Identification of one-parameter bifurcations giving rise to periodic orbits from their period function

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ABSTRACT

The motivation for this work is the following problem reported to us by our colleague the mathematician and neuroscientist Antoni Guillamon: "*Supose that* $\dot{x} = X_\mu(x) = X(x, \mu, \lambda)$ *is a good model for a realistic phenomenon, where* $x \in \mathbb{R}^n$, $\mu \in \mathbb{R}$ *is an experimentally controllable parameter, and assume that there exist other uncertain parameters gathered in* $\lambda \in \mathbb{R}^p$ *that need to be estimated in order to obtain a good description of the model. If it is possible to measure an observable magnitude, for instance the period* $T(\mu)$ *of some periodic orbits observed experimentally, and to know how it varies as* μ *is changed, perhaps it is possible to extract information on the uncertain parameters* λ*. This occurs, for instance, when studying neuron activities in the brain with the aim of determining the synaptic conductances* λ *that it receives. In the experiments, by injecting different external currents (which would correspond here to* μ*), people is able to extract information about the period of the oscillations of the voltage of the cell. So one has some measurements* $T(\mu_i, \lambda)$ *for* $i = 1, \ldots, q$ *and some kind of regression is needed to estimate* λ*. Therefore the analytical knowledge of* $T(\mu_i, \lambda)$ *is determinant to do this regression/estimation properly.*"

Now, the starting point for us is the following question: From the knowledge of the period function $T(\mu)$ of the one parameter family of periodic orbits appearing in the experiments, it is possible to identify the type of bifurcation which has originated the periodic orbits? Notice that if the answer is positive this would give important restrictions on the uncertain parameters $\lambda \in \mathbb{R}^p$ to be estimated.

Our results are restricted to the planar analytic case where the dependence of the differential equation on μ is also analytic. In this context, the "most elementary bifurcations" giving rise to periodic orbits are: the Andronov-Hopf bifurcation; the bifurcation from a semi-stable periodic orbit; the saddle-node loop bifurcation and the saddle loop bifurcation. In this work we obtain the dominant term of the asymptotic behaviour of the period of the limit cycles appearing in each of these bifurcations in terms μ when we are near the bifurcation.

The results show that, essentially, the principal term of the period of the periodic orbit arising from generic elementary bifurcations characterizes the bifurcation.

References

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