FUZZY ADAPTIVE SIGNAL PREDISTORTER FOR OFDM SYSTEMS

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ABSTRACT

This work develops an adaptive High Power Amplifier (HPA) predistorter that applies the fuzzy logic and fuzzy set theory. As Volterra and Fourier series models or Neural Networks, a fuzzy logic system (FLS) is a nonlinear function approximator and we demonstrate its ability to compensate the nonlinear distortions for orthogonal multicarrier transmitters and relay stations. When compared with Volterra or Fourier Series predistorters, the main features of the fuzzy predistorter are low computational complexity and simple design. The good performance of the proposed predistorter is compared with that of a Fourier Series predistorter via simulations applied to the DVB (Digital Video Broadcasting) standard.

1. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) systems are particularly adapted to terrestrial transmission, such as Digital Audio and TV Broadcasting, and other multipath channels. For instance, for Mobile Broadband Systems [1] in July 98, the IEEE 802.11 standardisation group decided to select OFDM [2] as the basis for their new 5 GHz standard, targeting a range of data rates from 6 up to 54 Mbps. Also, European research projects study the application of OFDM to offer capacities up to 150 Mbps to fully mobile users in various environments [3]. However, multicarrier systems are more sensitive than the single-carrier systems to the presence of non-linearities since their peak amplitudes are more important than those of single carrier. Such non-linear phenomena often result form driving the HPA near saturation in order to maximise power output and power efficiency. Power amplification is useful in broadcasting applications and when working in high frequency bands because of the greater attenuation. And therefore, maintaining the linearity requirement without significantly reducing power efficiency is a main issue in RF power amplifier design, and necessitates linearization of the amplifier response.

There are a number of linearization techniques available [4]. One of them is predistortion, which takes place at the transmitter and intends to reduce the effects of the HPA before the noise affects the transmitted signal. In general, predistortion schemes simplify (and in some cases eliminate) equalization at the receiver, which is interesting in point-multipath systems such as terrestrial TV broadcasting. Predistortion is categorized as data predistortion [4] and signal predistortion [5] depending on where the predistortion process is inserted.

Data predistortion involves predistorting the signal constellation. On the other hand, the so-called signal predistortion is commonly used to cancel the memoryless nonlinearities of HPA by inserting the predistorter after the pulse shaping filter and before the HPA. Signal Predistorters are information independent and are suitable for either base-band or intermediate frequency or even radio-frequency. Thus, they can be used both in transmission and in low cost non-regenerative repeaters. In spite of this, works on signal predistorters for OFDM are recent and limited in the literature [5-6]. This fact is one of the motivations for the present work. Additionally, digital TV, as a result of its increased data rates, suffers from the cliff effect. This causes the received signal quality to be very sensitive to variations and drifts the HPA characteristic. For this reason analytical predistortion is not a good solution and this work develops an adaptive digital predistorter with autocalibration capability. Finally, low complexity and low cost is a main issue because digital predistorters of OFDM signals involve high sampling rates and much of the effort has to be devoted to the analog to digital converters.

2. PROBLEM FORMULATION

The predistorter works in IF frequency; in this way, the system is suitable for both transmitter and relay station. Consider that \( x(t) = m_1(t) \exp(j \theta_1(t)) \) is the OFDM signal that must enter the HPA, where \( m_1(t) \) is the signal modulus. In order to avoid distortion \( x(t) \) is predistorted into \( y(t) = m_y(t) \exp(j \theta_y(t)) \) and then \( y(t) \) is applied to the HPA, resulting \( z(t) = A[m_y(t) \exp(j \theta_y(t)) + \phi[m_y(t)]]. \)

Since we require \( x(t)=z(t) \), the predistortion function must fulfill: \( m_1(t) = A^{-1}[m_y(t)] \) and \( \theta_1(t) = \theta_y(t) - \phi[m_y(t)] \).

Thus, the amplitude predistortion is accomplished by approximating the inverse function of the AM/AM response, \( A^{-1} \), and the phase predistortion is executed by approximating the AM/PM response of the high power amplifier, \( \phi \), then subtracting it from the input phase. Figure 1 depicts a simplified block diagram of the proposed adaptive predistorter, where \( e_{\text{mod}}(t) \) and \( e_{\text{phase}}(t) \) are the error signals employed for the adaptive estimation of \( A^{-1} \) and \( \Phi \) respectively.

To simplify the scheme, in figure 1 up and down converters have been omitted. Note, however, that the predistorted data is up-converted, amplified and filtered before transmission. At the same time, the output of the amplifier is down-converted and sampled for use in the predistorter adaptation. We assume that the predistorter/HPA interaction is memoryless. This assumption is valid because the SAW filters used in the IF and RF sections have group delays that are far less than a symbol time and thus introduce no ISI into the system. Finally, for a digital implementation of the predistortion A/D and D/A converters are to be placed at the input and output of the predistorter, as such they are not shown.

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respectively. The A/D stage performs IF sampling and clips the OFDM signal to \( \pm 3\sigma \), where \( \sigma \) is the standard deviation.

Several memoryless predistorters have been proposed for modeling nonlinear systems. These techniques include Volterra series, wavelet networks, neural networks, Fourier Series, etc. See [7-8] for a review. The key issue in adaptive system identification is to find the best model structure within which an optimal model has to be found by using an appropriate adaptive algorithm. Within the possible model structures, in next section we present the fuzzy logic system (FLS), which, in contrast to other techniques, does not perform a nonlinear black-box modeling. The FLS is a model-free universal approximator [9] that can be formulated as the linear combination of \( N \) kernels \( \theta_p(x) \) that are weighted by coefficients \( c_p \):

\[
y(x) = \sum_{p=1}^{N} c_p \theta_p(x) = c^T \phi(x)
\]

We note that \( N \) is known as the order of the system. In the designed FLS, each kernel directly controls the approximation of a portion of the predistortion function and this is one of its strongest points in comparison with other possible solutions.

### 3. FUZZY PREDISTORTER

Several memoryless predistorters based on Volterra Series, Fourier Series or Neural Network modeling can be found in the literature. Their goal is to approximate \( y(x) \) from a set of samples \( \{x, y(x)\} \) such that the error power \( E = \sum_{i=1}^{N} (x_i - y_i)^2 \) is minimized. Volterra modeling is not suitable for real-time implementation due to their computational complexity and slow convergence rate. On the other hand, Fourier Series modeling [7] presents faster convergence due to its exponential kernels instead of the polynomial kernels of the Volterra series modeling. Thus, Fourier Series modeling will be our basis for comparison. The authors leave the comparison with Neural Network predistorters for advanced stages of the present work.

Fuzzy logic and fuzzy set theory are used in a variety of fields and recently are being applied in engineering for function approximation purposes [9]. The most important advantage of using fuzzy basis functions or kernels, \( \theta_j(x) \), rather than polynomials, neural networks, etc., is that a linguistic IF-THEN rule is naturally related to a fuzzy basis function. In other words, the fuzzy basis functions provide a general framework to translate abstract concepts into computable entities: \( y=f(x,"\text{linguistic information}) \). In this application, the linguistic knowledge refers to any previous information about the predistortion curve.

A fuzzy system is a set of IF-THEN rules that map inputs to outputs. Each fuzzy rule defines a fuzzy patch or subset of the input-output state space \( XX \times YY \). An additve fuzzy system approximates a function \( f:XX \rightarrow YY \) by covering its graph with rule patches and averaging patches that overlap. Let us consider figure 2, where the function \( f \) represents in general terms both the AM/AM and the AM/PM predistortion curves. First, the input/output range has to be established, in this work both ranges are normalized so that \( X_{\text{min}} \) and \( Y_{\text{max}} \) are equal to 0.5 and \( Y_{\text{min}} \) and \( Y_{\text{max}} \) are 0. The input and output ranges are quantified by \( N \) equally spaced fuzzy sets, which, in order to obtain a low computational system, are designed as complementary triangles. In other words, in each region \( Ri \) the sum of two triangles is equal to 1.

![Block diagram of the proposed adaptive predistorter](image)

The correspondence between input and output fuzzy sets, \( A^i \) and \( B^j \) determines the set of IF-THEN rules

\[
R_j : IF \ x \ is \ A^i \ THEN \ y \ is \ B^j \ \ \ \ j=1\ldots N
\]

Data clusters can define or center the set of rules. However, the main advantage of the fuzzy system is to be able to establish the rules from expert knowledge and not only from data-based knowledge. For instance, the knowledge of the predistortion function (in analytical form or given by a set of input-output points) can be incorporated in the IF-THEN rules and provide a good initialization. Then, the fuzzy system is fine-tuned by means of a learning process in the same way as the other nonlinear models are. Additionally, if some knowledge of the HPA drifts is available, it can be easily incorporated into the rules.

This paper does not get into a detailed description of fuzzy sets and logic theory and we refer to [9-10] for a full explanation. A very widely used FLS is the one with singleton fuzzification, product implication, correlation-product inference and centroid defuzzification. This FLS is the one used in this work and can be formulated as expression (1), where the fuzzy basis functions or kernels are given by (2) and the weights \( c_p \) are the centers of output fuzzy sets \( B_p \). We note that expressions (1) and (2) are a direct result of the mathematics of fuzzy logic and of the use of complementary fuzzy sets

\[
\theta_p(x_n) = A_p(x_n)
\]

Regarding the computational complexity, the one of a FLS differs from Volterra or Fourier Series modeling in two aspects. First, \( N \) is the number of IF-THEN rules that are parallelly fired for each input sample. Thus, in contrast to Volterra or Fourier Series modeling, an increase in the order \( N \) does not imply an increase in the number of sequential operations to be carried out. Second, from (1) and (2) we note that the proposed fuzzy predistorter can be viewed as a collection of localized sub-systems. For instance, figure 2 shows that each input value \( x_n \) just activates 2 input and 2 output fuzzy sets; thus computational complexity is greatly reduced.

As commented previously, the parameters of the fuzzy system can be fine-tuned by means of an adaptive algorithm. If these parameters are the centers \( c_p \), both LMS (Least Mean Squares) or RLS (Recursive Least Squares) algorithms are suitable. Our goal is to obtain a simple and easy to implement DSP-system. Therefore, the LMS is chosen as equated in (3)

\[
c(n+1) = c(n) + \mu(c(x) - \theta(x_n))
\]
where $c(x_n)$ is either $e_{\text{modulus}}(t)$ or $e_{\text{argument}}(t)$, both depicted in figure 1, and employed depending on the predistortion function to be adapted. Note that if $x_n \in R_1$, then equation (3) simplifies to

$$c_k(n+1) = c_k(n) + \mu e(x_n) A_k(x_n) \quad k = l, l+1$$

As the designed FLS is based on decoupled or incoordinated kernels the system presents a fast convergence. If each fuzzy set $A_i(.)$ in figure 2 is partitioned into two disjoint triangular fuzzy sets, each one pertaining to a different region $R_l$ and $R_{l+1}$, then the convergence improves and is comparable to that of Fourier Series systems (which are faster than Volterra Series [7]). However, the rule set augments to 2(N-1) rules.

4. SIMULATIONS

This section shows some simulation results when triangular fuzzy sets are employed; thus, performing a piece-wise linear predistortion. Figure 3 plots the spectral density of the OFDM signal at the output of the HPA in two cases: with and without fuzzy predistortion. The fuzzy predistorter reduces the intermodulation products that are generated out of band by the non linearities of the HPA. Specifically, the intermodulation products at 4.3 Mhz from the central frequency are reduced in 8.3 dB. This improvement is useful to reduce the order or selectivity of the filters to be applied to the OFDM signal before going through the HPA.

Next figures compare the fuzzy predistorter with the Fourier predistorter, which bases on trigonometric Series. The distortion of AM/AM has been modeled in both predistorters with 8 coefficients while the AM/PM predistortion has been modeled with 16 coefficients.

The results of the Instantaneous Square Error (dB) for both predistorters are shown in Figure 4 and Figure 5 respectively. The simulations were made with 10 Monte Carlo runs, using 10 OFDM symbols each. The modus 2K of the DVB-T standard has been simulated. The A/D converter uses a sampling frequency equal to 2.68x, where $B_x=8$MHz, and introduces quantification noise that is modelled by a SNR = 30 dB. Therefore, the problem to solve is a noisy predistortion approximation. Initially, the weights are set to zero. The Instantaneous Square Error (ISE) of the AM/AM predistortion is similar for both systems (-37.7 for Fuzzy and -38.2 for Fourier) but the ISE of the AM/PM predistortion is better for the fuzzy predistorter (for fuzzy -33.57dB and -29.65 dB for Fourier). Although both models present the same convergence, note that Fourier modelling error power presents a greater variance or misadjustment.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Fourier</th>
<th>Fuzzy</th>
</tr>
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<tbody>
<tr>
<td>Basis Functions</td>
<td>$[N_{am}+N_{pm}]M+$</td>
<td>$[2M+2A+2S]*$</td>
</tr>
<tr>
<td>Estimation AM/AM and AM/PM predistortions</td>
<td>$[N_{am}+N_{pm}]M+$</td>
<td>$[2M+1A]^2*$</td>
</tr>
<tr>
<td>Adaptation of predistortion weights</td>
<td>$[N_{am}+N_{pm}]M^*$</td>
<td>$[3M+2A]^2*$</td>
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</tbody>
</table>

Table 1. Computational burden of Fourier and Fuzzy predistorters. (M: multiplication, A: addition, S:subtraction, $N_{am}, N_{pm}$ : number of coefficients for Fourier AM/AM and AM/PM predistorters).

In the AM/AM error curve some error peaks can be distinguished. These peaks are caused by the extremes of the Rayleigh distribution that presents the modulus of the OFDM signal. A feature of the Fuzzy predistorter is that it can diminish the magnitude of these peaks by associating a greater adaptation gain, $\mu$, to the weights in charge of approximating the predistorter in the high voltage zone.

Figures 6 compares the results offered by the fuzzy system with the AM/AM distortion and the AM/PM distortion of the HPA. Figure 7 shows the results obtained by the Fourier system.

Finally, Table 1 shows the computational load for each input sample $x_n$. In contrast to the Fourier predictor, note that the number of operations that carries out the fuzzy system is not sensitive to the approximation order.

5. CONCLUSIONS

This work studies the problem of signal predistortion when applied to DVB systems. The proposed low computational

![Figure 2. Fuzzy Logic system as function approximator](image-url)
solution starts from the fuzzy set and logic theory and it is compared against a Fourier Series predistorter when triangular fuzzy sets are used. The simulated fuzzy system performs a piece-wise linear predistortion. Further work will study other fuzzy set options and compare them against other possible predistorters such as those based on Neural Networks.