A First Estimate for a Pressure Retarded Osmosis-Driven Thermosyphon

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Pressure retarded osmosis (PRO) process and its significance for thermosyphon technology is discussed. In previous work the possibility to drive a thermosyphon by difference of densities from induced salinity gradients from evaporation at solar collectors with downward heat transfer was assessed. Nevertheless it was concluded that large evaporative areas or dilution with volatile compounds was mandatory. In this work it is shown that by taking advantage of the energy released during the spontaneous mixing of the low-concentration evaporative fraction and the high-salinity of the no-evaporated fraction which is generally referred as pressure-retarded osmosis (PRO) process, then the thermosyphon can runs with downward heat transfer (hot fluid flows down and cold fluid rises up) and using an evaporative surface area much more smaller and then eliminating the need for dilution with high volatile compounds.

Keywords. thermosyphon, salinity gradient, domestic hot water, pressure-retarded osmosis

I. INTRODUCTION

In a recent work the possibility to drive a thermosyphon by salinity gradients rather than traditional thermal gradients and then with downward heat transfer as alternative to the traditional upward heat transfer thermosyphon was assessed.[1]. In that approach, evaporation is promoted at the solar collector and as a result, the no-evaporated fraction with an increased salinity, and density, than the evaporated fraction sinks. The resulting downward heat transfer may be particulary attractive for domestic solar applications if one considers that solar energy is generally collected at the roofs and then needs to be transported inside houses. Unfortunately, to do this, the required surface area of evaporation was found too large if pure water was used, and then the dilution with a high volatile compound was mandatory.

In this work it is demonstrated that the salinity gradient form the induced evaporation could be harnessed in a better way, not by buoyancy (difference of densities) but for taking advantage of the energy released during the spontaneous mixing of the low-concentration evaporative fraction and the high-salinity of the no-evaporated fraction, in a process generally termed as pressure-retarded osmosis or PRO process.

Whereas osmosis has been researched for several solar applications, [2]-[9], nonetheless, the PRO process has received a very limited study for solar applications. e.g., for increasing solar pond efficiency., [10].

A. Pressure retarded osmosis

In short, pressure retarded osmosis or PRO process, is the technique based in harvesting the energy released during spontaneous mixing of two solutions with deferent salinities, [11]. The PRO process, since the 1970s, has been considered mostly as a sustainable energy source to desalination systems, or commercial power plant generation from the mixing of seawater and rivers or wastewaters with lower salinity. Unfortunately, it has been found, among other problems, that biofouling (or bacteria that clog the membrane structure and the and the feed channel) in those systems is of critical importance in reducing power generation next to nothing.

Although, it is certain that biofouling cannot be controlled -or at least economically controlled, in huge open-loop seawater systems where massive amounts of water are involved, nonetheless, this problem is not present in very small, compact and recirculating systems as could be a domestic thermosyphon where the extractable energy must be just enough to propel the fluid downward and overcoming the buoyancy as well friction losses.

II. THE PRESSURE RETARDED OSMOSIS-DRIVEN THERMOSYPHON

A. Momentum and thermodynamic considerations

To begin with, let us consider, for illustrative purposes, a possible pressure retarded osmosis-driven thermosyphon as schematically depicted in Fig. 1. Referring to this figure, a fluid loaded with a certain salinity (aqueous solution) is circulating in a closed-loop thermosyphon system. We assume water with a reference
FIG. 1: Physical model for analysis of the osmotic thermosyphon.

density \( \rho_o \) and salinity \( s_o \) (in % weight of salt). Then, after passing through the collector and being heated, some fraction is evaporated resulting in two streams with differences salinities. Let us call the low-salinity evaporated fraction with concentration \( s_1 \); and the high-salinity no-evaporated fraction with concentration \( s_2 \). Then both solutions are brought together and mixed (by using a proper semi-permeable membrane) and then the final mixture recovers its initial salinity \( s_o \) but with a temperature \( T_h \) higher than the initial temperature \( T_c \) before passing through the solar collector. From the mixing of two stream with different salinities, it is obtained an extractable energy from a PRO process, [12]-[15]. The condition here investigated is if this pressure could overcomes the buoyancy plus the friction losses and then propeling the hot water through the entire system. Mathematically this condition implies

\[
\Delta p_\pi > \frac{8fL \dot{m}_w^2}{\pi^2 \rho D^5} + (\rho_1 - \rho_2)gH
\]  

(3)

Where the first term is the pressure friction losses and the second one the hydrostatic buoyant pressure, being \( H \) the effective height of the pipes, \( g \) is gravity \( L \) is the total pipe length, \( f \) the pipe friction coefficient, \( \dot{m}_w \) the water mass flow, \( D \) the diameter of the pipe, \( \rho \) the average density of the fluid. By using Eq.(2), Eq.(3) may be rewritten as

\[
\Delta p_\pi > \frac{8fL \dot{m}_w^2}{\pi^2 \rho D^5} - \nabla T \rho \Delta T gH
\]  

(4)
On the other hand, the osmotic hydrodynamic pressure is related with the specific Gibss free energy of the mixing per volume of the total mixed solution \( \Delta G_v \) as

\[
\Delta p_r \approx \eta \Delta G_v
\]

where \( \eta \) is the efficiency of the PRO process. If one considers that for a given solution, \( c_D \) is the concentration of the high-salinity draw solution, \( c_F \) the concentration of the low-salinity feed solution, \( c_M \) the concentration of the mixed solution, and \( \phi \) the low-salinity feed solution volume fraction, then the Gibbs energy per volume is given by

\[
\Delta G_v \approx \frac{\Pi(c)}{c} \left[ c_M \ln(c_M) - \phi c_F \ln(c_F) - (1 - \phi) c_D \ln(c_D) \right]
\]

where \( \Pi(c) \) is a given reference osmotic pressure for a given concentration \( c \). For our case, these values can be related as: \( c_D \to s_2; c_F \to 0; c_M \to s_o \), and as we derive later, the feed fraction given by \( \phi = 1 - \frac{s_2}{s_o} \). Then, taking into account the limit \( x \ln x = 0 \) when \( x \to 0 \), Eq.(6) becomes

\[
\Delta G_v \approx \ln \frac{s_2}{s_o} \Pi_o
\]

and inserting into Eq.(5) we get

\[
\Delta p_r = \eta \ln \frac{s_2}{s_o} \Pi_o
\]

which inserted into Eq.(4) yields

\[
\frac{s_o}{s_2} = \exp \left( \frac{\nabla T \rho \Delta T gH}{\eta \Pi_o} - \frac{8fLW^2}{\pi^2 \rho D^3 \Delta T^2 \eta \Pi_o} \right)
\]

Taking into account that the water mass flow could be expressed as function of the power \( W \) as

\[
W \approx \dot{m}_w c_p \Delta T
\]

where \( c_p \) is the heat capacity, thus Eq.(9) may be rewritten as

\[
\frac{s_o}{s_2} = \exp \left( \frac{\nabla T \rho \Delta T gH}{\eta \Pi_o} - \frac{8fLW^2}{\pi^2 \rho D^3 \Delta T^2 \eta \Pi_o} \right)
\]

Now, by considering mass balance at the evaporator we know that

\[
\dot{m}_w s_o = [\dot{m}_w - \dot{m}_e] s_2
\]

where \( \dot{m}_w \) is the water mass flow entering the evaporator, and \( \dot{m}_e \) is the evaporation mass rate. We can rewritten Eq.(12) as

\[
\dot{m}_e = \dot{m}_w \left( 1 - \frac{s_o}{s_2} \right)
\]

where it is easy to see that \( \phi = 1 - \frac{s_2}{s_o} \). By taking into account Eq.(10), Eq.(13) can be rewritten as

\[
\dot{m}_e = \frac{W}{c_p \Delta T} \left( 1 - \frac{s_o}{s_2} \right)
\]

which inserted into Eq.(11) and after rearrangements of terms yields

\[
\dot{m}_e = \left[ 1 - \exp \left( \frac{\nabla T \rho \Delta T gH}{\eta \Pi_o} - \frac{8fLW^2}{\pi^2 \rho D^3 \Delta T^2 \eta \Pi_o} \right) \right] \frac{W}{c_p \Delta T}
\]

Many semiempirical formulations for evaporation of water as function of the surface area exist; but in view of several uncertainties, the simplest expression due to Shuttleworth, [16], seems preferable.

\[
A_s \approx \frac{4.34 \times 10^4 \cdot \lambda_v (m + \gamma)}{(mR_n + \gamma \delta_e (1 + 0.536 u_w))} \cdot \dot{m}_e \ (m^2)
\]

where \( A_s \) is the total surface area of evaporation (\( m^2 \)); \( m \) is the slope of the saturation vapor pressure curve (kPa/K); \( R_n \) the net irradiance (MJ/ (\( m^2 \))day); \( u_w \) the wind speed m/s; \( \delta_e \) the vapor pressure deficit (k Pa); \( \lambda_v \) the latent heat of vaporization (MJ/kg); and \( \gamma \) is the psychometric constant (kPa/K) given by, [17]

\[
\gamma = \frac{0.0016286 \cdot p}{\lambda_v} \ (kPa/K)
\]

with the pressure \( p \) in kPa.

On the other hand the vapor pressure deficit is given by

\[
\delta e = (e_s - e_a), \ or;
\]

\[
\delta e = (1 - e_r)e_s
\]

where \( e_s \) and \( e_a \) is the saturated vapor pressure of air and vapor pressure of free flowing air, respectively. Where, [17]

\[
e_s = 0.13 \exp \left( 21.07 - \frac{5336}{T_a} \right) \ (kPa/K)
\]

where \( T_a \) is the air temperature in kelvins.

and therefore

\[
m = \frac{de_s}{dT_a} = \frac{693.68}{T_a} \exp \left( 21.07 - \frac{5336}{T_a} \right) \ (kPa/K)
\]
FIG. 2: The required dedicated area of evaporation as function of the power with a reference osmosis pressure \( \Pi_0 = 1 \times 10^6 \) Pa., and some mechanical efficiencies in the conversion.

By calculating the evaporative mass \( \dot{m}_e \) from Eq.(15), then the evaporative area required can be evaluated with Eq.(16)

- Discussion.

To obtain some idea of the shape of the curves predicted by Eq.(16), we assume some typical values of the parameters: diameter of the pipe \( D = 0.02 \) m; a reference salinity similar than seawater i.e., \( s_0 = 3.5\% \); \( g = 9.8m/(s^2) \); \( c_p = 4.2 \times 10^3 \) J/(kg)(K) for a aqueous solution with similar properties than water, and \( \Delta T \approx 10 \) K; \( \nabla s \rho = 0.7\) kg/(m\(^3\))(/%); \( \nabla T \rho = -0.13 \) kg/(m\(^3\))(K) l pipe friction coefficient \( f = 0.012 \); \( H = 2 \) m; \( T_a \sim 298 \) K; \( m \sim 0.18 \) kPa/K; \( \lambda_v = 2.2\) MJ/kg; \( e_r \sim 0 \) (assuming continuous removal of vapor and then maximizing evaporation); and then \( \delta_e = e_s \) with \( e_s \sim 3\) kPa; \( u_w \sim 3 \) m/s; \( \gamma = 7.4 \times 10^{-2} \) kPa/(K) taking atmospheric pressure of 100 kPa. The resulting curves are shown in Fig. 2 considering an average irradiance \( R = 20 \) MJ/(m\(^2\))(day) and a reference osmotic pressure \( \Pi_0 = 10^6 \) Pa. According with this Figure, it is easy to see that for practical realizable collectors with areas no larger than 5-10 m\(^2\), the power which an osmotic thermosyphon could handle will be around 400 W. For the sake of comparison, the same figure is depicted in Fig. 3 compared with the required evaporative area if the thermosyphon is only driven by difference of densities by the salinity gradient.[1]

III. SUMMARY OF RESULTS AND CONCLUSIONS

The basis of a novel alternative approach for a thermosyphon driven pressure retarded osmosis (PRO) from salinity gradient induced by solar evaporation at collectors was discussed. It was shown that a pressure retarded osmosis-driven thermosyphon is able to work with downward heat transfer and reduced area of evaporation compatible with roofs and collectors and eliminating the need for dilution with volatile compounds.

NOMENCLATURE

- \( A_s = \) area of evaporation
- \( c_p = \) heat capacity
- \( e_s = \) saturated vapor pressure
- \( e_a = \) vapor pressure free flowing air
- \( \delta e = \) vapor pressure deficit
- \( m = \) slope of the saturation vapor pressure curve
- \( \dot{m} = \) mass flow rate
- \( \dot{m}_{we} = \) mass flow rate of evaporation
- \( \dot{m}_{wa} = \) mass flow rate of seawater
- \( \dot{m}_{w2} = \) mass flow rate of salinized seawater
- \( \dot{m}_s = \) mass flow rate of salt (diluted into the seawater)
- \( R_n = \) solar irradiance
- \( s = \) salinity %
- \( T = \) temperature
- \( T_a = \) air temperature at surface of seawater
- \( T_c = \) cold (bottom) temperature of seawater
- \( T_h = \) hot (surface) temperature seawater
- \( \Delta T = T_h - T_c \)
- \( W = \) power

Greek symbols
FIG. 3: Comparison between the required areas for evaporation if the thermosyphon is driven by the difference of densities by the salinity gradient or by a PRO process

\[ \eta = \text{PRO efficiency} \]
\[ \Pi = \text{osmotic pressure} \]
\[ \rho = \text{density of seawater} \]
\[ \gamma = \text{psychometric constant} \]
\[ \lambda_v = \text{latent heat of vaporization} \]

**subscripts symbols**

- \( o \) = nominal, reference value
- \( c \) = cold
- \( h \) = hot

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