Master thesis

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Topic: Increase of the body in white geometrical accuracy through fixtureless body shop

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<th>Units</th>
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<tbody>
<tr>
<td>$x_{\text{nom}}$</td>
<td>mm</td>
<td>Nominal dimension</td>
</tr>
<tr>
<td>$x_{\text{min}}$</td>
<td>mm</td>
<td>Lower limit</td>
</tr>
<tr>
<td>$x_{\text{max}}$</td>
<td>mm</td>
<td>Upper limit</td>
</tr>
<tr>
<td>$T_x, T_x^*$</td>
<td>mm</td>
<td>Lower and upper increments/decrements</td>
</tr>
<tr>
<td>$X, Y, Z$</td>
<td>-</td>
<td>Orthogonal directions</td>
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<tr>
<td>$\Phi$</td>
<td>°</td>
<td>Angle of a vector with respect to its preceding</td>
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<td>$H_x, H_y, H_\phi$</td>
<td>mm, °</td>
<td>Vector loop functions</td>
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<tr>
<td>$L$</td>
<td>mm</td>
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<td>nr. cmp.</td>
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<tr>
<td>$\Delta$</td>
<td>-</td>
<td>Increment</td>
</tr>
<tr>
<td>$t$</td>
<td>mm</td>
<td>Tolerance</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>mm, °</td>
<td>Population standard deviation</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mm, °</td>
<td>Population mean</td>
</tr>
<tr>
<td>$F$</td>
<td>mm, °</td>
<td>Assembly feature function</td>
</tr>
<tr>
<td>$S$</td>
<td>-</td>
<td>Tolerance sensitivity matrix</td>
</tr>
<tr>
<td>$s_{ij}$</td>
<td>-</td>
<td>S matrix component</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>-</td>
<td>Gradient</td>
</tr>
<tr>
<td>$P$</td>
<td>-</td>
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<td>$w_j$</td>
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<td>mm, °</td>
<td>Function to optimize</td>
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<td>$g(x)$</td>
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<tr>
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<td>-</td>
<td>Lagrange multipliers</td>
</tr>
<tr>
<td>$c(T)$</td>
<td>€</td>
<td>Global cost-tolerance sum function</td>
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<td>$N$</td>
<td>nr. meas.</td>
<td>Sample size</td>
</tr>
<tr>
<td>$y$</td>
<td>mm, °</td>
<td>Value of a measure</td>
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## Symbols and Abbreviations

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<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\bar{y}$</td>
<td>mm, °</td>
<td>Sample mean, central tendency of the measurements</td>
</tr>
<tr>
<td>$s$</td>
<td>mm, °</td>
<td>Sample standard deviation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>Risk</td>
</tr>
<tr>
<td>$t_{a,n-1}$</td>
<td>-</td>
<td>t-student coefficient</td>
</tr>
<tr>
<td>$k$</td>
<td>mm, °</td>
<td>Absolute precision</td>
</tr>
<tr>
<td>$p$</td>
<td>-</td>
<td>Mean-relative precision</td>
</tr>
<tr>
<td>$B$</td>
<td>€</td>
<td>Benefits</td>
</tr>
<tr>
<td>$C$</td>
<td>€</td>
<td>Assembly production costs</td>
</tr>
<tr>
<td>$I$</td>
<td>€</td>
<td>Generated incomes</td>
</tr>
<tr>
<td>$P_a$</td>
<td>nr. asm.</td>
<td>Total production</td>
</tr>
<tr>
<td>$y_a$</td>
<td>-</td>
<td>Assembly yield</td>
</tr>
<tr>
<td>$i_a$</td>
<td>€/asm</td>
<td>Incomes per assembly</td>
</tr>
<tr>
<td>$c_a$</td>
<td>€/asm</td>
<td>Cost per assembly</td>
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<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>BIW</td>
<td>Body in white</td>
</tr>
<tr>
<td>BS</td>
<td>Body shop</td>
</tr>
<tr>
<td>WC</td>
<td>Worst case</td>
</tr>
<tr>
<td>RSS</td>
<td>Root square sum</td>
</tr>
<tr>
<td>ProeK</td>
<td><em>Produktionseffizienz in der Kleinserie</em></td>
</tr>
<tr>
<td>MIG</td>
<td>Metal inert gas</td>
</tr>
<tr>
<td>USL</td>
<td>Upper specifications limit</td>
</tr>
<tr>
<td>LSL</td>
<td>Lower specifications limit</td>
</tr>
<tr>
<td>asm</td>
<td>Assembly</td>
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1 Introduction

1.1 Motivation and objectives

Nowadays, small series in the automotive manufacturing industry are becoming more and more important, mainly because of the development of the electric car and also due to the shorter life cycles of products. In such a competitive field, where technologies are continuously being improved and new alternatives presented, new and more flexible manufacturing methods are required.

Typical production methods in this industry present a lack of flexibility because of the use specific fixtures and jigs, which normally can just be used in one or few designs.

Since high investment costs are not bearable for small series, economic solutions need to be established, in order to satisfy both those manufacturers whose production volumes are small and those who develop new prototypes.

As a solution, the Chair of Production Engineering of E-Mobility Components (PEM) started the ProeK project (Produktionseffizienz in der Kleinserie) in which developed a flexible concept which enables shifting the intelligence from the jig into the part, as well as an automated laser welding process. This jigless body shop concept, apart from making the manufacturing more flexible, further makes it faster and reduces the investment costs.

Figure 1.1: Jigless structure prototype.

The basic idea is that the body parts are: first, cut by laser from metal sheets, and secondly bent to get the desired geometry. New design variations can be easily, fast and economically created modifying the CAD designs. Each piece integrates joints that act as self-positioning elements before welding, which is what makes the absence of proper fixtures possible.
Then comes the final part, the welding process, which is done by laser and solves the challenges and limitations that other methods such as spot welding and MIG welding present: distortion, high maintenance, need of flanges, etc. Moreover, it allows automation, agility and good union quality. To achieve this goals, a diverse set of laser welding parameters have been developed in previous works.

Within the Proek, **aim of this Master thesis**, as referred in its title, is to increase the *body in white* (BIW) geometrical accuracy through fixtureless Body Shop. In other words, **to deal with the tolerance issue** in this new manufacturing method presented by the PEM.

### 1.2 Work structure

As a starting point, apart from taking into account the previous thesis of the ProeK, a vast **literature review** concerning the following topics will be exposed in **chapter 2**: body shop, fixtures, tolerance chains and statistical methods.

In **chapter 3**, a study of the **influence of fixtures** to tolerance chains in the body shop as well as the relevance of the disuse of them to the BIW dimensional accuracy will be done.

Then, **tolerance compensating mechanisms** for fixtureless body shop will be developed, leading to new design requirements, considering the transference of fixtures functionalities into parts. This mechanisms will be presented in **chapter 4**.

As an **overview**, conclusions and suggestions for further investigation in the topic will be displayed in **chapter 5**:

In future works, having defined those features in chapter 4, a **tolerance management concept** for interlocked body parts should be developed, taking into account everything from the point of view of the assembly (both parts manufacturing and later mounting), and finally, as a last step, **practically implemented** within the ProeK-Project.
2 Literature review

In order to understand and successfully formulate the problem that is faced in this Master Thesis, it is required to previously define and study the elements that participate in it or that can be used as an approach to it.

These are the **body shop**, the **fixtures** used in it, the **tolerance chains** to take in consideration and the **statistical methods** to deal with them.

2.1 Body shop

The body shop (also referred as BS in this work) is the workshop of the automotive industry where the manufacturing process of assembling the **body in white** of a car is done. The **body in white** (or **BIW**) refers to the state of a chassis of a vehicle whose parts have just been mounted, i.e. before it is painted and gets any other parts (like the powertrain, for example) or subassemblies attached to it. In other words, the frame structure itself.

However, as well as diverse typologies of chassis regarding distinct objectives exist (light weight, high stiffness, etc.), a great amount of different manufacturing processes have been developed too, regarding also their own objectives (low cost, high quality, etc.). In most of the cases, frame typology and fabrication processes are designed and developed together to fit those objectives.

The fundamental aspect to be taken into account in any industry is the **market**. From it, the mentioned production objectives are derived. So, as it changes, they do change too. The automotive industry is constantly adapting to new realities, due to the variation of the factors that affect it. As external factors, environmental regulations and the changing trends of the customers’ demands stand out, focused today on a wider variety of products regarding different powertrains, for instance. This redounds in shifting from high volume, low mix production strategy into a high volume, high mix\(^1\), which means going from **large series to smaller series**.

The trouble is that the actual fabrication processes limits the overall flexibility in terms of product volume and applications. This implies adjusting or developing new manufacturing processes able to build more dedicated and flexible platforms, and makes it possible to face future product demand in short term.

With the focus on small series, the PEM Institute developed a frame building method that meets this search of flexibility and then the increasing amount of customer demands. This method and the classical manufacturing trends are compared in the following subsections.

\(^1\) Omar p. 178
2.1.1 State of the art

Nowadays, the most popular manufacturing process to build a body in white of a car consists of **joining previously stamped metal sheets**. Today, robots perform almost all manufacturing operations in advanced automotive factories.

- **Stamping:**

First of all, the vehicle body sheet panels are stamped using mechanical and hydraulic presses to form the vehicle different structures. These **stamped body parts** go to the joining process to form the vehicle **main sub-assemblies**.

![Figure 2.1: Example of stamped sheet panels.](http://www.htmetalstampings.com/automotive-die-stamping-industry-overview-a-73.html)

- **Joining process**

There are different technologies involved in the joining process, such as various welding technologies and adhesive bonding, and they are applied in each location depending on its concrete requirements.

For instance, **spot welding** is a very effective joining process that can be used with different materials and thicknesses, by using different electrodes and varying other factors such as current and time application. Its drawback is that it requires access by both sides of a joint, which means a problem in terms of accessibility with some designs.

Another commonly used technology is the **MIG welding**, which solves this problem as just one side access is needed. Its handicap is that it provides the area around the weld with a large...
amount of heat, which is not desirable as it causes deformations. Moreover, it requires a specific gas atmosphere to prevent the weld seam from contamination or oxidation.

On the other hand, the **adhesive bonding** presents added advantages that are making it have more and more applications in the automotive manufacturing industry. It permits to join dissimilar materials with folded joints, which increases its moment of inertia hence increasing its stiffness.\(^3\)

Once the different panels are joined, the BIW goes through the painting process, which conditions its surfaces with a corrosion resistance finish and spray paints it in several steps.

### 2.1.2 PEM Institute - ProeK

The main problem with this method is that it is only cost effective when implemented in large series production, and as exposed before, the market demands today an economical method to construct car chassis in smaller series.

The fundamental challenges that it faces when trying to bring it to the short series are that it does not allow enough **flexibility in the production**, as the stamping process, which implies high investment costs, is at the beginning of the whole process.

Consequently, this leads to a lack of **cost-effectiveness**, which, as said before, is basic. Vehicle shells are locked not only at the start of the manufacturing process, but through the product life too. Moreover, due to the own characteristics of the method, different designs will require different fixtures to hold the different panels in space, which results in a great amount of processing steps apart from larger number of non-value added efforts in stacking, de-staging, staging, and transporting tasks.

Lastly, final assembly processes depend on **manual operations** which require further tooling and **fixturing solutions** to allow facilitate the operators work.

A more integrated solution to the fixturing problem is to incorporate, during the stamping process, fixturing and clamping reference points and features through the panel shape and geometry, which ensure that the panels will only fit the right way.\(^4\)

Taking all these limitations into consideration the PEM Institute set the **ProeK** project, standing for *Produktionseffizienz in der Kleinserie*. As its name indicates, it looks for improving the efficiency in the production of small series, developing a concept that can adapt to process changes quickly and is also cost-effective, by transferring the functionalities from the proper jig into the chassis parts to be later welded through an automated laser welding process.

\(^3\) Omar p. 180  
\(^4\) Omar p. 183
The concrete idea is to transform the traditional designs and construction methods for structures explained above, into structures produced from metal sheets where the different parts are first cut and second bent with the desired geometry, and afterwards assembled without the help of any fixtures. As a puzzle, the tooth-cutout connections incorporated in the design of each part make them fit together in the correct position with high accuracy. These teeth and cutouts are distributed at the edges and surface of the parts respectively. With this solution the design of a structure could be changed as many times as desired, is completely flexible, and since the tools used in the production do not change, an idea could be realized overnight.\[^5\]

It may be true that there are other fabrication methods suitable for small series, like metal tubular frames or composite materials monocoques. The issue with them is that, although they are intended to be used in short series, they still do not end up to be economical enough. They are usually applied when there are great margins of profit, such as in luxury cars, or great investment available, such as in competition, which are not the case of study.

\[^5\] Elektromobilität p. 171–181
2.2 Fixtures

Workholding devices

Nowadays, the manufacturing processes are required to provide such high levels of accuracy and productivity that they cannot be lowered, in order to maintain competitiveness. Because of that, the use of workholding devices becomes vital. These are tools that allow to firmly attach elements in the adequate position during these manufacturing processes.\(^6\)

With them, it is possible to increase the productivity, by reducing the positioning time and the later verification, and by increasing the working speed given the firm and secure achieved subjection. Furthermore, the obtained accuracy with its use permits to create interchangeable pieces, i.e. pieces that comply with the same set of specifications and therefore can be assembled indistinctly the chosen unit, which redounds in a decrease of the discarded parts and so in global costs. The same way, it improves the safety at work and allows to dispense with the ability of the workers, enabling the machining of heavy and complicated shape pieces.\(^7\)

Aiming to fulfill these objectives, the workholding devices consist of four main elements: the body or structural element, whose fundamental purpose is to be the frame of the rest of the elements; the locating mechanisms, that let the workpiece be placed in the correct position; the clamping devices, in charge of restricting the movement of the pieces if the locating mechanisms were not capable enough to do so, in order to keep the required position, and finally the fixing elements, to attach the workholder to the machine.\(^8\) There can also be other elements such as bushes.

\(^6\) DTE  
\(^7\) Paragon  
\(^8\) Scallan
Jigs & Fixtures

Some fabrication processes require the operating tool to be guided and/or that the workpiece lies in a certain position and orientation, both with respect to other pieces and to the workbench.  

Within the workholding devices, there are the jigs and the fixtures, which are tools with a more or less specific design. On the one hand, a jig is a guidance device that provides the tool with the correct position to perform the manufacturing operation (for example templates), usually classified in drilling or boring jigs. In other words, a tool used to control the motion and position of another tool.

On the other hand, a fixture is a holding device whose main function is to positively locate the workpiece, and which can be used for a wider range of processes, such as milling, broaching, planning, grinding and turning.  It allows a secure mounting and ensures conformity, accuracy, precision, reliability and interchangeability of the finished parts, while reduces the requirement for skilled labor (indispensable in automated processes) and thus the risks, the working time and finally the labor costs.

---

9 Snyman
10 Scallan
11 Singh
Classification

Different jigs and fixtures can be applied in several distinct processes and thus classified. For instance, there can be found external and internal machining applications, such as flat, cylindrical or irregular-surface machining in the first case, and hole machining in the second. Moreover, there are also non-machining applications, for example, for assembly, inspection and finishing processes, among others.

Figure 2.3: Different manufacturing processes may need specific fixtures

The assembly fixtures are what is of interest in this work, especially the welding ones, but there are also fixtures for soldering and various mechanical-assembly processes (e.g. riveting, stapling, stitching, pinning). Normally, in the case of small series, specific auxiliary structures are built to place the parts and restrict their motion, or modular fixtures combining permanent tools are mounted. Having to create this complementary frame implies a high investment in terms of both money and time for each desired design.
Regarding this, the aim of the **ProeK fixtureless concept** is to prescind from this constructions and thus spare its additional cost. The concept is thought to be implemented in laser-cut and press-bent flat metal sheets and consists of connecting the different parts that will form the frame altogether like a puzzle through specific **interlocking joints**, which results in a **pre-assembled structure**, ready to be laser-welded (they act as **mechanical assembly fixtures**). This means that the fixtures are integrated in the pieces, so this is why the concept is considered to be **fixtureless**. As the fixtures remain with the assembly and do not need to be removed, operations such as transport are made more easily, because they do not interfere in it and keep “working” while the structure is being moved.

This interlocking joints idea is no other than to endow the pieces with **teeth and cutouts**, which interlock and maintain the parts together in position. “The basic geometry is a rectangle with the same nominal dimensions than the tooth that will fit inside, same length than the width of the tooth, and same width than the thickness of the metal sheet with which the male part is made. In order to create the different fits between the parts and look for the best compromise between assembly time and welding quality, two dimensions are added to these two basic

---

12 CAT07e; ETSEIB Motorsport, UPC
dimensions of the cutout, longitudinal increase and transversal increase.\textsuperscript{13} Some of the teeth include a round shape at the end, which is twisted to create a pair of burrs to hold the pieces together, and then bent to eliminate it. This is done manually by the operator.

Note that not all the teeth need to feature this semicircle, as for example one, two or three may be enough, depending on the case.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.5.png}
\caption{Detail of a tooth with the semicircle mechanism, and the result of the joint after removing this part.}
\end{figure}

\textsuperscript{13} MA Pérez
Figure 2.6: Assembly of two pieces through interlocking joints, and resulting joint once the round-shaped ends are removed. The burrs derived from this operation become the clamping elements.

Relating the ProeK’s proposal to the previous explained elements of the workholding devices, the own pieces with the teeth-cutouts joints act as the body, giving consistence to the assembly; the proper joints and also the edges of the pieces play the role of locating mechanisms, while some teeth, the ones that have the semicircle layout, work as manual clamping devices, which brings in economy and flexibility.

This fastening has to ensure enough resistance till the manufacturing processes to be done are performed, as well as other such as the transportation of the connected pieces and its manipulation by the operators.

There are no fixing devices to join the parts to the workbench, due to the fact that this method just looks for joining the parts together. To fix them to the workbench, other fixtures are needed.

The obvious benefits that this concept offers are simplicity, integration and lightweight, re-doudning in maneuverability, a reduction in the mounting time, the elimination of fixtures maintenance and finally economy. As a challenge to be solved, the tolerance issue must be studied.
2.3 Tolerance chains

Dimensional variations

As we live in the real world, there are a lot of factors that are out of our control, and those involved in each and every manufacturing process are not an exception.

These factors, or sources of variation during manufacturing, have diverse origins and thus demand different approaches to deal with them. Mainly, they are:

![Variation Sources Diagram]

Referred to the tooling, the wear of the tools as well as the use of fixtures play a major role in the dimensional accuracy issue, so that is why in the next chapter the effect of its use and disuse will be studied.

As for the material, it doesn't matter if it has been pre-processed or not, it will have imperfections in any case. On the one hand, if it is a raw material, then it will obviously have distortions, on the other, even if it is a labored material, for example, a flat metal sheet or a cylindrical block, it will have also been affected by several factors till it has been produced. Moreover, materials tend to present some kind of non-uniformities along themselves.

Another important factor to be taken into account that introduces uncertainty into the process is the human factor. Having to perform tasks manually, such as set up’s of the workpieces in their workplace, usually present more inaccuracies than if the process was done automatically.

Nevertheless, this does not imply that the equipment itself does not introduce variations: as it has to repeat several times the same movements, inequalities between two apparently equal parts after the same operation may appear. This can be attributed to the flexibility of the machine elements. Furthermore, using different units can contribute to that too.

Finally, environmental conditions, such as temperature and humidity, may have an effect in the processes.\(^{14}\)

\(^{14}\) Jackman, Stackup
Tolerances

Then, it is obvious that it is impossible to fabricate parts with the precise desired dimensions. These variations in the pieces are additive and may cause problems when mounting them together, i.e. not being able to fit in the assembly or resulting in an assembly with performance issues. As a consequence of this, there is material in which money was invested that needs to be discarded and/or assemblies under the required quality are obtained.\textsuperscript{15}

This fact makes having a way to predict the percentage of correctly built assemblies necessary, so the convenient decisions can be made. The idea is to control the variation factors to limit the dimensional discrepancies at a reasonable cost, but this does not mean that the parts have to be perfect.

These decisions go through choosing a right set of tolerances for the components that allow to have acceptable resulting assemblies. Tolerances are allowable variations in dimensions, the permitted sizes that features can have. They are the limits that discriminate between what is and what is not acceptable, and are normally expressed as increments and decrements respect the nominal dimension \((x_{\text{nom}})\) a feature should have, resulting in a lower \((x_{\text{min}})\) and an upper limit \((x_{\text{max}})\). The common way to express it is the following:

\[
x_{\text{nom}} \pm T_x \equiv [x_{\text{min}}, x_{\text{max}}]
\]

![Diagram showing tolerances](image)

Figure 2.7: The acceptance fraction (green zone) of the variable \(x\) goes from \(x_{\text{min}}\) till \(x_{\text{max}}\). It is expressed with reference to \(x_{\text{nom}}\) using the increments/decrements \(T_x^-\) and \(T_x^+\).

Usually, the lower limit is a smaller dimension than the nominal dimension and the upper is bigger, because the tolerances are a decrement (minus sign) and an increment (plus sign), but

\textsuperscript{15} GD&T
there may be the case where they are either increments or decrements, so the nominal dimension is not within the acceptance range:

![Tolerance representations](image)

Figure 2.8: Tolerance representations where the nominal dimension of the variable \( x \) is out of the acceptance range. This is not a problem, since \( x_{\text{nom}} \) is just a reference.

Size and geometric tolerance specifications define the deviation to be allowed for different geometric attributes, such as size, form, location or orientation. They describe relationships between part features and the permissible a range an attribute should be in. This features could be defined as identifiable geometries in the parts, mainly surfaces and edges, like protrusions, holes, slots, shaft axis, etc.\(^{16}\)

### 2.3.1 Tolerance stackup

The previously mentioned tolerance issue (section 2.2) refers to the problem that has to be faced when joining several parts to form an assembly, due to this accumulation or stackup of the component tolerances. It happens when a geometry depends on other features (what is being produced in individual processes), so one or more processes may affect these individual dimensions and therefore their tolerances.

#### 2.3.1.1 Assembly models

The way pieces are assembled and their real dimensions affect the whole assembly. To study how this happens, it is useful to dispose of mathematical models that represent assembly features depending on its component variables.

To obtain these models, a graphical tool called vector loop is used, which is more widely explained in Chase 1995. Starting from the design diagram of an assembly, vectors are drawn over to represent the component dimensions or cinematically variable dimensions, forming

\(^{16}\) Jackman, G.T.
a loop or chain. These loops represent the relationship between adjacent parts, showing how they stack and consequently contribute to the final assembly dimensions.

Vectors go from part to part through the points of contact, and characterize the rotation and translation related to the previous vectors. The variation of a vector that represents a component dimension is its specified tolerance. This way, the geometry of an assembly is reduced to just the parameters needed for a tolerance analysis.

There can be closed and open vector loops. In the first case, there is an element in the assembly that can be adjusted in some way and then close the loop. In the second, an assembly feature resulting from the components stack up is described, such as a gap or a clearance. The permitted variation of these features are defined by the assembly tolerance specifications, which ensure acceptable performance.

![Figure 2.9: Example of a 2D closed loop model of an assembly\textsuperscript{17}](image)

Once the loop is obtained, the constraints it represents can be expressed as an assembly equations system, i.e. a mathematical model of the assembly to work with.

In the case of the 2D loop above, three scalar equations are deduced, one for each dimension and one for the orientations.

Equations resulting from the sum of all the components of the vectors in the $X$ and $Y$ directions:

$$H_x = \sum_{i=1}^{n} L_i \cdot \cos \left( \sum_{j=1}^{i} \Phi_j \right) = 0$$

\textsuperscript{17} Chase 1995 p.13
\[ H_y = \sum_{i=1}^{n} L_i \cdot \sin \left( \sum_{j=1}^{i} \Phi_j \right) = 0 \]

Note that the sum of the relative angles till the current vector represent the angle in the global coordinate system.

Equations resulting from the sum of all the orientation angles of all the vectors:

\[ H_\Phi = \sum_{i=1}^{n} \Phi_i = 0^\circ \]

**Closed loops**

In a closed vector loop, the expressions result directly as implicit functions, as the dependent variable which refers to the adjustability of a component that permits the chain to be shut, is inside the chain itself.

**Open loops**

When there are not adjustable elements, the loop can be drawn open, representing the difference between the beginning of the first vector and the end of the last one the clearance or other variable feature to be limited. This situation implies that the assembly chain may be expressed as explicit functions. In any case, this variable can be included in the function (as a closing vector), and then express it as an implicit function as if it was a closed loop.
2 Literature review

2.3.1.2 Accumulation models

As soon as the assembly model is deduced, it is possible to operate with it in order to be able to study the relationship between the component variations and the resultant assembly variations.

Before doing that, though, an approach over how the tolerances stack up has to be chosen. Different mathematical models regarding different assumptions (and then, different purposes) can be found. The most widely used ones are the next:

**Worst Case model**

The *Worst case* or *WC model* is used to evaluate the extreme scenarios of the stackup issue. It assumes that the 100% of the produced components are within the specified tolerances and then calculates the maximum tolerances the 100% of the assemblies will be in, when using these mentioned components. To do that, it supposes that all the component dimensions will occur at their worst limit (nominal dimensions + tolerances) at the same time, by equaling the sum of the component tolerances to the total assembly variation:

\[
 t_{asm} = \sum_{i=1}^{n} t_i
\]

\[
 x_{nom,asm} = \sum_{i=1}^{n} x_{nom,i}
\]

Where:

- \(n\): number of components in the stack,
- \(t_i\): assigned/resulting tolerance of the \(i\)’th part, or maximum variation of \(x_i\) (\(\Delta x_{i,max}\)),
- \(t_{asm}\): resulting/assigned tolerance of the stackup (consisting of \(n\) parts), or its maximum variation (\(\Delta x_{nom, max}\)),
- \(x_{nom,i}\): nominal dimension of the \(i\)’th part,
- \(x_{nom,asm}\): resulting nominal dimension of the stackup.

The second equation refers to the fact that the nominal dimension of the combination of the parts is directly its sum.

---

18 Chase 1991 p.24
The advantage of using this model is that, given a set of component tolerances, it assures the conformity of the assemblies with their specified tolerances.

The counterpart is that then, the dimensions of the components have to be verified to confirm they strictly lie within the limits, which implies high cost. Moreover, as the worst situation is being studied, it lacks of flexibility. So, when an assembly tolerance specification is set, the needed component tolerances tend to be the tightest, compared with other accumulation models, and then redounds in higher production costs too. This phenomenon increases as the number of components of the assembly grows, because as a new part appears, the other components tolerances have to be tighten to let it have its own.

Despite having all these drawbacks, however, it is normally used as a reference and as a complement to other models.

**Root Sum Squared model**

Another very commonly used accumulation method is the Root Sum Squared model. It assumes that the variations of the dimensions can be described with normal distributions, which are fully defined by two parameters, the mean (μ) and the standard deviation (σ).

This assumption is usually made when little data is available to define the manufacturing processes (parts may have not been produced yet, processes and tooling have not been selected, etc.), and it supposes that most of the produced parts are centered within the tolerance range, as tolerances are expected to correspond to 6 standard deviations (6σ). Saying that means that the tolerance given to a component contains the information of the standard deviation of its presumed normal distribution and then it is sufficient to characterize it (being the nominal dimension the same as the mean, as the shape of the distribution is symmetrical).

![Figure 2.10](image-url)  
**Figure 2.10:** Normal distribution bell, with related acceptance fractions depending on the limits.

19 Chase 1991 p.29
The **variances of the components**, which are the squares of the standard deviations ($\sigma_i^2$), can then be added to estimate the **variance of the assembly**, it is, of the whole set of parts stacked up together. The same happens with the **means**\(^\text{20}\):  

\[
\sigma_{asm}^2 = \sum_{i=1}^{n} \sigma_i^2 \Rightarrow \sigma_{asm} = \sqrt{\sum_{i=1}^{n} \sigma_i^2} \\
\mu_{asm} = \sum_{i=1}^{n} \mu_i
\]

Where:

- $\sigma_i$: standard deviation of the $i^{th}$ component,
- $\sigma_{asm}$: standard deviation of the stack,
- $\mu_i$: mean of the $i^{th}$ component,
- $\mu_{asm}$: mean of the stack.

This way, if the previous assumptions are acceptable, it is possible to obtain the normal distribution that will approximately represent the resulting stackups/assemblies, $\mathcal{N}(\mu_{asm}, \sigma_{asm}^2)$. This method takes into account that, as there will be very few parts with actual dimensions near the tolerance limit, it is of low probability that the worst case combination may happen, and this is noticeable in the shape of the assembly distribution. Compared to the WC model, component tolerances can be loosen and thus, production costs decreased.

Notice that the formulas do not refer to tolerances yet. As said before, it is a common practice to associate the tolerances of the produced parts with **-3\(\sigma\) (lower tolerance)** and **+3\(\sigma\) (upper tolerance)**, so they define their production distribution.

---

\(^{20}\) Schenkelberg
Figure 2.11: Representation of the normal distribution of the $i^{th}$ part, fully characterized by its mean ($\mu_i$) and its standard deviation ($\sigma_i$). As a convention, the tolerances are referred to its $6\sigma$ range.

Once the distributions of all the components that take part in the stack are obtained, the distribution of it can be deduced:

Figure 2.12: Note that the shape of the assembly distribution is more flattened than the ones of the components, as its $\sigma$ will be greater than the greatest $\sigma_i$. 
Then, the sum provides the distribution of the stackup. From this point, given a set of assembly tolerances, the yield or acceptance fraction can be calculated or, the other way round, given a specified yield, the needed assembly tolerances to reach it can be deduced.

Summarizing (as shown in the following diagram):

Having characterized the normal distributions of all the components of a stackup (i.e. determining their mean and standard deviation), the Root Sum Square allows to infer the normal distribution of the parts assembled together. Then, it can be used as a tool to:

- a) Know the percentage of assemblies that will be considered valid (yield) given a minimum and maximum tolerances.
- b) Know the tolerances that have to be set in order to obtain a certain yield.

**Figure 2.13:** Relationship between the parts and the stackup normal distributions, and between the limits of acceptance of the assembly and the percentage of accepted units.

In the next chapter it is explained how these values are related.
Mid-case

Although the Root Sum Square model represents usually an accurate approximation, one of its limitations is that the real distributions of the parts may not adjust perfectly to normal ones, which may cause the number of actual rejects to be higher than the calculated amount.

In order to deal with these uncertainties and perform a more conservative estimate, it is recommended to average the calculated RSS assembly tolerances with the ones obtained with the Worst Case model.\textsuperscript{21}

\textsuperscript{21} Chase 1991 p.24
2.3.1.3 Tolerance analysis & Tolerance allocation

A vital part of the product design process consists of assigning tolerances to the components dimensions, so a balance between their manufacturing costs and the proper performance of the finished product is reached:

![Figure 2.14: Usually, tightening tolerances warrant a good quality, while relaxing them may reduce the production costs.](image)

Keeping a control over the dimensions and features of the components involved in an assembly allows to control the dimensions and features of the assembly itself. From that statement, two tolerance stackup management problems are derived, each one being the inverse of the other: Tolerance analysis and Tolerance allocation.

But before explaining them, it is convenient to unify the previously explained assembly and accumulation models:

Model unification

Continuing with the 2D case introduced before, the vector \( \mathbf{X} \) is defined. It includes all the variables that take part in the stackup loop, i.e. the nominal dimensions of both lengths and angles:

\[
\mathbf{X} = (x_1, ..., x_{2n}) = (L_1, ..., L_n, \Phi_1, ..., \Phi_n)
\]

Then the tolerances of these dimensions can be expressed as their variations:

\[
\Delta \mathbf{X} = (\Delta x_1, ..., \Delta x_{2n}) = (t_1, ..., t_{2n}) = \mathbf{T}
\]

Recovering the Assembly models implicit functions (2.3.1.1):
\[ H_x = 0; \quad H_y = 0; \quad H_{\Phi} = 0^\circ \]

They can be rewritten as \( m \) explicit functions where the **assembly features** \( (F) \) that are to be limited become the \( m \) dependent variables:

\[ F = F(X) = F(x_1, \ldots, x_{2n}) = (f_1, \ldots, f_m) \]

Once they are defined, the **tolerance sensitivity matrix** \( S \) can be obtained:

\[ S = \nabla F(X)|_X = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{2n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_{2n}} \end{bmatrix} = \begin{bmatrix} s_{1,1} & \cdots & s_{1,2n} \\ \vdots & \ddots & \vdots \\ s_{m,1} & \cdots & s_{m,2n} \end{bmatrix} \]

This matrix is the Jacobian of the \( F \) functions vector evaluated at the nominal dimensions of the independent variables \( X \). Its components \( s_{ij} \) contain the information of how a variation in a variable \( x_i \) affect and make an assembly feature \( f_j \) vary, as they are the partial derivatives of \( F \) with respect to \( X \). So it allows the variations of the features to be calculated regarding the variations of all the manufactured component dimensions (or their assigned tolerances).

This will be used as a **linearization** around the actual values of \( X \) that assumes that their variation \( (\Delta X) \) will affect the stackup feature variations \( (\Delta F) \) in a constant way (i.e. that the slope of the assembly function is nearly constant over the tolerance limits), which may be an accurate approximation when these variations are short enough compared to the nominal dimensions, or when the number of components is large enough to hide the effects.\(^{22}\)

Now is the time to unify the assembly model with the **Accumulation models** (2.3.1.2), by using the components of the matrix \( S \). This way, two possible inequations systems (depending on the sum being WC or RSS) are obtained:

\[
T_{asm} \geq \Delta F = \begin{cases} 
\sum_{j=1}^{2n} (s_{ij} \cdot t_j)^2 & \text{if RSS} \\
\sum_{j=1}^{2n} s_{ij} \cdot t_j & \text{if WC}
\end{cases} 
\]

\(^{22}\) Chase 1991 p.28
Where $T_{asm}$ is the vector containing the tolerances or the limit dimensions of the stackup features (the required **engineering design limits**):

$$T_{asm} = (t_{asm,1}, ..., t_{asm,m})$$

These inequations indicate that the variation on that features should be as great as or lower than the assembly tolerances. It is more common, though, to refer to the limit situation and express them as equations ($T_{asm} = \Delta F$) to work with them in the mentioned problems (analysis and allocation of tolerances).

This new model is valid for 1, 2 and 3D assemblies: it is just needed to insert the tolerance sensitivities $s_{ij}$ in the accumulation formulas as constant factors.\(^{23}\) In the 1D case, the $s_{ij}$ components equal to 1, as any variation in a length will obviously affect directly, as there are no angles involved.

Summarizing:

From the implicit functions $H(X)$, the explicit functions $F(X)$ are obtained by isolating the assembly features to be restricted (dependent variables).

Then, the sensitivity matrix $S$ is calculated as the Jacobian of $F(X)$ evaluated at the nominal dimensions of the manufactured components $X$.

Using its components $s_{ij}$ as **constant parameters** and the tolerances of the components $t_i$ as **independent variables**, a different function for the tolerance of each assembly feature $t_{asm,i}$ (dependent variables) is deduced. Their formulation will depend on the chosen sum case.

---

\(^{23}\) Chase 1999 p.21
Having gathered the two models to define two possible equations systems (WC and RSS), it is now possible to apply them in the tolerance problems:

Tolerance Analysis

Performing a Tolerance analysis involves deducing the distribution of a resulting assembly, from the information about how its components vary (expressed with their tolerance specifications, if normal distributions are assumed), applied in the assembly equations system.

It is as simple as solving the system, where the independent variables are the \( j \) component tolerances, which take the values of their specifications. Then, there are as many dependent variables \( (m) \) as equations, being the system consistent and exactly determined. The obtained results give an idea of the need or not of reassigning tolerances.

Tolerance analysis is used as a verifying tool for tolerance design, to check if the mentioned assembly feature variations \( (\Delta F) \) are within its tolerances \( (T_{asm}) \), given the tolerances of the components \( (t_j) \). If they are not (defective assembly), they should usually be tightened to satisfy the restriction; if they are, they may be loosen till \( T_{asm} \) to be able to produce parts with wider tolerances and thus lower manufacturing costs. Any action is taken in order to lead to design improvement.

It is a production procedure, as assembly variations are predicted regarding the parts that are currently being manufactured. It allows to estimate the yield or assembly acceptance fraction.

Figure 2.15: Tolerance analysis main idea.

A number of times, trial values of the component tolerances are used repeatedly to perform various tolerance analysis, to find the set that fit best (tolerance design). This inverse process can be done in a cleverer way using several algorithms, and is called Tolerance allocation.\(^\text{24}\)

\(^{24}\) Chase 1991 p.32
Tolerance Allocation

Allocating tolerances is a design procedure, so it is performed during the product development. It consists of distributing tolerances among the components that form an assembly, so after applying the required $T_{asm}$ limits on its resultant distribution, the yield should be the desired one.

In other words, tolerance limits for the assembly features are first decided, as well as the required acceptance fraction. Then, the assembly equations system is used, as in the Tolerance analysis, to determine the tolerances that the individual components should have.

In this case, though, the system results to be consistent but underdetermined, so infinite solutions can be found, each with different implications (cost, feasibility, etc.). It is because this time the assembly feature variations $\Delta F$ are the independent variables (which take the $T_{asm}$ values) and the component tolerances to be assigned are the dependent variables, and there are more of them than equations.

To get to a concrete solution, an allocation algorithm besides the original equation system is used:

These algorithms propose different ways to distribute the assembly tolerance among its parts, following various criteria. This will affect the decisions to be taken in terms of buying a set of tools and equipment or another. It is necessary, nevertheless, to differentiate between two types of component tolerances:

Figure 2.16: Tolerance allocation scheme.

Figure 2.17: Not all component tolerances are subject to Tolerance allocation.
Design tolerances are those on which the allocation methods can be applied, the ones that can be really allocated. On the other hand, fixed tolerances are those which for some reason cannot be modified, for example if they are vendor supplied or they are produced by a very specific machine or in a specific and immovable process, or they follow other design criterion.

The idea is to apply an allocation rule so that the accumulation of all the tolerances (both design and fixed tolerances) equals the specified assembly tolerance, which can be seen as the restriction to comply with. To explain that, a 2 component stackup problem is the best option, due to the easiness of representation in a Cartesian plane:

![Figure 2.18: Example of a 2 component 1D assembly tolerances combination.](image)

Where each axis represent a tolerance dimension, and every point on the plane, for example (T1,T2) or (T1',T2'), are combinations of these tolerance dimensions.

Then, depending on the selected sum case (WC or RSS), the restriction is drawn differently:

![Figure 2.19: Representation of the Worst case (left) and the Root sum square (right) restrictions.](image)

In the case of 2 or 3D, T1 and T2 should be multiplied by their sensitivities.
As represented with grey arrows, the aim of the allocation methods is to somehow move the current point on the plane somewhere on the green line, so the restriction is satisfied. This “somehow” and “somewhere” will depend on the chosen rule.

In the case of (T1,T2), the tolerances are reallocated just to comply with the assembly tolerance specifications increasing the cost minimally, while in the case of (T1’,T2’), these specifications are already satisfied, but there is still margin to reduce costs.

 Allocation Rules:

As displayed in Chase, 1999, Tolerance allocation methods for designers, there are three typical allocation methods (among others) based on different allocation rules, each having its advantages and disadvantages. They are:

- Allocation by proportional scaling:

The first step in any allocation method is to find or chose an initial set of component tolerances, selected according to machines and processes capabilities and/or design rules and standards. The achievable tolerances normally depend on the process but also on the nominal dimension of the piece, and the ones in the middle of the feasible range are usually chosen. This set is then evaluated by a Tolerance analysis in order to see if the assembly tolerance specification ($T_{asm}$) is satisfied, substituting these component tolerances in the equations system:

$$\Delta f_i = \begin{cases} \sum_{j=1}^{2n} (s_{ij} \cdot t_j)^2 & \text{if RSS} \\ \sum_{j=1}^{2n} |s_{ij}| \cdot t_j & \text{if WC} \end{cases} = t_{asm,i} \quad i = 1, ..., m$$

If it is not the case, they are scaled by means of a proportionality factor $P$, which graphically means that the initial design tolerances (T1,T2) are projected linearly (i.e. the projection goes through the origin (0,0)) till they reach the assembly tolerance limit, obtaining the new set (T1’,T2’):

---

25 Chase 1991 p.32
This way, the design **tolerances are incremented or decremented** (depending on if they are inside or outside the sum curve) the same proportional amount, so **they keep the same ratio** as the original ones. The intention is to avoid tightening a tolerance too much (making its manufacture more expensive), or to permit loosening all of them as much as possible:

\[
t_{asm,i} = \begin{cases} 
\sqrt{\sum_{j=1}^{k} (P \cdot s_{ij} \cdot t_j)^2 + \sum_{j=k+1}^{2n} (s_{ij} \cdot t_j)^2} & \text{if } RSS \\
P \cdot \sum_{j=1}^{k} |s_{ij}| \cdot t_j + \sum_{j=k+1}^{2n} |s_{ij}| \cdot t_j & \text{if } WC 
\end{cases}
\]

\[
\Rightarrow P \Rightarrow \begin{cases} 
t_j' = P \cdot t_j & j = 1, ..., k \\
t_j' = t_j & j = k + 1, ..., 2n 
\end{cases} \quad i = 1, ..., m
\]

After calculating the new set of tolerances, it is fundamental to make sure that it is between the feasible tolerance ranges of the processes used in the manufacturing.

- **Allocation by weight factors**

Continuing with the idea of the proportional scaling method, some variations can be introduced by means of assigning **weight factors** \(w_j\) to certain component tolerances. It allows to distribute the tolerance assembly among its parts according to any convenient consideration.

Assigning a **greater factor** to a component means that it will receive **greater allocation of the available tolerance**\(^{26}\). Then, it is logical to concede a larger weight factor to that component where a tight tolerance is more expensive to obtain. The greater the weight, the smaller

\[^{26}\text{Chase 1991 p.32}\]
the tolerance reduction (or greater the tolerance widening). Therefore, assign according to the difficulty in tolerance obtainment. 

The weight factors are calculated as a normalization of the weights $W_j$:

$$w_j = \frac{W_j}{\sum_{j=1}^{k} W_j}$$

Then, the weight factors are introduced in the equations as it follows, to obtain the scaling factor $P$:

$$t_{asm,i} = \begin{cases} 
\sum_{j=1}^{k} (P \cdot w_j \cdot s_{ij} \cdot t_j)^2 + \sum_{j=k+1}^{2n} (s_{ij} \cdot t_j)^2 & \text{if } RSS \\
(P \cdot \sum_{j=1}^{k} w_j \cdot |s_{ij}| \cdot t_j + \sum_{j=k+1}^{2n} |s_{ij}| \cdot t_j) & \text{if } WC 
\end{cases} \Rightarrow P$$

$$P \Rightarrow \begin{cases} 
t_j' = P \cdot w_j \cdot t_j & j = 1, ..., k \\
t_j' = t_j & j = k + 1, ..., 2n 
\end{cases} \quad i = 1, ..., m$$

Graphically, the original set of tolerances or point $(T1,T2)$ moves to the point $(T1',T2')=(w1 \cdot T1, w2 \cdot T2)$, which is then scaled to reach the tolerance limit:

Figure 2.20: Weight factor reallocation. Moving to point $(T1',T2')$ supposes having a new ratio between design tolerances. As before, for 2 and 3D cases, the sensitivities must be multiplied at each component of the points.
Allocation by least cost optimization

The previous methods provide ways to allocate tolerances based on rules that may lead to a better solution, but not necessarily to the optimal one, in terms of overall cost.

In order to apply an algorithm that minimizes the production costs of an assembly, a set of cost-tolerance functions is needed. The cost of elaborating a part with a certain tolerance may depend also on the manufacturing process as well as on its nominal dimensions:

\[ \text{cost} = f(\text{process, nominal dimension, tolerance}) \]

These functions have usually a curved shape, where the costs increase as the tolerances tighten (or decrease as they are relaxed), and are normally obtained empirically and then fitted to some generic function:

Figure 2.22: Usual shape of a cost-tolerance function, for a concrete process and nominal dimension.
Then, the problem to be solved is to find the optimum combination of component tolerances that minimizes the sum of their cost functions, while satisfying the assembly tolerance restriction. Having the functions defined, the idea then is that component tolerances are shifted, if necessary, to get to a mix which fulfills the stackup constraint and leads to minimum global production costs\textsuperscript{27}.

For example, imagine that two components take part in an assembly loop and their initial tolerances are T1 and T2, as represented in the previous graphic (supposing that have the same curve, and that their sensibilities are equal to 1, which may not be necessarily the case). Then, it is logical that, if they already comply with the restriction, relaxing T2 and tightening T1 would lead to an also complying combination at a lower cost. Because:

a) As the slope of the curve at (T2,C2) is almost horizontal, tightening the tolerance would mean a small increment in cost, while

b) Being the slope at (T1,C1) that vertical, the tolerance could be loosen and thus its production cost considerably reduced.

**Lagrange multipliers method**

An interesting mathematical method to carry this optimization is the Lagrange multipliers method, which permits to find the values of the variables that minimize or maximize a function (global costs sum) while being restricted by another one (assembly tolerance sum)\textsuperscript{28} or more.

It may be expressed as it follows:

$$\nabla f(x) = \lambda_1 \cdot \nabla g_1(x) + \lambda_2 \cdot \nabla g_2(x) + \cdots + \lambda_k \cdot \nabla g_k(x)$$

Where:

- $f(x)$: function to optimize,
- $g(x)$’s: functions of the restrictions that limit it,
- $\lambda$’s: multipliers, real scalars which make the equation valid.

\textsuperscript{27} Dong 1994

\textsuperscript{28} Chase 1999 p.14
An equation like this must be solved for each assembly feature \( i \), which have just a restriction \((T_{asm,i})\). Generally, there are \( k \) lambdas as there are \( k \) restrictions\(^{29}\), so in this case, there is just a restriction \((k=1)\) and the equation is reduced to:

\[
\nabla f(x) = \lambda \cdot \nabla g(x)
\]

In order to relate the previously used nomenclature to this method, the equations can be written as:

\[
\begin{align*}
    f(x) &= c_i(T) \\
g(x) &= \Delta f_i(T)
\end{align*}
\]

\[
\nabla c_i(T) = \lambda \cdot \nabla (\Delta f_i(T))
\]

or

\[
\nabla c_i(t_1, \ldots, t_{2n}) = \lambda \cdot \nabla (\Delta f_i(t_1, \ldots, t_{2n}))
\]

Where:

\( c_i(T) \): global cost-tolerance sum function, to be minimized,

\( \Delta f_i(T) \): previously found assembly functions (WC or RSS).

Note that the functions just depend on the tolerances, as it is obvious in the case of the assembly functions. In the case of the cost functions, it was said before that they depend on the nominal dimensions and in the manufacturing processes too. A different view for that is that they may be seen as parameters that change from a cost function to another, thus characterizing them.

The idea of the method is to reduce the \( 2n \) variables restricted problem to a \( 2n+k \) variables unrestricted problem. Then the equations present the next form:

\[
\left( \frac{\partial c_i(t_1, \ldots, t_{2n})}{\partial t_1}, \ldots, \frac{\partial c_i(t_1, \ldots, t_{2n})}{\partial t_{2n}} \right) = \lambda \cdot \left( \frac{\partial (\Delta f_i(t_1, \ldots, t_{2n}))}{\partial t_1}, \ldots, \frac{\partial (\Delta f_i(t_1, \ldots, t_{2n}))}{\partial t_{2n}} \right)
\]

For each assembly feature restriction \( i \), a system of \( 2n+1 \) equations to be solved appears, by equaling the components of the gradient vectors one by one, and solving the assembly restriction itself:

\[
\begin{align*}
    \frac{\partial c_i(t_1, \ldots, t_{2n})}{\partial t_1} &= \lambda \cdot \frac{\partial (\Delta f_i(t_1, \ldots, t_{2n}))}{\partial t_1} \\
    \vdots \\
    \frac{\partial c_i(t_1, \ldots, t_{2n})}{\partial t_{2n}} &= \lambda \cdot \frac{\partial (\Delta f_i(t_1, \ldots, t_{2n}))}{\partial t_{2n}} \\
    \Delta f_i(t_1, \ldots, t_{2n}) &= T_{asm,i}
\end{align*}
\]

\( t_1, \ldots, t_{2n}, \lambda \)
This system results to be **consistent and exactly determined**, with $2n+1$ unknowns to be calculated (the tolerances that take part in the assembly feature stackup plus the multiplier $\lambda$). The **solution will be the values of the tolerances** that make the value of the **cost function be minimum**, while complying with the assembly restriction. Evaluating the function at it may be useful to know the costs and compare to other suboptimal solutions. In the case of RSS, it is interesting to calculate the costs of the rejected assemblies and make a balance.

On the other hand, there are some limitations to the method. As with the proportional scaling and weight factors approaches, the **obtained solution must be checked** to see whether the tolerances are within the process ranges or not.

Moreover, the $m$ equations systems above $(i=1,\ldots,m)$ bring a solution for each assembly feature tolerance restriction. If two or more **assembly functions depend on shared variables** (component tolerances), the found solutions for both may imply different values for those variables. Then, iteration is needed to find a new solution that satisfies each requirement\(^{30}\).

As an alternative for cost-tolerance functions, when they are not accessible, **cost per reject data** could be used instead.

With all the methods, the chosen set of tolerances may obviously be different depending on the used accumulation sum, as different restrictions are applied.

\(^{30}\) Chase 1999 p.16
2.4 Statistical methods

In this section, some statistical procedures are explained to, on the one hand, provide with tools to implement a statistical study and, on the other hand, expose ways to deal with the tolerance issue.

2.4.1 Sampling

In the previous section, the distributions of the sizes of the components that are manufactured were supposed to be normal. With the aim of characterizing them, it is necessary to create pieces, measure them and then obtain the parameters to do the characterization \((\mu, \sigma)\). However, these values are not available, unless each and every manufactured piece was measured after being produced (all the pieces, i.e. the population).

Instead of that, a group of pieces is measured (sample) and their mean and standard deviation \((\bar{y}, s)\) will be used as an approximation for the population parameters, if a certain condition is fulfilled (explained below).

Mean

This parameter refers to the measure of central tendency of a set of measurements. It is the sum of them divided by their quantity:

\[
\mu \cong \bar{y} = \frac{\sum_{i=1}^{N} y_i}{N}
\]

Where:

- \(N\): number of samples, sample size,
- \(y_i\): \(i\)th measurement,
- \(\bar{y}\): sample mean, central tendency of the measurements,
- \(\mu\): population mean.

Standard deviation

The standard deviation provides information about the spread of the values around the mean. It is calculated as follows:

\[
s \cong \sigma = \sqrt{\frac{\sum_{i=1}^{N} (y_i - \bar{y})^2}{N - 1}}
\]
Where:

- $s$: sample standard deviation,
- $\sigma$: population standard deviation.\(^3^1\)

**Minimum sample size\(^3^2\)**

In statistics, sample size is the number of individuals taken out of a population that integrate the sample. The minimum sample size $N$ is the number needed so the data is considered to be representative of the population, being then the approximation $(\mu, \sigma) \approx (\bar{y}, s)$ acceptable.

The objective of determining the **appropriate sample size** is to estimate the parameters with the desired **confidence level**, while reducing costs and time for the study. A smaller sample size may result in obtaining non-representative parameters, and a bigger one may make the study more expensive or even not worth it, especially in the case of **small series**. It would make no sense to have to produce a massive number of pieces for a supposedly economic and ready-to-produce method.

The way to get this number is first measuring a sample of $n$ individuals and obtaining its mean $\bar{y}$ and standard deviation $s$. Then, choosing a **confidence interval** (usually 95%) and a precision is needed. The precision can be an absolute value or a relative one referred to the mean of the initial sample, and indicates the acceptable error that may be between the distribution obtained from the sample and the one from the whole population.

Finally, applying one of these formulas, the minimum sample size is obtained, and the representativeness of their mean and standard deviation guaranteed:

\[
N \geq \frac{t_{\alpha,n-1}^2 \cdot s^2}{k^2}; \quad N \geq \frac{t_{\alpha,n-1}^2 \cdot s^2}{p^2 \cdot \bar{y}^2}
\]

Where:

- $n$: initial sample size,
- $\alpha$: risk (e.g. 0.05),
- $t_{\alpha,n-1}$: $t$-student coefficient, with $n-1$ degrees of freedom & a confidence interval of $1-\alpha$, obtainable by tables

\(^{31}\) Lyman

\(^{32}\) ETSEIB
\( k \): absolute precision (e.g. 5μm),

\( p \): mean-relative precision (e.g. 1\%).

The \( \mu \) and \( \sigma \) parameters (and thus their distributions) of the manufactured pieces should be obtained for each piece design and machine.

It may happen that the minimum sample size \( N \) was bigger than the parts that are going to be produced, for example in the case of the ProeK. Consequently, it would be recommendable (when possible) to normalize designs, so they could be used in different projects.

This way, the distribution for a certain piece manufactured in a certain machine would still be valid independently of its destination project, so in the end, the number of produced parts would be bigger than \( N \) (and then characterized without cost overruns).
2.4.2 Yield of an assembly

As explained before, the yield of an assembly process is the predicted fraction of assemblies that will be inside the specified tolerance limits ($T_{asm}$), the percent of non-defective assemblies.

In the Worst case situation, assuming the mentioned hypothesis (all of the produced components are within the specified tolerances), the yield is ideally equal to 100%. For normal distributions (case of RSS), Microsoft Excel offers the possibility to calculate it, given the distribution parameters and the assembly limits:

![Diagram showing assembly distribution, limits, and yield calculation](image)

The formula is the following:

$$yield = (NORMDIST(\mu + T_{asm}, \mu, \sigma, TRUE) - 0.5) \cdot 2$$

Where the function NORMDIST returns the cumulative probability of the normal distribution defined by $\mu$ and $\sigma$, till the value $\mu + T_{asm}$, i.e. the probability that the measure is lower or equal to it. Then, 0.5 is subtracted (as the total area of the probability distribution is 1), to obtain the area under the curve between $\mu$ and $\mu + T_{asm}$. Finally the yield of the assembly is the double of this value, the area between $\mu - T_{asm}$ and $\mu + T_{asm}$ (the range between tolerances). Graphically:

Figure 2.23: a) $NORMDIST(\mu + T_{asm}, \mu, \sigma, TRUE)$ b) 0.5 c) $a - b$ d) yield = 2 · $c$

---

33 Schenkelberg
As commonly used reference values, the corresponding yields of multiple sigma limits are used: approximately 68.2% of the values fall within the range $\mu \pm \sigma$; 95.4% within $\mu \pm 2\sigma$ and 99.7% within $\mu \pm 3\sigma$.

Apart from being calculated, the yield can be imposed to:

a) **Decide the assembly limits** to specify ($T_{asm}$) given a normal distribution of the assembly (calculated by a Tolerance analysis):

\[
\begin{array}{c}
\text{ASSEMBLY DISTRIBUTION} \\
N(\mu, \sigma^2)
\end{array}
\quad\quad
\begin{array}{c}
\text{ASSEMBLY YIELD} \\
\%
\end{array}
\quad\quad
\begin{array}{c}
\text{ASSEMBLY LIMITS} \\
T_{asm}
\end{array}
\]

b) **Calculate the normal distribution** that the assembly would have to follow (to be later imposed in a Tolerance stackup) being the assembly limits already given:

\[
\begin{array}{c}
\text{ASSEMBLY LIMITS} \\
T_{asm}
\end{array}
\quad\quad
\begin{array}{c}
\text{ASSEMBLY YIELD} \\
\%
\end{array}
\quad\quad
\begin{array}{c}
\text{ASSEMBLY DISTRIBUTION} \\
N(\mu, \sigma^2)
\end{array}
\]
2.4.3 Tolerancing techniques

Given a set of assembly specification limits (upper spec. limit or USL & lower spec. limit or LSL), some techniques to improve (broaden) the yield can be implemented, with the aim of reducing global costs and ensure the required quality\textsuperscript{34}. They may be applied acting on the nominal dimensions as well as on the tolerances of the components that form the assembly.

First of all, obtaining the assembly features distributions is needed. After that, the relationship between the resulting mean (movable through adjustments) and the specification limits (fixed values) has to be observed. Then, one or more of the following operations can be done:

Mean centering

By changing the nominal dimensions of one or various components, the assembly mean can be shifted so it lies equidistant to the specification limits. As a consequence, there will be a greater area of acceptance (yield) and less scrap.

Selecting which elements to modify is a design decision: non-alterable parts (e.g. vendor supplied, as metal sheet thickness, etc.) must be discarded, then the most economical to vary should be chosen (in the ProeK case, changing the laser and bending programs may be enough).

If the assembly is already being produced, the least number of part types should be varied. If more than one element has to be modified, a convenient criterion to weight the variation among parts should be used.

Limit justification

Another mean-shifting method is limit justification. The idea is to move the mean towards a non-critical limit (in case there was), LSL or USL, and place it \( z \) times the standard deviation away from the critical limit, in order to get a \( z \cdot \sigma \) quality.

Spread reduction

In the situation where the assembly distribution is too flat (because the standard deviation is too high), tolerances of one or more components should be tighten, so the assembly spread is reduced by the components spread being reduced too.

\textsuperscript{34} Chase 1999
This is more expensive than just modifying nominal dimensions, as it may involve changing machinery and/or discarding more “defective” parts. The decisions on how to perform it can be done via *Tolerance allocation*.

![Diagram of distribution with labels](image)

**Figure 2.24:**
- a) Initial distribution
- b) Mean centering
- c) Upper limit justification
- d) Lower limit justification
- e) Spread reduction

All these methods may be combined with other that reallocate the mean and reduce the spread.
2.4.4 Monte Carlo simulation

The *Monte Carlo simulation* is a **tolerance analysis** tool that can be used for **non-normal distributions** and/or **nonlinear assembly functions**. As these distributions have to be well known, this method is usually applied after the production has started running, so they can be properly characterized, and when powerful computation resources are available. The process works as it follows:

First of all, assembly limits are specified, component distributions identified and the assembly function(s) formulated.

Then, a **random number generator** returns a set of tolerances for each component and, by means of the assembly function, the resultant assembly dimension is calculated. This simulations are done repeatedly, applying variations to the dimensions of each component, according to their own distributions.

Finally, after a sufficient number of simulations are performed, a **histogram** of the resultant assemblies is plot, and then **its distribution deduced**. The yield of the assembly can be now calculated.

With this method, the effects of how component variations influence the assembly can be observed.

The **drawbacks** of implementing this study in small series are, on the one hand, that the distributions of the components have to be very well defined, and that implies having produced a large amount of parts of each type and, on the other hand, that the required simulation resources may be also huge and then their costs unbearable (both in terms of money and time).

---

35 Chase 1991 p.29
3 Fixtures influence on tolerances

In order to study how the lack of fixtures affect the resultant assembly, a simple example is proposed. Two different designs (regarding the same functionality) will be compared: a typical tubular solution, and another one based on the ProeK interlocking joint concept.

The suggested assembly is a square frame with square cross section:

![Square Frame](image)

The shape of its parts will obviously depend on the solution applied:

**Tubular design**

A simple and conventional design to build the desired structure could be one that used square cross section pipes with the required dimensions, cut in angle and finally welded at the corners. To join the four parts, some type of fixtures are needed.

![Tubular Design](image)

*Figure 3.1: Detached cut pipes and corner fixture.*
Once the pieces are properly located, the welding process can start. However, it cannot be fully completed, as some seams are out of sight (they are below, in contact with the fixtures). Then, a minimum number of two welding steps is required to perform the assembly operation.

Figure 3.2: Tubular frame ready to be welded by one side.
ProeK design

Using the interlocking joint concept introduced in the ProeK, a second design is proposed. It consists of six laser cut and press bent metal sheets. Two bent pieces form the inner “wall” and other two the outer, containing the teeth of the joints, while the top and the bottom are two flat sheets that contain the cutouts:

![Diagram of the assembly](image)

**Figure 3.3:**
- a) Exploded view of the assembly
- b) Mounted assembly
- c) Welding position
- d) Assembly without round teeth

The teeth, both round and rectangular, are inserted in their correspondent slots. In the previous picture, the round teeth are represented in blue to express their included fixture functionality.
3 Fixtures influence on tolerances

Then, the teeth of the bottom are removed, the assembly turned to the welding position and the ones of the top removed too. At this point, the assembly is ready to be placed on the welding station and to be welded in a single step from above.

3.1 Tolerance stackup comparison

From these designs, their cross sections and upper views can be analyzed to get an idea of how the tolerances stack up and compare both solutions.

According to the previously exposed theory, the nominal lengths are here named with "L's" and the assembly features to be limited with "F's". To characterize them, a series of sub-indexes follow these letters and refer to the assembly type (t for tubular, p for ProeK), the space orientation (x, y, z) and a numeric index.

All these 2D cases are simplified in 1D cases, as the appearing angles are of 90º and then the dimensions can be fitted in the orthogonal axes. This way, the 1D vector loops are formed by the summing nominal dimensions and finally closed by the assembly features.

For the cross section, the loops and their equations result:

<table>
<thead>
<tr>
<th>CROSS SECTIONS</th>
<th>ProeK</th>
<th>Tubular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z axis:</td>
<td>$F_p, z = L_p, z, 1 + L_p, z, 2 + L_p, z, 3$</td>
<td>$F_t, z = L_t, z$</td>
</tr>
<tr>
<td>X axis:</td>
<td>$F_p, x, 1 = L_p, x, 1$</td>
<td>$F_t, x, 1 = L_t, x, 1$</td>
</tr>
</tbody>
</table>
And for the upper view:

### Upper View

<table>
<thead>
<tr>
<th>ProeK</th>
<th>Tubular</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="ProeK Diagram" /></td>
<td><img src="image" alt="Tubular Diagram" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Y axis:</th>
<th>Y axis:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{p,y,1} = L_{p,y,1}$</td>
<td>$F_{t,y,1} = L_{t,y,1}$</td>
</tr>
<tr>
<td>$F_{p,y,2} = L_{p,y,2}$</td>
<td>$F_{t,y,2} = L_{t,y,2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X axis:</th>
<th>X axis:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{p,x,2} = L_{p,x,2}$</td>
<td>$F_{t,x,2} = L_{t,x,2}$</td>
</tr>
<tr>
<td>$F_{p,x,3} = L_{p,x,3}$</td>
<td>$F_{t,x,3} = L_{t,x,3}$</td>
</tr>
</tbody>
</table>

In this design, the components $s_{ij}$ of the matrix $S$ are equal to 1 (as no angles apart from 0 or 90° are involved), and that is why the 2D problems could be simplified to 1D problems. This means that the equations of the tolerances will have the same form as the nominal dimensions equations (if WC), and then the tolerances will have a direct influence (because $s_{ij} = 1$, which in the end is no more than a "weight factor").

For example, with the first equation, related to the nominal dimensions:

$$F_{p,z} = L_{p,z,1} + L_{p,z,2} + L_{p,z,3}$$

The equation for the tolerances could be obtained:

$$\Delta F_{p,z} = s_{p,z,p,z,1} \cdot \Delta L_{p,z,1} + s_{p,z,p,z,2} \cdot \Delta L_{p,z,2} + s_{p,z,p,z,3} \cdot \Delta L_{p,z,3} = \Delta L_{p,z,1} + \Delta L_{p,z,2} + \Delta L_{p,z,3} = t_{p,z,1} + t_{p,z,2} + t_{p,z,3}$$
The same could be done using RSS:

\[ \Delta F_{p,x} = \sqrt{t_{p,x,1}^2 + t_{p,x,2}^2 + t_{p,x,3}^2} \]

This way, it can be seen how the specific manufacturing process (and then its associated tolerances) contribute to the tolerance stackup. Using this same example, one could compare the resulting tolerance of the height of the square beam if built from a tube or with press-bent-interlocked sheets (\( \Delta L_{p,z} \) vs \( \Delta F_{p,z} \)):

- In the case of the extruded pipe, the tolerances related to this dimension will be the typical ones obtained in an extrusion process (\( \Delta L_{p,z} \)). As usual, they will tend to be higher as higher the nominal dimension may be.
- For the interlocked sheets, the tolerances that will stack up will be the ones related to the thickness of the metal sheet (\( \Delta L_{p,z,1} + \Delta L_{p,z,3} \)) and the precision of the laser cut (\( \Delta L_{p,z,2} \)). Greater beams could be built with this methodology without incrementing their tolerances (maintaining the sheet thickness), because the laser cut precision would be independent from where it is performed, obtaining major lengths with the same low variations.

The same happens with the \( F_{p,x,1} \) tolerance: \( \Delta F_{p,x,1} \) depends only on the precision of the laser cut, while \( \Delta F_{l,x,1} \) on the extrusion die.

For the outer and inner dimensions, in the case of \( \Delta F_{p,y,1} \), \( \Delta F_{p,y,2} \), \( \Delta F_{p,x,2} \) and \( \Delta F_{p,x,3} \), the laser precision is also responsible of the final tolerances (where the slots are), while in the case of \( \Delta F_{l,y,1} \), \( \Delta F_{l,y,2} \), \( \Delta F_{l,x,2} \) and \( \Delta F_{l,x,3} \), the assembly tolerance lies on the precision of the cuts performed on the pipes. There could be the case, moreover, that a great variation of the pipes lengths (due to an unprecise cut) might lead to the assembly to be unfeasible. The fixtures here would have no influence as they are guided by the shape of the parts.

After this analysis, it is obvious that the manufacturing processes impose the designs (and then how the tolerances sum and stack up), and also the achievable tolerances (which also depend on the available equipment). So, for different designs, different solutions may apply better or worse. As every design is different, it may be interesting to study and compare for commonly used dimensions and shapes (beams, for example). This work is intended to be a guideline of how to implement these specific studies.

What has been made clear is that using laser cut to precisely locate pieces on an assembly is an intelligent idea, as in some parts tolerances can be made independent from nominal dimension (depending only on the laser accuracy).
In the next chapter, some mechanisms to compensate tolerances, for example the ones related to the press bending procedures, are presented.

Having now the assembly functions, and defining assembly limits, tolerance allocation methodologies, as seen before, could be applied:

![Diagram of allocation algorithm]

Figure 3.4: Allocation algorithm, based on the methodologies presented in the previous chapter.
The way to perform a benchmark is by comparing the benefits that can be obtained producing in different approaches. This is the case of the previous example, with extruded pipes on the one hand, and bent metal sheets on the other; it can also be applied for same manufacturing processes with different component tolerances.

The concept is as simple as calculating the benefits ($B$), subtracting the assembly production costs ($C$) to the generated incomes ($I$):

$$B = I - C = P_a \cdot y_a \cdot i_a - P_a \cdot c_a = P_a \cdot (y_a \cdot i_a - c_a)$$

The incomes are calculated as the total production ($P_a$) multiplied by the yield ($y_a$), i.e. the actual sellable assemblies, multiplied by the incomes per assembly ($i_a$). The total costs are the production multiplied by the cost per assembly ($c_a$).

Then, the two manufacturing approaches are compared calculating the divergence of benefits, assuming equal production and incomes per assembly:

$$\Delta B = B_1 - B_2 = P \cdot [(y_{a1} - y_{a2}) \cdot i_a - c_{a1} + c_{a2}]$$

The resulting sign of this operation will indicate the most convenient decision to make:

$$\frac{\Delta B}{P} = (y_{a1} - y_{a2}) \cdot i_a - c_{a1} + c_{a2} \quad \begin{cases} > 0 \Rightarrow Produce \ 1 \\ < 0 \Rightarrow Produce \ 2 \end{cases}$$

The chosen components tolerances (or manufacturing methods) will have their own related cost and, given a set of assembly tolerances, will determine the assembly yield. Then, iterating, the best solution among the feasible ones can be obtained.
4 Development of tolerance compensating mechanisms for fixtureless BS

As seen along this whole work, there is a phenomenon that affects all assemblies, just because of the fact of being formed be the addition of imperfect parts: the tolerance stackup.

While conventional assemblies use fixtures to constrain their pieces during the joining operations (welding, riveting, bolting, etc.) the fixtureless ones require integrated solutions that ensure the right mounting and assembling. This integration can be understood as a transference of the fixture functionalities to the parts. Some ideas to do that are here presented:

4.1 Use of baseplates as global reference

A good approach to keep the global dimensions under control, and avoid having a large amount of pieces that stack up, is to design parts that act as a global reference framework.

To do that, it is opportune that most of the other parts are restricted by it in some way, so the relationship between these pieces will be conducted by the references present on the baseplate. Therefore, these parts have to present minimum inaccuracies, so it is convenient that they are manufactured using the most precise processes. Trying to embrace almost all the other pieces makes them usually the bigger ones.

Figure 4.1: Example of the previous chapter structure reference baseplate, a flat laser cut part.

Using the previous fixtureless square frame as a practical example, there is a flat sheet containing several slots for the interlocking joints which plays the role of a global reference
Development of tolerance compensating mechanisms for fixtureless BS baseplate (in fact the upper and lower parts play both this role). It is designed flat so bending operations, which may introduce relatively big inaccuracies, are not present. The slots, which act as position references/constraints, benefit from the high precision of the laser cut operation. These slots guide and locate in the right place the four L-shaped pieces (inner and outer “walls”).

It also provides structural stiffness and helps to perform transportation operations, because all the weight of the assembly lies on it.

### 4.2 Folds and joints distribution

When talking about press-bending accuracy, there are two topics to be taken into account: position and angle bending error. Angle deviation may not be relatively significant, as will be seen in next subsections. Position deviation, however, may make the interlocking joint assembly simply unfeasible. Some advice to avoid this situation are:

- Distribute the bending operations **among the parts**, avoiding the accumulation of consecutive folds (and then their position errors) in the same parts: it is easier to join **two L-shaped pieces** (with **one fold** each), than **one U-shaped piece** (with **two consecutive folds**) with a flat piece:

  ![Figure 4.2:](image)

  **Figure 4.2:**
  
  a) Two L-shaped parts may form an easily mountable beam.
  
  b) A U-shaped profile may present excessive bending error accumulation.
- Distribute the interlocking joints so the **joining move** is performed in **just one direction**. Having to do it in more than one direction may imply forcing the pieces:

![Interlocking joints diagram]

*Figure 4.3: Example of a hard mounting design and comparison of the required moves to interlock the three presented layouts.*

- **Avoid** designing assemblies where **more than 3 parts** interlock together (all with all). This may derive in high complexity, which would demand a very accurate bending process, and in having to interlock pieces in more than one direction. Instead of using tooth-cutout joints everywhere, small weldable flanges could be used (as presented in the next section).
4.3 Combination of tooth-cutout and folded joints

Sometimes, when a bending operation is involved in the manufacturing of a piece, it is a clever solution to combine the interlocking joint system with folded joints, or flanges, because interlocking joints require high precision to fit correctly and the bending operation may introduce a higher dimensional variation.

![Figure 4.4: Two L-shaped folded profiles, the second one containing a flange. Resulting beam.](image)

The folded joint could compensate this deviation and make the assembly still feasible. Moreover, it presents the characteristic that it can be welded indistinctly from both sides, so it may allow to perform the laser welding of the assembled parts from one side in a single step:

![Figure 4.5:](image)

a) A beam using two interlocking joints like this needs to be turned in order to perform all the laser welds.

b) Combining an interlocking joint with a flange in a clever way may make the welding operation possible in a single step.
Furthermore, the flange bending does not have to be perfect, and in fact it needs some gap for degassing during the welding operation.

Again, taking the example of the square frame, there is a combination of interlocking joints with flanges:

![Diagram](image1)

**Figure 4.6:** Cross section of the example of the previous chapter structure, fully weldable in one step form above.

In order to ensure that the Z-dimension (vertical direction on the figure) is within the tolerance range, a set of *stoppers* are placed at the lower part of the vertical pieces to control the distance between the lower and upper sheets, assuring this assembly feature to be within the required tolerances. These *stoppers* are part of the pieces and are materialized as round tops during the laser cut of the sheets:

![Diagram](image2)

**Figure 4.7:** Lateral “wall” of the proposed beam. It features three teeth on the top and one tooth, two flanges, and two *stoppers* on the bottom (at the ends).
They allow the required degassing gap and assure that the bent flange will not touch the next piece thus affecting the final dimensions of the assembly. This is done this way because, one more time, the precision of the laser is better than the one of the bending process.

Figure 4.8: The same piece joined to the upper and lower flat pieces (from three different views).

The bottom-centered teeth has just a clamping function, not a locating one, so its fit is designed wider than the fit of the upper teeth.

In conclusion, combining teeth-cutouts with flanges provide, on the one hand, proper positioning between parts and self-clamping, and flexibility that helps the assembly feasibility on the other, obtaining quality welds in both cases.
4.4 Flexibility management

The flexibility of the metal sheets is a factor that should be thought as an advantage for the mounting operation, and therefore favored in certain situations.

Having the possibility of forcing by hand the bent parts makes the angle bending tolerances become less important. Since there is a part that acts as a baseplate, it will force the parts to have the right location in the assembly and then make it have the right dimensions.

A way to promote flexibility is, for example in L-shaped pieces, designing long and narrow “arms”, or making the shorter “arm” as long as possible. This also results in best tolerances (depending on the bending procedure) due to the larger lever arm.

![Figure 4.9: Two different proposals: in the left one, the L-shaped folded profiles are easier to manually bend to adjust them to their right position (guided by the interlocking joints), when compared to the L-profiles of the proposal in the right (the bigger ones).](image)

Related to the distribution of the tooth-cutout joints, it is interesting to avoid locating them at the very end of the “arms”, to gain flexibility and perform the needed adjustments, which can be compensated placing a flange at the end (relaxing the required precise fit):
4.5 Clamping considerations

There may be some situations where the use of external fixtures may be considered, at least as a last solution to an eventual tolerance stackup excess.

The design of an assembly should permit, when possible, the use of external clamps, although it was not the original idea, to avoid the waste of acceptable assemblies discarded by a single unfeasible operation.

Imagine a square beam that had to be welded to two flanges. It is convenient to guarantee the appropriate welding gap and, if needed, the access of the required fixture:

Figure 4.11: With this design, inside flanges cannot be adjusted to obtain the required welding gap (extruded shapes are here used as an example just to simplify the illustration).

In this case, the flanges go inside the beam and there is no way to modify the gap between them and the inner walls, which may be desirable for a proper degassing. A too wide gap will make the welding not possible. A better solution may be:
Figure 4.12: In this solution the flanges embrace the beam and are reachable from the exterior.

Figure 4.13: The laser welding procedure is the same, the possibility of varying the required gap is not.
5 Summary and outlook

The main objective of this work has been to provide with a general methodology to face the tolerance topic in the ProeK that could be implemented with the aim of incrementing the accuracy of the assemblies produced with this manufacturing method.

To do so, a wide literature review has been done. It consisted first of deepening in and understanding the body shop and fixtures world, as an early basic approach, to later study the more theoretical topics of tolerance chains and statistical methods.

From the tolerance chains review, a generic mathematical model gathering both assembly characterization models and tolerance accumulation models has been formulated. This model can be applied to any type of assembly, and relates the behavior of its tolerances to the ones of its constituent components (how they stack up).

Having this mathematical representation of the assembly, two type of engineering studies may be carried out: the tolerance analysis and the tolerance allocation. The first one consists of simply evaluating the assembly based on the values of the components tolerances (the variables of the problem). The second, the most interesting one in the design and manufacturing planning phase, consists of intelligently distribute the tolerances among the components given a set of assembly requirements. Three allocating procedures have been here presented: proportional scaling, weight factors and least cost optimization. To perform the last one, obtaining cost-tolerance functions is essential, which could be object of further investigation. It is vital to be aware that they may be very dependent on the machinery, the part design and its nominal dimension.

These methodologies are ready to be implemented, as soon as the assembly model and its inputs are characterized. These inputs (and also the outputs) are usually statistical distributions. The observed literature review on statistical methods has supplied with various tools to do so (sampling to get distributions, calculation of assembly yields), but also to take action and improve the production, using tolerancing techniques. Moreover, the possibility of using the Monte carlo simulation is indicated, but this would be enough work for another complete thesis.

In order to study the influence of having or not fixtures in the tolerance stackup, a basic tolerance analysis, applying the previously mentioned assembly model, has been executed. The resulting conclusions have been that laser cut may be used to locate parts of an assembly with precision, because the tolerance of the cut does not depend on how far it is performed from the previous cut.

On the other hand, as the design of the assemblies and also the achievable tolerances are directly imposed by the manufacturing processes, no conclusive comparison can be made with such a simple study. They impose the component tolerances (depending on the equipment)
but also how they stack up. For different designs, different solutions may apply better or worse. For this reason, future research should consider implementing the comparison in real and more complex assemblies, which would be more representative. The aim of this work is to provide with guidance on how to implement these studies. Furthermore, an algorithm to perform a tolerance allocation comparison regarding all the shown allocating procedures has also been presented here.

In the final part of this thesis a series of design recommendations has been displayed. The purpose of these recommendations is to cope with the inaccuracies introduced by the specific manufacturing processes that are present in the ProeK methodology.
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