model, the higher the downstream distance, the higher the wake diameter and the lower the velocity deficit (the higher the wake recovery). Additionally, the parameter $\kappa$ also plays an important role in the model. This wake model must be tuned with this $\kappa$ parameter that corresponds to a wake decay constant. According to [4], typical values range from 0.01 to 0.5 depending on the ambient turbulence, topographical effects, and turbine operation. For example, for high turbulences, mixing of the wake results in faster wake recovering leading to higher $\kappa$ values.

This model neglects other velocity components, spanwise velocity component and vertical velocity component are set to 0, and therefore the wind speed always remains parallel to nacelle axial dimension, i.e. $x$ direction.

Undoubtedly, the success of this model relies on its low computational cost thanks to its simplicity, combined with its accuracy for that little cost. A comparison between this model and the models explained in Subsections 2.1.2, 2.1.3, and in Section 2.2 can be seen in Figure 2.1. Since the Park model has to be tuned by assigning a wake decay constant, the constant chosen for Figure 2.1 was $\kappa = 0.45$ in order to have the best matching in the far wake with SOWFA’s simulation, which is, in principle, the most reliable model.

![Figure 2.1: Comparison between different wind farm wake models in the prediction of the averaged streamwise velocity component with respect to the downstream distance [4].](image)

2.1.2 Dynamic Wake Meandering Model (DWM)

The dynamic wake meandering model [5] is considered a medium-fidelity model in comparison to low-fidelity models like the static park model or high-fidelity models like SOWFA (see Subsection 2.2). Dynamic wake meandering models calculate wake deficits and the meandering movement of the wake center as wake develops after wind turbine interaction. These models simulate unsteady wake effects and can be used to know how wakes affect magnitudes of interest.
like power generation or loads, while at the same time keep an acceptably low computational cost, especially when compared to high-fidelity models (the ones used for this work).

Some works like [20] show that DWM models are in good agreement with LES (large-eddy simulation) simulation results and even field data (e.g., data for North Hoyle and Lillgrund wind farm, which also appear in other papers in the literature).

DWM models represent a main advantage with respect to static park model. The former give a more realistic representation of the far wake at still reasonably low computational cost (they can be used in common desktop computers with only several minutes of computational times). In [21] it is remarked that DWM models present complications and unsuitabilities with respect to its use for feedback control, since wind turbines wake are calculated one at a time, without providing a continuous flow. "This complicates the use of this model for dynamic wind farm control."

### 2.1.3 Actuator Disk Model

Firstly, it is necessary to clarify that the concept ‘Actuator Disk Model’ applies to both different frames. On the one hand, an actuator disk model can refer to the way wind turbines are modeled for calculations or to perform simulations. This concept differs, for example, with the concept of actuator line model explained in SubSection 2.2. On the other hand, in this subsection the concept refers "to the flow field computed using the Navier-Stokes equations where the turbines in the wind farm are represented as actuator disks" [4].

Figure 2.3: Medium-fidelity actuator disk wake model simulation in a two-turbine array [6].
Chapter 2. Wind Farm Wake Models & CFD Simulations

The bases of the actuator disk model can be found in [22,23]. This model solves "the unsteady, axisymmetric Navier-Stokes equations by using the streamfunction ($\Psi$) - vorticity ($\Omega$) formulation assuming the flow is incompressible and inviscid" [4]. The main difference between high-fidelity models and this actuator disk model is that blades are not considered during the simulation, in its place there is an actuator disk. This implies that wakes are not reliable in the near wake region. The model however predicts the wake characteristic in the far wake, after several D distances downstream (see Figure 2.3). For better near wake precision, tip vortices that results from blade profile geometries must be taken into account (e.g. high-fidelity CFD simulations).

2.2 CFD Simulations Tool: SOWFA (OpenFOAM & FAST)

The tool used for obtaining the data needed for this work was the Simulator fOr Wind Farm Applications (SOWFA). SOWFA is a high-fidelity large-eddy simulation tool developed at the National Renewable Energy Laboratory (NREL) for wind farm studies [24]. SOWFA is a CFD solver based on OpenFOAM (OpenCFD Ltd., Bracknell, UK) coupled with NREL’s FAST wind turbine simulator [25]. An overview of the principal components of SOWFA can be seen in the next plot [26]:

![Figure 2.4: Overall vision of SOWFA tool composed of: WRF (Weather Research and Forecasting), OpenFOAM solver, and NREL’s FAST.](image)

Essentially, SOWFA combines a numerical solver such as OpenFOAM, which is able to simulate with high fidelity the turbulent evolution of the fluid flow over time, with FAST tool, which incorporates into the simulation the perturbation that wind turbines produce on the flow. In general terms, the combination of the two tools leads to SOWFA tool [26], see Figure 2.5. SOWFA solves the three-dimensional incompressible Navier-Stokes equations and accounts for the potential temperature equations, among other fundamental equations and models, such as wake models. It also considers thermal buoyancy and Coriolis force effects due to earth rotation.

As stated, SOWFA has embedded FAST tool to incorporate to the study the presence of wind turbines. The effects and perturbations that wind turbines apply to the fluid are calculated by means of the so-called actuator line (AL) model [27]. This method discretizes spanwise each wind turbine blade into different finite segments with constant airfoil shape, chord and twist, and incoming wind flow (see Figure 2.6). Then, with the help of look-up aerodynamic tables, lift, drag, and aerodynamic moments are calculated for each simulation time step, so as to know the action of the wind on the blades. The reaction of the blades on the wind is logically also considered and applied. Further details about SOWFA can be found in [28].
According to the literature, SOWFA presents good agreement between real wind farm data and simulated data, as checked and explained in many literature pieces, for example [4, 28, 29]. Therefore, the data obtained from the CFD Simulations performed for this work are taken as the basis of the work, and are considered as real data in terms of reliability.

Regarding the collection of data, NREL’s FAST provides the necessary frame to collect the most important magnitudes that characterize the behavior, specially the structural one, of wind turbines. For this work, some physical magnitude gain particular interest, namely: wind turbine yaw position, rotor power, rotor rotational speed, generator torque, thrust force, lateral force, yawing, tilting, and bearing moments, among others. Additionally, and of utmost importance, SOWFA and OpenFOAM allow by specifying certain contours/surfaces to extract the velocity flow data at any point in the 3D discretized space. For this study, the 3 components of the velocity vector have been recorded.

Since SOWFA is a high-fidelity fluid simulator solver, the computational time required is pretty high. Simulations last for days even in supercomputing units, and of course depending
on the size of the wind farms (e.g. spatial dimensions or number of wind turbines), simulation time step (related to the convergence of the solver), simulation total time, and data acquisition volume. For this Thesis, the simulations were performed mainly at the supercomputing units at the LWE (Wind Energy Institute, ‘Lehrstuhl für Windenergie’) at the TUM (Technical University of Munich). The supercomputer units used were Intel® Xeon® Processor E5-2690 v3 Haswell. Additionally, some simulations were performed in the Leibniz Supercomputing Centre (‘Leibniz-Rechenzentrum’, LRZ), which has an impressive speed of 2.90 petaFLOPS (Top #30 in the world).

2.3 CFD Simulation Configuration

Now that the main features of the SOWFA tool are presented, this section focuses specially on the particular configuration used for this work.

2.3.1 CFD Simulation Setup

The CFD simulation setup is essentially composed of 2 wind turbines in a 3D dimensional space as illustrated in Figure 2.8.

Figure 2.8: Layout of the two-turbine array used in the CFD simulation.
In the baseline configuration, the two wind turbines are perfectly aligned with respect to a vertical XZ-plane. The rotor plane is contained in the vertical YZ-plane. As seen in Figure 2.9 (XY-plane), the discretization distribution is not homogenous so as to focus the data recording on the region between wind turbines and on the region after wind turbine #2, i.e. the wind turbines wakes.

![Figure 2.9: Layout and mesh of CFD simulation.](image)

The dimensions of the wind turbines simulated correspond to the dimensions of a full-scale model, but scaled to a smaller one. The original full-scale model of reference is the NREL 5-MW baseline turbines [30], and the scaled one is the G1 scaled model [12,13]. The main characteristics of the G1 model used for the CFD simulations are highlighted in Table 2.1:

<table>
<thead>
<tr>
<th>Wind Turbine</th>
<th>G1 scaled model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor orientation</td>
<td>Upwind wind turbine</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>1.1 m</td>
</tr>
<tr>
<td>Hub height</td>
<td>0.825 m</td>
</tr>
<tr>
<td>Number of blades</td>
<td>3</td>
</tr>
<tr>
<td>Rated rotor speed</td>
<td>850 rpm</td>
</tr>
<tr>
<td>Nacelle tilt angle</td>
<td>0°</td>
</tr>
<tr>
<td>Rotor cone angle</td>
<td>0°</td>
</tr>
<tr>
<td>Control</td>
<td>Variable speed, Pitch, and Yaw</td>
</tr>
</tbody>
</table>

The simulated-scale model’s diameter is $D = 1.1 \text{ m}$, different to the the full-scale wind turbine, which has a diameter $D = 126.4 \text{ m}$. In order to keep conditions of dynamic similarity, also the rotational speed of the scaled model is altered accordingly. The rated rotor speed of the full-scale model is $\omega = 12 \text{ rpm}$, while for this work the values used was $\omega = 850 \text{ rpm}$.

Due to limitations regarding data collection, velocity components of the flow will only be recorded on two specific planes (unless otherwise stated), which have been, in turn, discretized. The characteristics of the two planes are shown in Table 2.2. In total, the number of points available in the grid for collecting data is close to 167,000. Considering the three components of the velocity, the total number of velocity values that are collected is: $n_x \approx 500,000$.

Regarding the characteristics of the flow, SOWFA allows to choose an initial flow condition that is applied to the spatial simulation domain chosen. In accordance with the coordinate system chosen, the streamwise velocity component $u$ coincides with x-dimension, the spanwise velocity component $v$ coincides with y-dimension, and the vertical velocity component $w$ coincides with z-dimension. For this work, a constant flow of $5.8 \text{ m/s}$ in the streamwise direction was chosen. The other two components of the initial flow condition were set to be 0. In terms of turbulence, the
Table 2.2: Planes where CFD simulation velocity components of the flow were collected.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Dimensions</th>
<th>Fixed coordinate</th>
<th>Spatial Resolution</th>
<th>Total points</th>
</tr>
</thead>
<tbody>
<tr>
<td>XY-plane</td>
<td>$\Delta x = 6.5D$, $z = 0.825\text{ m}$</td>
<td>$\delta x \approx 0.012D$</td>
<td>$\approx 68,000$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Delta y = 1.5D$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XZ-plane</td>
<td>$\Delta x = 6.5D$, $y = 4.5\text{ m}$</td>
<td>$\delta x \approx 0.0115D$</td>
<td>$\approx 98,000$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Delta y = 2D$</td>
<td>$\delta y \approx 0.012D$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

incoming flow was chosen without any turbulence. However, the mere presence of the wind turbine alters the flow from the first simulation instant, and this turbulence is, of course, taken into account by SOWFA. Only the turbulence of the incoming flow is the one that was set to 0. This aspect of including turbulent inflow in the simulations is one of the future steps to be carried out.

### 2.3.2 Obtained data

Once the CFD simulations are appropriately run, the data obtained needs to be arranged and processed. The mainly important values that are obtained are those of the velocity field in all the domain subject of study. This velocity field is composed of the 3 components of the velocity in the 3D space, i.e. $u, v, w$. That is:

$$\vec{v} = (u, v, w), \quad \vec{v} \in \mathbb{R}^3 \quad (2.4)$$

Apart from those, there are many output magnitudes that can be obtained from the CFD Simulation. These magnitudes are calculated and provided by the FAST software. A subset of them is presented in the following (according to the nomenclature in [31]):

- $RotPwr_j$ : Rotor Power
- $LSShftFxa_j$ : Thrust Force
- $LSShftFys_j$ : Lateral Force (wrt rotor plane)
- $LSSTipMzs_j$ : Yaw Moment
- $LSSTipMys_j$ : Tilting Moment
- $YawBrMzn_j$ : Bearing Moment
- $YawPzn_j$ : Yaw angle

Where $j$ corresponds to the $j$-th wind turbine. The work has been focused essentially on the Rotor Power output as mainly important output.

In order to adapt these values to the desired State Space formulation needed for the method that computes the Reduced-Order Model (widely explained in Chapter 4), the following quantities are introduced.

**Snapshot Matrix**
2.3 CFD Simulation Configuration

The Snapshot Matrix is the matrix that gathers all the velocity components recorded for each time snapshot of the simulation:

$$X = [X(t_1) \ X(t_2) \ \ldots \ X(t_{n_s})] \in \mathbb{R}^{n_x \times n_s}, \quad (2.5)$$

where \(n_x\) represents the number of grid points from which data is collected (the number of states in the state-space formulation), where \(n_s\) represents the number of snapshots, and where \(X(t_i)\), \(i = 1, 2, \ldots, n_s\) represents the snapshot velocity vector at time \(t_i\), which in turn is composed of the 3 components \(u, v, w\) of the velocity recorded at that time \(t_i\). This snapshot vector is arranged by pure convention and without any expected effect on the results according to 2.6.

Therefore, the Snapshot Matrix from 2.5 is, in its expanded version:

$$X(t_i) = \begin{bmatrix} u_{t_1} \\ \vdots \\ u_{t_{n_s}} \\ v_{t_1} \\ \vdots \\ v_{t_{n_s}} \\ w_{t_1} \\ \vdots \\ w_{t_{n_s}} \end{bmatrix} \in \mathbb{R}^{n_x \times 1} \quad (2.6)$$

Therefore, the Snapshot Matrix from 2.5 is, in its expanded version:

$$X = \begin{bmatrix} u_{t_1} & u_{t_2} & \ldots & u_{t_{n_s}} \\ v_{t_1} & v_{t_2} & \ldots & v_{t_{n_s}} \\ w_{t_1} & w_{t_2} & \ldots & w_{t_{n_s}} \end{bmatrix} \in \mathbb{R}^{n_x \times n_s} \quad (2.7)$$

Input (or Control) Vector

The Input (or Control) vector is the vector that contains the input signal with which the system is altered. The interaction of these magnitudes with the system are the object of study. These magnitudes are normally chosen to be externally regulable in order to be able to freely vary them independently of the reaction of the system.

$$U(t) = [U(t_1) \ U(t_2) \ \ldots \ U(t_{n_u})] \in \mathbb{R}^{n_u \times n_s}, \quad (2.8)$$

where \(n_u\) represents the number of different input variables.

In this work, since the study is essentially focused on the yaw variations, the yaw angle of the wind turbines are the input variables to the system. In fact, typically, \(n_u = 1\) since this work is focused on studying the effect of the yaw position of the first wind turbine upstream.
Output Vector

The Output vector is the vector that contains the magnitudes chosen to be the output of the system object of study. These magnitudes are those who are expected to be altered by the inputs of the system.

\[ Y(t) = [Y(t_1) \ Y(t_2) \ \ldots \ Y(t_{ns})] \in \mathbb{R}^{n_y \times ns}, \tag{2.9} \]

where \( n_y \) represents the number of different output variables of the system.

For this work, there are many output variables that can be selected, as described in Subsection 2.3.2. However, the output variables that gain much more importance for this study, and in general for a successful integration of wind farms in the electric grid, are the power outputs.

Lastly, it is worth highlighting that the layout chosen for this work, especially regarding the number of turbines (which is just two in this work), has been selected in this way in order to reduce the amount of computation needed, since the main focus of the work was developing reduced-order models and studying all the implications and repercussions present in their development. Therefore, the number of turbines, or the layout itself, or even the turbulence of the input inflow are parameters that held high interest for future studies and, in fact, the reduced-order models developed in this work are designed so as to be capable of incorporating this further improvements.
System Identification

This chapter deals with a vital issue always present in engineering problems when trying to obtain models from systems, namely System Identification. To address this matter, this Chapter firstly introduces the fundamentals of system identification in Section 3.1. The approach is particularly oriented to the type of problem studied in this work. Afterwards, Section 3.2 and Section 3.3 discuss in detail, by following renowned literature, a procedure to fulfill the necessary criteria to be able to identify appropriately physical systems.

3.1 Fundamentals

The topic of system identification has been widely studied over the recent years; particularly, in the last decades as control theory has mainly evolved. However, what in this chapter is explained corresponds to the particular application of those theory and practices to the subject object of study of this Thesis. Specifically, the author has mainly looked at references [7,8,32–34].

As a general concept, in order to identify systems appropriately it is necessary to have a pertinent excitation signal that can let the user extract the characteristics of the system. Indeed, the input signal is clearly the only possibility to affect and interfere with the system in order to obtain the information about its behavior. These excitation input signals have to fulfill a series of features that will be described below these lines. However, when characterizing non-linear systems, it is particularly important to take into account extra requirements.

When looking at real-life systems and applications, such as wind farms, there are many constraints that limit the type of signals that can be chosen as input signal to excite the system. One of them is the range of values that the input signal is expected to have during realistic operation. For instance, yaw angles are limited to a range of values between a minimum and a maximum, i.e. $u_{\text{min}}$ and $u_{\text{max}}$, but also the yaw rate is limited since real wind turbines can not yaw at any yaw rate. This is particularly important for the design of the yaw control input signal, and particularly if we compare it to studies that use other inputs such as pitch variation, where pitch rates are allowed to change much faster. Normally, it is advised to cover the operational input range of interest, since interpolation is typically much more accurate than extrapolation.

Besides, another important aspect in regard to an appropriate system identification is the frequency spectrum of the input signal. The spectrum of the input signal determines the different frequencies at which the system is going to be excited. Logically, the quality of the model will be higher at those frequencies that are strongly excited with the excitation input signal, compared to those frequencies that are less excited.

Therefore, the purpose of the model is of high importance. Operating conditions and frequencies’ working range characterize, amongst other factors, the purpose of the model. If the model is intended, for instance, to be used on static behavior without many fluctuations, then a signal mainly compounded of low-frequency components would be a better choice than another with a spectrum shifted to high frequencies. For the case of this work, the main low-frequency
dynamics of the flow are of major interest rather than being able to predict with precision the high-frequency turbulent behavior. In fact, since this work develops reduced-order models, the highly fluctuating turbulences are clearly beyond the scope of these ROMs.

Additionally, the simulation time also represents a limitation since the computational available time on the supercomputer is limited; idling problems start to appear when too much computational effort is required, etc. In general, simulation time is desired to be the longest possible to have the highest possible variety of operational conditions and frequency range excitation.

These aspects can be summarized in the following list:

- **Range of input amplitudes**: \( u_{\text{min}} \) to \( u_{\text{max}} \).
- **Length of the input sequence** \( N \): related to simulation time available.
- **Frequency spectrum of the excitation signal**: characterized by the PSD (power spectral density).
- **Purpose of the model**: operating conditions (operating range and frequency range).

In relation to defining which specific signals can be used, a classification of [8] provides many classic signals that can be used to identify systems. The signals presented have the advantage to be characterized with few parameters and to be deterministic in the sense that they can easily be replicated:

- **Constant**: Only suitable for identification of the the static gain. Constant signals are not suitable for identification because they do not excite any dynamic.
- **Impulse**: Not recommended for identification.
- **Step**: Suited for identification. It emphasizes low frequencies.
- **Rectangular**: Well suited for identification. Depending on the frequency of the rectangular signal a desired frequency range can be emphasized.
- **PRBS (Pseudo Random Binary Sequence) signals**: Well suited for identification. They mimic white noise in discrete time and can be obtained in a deterministic way. These PRBS signals tend to excite all frequencies equally well. See Figure 3.4 in Section 3.2 for a power spectral density diagram based on an example of a PRBS signal of Figure 3.3.

### 3.2 PRBS (PseudoRandom Binary Sequence)

The PRBS (PseudoRandom Binary Sequence) is a classical excitation signal used for system identification. This type of signal is a deterministic approximation of white noise in discrete time [7]. In this document we have generated this signal in a similar way to what is explained in [7]. The properties of such a signal are explained in detail in [7,35].

Such a PRBS signal generation can be explained based on a virtual *shift register* of \( n \) bits representing \( n \) stages. The input of this shift register is the results from an operation with an XOR logic gate based on two inputs take from two fixed positions in the register and one output that reinserted in the register as input. The output of the shift register corresponds to the last bit of the register, and by collecting the evolution of this last bit over time a cyclic PRBS is built. The reader is reminded that the an XOR gate functions according its truth table (see Table 3.1).
Table 3.1: XOR gate truth table.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

For the operation with the XOR logic gate, two bits from the register are taken. The specific bits to consider depend on the number of bits of the register \( n \). This can be seen in Table 3.2.

Table 3.2: Bits combination for shift register inputs for PRBS signals of full-length \( N \).

<table>
<thead>
<tr>
<th>No. of Stages</th>
<th>Feedback Law</th>
<th>Length = ( 2^n - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 XOR 2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1 XOR 3 or 2 XOR 3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>1 XOR 4 or 3 XOR 4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>2 XOR 5 or 3 XOR 5</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>1 XOR 6 or 5 XOR 6</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>1 XOR 7 or 3 XOR 7 or 4 XOR 7 or 6 XOR 7</td>
<td>127</td>
</tr>
<tr>
<td>8</td>
<td>1 XOR 2 XOR 7 XOR 8</td>
<td>255</td>
</tr>
<tr>
<td>9</td>
<td>4 XOR 9 or 5 XOR 9</td>
<td>511</td>
</tr>
<tr>
<td>10</td>
<td>3 XOR 10 or 7 XOR 10</td>
<td>1023</td>
</tr>
<tr>
<td>11</td>
<td>2 XOR 11 or 9 XOR 11</td>
<td>2047</td>
</tr>
</tbody>
</table>

If for a given number of bits of the register \( n \) (column 1 in Table 3.2), the corresponding bits highlighted in column 2 of Table 3.2 are chosen, then the length of the signal is said to be full-length. If this is the case, the PRBS signal is complete, its length is the one displayed in Table 3.2, and it can be used to excite all the frequencies available for such a signal of length \( N = 2^n - 1 \). After one complete generation of this full-length signal, since it is cyclic, another exact one would be produced. If the concrete bits for the calculation with the XOR gate specified in Table 3.2 for each \( n \) are not chosen, then the signal would not be full-length and much smaller cycles would be produced. In this case, such a PRBS could have almost no sense.

Additionally, if the length of a desired signal does not have a length that obeys the formula \( N = 2^n - 1 \), the designer can choose between two possibilities. On the one hand, to choose a signal of full-length with an \( N \) smaller than the desired one, that is: \( N < N_{\text{design}} \). In that case the design signal would include one full-length signal and a beginning a the second cycle of that signal this \( N < N_{\text{design}} \). On the other hand, the designer can choose a signal with \( N > N_{\text{design}} \). In that a full-length signal would not be possible.

The bits of the register can be arbitrarily initialized at any initial values except from the 0 initialization, which would imply a constant zero output. Except from that one, all other \( 2^n - 1 \) possibilities with a register of \( n \) bits are allowed. Since the signal is cyclic, the initialization of the register only affect to the position in which these PRBS cyclic signals start.

To see the process of generation of the signal, the reader is referred to Figure 3.1.

The process of obtaining a real PRBS signal from the output of the register as seen in Fig-
Figure 3.1: Shift register of \( n \) bits used for the generation of PRBS signals.

Figure 3.2: PRBS signal example obtained with the method explained in [7].

Lastly, an example of a PRBS generated by the author of this work is shown in Figure 3.3 with its correspondent power spectral density (PSD) analysis performed for that signal (Figure 3.4).
3.3 APRBS (Amplitude modulated PseudoRandom Binary Sequence)

This section covers the explanation of the extension of an PRBS to an APRBS. According to renowned literature like [7, 8, 33, 34], for non-linear system identification PRBS must be extended to APRBS (Amplitude modulated PseudoRandom Binary Sequence). Since PRBS consists only in a binary sequence between two single values, i.e. \( u_{\text{min}} \) and \( u_{\text{max}} \), PRBS are not completely suited to for non-linear systems as they do not excite the system over the intermediate range from \( u_{\text{min}} \) to \( u_{\text{max}} \). Therefore, for non-linear systems, APRBS intends to excite not just the frequency domain (which comes from the PRBS signal), but also the amplitude domain, i.e. the behavior of the system over that amplitude domain.

To design an APRBS, a previous PRBS signal is designed following the guidelines related in Section 3.2. Once the PRBS is generated, the range from \( u_{\text{min}} \) to \( u_{\text{max}} \) is divided by the number of horizontal levels that the original PRBS signal has. This value gives the minimum jump between two levels of the APRBS signal. The different possible level values \( a_i \in [u_{\text{min}}, u_{\text{max}}] \) are given by this formula:

\[
a_i = u_{\text{min}} + i \ast \frac{u_{\text{max}} - u_{\text{min}}}{\text{PRBS levels}} - 1, \quad i = 0, \ldots, \text{PRBS levels} - 1
\]

(3.1)

Finally, this amplitudes \( a_i \) are assigned randomly to each horizontal level of the original PRBS signal to finally shape the desired APRBS signal.

Lastly, since these APRBS signals are going to be used to establish the yaw excitation input signal, ramps corresponding to the yaw rate of the wind turbines must be taken into account. An example of an APRBS designed as explained is shown in Figure 3.5 with with its corresponding PSD in Figure 3.6.

Clearly, as a result of the random assignment, there exist multiple APRBS generated signals that could potentially be classified according to an additional criterion. As highlighted by [8], this last aspect remains open for further improvement, although typically the longer the simulation, the lesser the adverse effect of randomness. For the work described in this document, we have tried, in order to produce different excitation situations to the system, to maximize the number of different possible jumps between levels in order to avoid equal jump sizes that might excite the system similarly. Also, the author has given priority to those APRBS generated signals whose jumps between levels prompt the system to change its performance noticeably.

Figure 3.4: Power spectral density of PRBS signal example with \( N = 31 \) and \( T_h = 1 \) s.
Another available option would be to assign levels randomly from values between $u_{\text{min}}$ and $u_{\text{max}}$ without equal spacing. In this case, the distribution of amplitudes and frequencies of the resulting test signal would probably not be, of course, equally distributed. However, for sufficiently long signals, this effect would have lower impact on the excitation.

3.3.1 Parameters APRBS

The remaining of this Chapter 3 deals with the particularities of the characteristic parameters selection for APRBS signals. Firstly, the already introduced Hold time $\lambda$ or $T_h$, i.e. the temporal value assigned to each one of the bits generated for the construction of the PRBS and APRBS signal, plays a crucial role in system identification. Given the length of a desired input APRBS signal, the hold time $T_h$ determines how many different levels can the APRBS signal have, which in turn is related to how well can the system be excited by the frequencies present in the excitation input signal. Fixing a specific signal’s length has been a certainly usual practice because of the computational limitations for the simulations of this work.

The example in Figure 3.7 extracted from [8], shows the importance of the Hold time $T_h$. For too small $T_h$, the system will not have enough time to approach any stationary conditions for a given input value $u_k$. Therefore, although many changes in terms of excitation are given to the system, the input signal does not allow the system to fully deploy its behavior. This phenomenon is illustrated in Figure 3.7c. On other other hand, if too large $T_h$ is prescribed, a lower number of operational amplitudes will be covered for a given length. This means that
3.3. APRBS (Amplitude modulated PseudoRandom Binary Sequence)

the exciting spectrum of the input signal $u_k$ would be of worse quality for the purpose of it, emphasizing the low-frequency spectra to the detriment of a wider frequency range to cover. This phenomenon is illustrated in Figure 3.7d.

In the experience of [8] "it is reasonable to choose the minimum hold time of the APRBS about equal to the dominant (largest) time constant of the process", that is:

$$T_h \approx T_{max}. \quad (3.2)$$

Since time constants are not known a priori and depend on the particularities of each system (in this case a system composed of two turbines under the influence of clearly different inflow conditions), the hold times have been estimated successively trying always to arrive to a compromise between the suggestions in [8] for $T_h$ and a sufficient number of excitation amplitudes given the available simulation time.

The list of parameters to take into account is, eventually, extended to:

- **Range of input amplitudes**: $u_{min}$ to $u_{max}$.
- **Length of the input sequence $N$**: related to simulation time available.
- **Frequency spectrum of the excitation signal**: characterized by the PSD (power spectral density).
- **Purpose of the model**: operating conditions (operating range and frequency range).
- **Distribution of input amplitudes**: $a_i \in [u_{min}, ..., u_{max}]$.
- **Hold time $T_h$ ($\lambda$)**: crucial to have an optimal balance between simulation time, frequency range excited, and different operating conditions within the range of input amplitudes.
Figure 3.7: Excitation signals for nonlinear dynamic systems example: a) binary PRBS, b) APRBS with appropriate minimum hold time, c) APRBS with too short a minimum hold time, d) APRBS with too large a minimum hold time. Adapted from [8].
Chapter 4

ROM Obtention

This chapter contains the theoretical description of the algorithms used to obtain ROMs (Reduced-Order Models). Firstly, Section 4.1 tackles with the different criteria that are expected for the reduced-order models developed for the purpose of this work. In Section 4.2, different methods existing in the literature to compute and obtain reduced-order models are presented and compared according to the suitability of their own features in relation to the desired features of the ROMs for this Thesis. Section 4.3 explains further in detail the concrete method used to obtain the ROMs of this Thesis, the IOROM (multiple-Input multiple-Output Reduced-Order Model) [4, 6, 11, 36]. Lastly, Section 4.4 proposes a future improvement in terms of broadening the range of application of the reduced-order models developed in this work.

4.1 Reduced-Order Model Criteria

To select the most appropriate method from all the methods available, a certain number of criteria to fit the characteristics of the problem of this work must be set. Thus, the main criteria that the used method must have are the following:

- **Low computational cost**: As already specified, the amount of data that the model will have to deal with is enormous (around $10^5$ or $10^6$ states). Therefore, the algorithms necessary for the calculation of the reduced-order model are particularly required to be of the minimal possible computational cost. Additionally, since the models are oriented to be used for real-time control, it is also necessary to work with a reasonable amount of data that does not avoid the use of these models for control.

- **I/O**: Some proposed methods are not designed in principle to deal with input and output relations. However, the desired reduced-order model generator method is expected to include the information of both the inputs to the system (e.g. yaw position) and the outputs from the system altered by these inputs (e.g. power outputs). Some method that do not consider in principle input/output relationships is POD. On the other side, Balanced Truncation is thought to establish relations between inputs and outputs.

- **Physical meaning of reduced-order states**: One particular aspect of this work, compared to many other studies about reduced-order modeling, is that this work focuses on keeping accessible the physical meaning of the states at any time. Typically, other works focus on relating inputs and outputs without keeping so much interest in the physical meaning of the states of the state-space representation. The physical meaning helps to understand the changes in the states and to know the evolution and general state of the system at any time.

This could be potentially useful for control applications in case of, for example, a feedback state observer (see Figure 4.1). The states of the observer (the estimated states), which are the states of the models developed in this work, are aimed to correspond to the states of the actual plant, i.e. the wind speeds within the wind farm (in this work). For such
Figure 4.1: Generic State Observer diagram.

an implementation of a feedback state observer, having a model where the states of the state observer have physical interpretation is useful in order to know how the observer is evolving. This approach is also valuable because the states not only keep the physical meaning, but this physical meaning is typically measured in real-life applications, by means of, e.g. LiDARs or anemometers, among others. Therefore, selecting physical meaningful states that are typically measured allows to keep track of the evolution of the models with higher convenience.

- **LPV (Linear Parameter Varying) adapted**: Since the system under study is composed of high turbulences, abrupt changes induced by the wind turbines, wakes, shadow effects, etc., highly non-linear characteristics are present (they appear clearly much more locally, being the global behavior much less non-linear). Therefore, changes such as variations in freestream average wind speed or in wind speed incoming direction are expected to drop the accuracy of the models considerably. If fact, these variations are absolutely frequent in real-life applications. Hence, an approach that considers usual operational conditions for wind farms must take these elements into account.

To avoid this loss of accuracy in the developed models, a *family of models* can be obtained, where each of them is gotten under different conditions. Essentially, this is the basis of Linear Parameter Varying (LPV). However, for simplicity reasons, the models developed in this work do not consider LPV and the variations mentioned. Nevertheless, since it is an important criteria for the selection of the reduced-order model method, it has been taken into account and, indeed, details are given in Section 4.4 regarding a potential expansion of the proposed reduced-order modeling method of this work (Section 4.3) where LPV is considered.

- **Adjoint free**: According to [4, page 49], it is beneficial to have models that do not require the so-called model adjoint (as work in [37] do). "The goal of this chapter is to avoid the use of such adjoints so that the proposed method can be applied to either experimental data or to simulation models."

### 4.2 Methods to obtain ROM

There are a variety of methods in the control and fluid literature that attempt to construct low-order models to describe the dominant dynamics of a high-dimensional system. In this
subchapter, some methods to compute low-order model that can describe high-order systems are briefly explained. This subchapter is focused on the following model reduction procedures:

- POD (Proper Orthogonal Decomposition)
- DMD (Dynamic Mode Decomposition)
- Balanced Truncation

There are other methods of also high importance, but this work will not deal with them. Some of them are:

- BPOD (Balanced Proper Orthogonal Decomposition)
- N4SID (Numerical Algorithm for Subspace State Space System Identification): This method allows to reflect input/output relationships according to experiments performed to the system. In this approach the states retain no physical meaning [38,39].
- ERA (Eigensystem Realization Algorithm)

### 4.2.1 POD (Proper Orthogonal Decomposition)

Proper Orthogonal Decomposition (POD) is method that allows to extract the main dynamics from the evolution of a system over time, i.e. the dominant characteristics. Applied to this study, POD and POD modes (explained in the following) provide a description of the flow patterns whose weighted superposition gives the overall evolution of the system. POD method has been used in the literature for studies regarding wind warm wake dynamics [40,41].

For a given system where the evolution of the state of the system depends on the state itself and on inputs to the system, that is

\[
\dot{x}(t) = f(x(t), u(t)),
\]

or linearized looking for a simpler and more manageable state-space representation

\[
\begin{cases}
\dot{x}(t) = Ax(t) + Bu(t)
\end{cases},
\]

one can arrange the different measurements at different moments in time in a snapshot matrix, i.e.

\[
X = [x(t_1) \ x(t_2) \ \ldots \ x(t_m)], \ X \in \mathbb{R}^{n \times m},
\]

where \( m \) is the number of snapshots and \( n \) is the number of samples or states taken from the system. Then, a SVD (Singular Value Decomposition) of this \( X \) snapshot matrix, i.e.

\[
X = U_{SVD} \Sigma_{SVD} V^{T}_{SVD},
\]

where

\[
U_{SVD} \in \mathbb{R}^{n \times m}
\]
\[
\Sigma_{SVD} \in \mathbb{R}^{m \times m}
\]
\[
V_{SVD} \in \mathbb{R}^{m \times m}
\]
gives that the columns of matrix $U_{SVD}$ correspond, indeed, to the POD modes of the system and the diagonal values of the diagonal matrix $\Sigma_{SVD}$ represent a measure of energy (or weight, or importance) of each of these POD modes in the system.

$$U_{SVD} = [\phi_1 \ \phi_2 \ \ldots \ \phi_k \ \ldots \ \phi_r], \ k = 1,\ldots, r,$$

where each $\phi_k$ represents the $k$-th POD modes from the system.

Examples of how these POD modes look like for a simulation of the type of this work and of how the energy content of each one of these POD modes is related or evolves for different POD modes can be seen in Figure 4.2.

If the system is appropriately identified (see Chapter 3), these POD modes should be able to reasonably reproduce the behavior of the system for other inputs $u(t)$ of similar kind. This topic is subtle so the reader is referred again to Chapter 3 for better insight.

In order to produce a model reduction by means of Proper Orthogonal Decomposition just a number $r$ of POD modes (the order of the reduced-order model) are selected. This is done by evaluating the difference between the original evolution of $x(t)$ compared to the evolution of a reduced-order projection. This difference is evaluated over the whole time period of interest in the following mathematical expression:

$$\int_{0}^{T_{end}} \| x(t) - P_r x(t) \|^2 dt,$$

where the states $x(t)$ are considered in a time interval between 0 and $T_{end}$.

$P_r$ is taken from the POD modes of the system, i.e.

$$P_r = \sum_{k=1}^{r} \phi_k \phi_k^*,$$

where $\phi_k$ is the $k$-th POD modes of the system.

Lastly, the qualitative particularities of this method is summarized under these lines:

- **Advantages:**
  - Low computational cost: Proper Orthogonal Decomposition can handle in acceptable time systems, like the one studied in this work, with between $10^5$ and $10^6$ states.
  - LPV (Linear Parameter Varying) adapted: POD method can be used with LPV ([42]).
  - Adjoint free: Proper Orthogonal Decomposition is adjoint-free.

- **Disadvantages:**
  - I/O: As seen in the formulation, there is no relation between inputs and outputs in the classic formulation of Proper Orthogonal Decomposition (POD). Therefore, this formulation itself is not sufficient for the intended use of the models developed in this work.
  - As said in [43], "POD modes are characterized by spatial orthogonality, but they have multi-frequential temporal content. This means that different wake dynamics acting at completely separated time scales, thus connected to different phenomena, cannot be distinguished if they have comparable energy." However, despite POD having
different phenomena that cannot been distinguished, the energy content still justifies whether a POD modes is retained or not.

4.2.2 DMD (Dynamic Mode Decomposition)

Dynamic Mode Decomposition [44–46] is a method that tries to fit a discrete-time linear system to a set of measurements or samples recorded and to reduce its order by projecting the information according to each discrete time onto a reduced-dimensional subspace.

Therefore, for a discrete system like the one in Eq. 4.11

\[ x_{k+1} = f(x_k), \]

where \( x_k \in \mathbb{R}^{n \times m} \) represents each state of the system at discrete time \( k \), DMD tries to achieve the following linear relation:

\[ x_{k+1} = Ax_k. \]

If the different measurements \( x_k \) for a temporal period such that \( k = 1, \ldots, m \), where \( m \) corresponds to the number of snapshots, are arranged in a snapshot matrix, we get:

\[
\begin{align*}
X_0 &= \begin{bmatrix} x_1 & x_2 & \cdots & x_{m-1} \end{bmatrix} \in \mathbb{R}^{n \times (m-1)} \\
X_1 &= \begin{bmatrix} x_2 & x_3 & \cdots & x_m \end{bmatrix} \in \mathbb{R}^{n \times (m-1)},
\end{align*}
\]

leading to a more compact representation of Eq. 4.12:

\[ X_1 = AX_0. \]

From this formulation, matrix \( A \) could be directly computed in the following way:

\[ A = X_1X_0^\dagger, \]

where \( \dagger \) indicated the pseudoinverse matrix (more details are given in SubSection 4.3.3).

To achieve a reduction in the order of the system, the states \( x_k \in \mathbb{R}^n \) are projected onto a lower-dimensional subspace \( z_k \in \mathbb{R}^r \). The relation between these states is:

\[ z_k = Q^T x_k, \]

where \( Q \) is the so-called projection subspace matrix such that \( Q \in \mathbb{R}^{n \times r} \) being \( r \) the order of the reduced-order model.

If we reformulate Eq. 4.11 by introducing the new change of variables, we get:

\[ z_{k+1} = Q^T AQz_k := Fz_k, \]

where matrix \( F \) is the state (or system) matrix of the reduced-order system that describes the intrinsic dynamics (for zero-input signal) of the reduced-order system.

A typical selection for this projection subspace matrix \( Q \) are the POD modes of \( X_0 \) (see previous SubSection 4.2.1 and detailed explanation in Section 4.3). That is:

\[ Q = U_r^{SVD} := U_r, \]
where $U_r$ represents a submatrix of $U_{SVD}$ (see Eq. 4.8 or Eq. 4.5) containing the first $r$ POD modes.

Once this is introduced, matrix $A$ can be approximated by:

$$A \approx QFQ^T = U_r F U_r^T.$$  \hfill (4.20)

If an eigenvalue decomposition is performed on matrix $F$, we get:

$$A \approx (U_r T) \Lambda (T^{-1} U_r^T),$$ \hfill (4.21)

where $U_r T$ are the DMD modes of the original system matrix, although approximated as said by taking just $r$ POD modes from the original system. The eigenvalue matrix $\Lambda$ provides the specific temporal frequency for each DMD mode.

"DMD modes may be non-orthogonal, but each DMD mode captures a single spectral contribution, thus an isolated physical phenomenon. DMD modes are characterized as for their spectral content (i.e. frequency and spatial wavelength), growth rate (i.e. a stable mode is going to be damped by proceeding in space and time, conversely an unstable mode will be amplified and add kinetic energy to the system), and energy. All those information are essential to highlight the main contributions to wake morphology and dynamics, which in turn affect power production and fatigue loads of wind turbines. DMD modes can be sorted accordingly to different objective functions, rather then their merely energy contribution to wake dynamics, as for POD. DMD modes can be sorted as a function of their spectral contribution and characteristic length scale. This feature can turn out to be very effective for investigations on wake interactions and added fatigue loads of waked wind turbines. Indeed, the overall life cycle of a wind turbine is mainly affected by dynamic loads acting within a well defined spectral range, which neglects very small and very large scales of the turbulent atmospheric wind with respect to the rotor diameter. In the same spirit, DMD modes can be sorted as a function of their growth rate. This criterion is useful for design of wind farm layout, for which only the DMD modes with a relevant energy at a certain downstream distance are considered. Therefore, accordingly to the specific criterion adopted, only a subset of DMD modes is selected in order to formulate a reduced order model (ROM)." [43].

Lastly, the qualitative particularities of this method is summarized under these lines:

- **Advantages:**
  - Low computational cost: Dynamic Mode Decomposition can handle in acceptable time systems, like the one studied in this work, with between $10^5$ and $10^6$ states.
  - Physical meaning of reduced-order states: Dynamic Mode Decomposition’s states retain its physical meaning.
  - Adjoint free: Dynamic Mode Decomposition is adjoint-free.

- **Disadvantages:**
  - I/O: Despite its classic formulation establishing no relation between inputs and outputs, DMD has been extended to DMDc (Dynamic Mode Decomposition with Control) which does include a full characterization including inputs, outputs, and states (see [47]).
  - LPV (Linear Parameter Varying) adapted: DMD has not been extended to LPV (to our knowledge) [4].
4.2.3 Balanced Truncation

While Proper Orthogonal Decomposition (POD) method is not intended to consider input/output relationships, Balanced Truncation \([48–51]\) does incorporate this characteristic. The idea behind Balanced Truncation is fully related to the control engineering concepts of controllability and observability, giving to this method a different perspective compared to methods, like POD, that look in the shape and energy of different patterns in the states (e.g. flow velocity field).

It is recalled that a system is said to be controllable if, for any initial state \(x(t_0) = x_0\), an appropriate series of inputs \(u(t), t_0 \leq t \leq t_1\), allow the system to reach any other state \(x(t_1) = x_1\) in a finite time interval. Also, a system is said to be observable if, by knowing \(u(t)\) and \(y(t)\), \(t_0 \leq t \leq t_1\), for a finite time interval, any initial state \(x(t_0) = x_0\) can determined.

So, for a classical time-invariant continuous-time linear system represented by a state-space representation, like the one in \((4.22)\)

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

Balanced Truncation aims, in general sense, to remove from the model the least controllable and/or least observable subpart (or subspace) of this state-space representation. This is achieved by establishing a "degree of controllability and observability" of different subspace with the state-space, being the more controllable and more observable subpart the one that will constitute the reduced-order model. For that, Balanced Truncation requires the computation of the controllability and observability gramians \([52]\). These matrices give insight about the influence of the states on the inputs and outputs of the system. For the previous system they are calculated in the following way:

\[
\begin{align*}
AW_c + W_cA^* &= -BB^* \\
A^*W_o + W_oA &= -C^*C
\end{align*}
\]

where \(W_c\) is the controllability gramian and \(W_o\) is the observability gramian.

"The controllability gramian \(W_c\) specifies the minimum control energy required to reach any specific state. Therefore, those states that require less energy to reach are more controllable and hence have a greater influence on the input/output dynamics. Similarly, the observability gramian specifies the energy in the output measurement when the system evolves from a given initial state (with zero input). States that produce more energy in the output are more observable and hence have a greater influence on the input/output dynamics." \([4]\).

Each of these gramians are defined in certain directions of the state space. Despite these coordinates defining the directions where the strongest states are aligned, it is not evident to choose those states that are both controllable and observable, since some will have a higher degree of controllability rather than observability, and vice versa. Therefore, a coordinates transformation can be performed to both of them and it is shown that there is one that can be found where both controllability gramian \(W_c\) and observability gramian \(W_o\) have the same coordinate system and in which the grammians are equal and diagonal. This new transformed gramians are said to be balanced. This property allows to have those states with higher degree of both controllability and observability.

So, considering a coordinate transformation \(T\) such that the new state \(\tilde{x} = Tx\), the following
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A transformed equivalent state-space can be found

\[ \begin{cases} \dot{x}(t) = \tilde{A} \tilde{x}(t) + \tilde{B} u(t) \\ y(t) = \tilde{C} \tilde{x}(t) + D u(t) \end{cases} \]  
\hspace{4cm} (4.24) 

with

\[ \tilde{A} = TAT^{-1} \]  
\hspace{4cm} (4.25) 

\[ \tilde{B} = TB \]  
\hspace{4cm} (4.26) 

\[ \tilde{C} = CT^{-1} \]  
\hspace{4cm} (4.27) 

The transformed gramians would be:

\[ \tilde{W}_c = TW_c T^T, \]  
\hspace{4cm} (4.28) 

and

\[ \tilde{W}_o = T^{-T} W_o T^{-1}. \]  
\hspace{4cm} (4.29) 

Lastly, and applying the mentioned condition of equality, we get

\[ \tilde{W}_c = \tilde{W}_o = \Sigma = \text{diag}\{\sigma_1, ..., \sigma_n\}, \]  
\hspace{4cm} (4.30) 

where \( \{\sigma_1, ..., \sigma_n\} \) are the Hankel singular values that are independent of the coordinate transformation and are ordered such that \( \sigma_1 \geq \sigma_2 \geq ... \geq \sigma_n \) implying that the higher the value of \( \sigma_i \), the higher the energy of that state in the system.

The system of Eq. 4.24 can be rewritten as

\[ \begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{pmatrix} u(t), \]  
\hspace{4cm} (4.31) 

where only a subsystem is obtained after applying different criteria in order to determine the number of states retained, i.e. the order of the reduced-order model, [53]. This subsystem constitutes the reduced-order model by Balanced Truncation (see Eq. 4.32) and is expected to retain the high energy states and discard the low energy states, and hence keeping the main characteristics of the original model.

\[ \begin{cases} \dot{x}_1(t) = \tilde{A}_{11} x_1(t) + \tilde{B}_1 u(t) \\ y(t) = \tilde{C}_1 x_1(t) + D u(t) \end{cases} \]  
\hspace{4cm} (4.32) 

Lastly, the qualitative particularities of this method are summarized under these lines:

- **Advantages:**
  - I/O: Balanced truncation can deal with input/output relationships (see Eq. 4.32).
  - Adjoint free: Balanced truncation is adjoint-free.

- **Disadvantages:**
4.3 IOROM

(multiple-Input multiple-Output Reduced-Order Model)

4.3.1 State-Space representation

As already explained, one of the objectives of this work is to generate discrete-time linear relation between states, inputs, and outputs in the so called: State-Space representation. In general terms, the system that is being modeled obeys to the following conceptual relation, as stated in Eq. 4.33.

\[
\begin{align*}
    x_{k+1} &= f(x_k, u_k) \\
    y_k &= g(x_k, u_k)
\end{align*}
\]  

(4.33)

where \( x_k \in \mathbb{R}^{n_x} \) is the so-called state space vector (in this case at discrete time \( k \)), where \( u_k \in \mathbb{R}^{n_u} \) is the input vector, and where \( y \in \mathbb{R}^{n_y} \) is the output vector. The values and meaning of \( n_u \) and \( n_y \) may vary along this work depending on the analysis performed, but since the focus of this Thesis is controlling the Yaw angle of Wind Turbine 1, normally \( n_u = 1 \), corresponding the input vector to the yaw angle control law given to the CFD simulation. Regarding the output vector, also the focus of this work is mainly the Power Output of both Wind Turbines, therefore normally one would have \( n_y = 2 \), enclosing the output vector the two power output data vector.

This discrete non-linear system encloses what in fact is simulated, a system where the next states and outputs are non-linear with respect to the successive inputs and states. However, and as it is done in the majority of engineering problems, in this work the system is going to be treated as a linear system with time-invariant relations, i.e. a time-invariant discrete-time linear system, see Eq. 4.34.

\[
\begin{align*}
    \{x_{k+1}\} &= [A]\{x_k\} + [B]\{u_k\} \\
    \{y_k\} &= [C]\{x_k\} + [D]\{u_k\}
\end{align*}
\]  

(4.34)

where \( A \in \mathbb{R}^{n_x \times n_x} \) is the so-called "state (or system) matrix", where \( B \in \mathbb{R}^{n_x \times n_u} \) is the so-called "input matrix", where \( C \in \mathbb{R}^{n_y \times n_x} \) is the so-called "output matrix", and where \( D \in \mathbb{R}^{n_y \times n_u} \) is the so-called "feedthrough (or feedforward) matrix".

Alternatively, Eq. 4.34 can be rewritten by gathering all terms for every discrete time \( k \) from \( k = 1 \) to \( k = n_s - 1 \) to form an equivalent expression:

\[
\begin{align*}
    X_1 &= [A]X_0 + [B]U_0 \\
    Y_0 &= [C]X_0 + [D]U_0
\end{align*}
\]  

(4.35)
and compacted,

\[
\begin{bmatrix}
X_1 \\
Y_0
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_0 \\
U_0
\end{bmatrix}, \tag{4.36}
\]

where,

\[
X_0 = \begin{bmatrix} x_1 & x_2 & \ldots & x_{n_\text{s}-1} \end{bmatrix} \in \mathbb{R}^{n_x \times (n_x - 1)} \tag{4.37}
\]

\[
X_1 = \begin{bmatrix} x_2 & x_3 & \ldots & x_{n_\text{s}} \end{bmatrix} \in \mathbb{R}^{n_x \times (n_x - 1)} \tag{4.38}
\]

\[
U_0 = \begin{bmatrix} u_1 & u_2 & \ldots & u_{n_\text{s}-1} \end{bmatrix} \in \mathbb{R}^{n_u \times (n_x - 1)} \tag{4.39}
\]

\[
Y_0 = \begin{bmatrix} y_1 & y_2 & \ldots & y_{n_\text{s}-1} \end{bmatrix} \in \mathbb{R}^{n_y \times (n_x - 1)} \tag{4.40}
\]

The model shown so far is a full-order model, since all the information collected in the different snapshots is present in the model (states \(x_k\)) and since matrices \(A, B,\) and \(C\) are matrices that have at least one of row or column with length \(= n_x\). In case of matrix \(A\), for example, the computation and processing of such a matrix is absolutely computationally prohibitive. If we take the values from Table 2.2 regarding the numbers of points for data collection, and we consider the three velocity components, with simple arithmetic it is easy to calculate that such a matrix would have dimensions of approximately \(A \in \mathbb{R}^{500000 \times 500000}\). This, in turn, for a Matlab® single precisions of 32-bits, such a matrix implies a size of more than 1500 GB. This is clearly outside of any reasonable computational consumption.

In order to construct a reduced-order model, a full-order model of reference is taken, such as that of Eq.4.34, and a projection of the original states \(x_k\) onto a chosen for the purpose subspace is applied.

\[
z_k = Q^T x_k, \quad z_k \in \mathbb{R}^r, \tag{4.41}
\]

where \(Q\) is the so-called projection subspace matrix such that \(Q \in \mathbb{R}^{n_x \times r}\) being \(r\) the order of the reduced-order model.

Additionally, projection subspace matrix \(Q\) is chosen such that it is a (real) orthogonal matrix formed by orthogonal unit vectors (i.e., orthonormal vectors) arranged in columns, which means that

\[
Q^T Q = I. \tag{4.42}
\]

Since projection subspace matrix \(Q\) is not a square matrix and given that \(r \leq n_x\), the orthonormality property happens only for the columns of \(Q\). Therefore, the following property that holds for square orthogonal matrices, does not hold in this case:

\[
QQ^T \neq I. \tag{4.43}
\]

With this projection from the original states \(x_k\) to the new reduced-order states \(z_k\), Eq.4.34 can be re-written as

\[
\begin{align*}
\{z_{k+1}\} &= [F]\{z_k\} + [G]\{u_k\} \\
\{y_k\} &= [H]\{z_k\} + [D]\{u_k\}
\end{align*} \tag{4.44}
\]

where the following relation between matrices \(F, G,\) and \(H\) and matrices \(A, B,\) and \(C\) is established:
\[ F = Q^T A Q, \quad F \in \mathbb{R}^{r \times r} \]  
\[ G = Q^T B, \quad G \in \mathbb{R}^{r \times n_u} \]  
\[ H = C Q, \quad H \in \mathbb{R}^{n_y \times r} \]  

If Eq. 4.44 is written by gathering all reduced-order states into one matrix, as it was done with Eq. 4.34 in Eq. 4.48, we get:

\[
\begin{align*}
Z_1 &= [F]Z_0 + [G]U_0 \\
Y_0 &= [H]Z_0 + [D]U_0
\end{align*}
\]  
\[(4.48)\]

If it is compacted as it was done with Eq. 4.34 in Eq. 4.36, we finally have:

\[
\begin{bmatrix}
Z_1 \\
Y_0
\end{bmatrix} = 
\begin{bmatrix}
F & G \\
H & D
\end{bmatrix} 
\begin{bmatrix}
Z_0 \\
U_0
\end{bmatrix}.
\]  
\[(4.49)\]

This final system of equations representing a reduced-order model is what this work mainly wants to achieve.

### 4.3.2 Projection Subspace matrix \( Q \)

Once that the theoretical formulation of the state-space representation for both the full-order model and the reduced-order model has been related, now it is time to derive the procedure to go from the recorded data to the reduced-order model. This implies computing the unknown matrices \( F, G, H, \) and \( D \) since, as said in the previous subchapter, the state-space representation with matrices \( A, B, C, \) and \( D \) is prohibitive in terms of computational costs. It is also necessary to compute the projection subspace matrix \( Q \) used to project the full-order model states to obtain the reduced-order model states.

A useful choice for the projection subspace matrix \( Q \) is obtained by means of a Galerkin Projection \[4,55,56\]. The Galerkin projection consists, as wanted, in a projection of the original states \( x_k \in \mathbb{R}^{n_x} \) to a reduced-order states \( z_k \in \mathbb{R}^r \). This projection is conceptually performed with a separation of variables where the original states \( x_{k,i}, i = 1, \ldots, n_x \) are calculated by a term that depends on the space and another term that depends on the time. This can be seen schematically in Eq. 4.50.

\[ x_{k,i}(\vec{p}, t) = \phi(\vec{p})c(t), \]  
\[(4.50)\]

where \( \vec{p} \) corresponds to a spatial position, i.e. \( \vec{p} = (x, y, z) \), \( c(t) \) corresponds to a here undefined function that contributes with the temporal component, and where \( \phi(\vec{p}) \), function of the position, corresponds directly to the POD modes of the Snapshot matrix (see Eq. 2.5) according to the Galerkin Projection.

The projection subspace matrix \( Q \in \mathbb{R}^{n_x \times r} \) is the element that contains the functionality of \( \phi(\vec{p}) \), i.e. the change of variables from the full-order dimensions to the reduced-order dimensions. Therefore, this projection subspace matrix \( Q \) is taken as the POD modes from the SVD of the Snapshot matrix. To be precise, since the original system (see Eq. 4.48) relates the first \( n_x - 1 \) snapshots with the second \( n_x - 1 \) snapshots, the POD modes are computed from that submatrix \( X_0 \). That is, given

\[ X_0 \rightarrow SVD \rightarrow X_0 = U\Sigma V^T, \]  
\[(4.51)\]
Chapter 4. ROM Obtainion

the POD modes from $X_0$ correspond to the columns of SVD matrix $U$.

However, since the objective of this work is the reduced-order modeling, only the first $r$ POD modes from $U$ are chosen. This implies that only an approximation, which is expected to be reasonably acceptable, of $X_0$ can be achieved. This reduction to the reduced-order model order $r$ allows to express $X_0$ in the following terms:

$$X_0 \simeq U_r \Sigma_r V_r^T,$$

where $U_r$ corresponds to the truncation of the first $r$ POD modes from the matrix $U$, i.e.

$$U = [U_r \ U_{(n_s-1)-r}] , \ U_r \in \mathbb{R}^{n_s \times r},$$

where $\Sigma_r \in \mathbb{R}^{r \times r}$ corresponds to a square diagonal matrix with the first $r$ singular values from the SVD of $X_0$, and where $V_r^T \in \mathbb{R}^{r \times n_s}$ corresponds to the truncation of the first $r$ rows from SVD matrix $V^T$.

Therefore, for a given reduced-order model order $r$, we set

$$Q = U_r , \ Q \in \mathbb{R}^{n_s \times r}.$$  

It is important to point out that the projection subspace matrix $Q$ is considered as part of the model, although it does not strictly belong to the model’s state-space representation, since it is required for converting the reduced-order states to the full-order states, and vice versa.

4.3.3 Reduced-Order Model Matrices: $F$, $G$, $H$, $D$

Once the projection subspace matrix is found, the remaining parameters to characterize the reduced-order model wanted are the matrices $F$, $G$, $H$, $D$. To find them, the following approach from [4] is used. This approach consists in finding the matrices that minimize a certain quantity or error regarding the full-order system. It is true that the state-space representation that this work proposes, and that this work explains to obtain, is a reduced-order state-space representation. However, to calculate an error that shows how good this model is, it is logical to work with the full-order state-space representation, instead of reduced-order state-space representation. Nevertheless, the full-order state-space representation is, of course, obtained from the reduced-order state-space representation.

The mentioned approach evaluates the difference between the left-hand side of Eq.4.36 with respect to the right-hand side of that equation. That is:

$$\begin{bmatrix} X_1 \\ Y_0 \end{bmatrix} - \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_0 \\ U_0 \end{bmatrix}. $$

The error function chosen is the so-called Frobenius Norm, or Hilbert-Schmidt norm, or directly $L_{2,2}$ norm. For a given matrix $A$, the Frobenius norm of $A$ is:

$$\|A\|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2},$$

where $a_{ij}$ corresponds to each element $(i, j)$ of matrix $A$. 

If we apply this formulation to expression 4.55, and we consider the relations between reduced-order model matrices $F$, $G$, $H$, and $D$ with respect to full-order model matrices $A$, $B$, $C$, and $D$ (see equations from Eq. 4.79 to Eq. 4.81), and we consider the property of orthogonality of projection subspace matrix $Q$ stated in Eq. 4.42, we can reformulate the expression in 4.55 in the following terms and define the so-called Model Error:

$$
\text{Model Error} := \| X_1 Y_0 - [QFQ^T\ 0 \ 0] [QG^T \ H \ D] [Q^T \ 0 \ 0 I_{ns}] [X_0 U_0] \|^2_F 
$$

$$
= \| X_1 - [QFQ^T \ 0 \ 0] [QG^T \ H \ D] [Q^T \ 0 \ 0 I_{ns}] [X_0 U_0] \|^2_F \quad (4.57)
$$

This error essentially computes the difference between the future states and the system outputs with the calculation results from which they are obtained. For the full-order model, this calculation would lead to a Model Error $= 0$ since the projection subspace matrix $Q$ would have $r = n_s$ and all the POD modes from the original system would have been selected. However, if we start reducing the reduced-order model order, we should generally start having higher Model Errors and more inaccurate reduced-order models.

Therefore, and in order to minimize this error, and have the highest accuracy for a certain number of POD modes (or reduced-order model order), a minimization to find matrices $F$, $G$, $H$, and $D$ is carried out. This is:

$$
\min_{[F \ G \ H \ D]} \| X_1 - [Q 0 \ 0 \ I_{ns}] [F G \ 0 \ 0 \ I_{ns}] [Q^T \ 0 \ 0 I_{ns}] [X_0 U_0] \|^2_F 
$$

$$
= \| X_1 - [Q 0 \ 0 \ I_{ns}] [F G \ 0 \ 0 \ I_{ns}] [Q^T \ 0 \ 0 I_{ns}] [X_0 U_0] \|^2_F \quad (4.58)
$$

The result of this minimization, as well as a direct isolation of the reduced-order model matrices from Eq. 4.49, results in the following expressions for optimal matrices $F$, $G$, $H$, and $D$:

$$
\begin{bmatrix}
F & G \\
H & D
\end{bmatrix}_{opt} = \begin{bmatrix}
Q^T X_1 \\
Y_0
\end{bmatrix} \begin{bmatrix}
Q^T X_0 \\
U_0
\end{bmatrix}^\dagger 
$$

$$
(4.59)
$$

where $\dagger$ corresponds to the most widely used type of pseudoinverse matrix, i.e. the Moore-Penrose pseudoinverse [57–59], needed to pseudo-invert matrices that are not square matrices.

Alternatively, if we apply the chosen selection for $Q$ as stated in Eq. 4.54, we obtain

$$
\begin{bmatrix}
F & G \\
H & D
\end{bmatrix}_{opt} = \begin{bmatrix}
U_r^T X_1 \\
Y_0
\end{bmatrix} \begin{bmatrix}
U_r^T X_0 \\
U_0
\end{bmatrix}^\dagger 
$$

$$
(4.60)
$$

And, alternatively, if we use the orthogonality property described in Eq. 4.42 (not the case for Eq. 4.43) considering that the POD modes of $X_0$ are orthogonal columnwise, it is obtained that

$$
X_0 = U_r \Sigma_r V_r^T \rightarrow U_r^T X_0 = \Sigma_r V_r^T 
$$

$$
(4.61)
$$

which in turn results in

$$
\begin{bmatrix}
F & G \\
H & D
\end{bmatrix}_{opt} = \begin{bmatrix}
U_r^T X_1 \\
Y_0
\end{bmatrix} \begin{bmatrix}
\Sigma_r V_r^T \\
U_0
\end{bmatrix}^\dagger 
$$

$$
(4.62)
$$
4.3.4 Order Selection for Reduced-Order Models

The last stage of this IOROM method is to specify ways in which the reduced-order model order can be chosen. In this subsection a couple of different approaches are proposed.

Relative energy POD modes

The first possibility is to select the order of the reduced-order model based on how much energy is retained from the original system for a selection of the first, from 1 to \(r\), POD modes. From SubSection 4.2.1 it is recalled that, when talking about "energy", the writer is referring to the magnitudes contained in the diagonal of matrix \(\Sigma_{SVD}\) (see Eq.4.6). Theses magnitudes, as explained, represent how much "importance" does every POD modes have with respect to the original system. To clarify this concept, matrix \(\Sigma_{SVD}\) can be expressed, once only the first \(r\) POD modes are selected, as

\[
\Sigma_{SVD}^r = \begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_r \\
\end{bmatrix},
\] (4.63)

where each \(\sigma_i, i = 1, ..., r\), corresponds to the \(i\)-th singular value of \(\Sigma_{SVD}^r\).

An example of how the energy content of each POD modes varies with respect to each POD mode can be seen in Figure 4.3. Also, if we compute the accumulative relative energy content for a each range of POD modes from 1 to \(r\), Figure 4.4 is obtained. From these figures it is easy to see that in order to retain a high percentage of accumulative relative energy a really high number of POD modes would have to be selected. So, almost no such reduced-order model would be obtained. Therefore, typically, this criterion of percentage of retained energy is not used to select the order of the reduced-order model.

Comparison of CFD data vs ROM reconstructions

Another option, and the one mainly adopted in this work, consists in comparing the data obtained from CFD simulations with reconstructions by using the ROMs. This terminology of reconstruction, used in the rest of the document and particularly in Chapter 5, refers to the use of the reduced-order model in order to try to reproduce the system behavior from which it was obtained.

Once the ROMs are obtained, i.e. the matrices \(F, G, H,\) and \(D\) are obtained and also the appropriate projection subspace matrix \(Q\), if we start from an initial reduced-order state \(z_0\), we can construct both the future reduced-order states \(z_{k}^{\text{ROM}}\) and the future approximated system outputs \(y_{k}^{\text{ROM}}\) (e.g. power outputs) by means of Eq.4.44 for the \(u_k\) used in the CFD simulation, that is:

\[
\begin{aligned}
z_{k+1}^{\text{ROM}} &= Fz_{k}^{\text{ROM}} + Gu_k \\
y_{k}^{\text{ROM}} &= Hz_{k}^{\text{ROM}} + Du_k.
\end{aligned}
\] (4.64)

Then, if we reverse Eq.4.41, we get:

\[
x_{k}^{\text{ROM}} = Qz_{k}^{\text{ROM}}, \quad x_{k}^{\text{ROM}} \in \mathbb{R}^{n_e},
\] (4.65)
which allows to approximately reconstruct the velocity field from the CFD simulation. How accurate these reconstructions of the wind farm flow and of the system outputs (mainly power outputs) are compared to the original data will reveal the accuracy of the reduced-order model.

So as to try to estimate this accuracy we can employ the Frobenius norm introduced in Eq. 4.56. For the case of the velocity field, the difference between the original states $x_k$ and the reconstructed states $x_k^{ROM}$ is computed introducing Eq. 4.65:

$$x_k - x_k^{ROM} = x_k - Qz_k^{ROM}. \quad (4.66)$$

In compact form, the Frobenius norm of this difference has the following expression:

$$\|X_1 - QZ_1^{ROM}\|_F. \quad (4.67)$$

In order to have an non-dimensional quantity to compare different models and in order to avoid divisions by zero, the following expression is used to calculate the error associated with the CFD data flow and the reconstructed data by means of the ROM:

$$\text{Velocity Field Error} := \frac{\|X_1 - QZ_1^{ROM}\|_F}{\|X_1\|_F}. \quad (4.68)$$

If we proceed in the same manner but for the case of the system outputs (like power outputs), the error would be calculated with the following difference:

$$y_k - y_k^{ROM}. \quad (4.69)$$

This time, since the divisions by zero will be avoided, we can compute a classical percentage error with respect the values considered to be real, in this case the CFD simulation outputs. The output’s error is defined by following expression:

$$\text{Output's Error} := \frac{y_k - y_k^{ROM}}{y_k}. \quad (4.70)$$

It is not rare that in a range of acceptable orders for the reduced-order model order (e.g. between 1 and 50, or 1 and 100) the number of POD modes of the ROM where the Velocity Field error is minimum does not coincide with the number of POD modes of the ROM where the Output’s error is minimum. Therefore, it is the task of the engineering to determine where to choose the order $r$ of the ROM to balance the tradeoff.

Lastly, it must be said that logically typically by increasing the order of the reduced-order model, i.e. the number of POD modes selected, these errors decrease. Essentially, this happens due to a lower reduction from the original system and due to a lower loss of information in the process of projecting from the original states $x_k$ to the reduced-order states $z_k$. Additionally, it is important to clarify that lower errors (and normally higher number of POD modes) may entail overfitting. In that case the system would be trying to reproduce with excessive accuracy the turbulence of the original system rather than the dominant general dynamics of the system. Therefore, the model might be less accurate for other cases with other different inputs to the system. All in all, the reduced-order model is expected to be used for control strategies. Hence, an excessive accuracy to one specific system (the one simulated to create the model) is not beneficial for a model expected to be used in various cases.
4.4 LPV (Linear Parameter Varying) - Future Upgrade

This section about Linear Parameter Varying (LPV) is added to this work because of its importance for a thorough design of reduced-order models applied to wind farm control. As introduced in Section 4.1, Linear Parameter Varying allows to broaden the range of application of the reduced-order models developed in this work without changing the structure of the reduced-order model calculation itself. LPV allows to obtain a family of models each of them thought to a particular operating condition. The mathematical approach of LPV is the main focus of this section [4, 6, 60].

Given an operating condition characterized generically by \( \rho \), the procedure explained in Section 4.3 can be generalized for every \( \rho^j, j = 1, ..., J \), being \( \rho^j \) each different operating condition. As introduced in Section 4.1, these different operating conditions can be different freestream average wind speeds or different wind speed incoming direction. Therefore, with LPV, each of these conditions, that in fact can be a combined condition, e.g. specific average wind speed and specific wind speed direction, would be associated to a \( \rho^j \). The time-invariant discrete-time linear system in Eq. 4.33 would be now a parameter-variant discrete-time linear system:

\[
\begin{align*}
x_{k+1} &= f(x_k, u_k, \rho^j) \\
y_k &= g(x_k, u_k, \rho^j)
\end{align*}
\]  

(4.71)

If we continue the same procedure of SubSection 4.3.1, we get:

\[
\begin{align*}
\{x_{k+1}\} &= [A(\rho^j)]\{x_k\} + [B(\rho^j)]\{u_k\} \\
\{y_k\} &= [C(\rho^j)]\{x_k\} + [D(\rho^j)]\{u_k\}
\end{align*}
\]  

(4.72)

Gathering and compacting Eq. 4.72:

\[
\begin{bmatrix}
X_1(\rho^j) \\
Y_0(\rho^j)
\end{bmatrix} = 
\begin{bmatrix}
A(\rho^j) & B(\rho^j) \\
C(\rho^j) & D(\rho^j)
\end{bmatrix}
\begin{bmatrix}
X_0(\rho^j) \\
U_0(\rho^j)
\end{bmatrix},
\]  

(4.73)

where,

\[
X_0(\rho^j) = [x_1(\rho^j) \ x_2(\rho^j) \ ... \ x_{n_x-1}(\rho^j)] \in \mathbb{R}^{n_x \times (n_x - 1)}
\]  

(4.74)

\[
X_1(\rho^j) = [x_2(\rho^j) \ x_3(\rho^j) \ ... \ x_{n_x}(\rho^j)] \in \mathbb{R}^{n_x \times (n_x - 1)}
\]  

(4.75)

\[
U_0(\rho^j) = [u_1(\rho^j) \ u_2(\rho^j) \ ... \ u_{n_u-1}(\rho^j)] \in \mathbb{R}^{n_u \times (n_u - 1)}
\]  

(4.76)

\[
Y_0(\rho^j) = [y_1(\rho^j) \ y_2(\rho^j) \ ... \ y_{n_y-1}(\rho^j)] \in \mathbb{R}^{n_y \times (n_y - 1)}
\]  

(4.77)

And applying the order reduction:

\[
\begin{align*}
\{z_{k+1}\} &= [F(\rho^j)]\{z_k\} + [G(\rho^j)]\{y_k\} \\
\{y_k\} &= [H(\rho^j)]\{z_k\} + [D(\rho^j)]\{y_k\}
\end{align*}
\]  

(4.78)

where the \( F, G, \) and \( H \) depend now on \( \rho^j \):

\[
F(\rho^j) = Q(\rho^j)^T A(\rho^j) Q(\rho^j), \quad F \in \mathbb{R}^{r \times r}
\]  

(4.79)

\[
G(\rho^j) = Q(\rho^j)^T B(\rho^j), \quad G \in \mathbb{R}^{r \times n_u}
\]  

(4.80)

\[
H(\rho^j) = C(\rho^j) Q(\rho^j), \quad H \in \mathbb{R}^{n_y \times r}
\]  

(4.81)
Lastly it is also important to clarify the so-called State Consistency issue, presented in [4]. The projection subspace matrix \( Q \) that relates the full-order model states with the reduced-order model states must be invariant with respect to any operating condition \( \rho^j \), i.e.

\[
Q(\rho^j_k) = Q, \quad Q \in \mathbb{R}^{n_x \times r}. \tag{4.82}
\]

The reason for that is that independently of the operating conditions, the meaning of the reduced-order model states \( z_k \) and, particularly, the full-order model states \( x_k \) must be the same. Therefore, consistency would be lost if different projection subspace matrices \( Q(\rho^j) \) were calculated.

To avoid this problem, [4] explains how to compute an overall projection subspace matrix \( Q \) for all operating conditions \( \rho^j \). Essentially, this consists in building a global snapshot matrix \( X_0 \) such that

\[
X_0 = \begin{bmatrix}
X_0(\rho^j=1) & X_0(\rho^j=2) & \ldots & X_0(\rho^j=J)
\end{bmatrix} \in \mathbb{R}^{n_x \times (n_x-1)J}, \tag{4.83}
\]

and then computing the POD modes from that Eq. 4.83 and, finally, obtaining the projection subspace matrix \( Q \) the same way as before, i.e. selecting the first \( r \) POD modes from matrix \( U \) after SVD calculation applied to \( X_0 \) (see Eq. 4.51, 4.52, 4.53, and 4.54).

Lastly, an important note regarding the different operating conditions must be made. In this section \( \rho^j \) has been considered constant in time (time-invariant) for each operating condition. However, \( \rho^j \) can be time-variant as \( x_k, u_k, \) or \( y_k \) are, that is

\[
\rho^j = \rho^j_k. \tag{4.84}
\]

For further details about this possibility, the reader is referred to [4, 6].
Chapter 4. ROM Obtention

(a) Example of Mode 1 for the Streamwise velocity component at Hub Height.

(b) Example of Mode 10 for the Streamwise velocity component at Hub Height.

(c) Example of Mode 100 for the Streamwise velocity component at Hub Height.

(d) Example of Mode 250 for the Streamwise velocity component at Hub Height.

(e) Example of Mode 425 for the Streamwise velocity component at Hub Height.

Figure 4.2: Example of different POD modes for the Streamwise velocity component U at Hub Height.
4.4. LPV (Linear Parameter Varying) - Future Upgrade

Figure 4.3: Representative case of the evolution of the Relative energy of each POD mode.

Figure 4.4: Representative case of the evolution of the Accumulative Relative energy for each given range of POD modes from 1 to $r$. 
Results: Simulation data analysis and IOROM application

The structure of this Chapter about results is the following: Section 5.1 introduces in the first place an additional consideration that must be taken into account regarding the implications of having a yawing motion as input of the reduced-order model. Subsequently, Section 5.2 and Section 5.3 explain in detail the results from the CFD simulations, its post-processing, the obtention of the ROMs from the CFD data, the comparison between CFD and ROMs’ predictions, and validation cases to test the accuracy of the obtained ROM.

For each simulation explained in this Chapter, the following steps are presented in the following order:

- **Initial conditions of the simulation.**
- **CFD data study:** flow visualization, outputs visualization (mainly, power outputs).
- **Analysis of the SVD elements:** POD modes spatial distribution, singular value energy distribution.
- **ROM’s order selection.** Criteria: Singular values, Errors.
- **Flow and Outputs reconstructions:** reconstructed flow visualization, reconstructed outputs visualization, instantaneous errors, time-averaged errors.

The reader is additionally remained that, unless otherwise stated, although normally the figures presented to illustrate the usage of the developed IOROMs are for the streamwise velocity component and the horizontal hub-height plane (XY Plane), the other two velocity components for that plane and the other three components belonging to the vertical plane are always calculated and included in the model.

### 5.1 Yaw motion implications regarding IOROM

This section addresses the difficulties that were found during this work regarding the application of the developed IOROM (explained in Chapter 4) with an input control signal such as the yaw angle of the wind turbine. The main problem that must be identified was the non-linearity and non-proportionality between the yaw angle and the power output. The non-linearity is an issue that is usually present in engineering problems and processes. Specially those with high complexity as fluid flows, in this case wind farm wakes. Therefore, an approximation of non-linear systems with linear systems, such as the state-space representation and the models explained in Chapter 4, always implies a loss in accuracy. Of course, if the range of operation is not excessively large, the approximation can hold and even provide high forecasting quality of the model. A simple but close to reality relation between yaw angle and power output can be explained with
Chapter 5. Results: Simulation data analysis and IOROM application

Figure 5.1: Relation between the inflow wind speed orthogonal to the rotor disk with respect to the yaw misalignment.

the help of Eq. 5.2 and Figure 5.1.

\[ P(t, \gamma) = \frac{1}{2} \rho A U_{\perp}(t, \gamma)^3 C_p = \frac{1}{2} \rho A U(t)^3 \cos(\gamma)^3 C_p. \]  

(5.1)

where \( P(t, \gamma) \) corresponds to the power depending on the time instant \( t \) and the yaw angle \( \gamma \), where \( \rho \) corresponds to the air density (usually 1.225 kg/m\(^3\) at sea level in the standard atmosphere), where \( A \) corresponds to the rotor disk area, where \( U_{\perp}(t, \gamma) \) corresponds the velocity speed upstream and orthogonal to the rotor disk (the one that leads to the power production), and where \( C_p \) corresponds to the power coefficient.

Apart from the non-linearity, there is another even more important aspect for this particular case of yaw variations related to the non-proportionality between yaw angle and power output, namely, the fact of the system being symmetric, that is:

\[ P(\gamma) = f(|\gamma|), \]  

(5.2)

which makes the system unrepresentable with linear systems. This equations means that for either positive or negative yaw angle \( \gamma \), the system behaves in the same way. For example, for the case of the second wind turbine in the two-turbine array showed in Chapter 2, either a positive or a negative change in the yaw angle would produce the same increase in its power output. Therefore, in order to still use the linear state-space representation model explained in Chapter 4, the problem will be divided into two parts, the positive side and the negative side of the yaw angle range of operation. For each case a simulation will be performed and the corresponding models will be extracted. Since the behavior of wind turbine wakes, and wind turbines in general, are really similar for both positive yawing and negative yawing, an objective in this regard is comparing the two simulations in order to see if such expected behavior occurs. Thus, another future, open improvement proposed in this work would be to combine the two models obtained by working with them together in a simulation with an input signal that covers the positive and negative operational regions.

This particularity of the symmetry and non-linear behavior exists conceptually even for small changes in the input variable (yaw angle of WT1 in this case). This is not the case for other input variables such as pitch, which for small changes around the operating conditions there exist a proportionality between the power output and the pitch angle in the sense that an increase in the angle produces a decrease in power, and a decrease in the angles produces an increase in the power; which is not the case for yaw, as explained.

For this work, the yaw angle range chosen to work with covers a range that goes from 0\(^\circ\) to +25\(^\circ\) for the positive side, and from 0\(^\circ\) to −25\(^\circ\) for the negative side.
5.2 Simulation: Positive-side

This initially presented simulation corresponds to the main simulation of this Chapter 5. As explained, the problem has been divided into two parts, one for case for only positive yaw angles (referred as ‘positive side’ onwards) and another for the negative side (referred as ‘negative side’ onwards); being the latter studied in Section 5.3. Therefore, this Section deals with the ‘positive side’.

The objectives of this simulation are to show the prediction capacity of the IOROM theory explained in this work.

5.2.1 Initial conditions of the simulation

The environment of this simulation is the one explained in Chapter 2, particularly in Section 2.3. Therefore, the data was collected at the two planes explained, one horizontal plane at hub height and another vertical plane aligned with the streamwise velocity component. The yaw control input used in this simulation is the one showed in Figure 5.2.

![Figure 5.2: Yaw control law input for the CFD simulation. The first 4.5s have been set to 0° in order to let the simulation start from an almost stationary situation.](image)

This yaw control law signal was generated following the APRBS procedure explained in Chapter 3. The length of the baseline PRBS signal is $N = 31$, and $T_h = 1$ s as well. In order to create the signal of Figure 5.2, a first APRBS was design and a copy of it was added at the end of the first one to reach the 50 s time.

The simulation is performed during a simulation time of almost 50 s (49.86 s), with a simulation time step of 0.002 s and a snapshot time step of 0.01 s, resulting in approximately 5000 snapshots of data. Compared to one of the reference documents (e.g. [4]), in this work the simulation time step is 14 times larger than the one used with that author (this calculation takes into account the temporal scale between our scaled-model and the full-scale model explained in SubSection 2.3.1), which implies lower snapshot frequency. However, the simulation length used in this work, 50 s, is approx. 2 times larger, resulting in an overall collection of data between 2 and 3 times that of this author. Lastly, the obtained CFD data correspond to 167249 grid points resulting in a total state-space vector length of 501747 (number of states per snapshot).

5.2.2 CFD data post-processing

The first figures that can be shown to explain the behavior of the system due to the excitation of the input signal are shown in Figures 5.3 to 5.8. The first four figures show, for a specific
instant in time, a snapshot of the streamwise velocity component of the flow. The first one (see Figure 5.3) is taken at the beginning of the yawing motion so as to show the behavior of the flow at stationary conditions. Clearly, due to the turbulence, there is no such stationary condition in the strict sense. However, once the power outputs are reasonably stabilized over time, considered to happen at 4.5 s in this work, the simulation is considered to be in stationary conditions. The second one (see Figure 5.4) has been taken to illustrate a typical intermediate state of the simulated flow with high yaw misalignment. The fact of yawing the first wind turbine, approx. 20° at that instant in time, makes the wake deflect as expected and makes it leave an open region where unaffected freestream wind can provide higher inflow conditions to the second wind turbine, which in turn increases its power as the figure shows.

Figures 5.5 and 5.6 are introduced to show the effect of the delay in the movement of the flow. Essentially, because of the physical need of the flow to travel the distance from wind turbine 1 to wind turbine 2 (4D distance), the action on WT1 is not instantaneously reflected neither on the flow nor on the power output of WT2. In Figure 5.5, WT1’s yaw angle is already at its maximum for this simulation, 25°, however the flow is not completely deflected. After some seconds, see Figure 5.6, the flow is perfectly deflected and the wake-steering effect can be clearly seen, WT1’s power is almost stationary, and WT2’s power output is near to the stationary. At this point, the increase in total power output represents a 16% more with respect to the initial total power output with no misalignment. Additionally, it is also worth highlighting that the are transitory conditions where a peak in power is attained, such as the one at 38.5 s, due to a quick increment in WT1’s power when the yaw angle is reduced while at the same time the wake is still deflected. These values are not considered as relevant since they come from transitory, untenable conditions.

Lastly, Figures 5.7 and 5.8 are illustrated to show that the spanwise velocity component and vertical velocity component are also recorded during the simulation. The results are consistent with what expected: predominant negative spanwise component according to the direction of deflection of the wake (negative coordinate y), and a whirling wake as a reaction to the clockwise rotation of the wind turbine rotor (if viewed from streamwise direction).

To better observe the power output of WT1, WT2, and the total power output of the two-array wind farm, a plot for this purpose (see Figure 5.9) is provided.
5.2. Simulation: Positive-side

Figure 5.3: CFD data Streamwise velocity component U at Plane XY at hub height at 0.01s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.

Figure 5.4: CFD data Streamwise velocity component U at Plane XY at hub height at 10.75s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.
Figure 5.5: CFD data Streamwise velocity component U at Plane XY at hub height at 32.34 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.

Figure 5.6: CFD data Streamwise velocity component U at Plane XY at hub height at 37.40 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.
5.2. Simulation: Positive-side

Figure 5.7: CFD data spanwise velocity component $V$ at Plane XY at hub height at 37.40 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.

Figure 5.8: CFD data Vertical velocity component $W$ at Plane XY at hub height at 37.40 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.
5.2.3 SVD information

Once the data from the CFD simulation is collected and analyzed, the procedure explained in SubSection 4.3.1 in Chapter 4 can be applied. The first information that is obtained concerns the Singular Value Decomposition (SVD) from the snapshot matrix. According to Eq. 4.83 and considering the removed first 4.5 s of simulation, the dimensions of this matrix for this simulation are: 501747-by-4537.

From the SVD decomposition, the POD modes are obtained and the relative energy of each POD mode too. The POD modes, as explained in Section 4.2, represent a spatial, atemporal, orthogonal projection of the flow (the states of the models) in a way that the superposition of all these modes, ‘weighted’ with its corresponding singular values, can construct the whole flow. Some examples of the 4537 POD modes are shown in Figure 5.10. It can be observed that the first POD modes of the SVD represent the non-turbulent behavior, while higher order POD modes present patterns containing much more turbulent behavior. As it can be seen, each POD mode (with dimensions 501747-by-1) can be consistently drawn and interpreted in space. In these plots, only the part of the POD mode corresponding to the Streamwise velocity component at the horizontal plane XY has been shown. Anyway, the other 5 parts (other two velocity components for that plane, and the other three components belonging to the vertical plane) are calculated in the same operation and could have been equally plotted.

Apart from the POD modes, the singular values have also some interest. As said, they represent the energy of each POD mode. This can be understood as the relative weight that each POD mode spatial pattern has in composing the original snapshot matrix. Two illustrate with information two figures have been provided (see Figure 5.11 and Figure 5.12). The first one shows the relative energy of each POD mode compared with the total energy present in all singular values. According to how the SVD computation is performed, these values are always sorted in descending order. The second figure shows the accumulative energy obtained.
5.2. Simulation: Positive-side

(a) POD mode number 1 Streamwise velocity component at Hub height (Plane XY).

(b) POD mode number 3 Streamwise velocity component at Hub height (Plane XY).

(c) POD mode number 7 Streamwise velocity component at Hub height (Plane XY).

(d) POD mode number 105 Streamwise velocity component at Hub height (Plane XY).

(e) POD mode number 444 Streamwise velocity component at Hub height (Plane XY).

Figure 5.10: POD modes number 1, 3, 7, 105, and 444 for the Streamwise velocity component at Hub Height (Plane XY).
when adding each next POD mode. Lastly, as mentioned in SubSection 4.3.4, this information typically does not help in order to choose the order of the reduced-order model, which is the topic of the next subsection.

5.2.4 ROM order selection

In this subsection, the order selection of the reduced-order model is explained. The tools for this purpose are explained in SubSection 4.3.4 and consist in evaluating the so-called Velocity Field Error and the Output’s Error. This study has been performed in a range from $r = 1$ to $r = 500$, but not further to keep a certain reduction in the ROM. The result of this calculation are presented in Figure 5.13 and Figure 5.14.

These graphs allows to select more easily the order of the reduced-order model. In both figures, there is clearly a decrease in both errors around the 50 POD modes included in the model. More specifically, the minimum error takes place with 45 POD modes for both velocity field error and power outputs’ error, and, therefore, 45 POD modes is the order chosen for the model. This circumstance is neither necessary nor usual, but in this case both error coincide for a ROM order of 45 POD modes. It must be said that this work proposes a series of methods to try to select the order of the system. However, this is not a closed set, other methods or approaches can perfectly be used. This point could be potentially systematized, although in this work a direct inspection approach has been chosen.

Additionally, a strange phenomenon observed in this figure is also noticed. The error behavior in the first tens of POD modes seems to increase or decrease in sort of packages. This peculiarity might be related to a physical phenomenon, although there is no clear explanation to it. This will be object of study for future improvement of this work.
5.2. Simulation: Positive-side

Figure 5.13: Power outputs’ Percentage Error calculated with Eq. 4.70 with respect to the order of the model for a range from 1 to 500 POD modes.

Figure 5.14: Velocity Field Error calculated with Eq. 4.68 with respect to the order of the model for a range from 1 to 500 POD modes.
Chapter 5. Results: Simulation data analysis and IOROM application

Figure 5.15: Reconstructed Streamwise velocity component U at Plane XY at hub height and Power outputs at 10.69 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are computed by means of an IOROM with 45 POD modes.

5.2.5 IOROM Reconstructions

In the previous subsection the steps to obtain the reduced-order model have been taken, i.e. the model with 45 POD modes. Once the model is selected, there are two tests to perform to value the quality and accuracy of it. Firstly, the model is used to reconstruct the original flow and power from which it was obtained. Then, the model is validated with another input signal in a new simulation to see if the model is able to produce enough accuracy with other simulation data.

For the reconstruction, the model is built and used by computing Eq. 4.64. The initial states used to initialize the system for its usage is the same initial state of the snapshot matrix, i.e. the first column of the snapshot matrix (corresponding to the instant 4.5 s). Since this initial state is expressed in full-order coordinates, Eq. 4.65 need to be used to project the full-order state vector to the reduced-order state vector, whose dimensions match with the obtained matrices $F$, $G$, $H$, and $D$. With Eq. 4.64 the predicted, future states and power output can be computed. Lastly, the future reduced-order states are converted back to full-order states so that they can be understood (physical meaning). This is done by means of Eq. 4.65, as with the initial state.

Two examples of such flow reconstruction can be seen in Figure 5.15 and Figure 5.16. Logically, since a low number of POD modes are selected, the model retains the main dynamics of the system without taking into account the high-frequency dynamics of the system such as vortices and turbulences, as expected and desired.

Additionally, Figure 5.17 and Figure 5.18 are provided to show a pairwise comparison between the flow at two instants of time. The effect highlighted before is also present here. The reduced-order model reconstructs the main features of the flow, except from the detailed turbulences which are diffused.
5.2 Simulation: Positive-side

Figure 5.16: Reconstructed Streamwise velocity component $U$ at Plane XY at hub height and Power outputs at 42.54 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are computed by means of an IOROM with 45 POD modes.

Figure 5.17: Comparison between the Reconstructed and CFD Streamwise velocity component $U$ at Plane XY at hub height at 32.34 s. The reconstruction was computed by means of an IOROM with 45 POD modes.
Figure 5.18: Comparison between the Reconstructed and CFD Streamwise velocity component $U$ at Plane XY at hub height at 37.40 s. The reconstruction was computed by means of an IOROM with 45 POD modes.

Power output reconstructions are clearly worth comparing as well. Since the reduced-order models are expected to be used for control, the outputs of the system, in this case power outputs, will be the elements that will mainly guide the control algorithms. Such a comparison between the power outputs collected during the CFD simulation and the reconstructed power outputs obtained with the model is shown in Figure 5.19. Additionally, a brief statistical description is provided (see Table 5.1).

Table 5.1: Descriptive Statistics for the Output’s Error for the Comparison of the Power Outputs between CFD data and Reconstructed power outputs.

<table>
<thead>
<tr>
<th></th>
<th>Power Output WT1</th>
<th>Power Output WT2</th>
<th>Power Output Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.91%</td>
<td>7.11%</td>
<td>1.75%</td>
</tr>
<tr>
<td>std</td>
<td>0.75%</td>
<td>5.68%</td>
<td>1.51%</td>
</tr>
</tbody>
</table>

The match for the power output of WT1 is really accurate, with a mean value of the percentage error lower than 1% with also low deviation. The power output of WT2 is clearly more difficult to predict, specially when the misalignment reaches its maximum value for this simulation, 25°. However, at the end, the results for the total power output, which in fact should be the most interesting figure, has a mean error of around 2% with 1.5% of standard deviation, which is reasonably acceptable prediction.

Lastly, some qualitative measurements regarding flow reconstructions are also provided. This is the case of Figure 5.20 and Figure 5.21. These figures represent the so-called time-averaged error of the magnitude of interest at any grid point over time. For this calculation, the difference between the reconstruction and the original CFD simulation value has been divided by the
5.2. Simulation: Positive-side

Figure 5.19: Comparison of the Power Outputs between CFD data and Reconstructed power outputs. The reconstruction was computed by means of an IOROM with 45 POD modes.

freestream velocity, i.e. 5.8 m/s instead of calculating a pure percentage error, to avoid division by zero or unrealistic errors that could be produced, even for the Streamwise velocity component, at points of really low velocity, for example, in the proximity of the nacelle. The results obtained can be considered as really adequate since the absolute value of the errors displayed on those plots are below the 2% of the freestream velocity.

5.2.6 IOROM Validation

After showing that the model works acceptably well and predicts with a certain degree of accuracy both the flow and power output, it is time to validate the model in another simulation with another different input signal to assure that the model is not overfitted for the simulation which it was created with, but that is can be used with any other input signal within the operational range expected (see details regarding the importance of the operational range in Chapter 3). Such another signal (see Figure 5.22) has been used to validate the model.

This yaw control law signal was also generated following the APRBS procedure explained in Chapter 3. The length of the baseline PRBS signal is $N = 31$ and $T_h = 1$ s. The difference with respect the original signal used to excite the system to obtain the model is a different initialization of the register (see Section 3.2 for more information) used to create the PRBS signal that results in the APRBS signal with ramps.

The most interesting validation is the accuracy of the system in predicting the power outputs. For this purpose, the model has been used as explained in the previous chapter, but with this new input signal of the validation case. The forecasted results that the models gives are compared to the power outputs collected from this new simulation of validation. This comparison can be visualized in Figure 5.23. Additionally, a brief statistical description is provided (see Table 5.2).
Figure 5.20: Time-averaged error for the Streamwise velocity component at the horizontal plane at hub height XY.

Figure 5.21: Time-averaged error for the Streamwise velocity component at the vertical plane XZ.
5.3 Simulation: Negative-side

The purpose of this simulation is to address the obtention of the model for negative yaw angle regions. This goes in line with what explained in Section 5.1 regarding the existing non-linearity in the system due to its symmetry. As well as in the previous case, this negative-side simulation includes a detailed, but reduced in this case, description of the simulation. Nevertheless, the purpose of this negative-side simulation falls more on comparing the behavior of the positive side with respect to that of the negative side than on extracting and validating a model based on the negative side simulation.

5.3.1 Initial conditions of the simulation

The environment of the simulation corresponds to the same one of the positive-side simulation, i.e. data collected at the two planes explained. The yaw control input used in this simulation is the also the one in shape of the positive-side simulation but with negative values. See Figure 5.24.

As before, this yaw control law signal was generated following the APRBS procedure explained in Chapter 3. The length of the baseline PRBS signal is \( N = 31 \), and \( T_h = 1 \text{s} \) as well. In order to create the signal of Figure 5.24, a first APRBS was design and a copy of it was added.
Chapter 5. Results: Simulation data analysis and IOROM application

5.3.2 CFD data post-processing

The behavior of the system due to the excitation of the input signal is absolutely similar to that of the positive-side simulation, e.g., same effects regarding delay in the flow, or stabilization of the power, or wake deflection. In this case, however, since the yaw angles cover the negative range the wake, it is logically deflected in the opposite direction. In Figure 5.25 such a behavior can be seen for a particular instant in time. It can be noticed that the power outputs have the same sign (obviously positive, i.e., production of power) as the positive-side simulation. Therefore, the non-linearity phenomenon discussed in Section 5.1 holds.

Lastly, the spanwise velocity component and vertical velocity component are also recorded during the simulation, although no plots are provided given the principal interest of this negative-side simulation. The results are also consistent with what expected: predominant positive spanwise component according to the direction of deflection of the wake (positive coordinate y), and a whirling wake as a reaction to the clockwise rotation of the wind turbine rotor (if viewed from streamwise direction).

What is worth comparing for this simulation is the power outputs of the positive-side simulation vs the power outputs of this negative-side simulation. Such a comparison is illustrated in Figure 5.26. As expected, the behavior of the whole system composed of both positive and
negative operational range is almost symmetric. For WT1 the matching is almost perfect, it only slightly differs owing to the turbulence which is sensitive to little variations. For the case of WT2, the difference is noticeable although not considerable. The main reason that might lead to this difference is the motion of the whirling of the wake. Its initial direction of rotation is fixed since it relies on the direction of rotation of the wind turbine, which is also fixed (always). Therefore, a wake whirl that does not depend on the yaw angle causes different perturbations in each the two cases. Oppositely, the wake deflection depends on the value yaw angle. Hence, this deflection is practically the same.

5.3.3 SVD information

For this simulation, the analysis of the snapshot matrix through a SVD decomposition has also been considered. In this subsection, the POD modes of the negative-side simulation are shown (see Figure 5.27). POD modes exhibit similar patterns as the ones from the positive-side simulation: first POD modes, non-turbulent behavior; higher order POD modes, much more turbulent behavior. With regard to the singular values (energy of each POD mode of the system), Figure 5.28 and Figure 5.29 are provided to show the relative energy of each POD modes and the cumulative relative energy, respectively. As before, these figures are not very conclusive in relation to reduced-order model’s order selection.

5.3.4 ROM order selection

In this subsection, the order of the reduced-order model is chosen with the same procedure as it was done for the previous simulation in subsection 5.2.4, i.e. by evaluating the Velocity Field Error and the Output’s Error for a series of ROM’s order (e.g. in the range from \( r = 1 \) to \( r = 500 \)). The result of these calculations are presented in Figure 5.30 and Figure 5.31.

These graphs show similar relation between an increased ROM’s order and both errors. As a general trend, the higher the number of POD modes that the model uses (ROM’s order), the lower the error of both flow and power. However, in this case, as it normally occurs, the local minima that can be found all over the plot does not have to correspond between graphs. This obviously complicates the decision regarding the ROM’s order. In this case, the order of the reduced-order model has been chosen by looking at the Output’s Error (power outputs’ error) since the reduced-order models are said to be used for control, for which the outputs of the system gain higher relevance. Thus, a local minimum is identified in Figure 5.30 at 71 POD modes. The Velocity Field Error is also low with 71 POD modes compared to other orders around it.

![Figure 5.24: Yaw control law input for the CFD simulation. The first 4.5s have been set to 0° in order to let the simulation start from an almost stationary situation.](image)
Chapter 5. Results: Simulation data analysis and IOROM application

Figure 5.25: CFD data Streamwise velocity component U at Plane XY at hub height at 37.40 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are recorded from the CFD simulation.

Figure 5.26: CFD data Power outputs Comparison between the negative-side simulation and the positive-side simulation.
Figure 5.27: POD modes number 1, 5, 8, 100, and 485 for the Streamwise velocity component at Hub Height (Plane XY).
Figure 5.28: Relative energy of each POD mode.

Figure 5.29: Cumulative relative energy of POD modes for a range from 1 to $r$.

Figure 5.30: Power outputs’ Percentage Error calculated with Eq. 4.70 with respect to the order of the model for a range from 1 to 500 POD modes.
5.3. Simulation: Negative-side

Figure 5.31: Velocity Field Error calculated with Eq. 4.68 with respect to the order of the model for a range from 1 to 500 POD modes.

5.3.5 IOROM Reconstructions

Once the model is selected, it is time to test the accuracy of it in reconstructing and predicting the behavior of flow and power for a given input and initial states (initial conditions). The model is therefore used in this subsection to reconstruct the original flow and power from which it was obtained. For these reconstructions, the model is built and used by computing Eq. 4.64. The initial states used to initialize the system for its usage is also the same initial state of the snapshot matrix, i.e. the first column of the snapshot matrix (corresponding to the instant 4.5s) and the input signal is the input yaw control law of Figure 5.24.

One example of the results of the reconstruction using this negative-side model is presented in Figure 5.32 for a specific instant in time. As expected, the reconstruction of the flow and power look similar to that of the positive-side model. To compare the flow field another plot is provided (see Figure 5.33). The results for this plot seem to be quite precise, reconstructing the main features of the flow and not reproducing the high turbulent dynamics. Lastly, regarding the power outputs, they have been compared both graphically (see Figure 5.34) and with a brief statistical description (see Table 5.3).

Table 5.3: Descriptive Statistics for the Output’s Error for the Comparison of the Power Outputs between CFD data and Reconstructed power outputs.

<table>
<thead>
<tr>
<th></th>
<th>Power Output WT1</th>
<th>Power Output WT2</th>
<th>Power Output Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.95 %</td>
<td>6.36 %</td>
<td>1.52 %</td>
</tr>
<tr>
<td>std</td>
<td>0.72 %</td>
<td>4.78 %</td>
<td>1.25 %</td>
</tr>
</tbody>
</table>
Figure 5.32: Reconstructed Streamwise velocity component U at Plane XY at hub height and Power outputs at 8.00 s. The yaw angle motion showed corresponds to the input to the CFD simulation. The power outputs are computed by means of an IOROM with 71 POD modes.

Figure 5.33: Comparison between the Reconstructed and CFD Streamwise velocity component U at Plane XY at hub height at 19.45 s. The reconstruction was computed by means of an IOROM with 71 POD modes.
Figure 5.34: Comparison of the Power Outputs between CFD data and Reconstructed power outputs. The reconstruction was computed by means of an IOROM with 71 POD modes.

The results obtained present similar trends with respect to the ones of the positive-side simulation. For the power output of WT1, the matching is really accurate with some precision losses at the extremes of the operational range, namely, $0^\circ$ or $+25^\circ$. The power output of WT2 is always more difficult to predict owing to the high turbulence of the wake that it encounters, specially when the misalignment reaches its maximum value for this simulation, $25^\circ$, as well. Now quantitatively, the errors from Table 5.3 are really similar as well. Indeed, this model has slightly lower errors for the three sets (WT1, WT2, both WTs), but it is also obtained with a higher number of POD modes.
Chapter 6

Conclusions & Outlook

6.1 Conclusions

As it was firstly addressed in Chapter 1, wind energy is continuously evolving and improving. In this regard, this work tried to help in developing reduced-order model that could capture the main dynamics of complex, simulated system such as wind farms and obtain simpler model with much lower computational cost. The model are expect to be used for the design of control strategies that could improve the overall performance of wind farms. This control strategies may imply, for instance, working in an individual suboptimal operating conditions that in turn results in a global improvement of whole farm. As already proposed, this global improvement can be obtained by maximizing a certain figure of merit that could potentially be based on fatigue and/or critical forces or moments and/or power productions, amongst others.

According to the results observed, this goal is achieved to a considerable extent. The system studied (composed in fact of two sides, namely both the positive-side and negative-side simulations) showed high agreements even for low number of POD modes when compared to the results obtained by using CFD simulation data. This shows that the methodology implemented originally for pitch control, also work for the case of yaw control with success. Clearly, as explained in Chapter 5, the matching between reconstructed power output and CFD power outputs is better for the case of wind turbine 1. The reason for that is that the first wind turbine operates under ideal conditions without any initial turbulence from the inflow wind. Therefore, its performance is less non-linear and less variable, and thus easier to predict. However, it is for wind turbine 2 where the higher imbalances take place. Turbulences are much more sensitive to little variations and its forecasting entails more complexity. In fact that was observed for the case of the comparison between the power outputs of the positive-side and negative-side simulations with no intervention of the model at that particular case.

Additionally, system identification also played an important role during this work. A proper excitation of the system was necessary, and the approach of computing an APRBS signal with ramps (to include the realistic yaw rates) from the classical formulation of APRBS signal based on a PRBS signal resulted satisfactory.

Furthermore, different procedures to obtain ROMs have been studied. At the end, the IOROM procedure selected fulfilled several important criteria for this work, namely: having low computational costs, keeping a relationship between inputs and outputs of the system, retaining the physical meaning of states, being extensible to LPV (Linear Parameter Varying), and being adjoint free.

As explained during this work, the models are developed under certain operational conditions. Clearly, the engineer must be aware of the conditions that can affect the system to be able to determine a finite set of conditions for which each model is oriented. In the study of this work, this set can be summarized in:
Chapter 6. Conclusions & Outlook

- 2 Wind turbine aligned with respect to the incoming flow
- Constant freestream wind speed
- No change in wind speed direction
- No turbulence of incoming flow

All in all, the models developed in this work are able to predict within seconds the behavior of wind farms whose performance takes days to be known through CFD simulations. Besides, they present successful agreement with respect to the results obtained via CFD simulations in terms of both flow and power output reconstruction. In the next section several improvements are also presented as future upgrades for the IOROMs obtained from CFD data of this work.

6.2 Future Studies & Outlook

This section about future studies and outlook will propose several aspects in which this work can progress and improve in the future in case this topic is studied further.

One aspect that can be clearly improved for future studies is the collection of the data. In this work data were collected just at two planes: one horizontal plane at hub height and other vertical plane passing through the centers of the two wind turbines of the wind farm (and also perfectly aligned with the incoming flow). As it was done in other works like [4,11,61], data could have been collected in a 3D space, therefore in several horizontal and vertical planes. The effect of this possible improvement has not been tested in this work. Nevertheless, it seems logical that a 3D characterization could provide more understanding of the existing phenomena in wind turbine wakes compared to data from just 2 planes. However, the results obtained in this work seem to be acceptable in terms of the evolution of the system when yaw position is changed over time. Anyway, the objective of this study of developing models that could identify and reconstruct with low computational effort systems with high complexity (like flow turbulence) is achieved, although the behavior of the system (for given inputs) may not be fully known due to this potential lack of data.

Other aspect to consider for an evolution of this work is other and more complex layouts. The layout proposed in this work consisted in 2 wind turbines perfectly aligned with respect to the incoming flow. Therefore, other layouts involving more wind turbines and at different positions could broaden the application of the models developed in these study. Studies like [9,10] explore results obtained with SOWFA with different layouts, e.g. repositioning of the downwind wind turbine and its effects, or higher number of wind turbines (and hence more complex wind farms) (see Figures 6.1 and 6.2). However, they do not extract models from these studies nor include order reduction. Thus, the introduction of the techniques explained in this work could be applied to these other possibilities (different number of wind turbines and/or different layouts) and achieve some interesting operating points highlighted in those studies.

The turbulence of the incoming flow could also play a role. In comparison to the power data obtained in [11], the power output data obtained from the simulations of this work and the power output reconstructions using the developed models are pretty undisturbed since the original flow does not have any initial turbulence and the freestream wind speed does not vary over time (see Figure 6.3). This turbulence effect is clearly present in real-life operation, and therefore a model to be used in these conditions should be robust to wind speed fluctuations.
6.2. Future Studies & Outlook

Figure 6.1: Effect of repositioning (and yawing and tilting) on different magnitudes for a similar layout as the one in this work [9].

Another aspect to take into account for the future is the introduction of LPV (Linear Parametric Varying) explained in Chapter 4 in Section 4.4. This upgrade allows the models developed in this work to be used under other operational conditions than those under which the models of this work were developed. This is particularly important for systems like this one where the operating conditions can vary considerably, reducing in turn the forecasting capacity of the developed model. Examples of this variations in the flow of wind farm wakes are, among others, changes in:

- the direction of the wind during operation of wind farms, or
- the speed (average speed) of the wind, or even
- the turbulence of the wind itself.

An example of the application of LPV can be seen in [6] (see Figure 6.4).
Figure 6.2: Layout studied in [10], where global wind farm power output was improved through optimal control.

Figure 6.3: Power variations for the same layout of this work with varying pitch under turbulent conditions [11]. Note (bottom) that there are regions, e.g. between instants 1200s and 1300s, where power variations are observed, due to turbulent conditions, that would not be observed in case of non-turbulent conditions.
Figure 6.4: Comparison between CFD data and IOROM with LPV as it was done in [6]. Note that (up) the operating condition $\rho^j = Re$ (Reynolds number) is in this case $\rho^j_k = Re_k$ and that (middle) the input variables is $a_2$ (induction factor). The power output (bottom) can be matched with good accuracy.
Bibliography


