Master Thesis

M.Sc. Energy for Smart Cities

Estimation of the net charging demand from privately owned electric vehicles using Game Theory

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Abstract

Electric vehicles are growing at a significant rate in the world and that makes it essential for modern day electricity networks to be prepared for their integration. A common approach of preparing the network for any kind of demand is to be able to predict or estimate the same based on data and simulations using optimization techniques.

This work was aimed at the same in two distinct parts. In the first part, game theoretic methods were tried to be applied to an existing multi agent probabilistic model estimating net demand from electric vehicle. Owing to the complexity of the undertaking, it was decided to only include a payoff based allocation of electric vehicle charging scenarios to estimate electric vehicle demand which accounted for all scenarios rather than all vehicles charging in a single scenario. In the second part, a smaller scenario of an affluent household with two electric vehicles and typical mobility pattern was formulated. Game theory solution concept of Nash Equilibrium was used to optimize the charging of both electric vehicles over a week of usage.

The results from the first part, displayed an overall reduction in maximum loads while there were certain shifts in loads observed as well. As an exercise without any inherent optimization mechanism the overall results from this segment were inconclusive. The results from the second part, demonstrated needs for charging the EVs shifting to off-peak hours and charging of vehicles, a maximum of 1-2 times per week based on user range anxiety, game theoretic competition and mobility needs. Further, savings from charging at off-peak tariffs based on time of use electricity tariffs were also evaluated.

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Glossary

AC – Alternating Current
BEV – Battery Electric Vehicle
DC – Direct Current
EV – Electric Vehicle
EVSE – Electric Vehicle Supply Equipment
ICE – Internal Combustion Engine
IEEE – Institute of Electrical and Electronics Engineers
IESD – Iterated Elimination of Strictly Dominated Strategies
HEV – Hybrid Electric Vehicles
PHEV – Plug-in Hybrid Vehicles
PPF – Probabilistic Power Flow
SOC – State of Charge
SAE – Society of Automotive Engineers
TOU – Time-of-use

1. Preface

This project came about as a master thesis based on available positions at CITCEA- UPC for master thesis students in the field of electric vehicles. A need was felt to predict the net demand from Electric Vehicles (EV) as the same can help the grid prepare for scenarios in which electric vehicles are the norm and wide scale EV penetration can thus be facilitated. Previous attempts had been made to estimate the net load from privately owned EVs and the motivation behind undertaking this study is to see if the application of a new approach namely game theoretic methods can better help in simulating the load from EVs and present results which are closer to the actual data. It was decided to utilize data and algorithms from [1] to run simulations and compare results to obtain an understanding as to which method is better suited and delivers results closer to real life scenarios. While that does form a part of this work, the complete implementation of the estimation of the electric vehicle demand using the previous work by applying game theory to the agent based probabilistic model, could not be possible due to reasons of complexity.

Further discussions with the supervisor resulted in the project evolving to a two-step undertaking. In the first step, it was decided that the model from [1] would be modified to account for all kinds of charging strategies simultaneously. In the second step, it was decided to formulate a household scenario comprising of two EVs and using game theory to predict the schedule and demand of electric vehicle charging.

Topics which were studied as part of this endeavor to apply game theoretic methods to electric vehicle charging include, types of games, game theory logic and algorithms, solution concepts including Nash equilibrium, pareto optimality amongst other. It will be attempted to use Nash equilibrium as the solution concepts to identify optimal strategies which suit all players in the given scenario.

Initial studies during the internship carried out previously involved research on electric vehicles and game theory to build a foundation for the master thesis. This research is presented in part to give the reader an understanding and background on the necessary technical aspects of this work.

Thus, in totality, this master thesis work brings together electric vehicle technology, demand modeling and game theory under a single umbrella to obtain useful insight in this domain.

2. Introduction

At the outset of this project it was decided to attempt to utilize game theory to model the net electric vehicle charging demand from privately owned EVs. In the technical section of this report, we will at first go through the basics of electric vehicle technology, the state of art of the technology and infrastructure pertaining to EVs, identify all the stake holders in the EV scenario and look at regulations and initiatives in different countries regarding EVs. Thereafter, we will proceed to survey technical aspects which are involved in this study and are necessary to formulate the algorithms for this work. This will include details on the electric vehicles used, research on demand models, applications of game theoretic concepts in the electric power sector and more specifically to electric vehicles and their charging and game theoretic modeling concepts. Post this we present an approach to create models and simulations based on our learning and available data. First up, in the next sections we go through the basics of electric vehicle technology.

2.1. Introduction to Electric Vehicles

An electric vehicle is an automobile which uses electric motors or traction motors for propulsion of the vehicle. EVs technically include both rail and road transport, surface and underwater vessels as well as air transport mediums but for this report the term electric vehicle or EV will refer to just road transport. The electrification of all modes of transportation is one of the key approaches to tackle the issue of climate change. The continual adoption of EVs into markets worldwide involves multiple aspects bringing together impacts on the power grid, development of powertrain, battery and charging technologies, as well as policy and regulation in different part of the globe. In this section, we will look at these multiple aspects in detail and understand the current state of affairs in these aspects.

The focus on EV development has been on powertrains, batteries and charging equipment or electric vehicle supply equipment (EVSE). In order to meet various requirements of the automobile industry such as fuel economy, different powertrain setups are tried for hybrid vehicles such as series, parallel and series-parallel configurations. Advances have also been made on the electric motor used to drive the vehicle or to support operations. All these developments have resulted in vehicles with better fuel economy and higher efficiency. Battery technology meanwhile, has evolved from lead acid to nickel based and now to lithium ion based batteries in a quest to develop batteries which have higher energy density and higher power density along with properties of being lightweight and durable. Similarly, charging stations have progressed from slow chargers to fast chargers to address the limitation of low range on EVs. All these developments in EV technology have resulted in a faster adoption of EVs globally, more so in developed nations and nations which are leading the way in implementation of EVs. This can be seen in Fig. 1 from the Global EV Outlook 2016 report by the

International Energy Agency which is an autonomous energy consortium with 29 member countries.

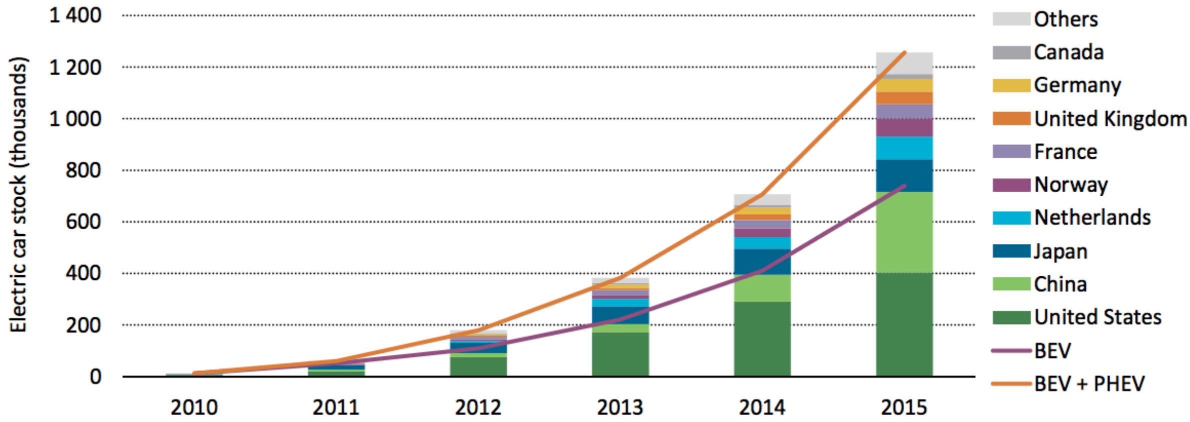

Figure 1: Global Growth of Electric Vehicle Stock from 2010 to 2015 [2]

In the following sections, we will cover the aspects of basic EV technology, categories of EVs, batteries, charging infrastructure and EVSE, and policy and regulation regarding EVs. To get some better perspective and understanding of the EV scenario globally, we will also look at some number and progression of different parameters over years of research and development. Let us now look at the different categories of EVs and their characteristics.

2.1.1. EV Technology Basics

In terms of EV technology, there is a distinct differentiation between the technology employed in traditional internal combustion engine (ICE) powered vehicles and the different types of EVs i.e. hybrid electric vehicles (HEVs), plug-in hybrid electric vehicles and battery electric vehicles (BEVs). It is assumed that the reader is familiar with the operation of traditional ICE powered vehicle and in this section, we will only describe the additional technology that goes into EVs. While HEVs and PHEVs retain the ICE components but add certain electric drive components, BEVs include only the electric drive components. In this section, we will discuss the components of BEVs as these will be more or less applicable to the other types of EVs. The other types of EVs and their functioning and layout is described in the next section.

A Battery Electric Vehicle or BEV (also called a pure electric vehicle) consists of the following three component systems:

1. The drive system: A traction motor forms the central part of the drive system of an EV. The most efficient design is to place the motors directly at the wheel. These are then referred to as wheel motors. Three types of motors are primarily used for the EV applications. These are: DC brush type motor, DC brushless permanent magnet motor

and AC induction motor. A detailed discussion of these types of motors is beyond the scope of this report.

2. The battery system: An electric vehicle's battery determines its range, acceleration capability and recharge specifications.

3. The control system: The control system is responsible for overlooking the operation of the electric vehicle. It comprises of a microprocessor just like a computer and is often referred to as the on-board computer. Based on feedback signals and employing a whole range of power electronics the control system controls the functioning of different components of the EV.

The above components can be said to be common to both PHEVs and BEVs however their placement and functioning maybe significantly different. Let us now proceed to look the different types of EVs mentioned previously and understand the differences between them.

2.1.2. Types of Electric Vehicles

EV classification depends primarily on the extent to which electricity is their main energy source. Based on this premise EVs are broadly categorized as follows:

2.1.2.1. *Hybrid Electric Vehicles (HEVs)*: HEVs are powered by both a fossil fuel and electricity but the electricity in this case is generated by a certain function of the vehicle itself such as regenerative braking. A typical mode of operation involves the HEV operating through the electric motor and then the engine takes over once the load on the vehicle increases. The overall pattern of drive control i.e. electric or ICE based is governed by an onboard computer which is programmed to optimize the switches from electric to ICE and vice versa for best fuel economy and optimum performance. Examples of these kind of vehicles includes the HONDA Civic Hybrid and the Toyota Camry Hybrid.

Note: As this kind of EV has no scope of connection to the grid, it is not important for our studies. It has been mentioned here to just give the reader a wholesome overview of the electric vehicle technology.

2.1.2.2. *Plug-in Hybrid Electric Vehicles(PHEVs)*: PHEVs as the name suggest can charge the battery through both a function of the vehicle i.e. regenerative braking as well as by connecting to a charging outlet. These vehicles are also sometimes referred to as range extended EVs as the ICE can recharge the battery as it gets low, thereby by extending its range. These EVs may chose electricity as the primary energy source or the fossil fuel. A good example of the same is the Toyota Prius which uses petrol as the primary energy source while the Mitsubishi Outlander utilizes electricity as the primary energy source.

Note: As mentioned earlier, hybrid vehicles (both normal and plug-in) have experimented with different kinds of powertrain layouts over the years. These different layouts are shown below in an image from [3] just to give a basic understanding to the reader. These will not be discussed in detail in this report as the they are not important for estimating the net charging demand.

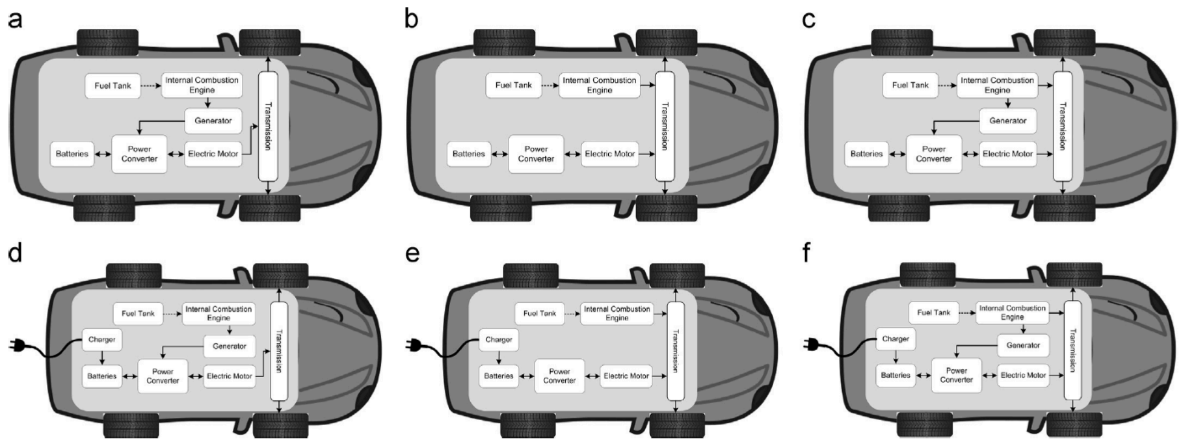

Figure 2: Series, Parallel and Series-Parallel HEVs(a,b,c) and PHEVS(d,e,f) [3]

2.1.2.3. *Battery Electric Vehicles(BEVs)*: Battery electric vehicles are fully powered by their onboard batteries which can be charged by plugging into charging outlets. These vehicles also employ techniques like regenerative braking to charge the battery but aren't primarily dependent on it and depend solely on grid energy for charging the batteries. Fig 3. sourced from [3] demonstrates the layout of a BEV. Common examples of BEVs include the Tesla Model S, Nissan Leaf and the BMW i3.

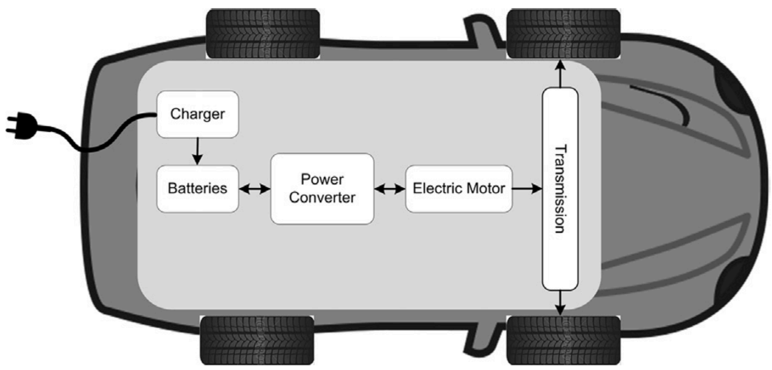

Figure 3: Typical powertrain layout for BEVs [3]

2.1.3. Battery Technology

The battery is the central component of an EV and the increasingly the primary energy source in PHEVs and the primary energy source in BEVs most definitely. The technology for energy storage and batteries has posed a lot of technical challenges to researchers and it has been a major barrier in the widespread adoption of EVs. There are still some constraints on present EV battery technology, which becomes the barrier for wider EV uptake. The present EV battery technology has relatively low energy density which affects the overall range of the EV and thus making EVs a less favorable option compared to traditional ICE powered vehicles. Additionally, the cost of batteries is high which results in BEVs and PHEVs being considerably more expensive than an ICE vehicle. Apart from this there are also concerns about the degradation of the battery over its life cycle and certain safety features. Research and development over the past years has been focusing on increasing energy density and reducing battery cost while addressing the above concerns. The following chart from the Global EV Outlook 2016 demonstrates the evolution of energy density of EV batteries and how they have gotten cheaper over the years.

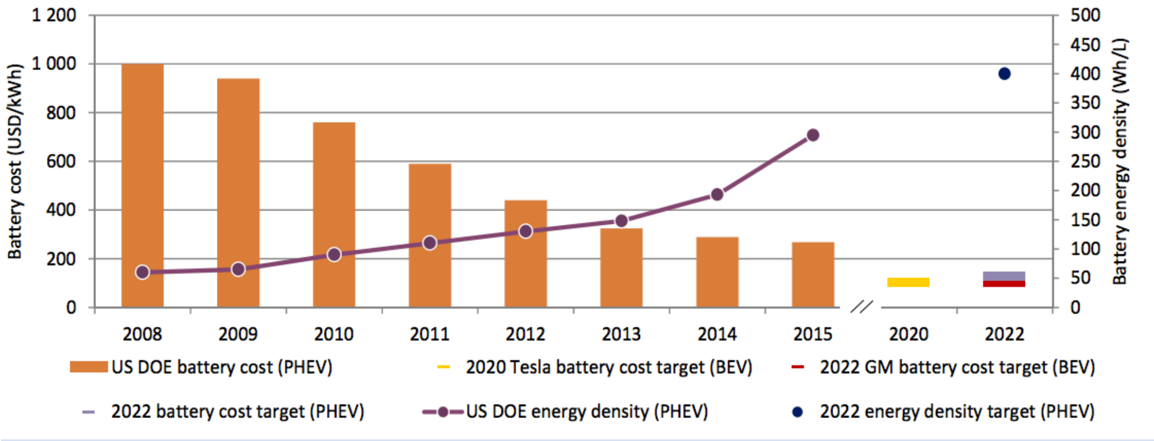

Figure 4: Battery Density and Cost Evolution [2]

The reason for the increase in energy density and fall in prices of EV batteries is the tremendous advancements which have been made in battery technology in order to achieve an end goal of a battery with high energy density, high power density, cheap and durable. The evolution of battery technology started from the use of lead-acid battery in automotive applications. These were soon replaced by nickel based batteries which included nickel-cadmium(Ni-Cd) and nickel-metal hydride(Ni-MH) which had much higher energy density than the lead-acid battery. However, these batteries had drawbacks such as poor charge and discharge rates and efficiency, which are essential for EV applications. Furthermore, the Ni-Cd batteries were found to be toxic and harmful for the environment. Around the same time the ZEBRA battery (sodium-nickel chloride) was introduced into the EV industry. These batteries have high energy and power density but can only operate at very high temperatures.

Post the era of these batteries, lithium based batteries were introduced as EV batteries and marked the beginning of a new era in EVs. These batteries are one of the most promising in this field with high energy and power density, lightweight, cheap and non-toxic and with fast charge rates. Due to these characteristics, Lithium based batteries are the most common choice amongst EV manufacturers currently. For instance, lithium ion battery packs are used in the Tesla Model S, Nissan Leaf, Mitsubishi i-MiEV and the Chevrolet Volt, the most preferred EV choices currently amongst customers. Some categories of Lithium-based batteries are lithium-ion(Li-ion), lithium-ion polymer(LiPo) and lithium-iron phosphate($LiFePO_4$). It is widely acknowledged that the lithium based battery technology holds the potential to be the ideal battery for all future EV applications.

Other battery technologies currently in experimental phases but known to have promise are lithium-sulfur(Li-S), zinc-air(Zn-air) and lithium-air(Li-air) of which Li-air and Zn-air have very high energy densities and are currently in prototype stages of development in research. Fig. 5 illustrates the evolution of different EV battery technologies overtime.

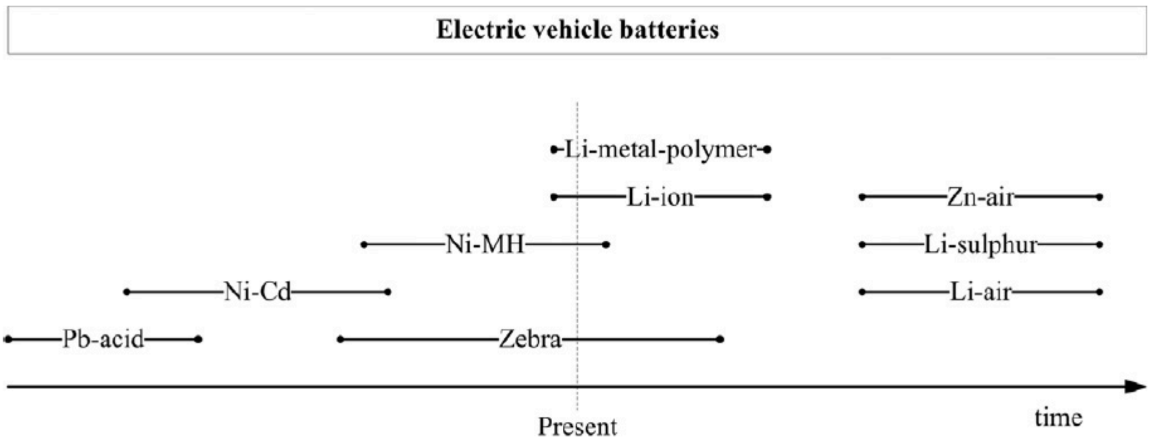

Figure 5: Development timeline of EV battery [3]

In the next section, we will look at EV charging infrastructure and the state of the art in the technologies pertaining to EV charging and different modes of charging.

2.1.4. EV Charging Infrastructure and Charging Technologies

An EV charger or EVSE forms the essential interface between an EV and the electric grid. A charger is necessary because the grid supply is in alternating current (AC) form while the onboard electronics and battery are in direct current (DC) form. The EV charger is thus designed to rectify the high-power levels in AC to a suitable DC level which can then be used to charge the battery. It is often designed as an AC/DC converter or rectifier. In certain modern applications such as fast charging, a DC/DC converter is added to the design for enhanced energy conversion. Based on their power levels and how quickly they can charge a vehicle, chargers are often very plainly categorized as slow and fast chargers which includes private and public charging points. In Fig. 6 the charts sourced from the Global EV Outlook 2016 show the number of charging stations in different countries to give an idea about how established the EV infrastructure is in various countries.

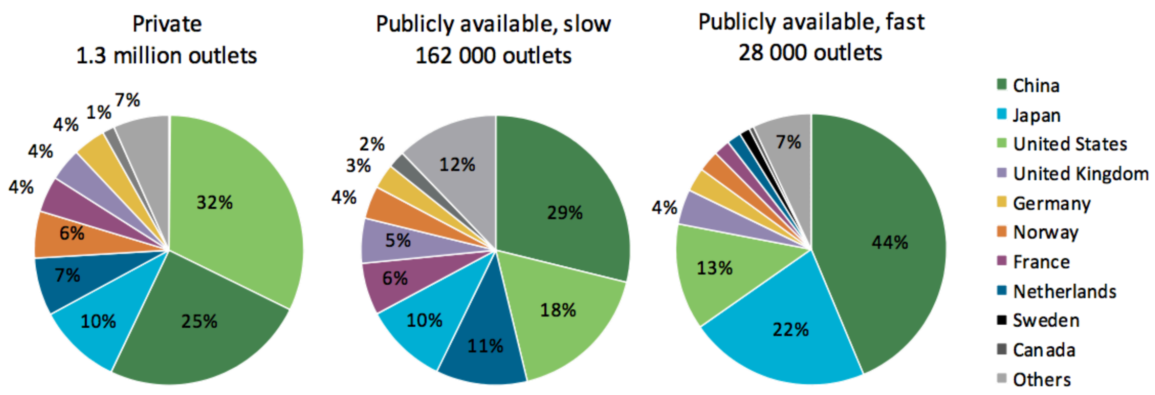

Figure 6: Global Charging Infrastructure Overview

For a more thorough categorization we look to established international standards such as the SAE EV standard with reference to SAE Electric Vehicle Conductive Charge coupler standards SAE J1772. These divide the EVSE into three levels (Level 1, Level 2 and Level 3) each for AC and DC. With reference to this standard AC charging utilizes the on-board charger of the vehicle while DC charging is performed with off board equipment. For example, AC Level 1 is applicable to slow charging for overnight durations from a 120 V_{AC} single phase network. AC Level 2 is rated at 240 V_{AC}. Similarly, DC Level 1 and DC Level 2 operate at 200-450 V_{DC} with charging powers of 36 kW and 90kW respectively. Most of these standards function at a charging current of up to 80A. DC chargers are the ones more commonly known as fast chargers and can charge a BEV to up to 80 percent State of Charge(SOC) in 30 minutes. AC Level 3 and DC Level 3 are not standardized yet, but proposed power levels for these are 20kW and 240 kW respectively [4]. While the SAE J1772 is more applicable for North America, in Europe the standard referred to for European specifications is IEC 61851.

Chargers adhering to different standards and developed by different manufacturers have different plugs and need adapters and standards to be able to switch from one form to another. Besides referring to standards such as above, EV manufacturers have come up with their own patented technologies which operate at different power levels.

A prime example of the same is the Tesla Supercharger. Tesla supercharging stations can charge with up to 145 kW of charging power which is distributed between two adjacent cars, with a maximum allocation of 120 kW per car. These hi-tech charging stations provide direct current at high charging power straight to the battery bypassing the onboard charging power supply. The high charging power of the Tesla supercharger network allows a charge of up to 100% SOC in 75 minutes.

Other common charging stations include the CHAdeMO standard which is a level 3 DC fast charging station which was developed by the CHAdeMO Association formed by the Tokyo Electric Power Company, Nissan, Mitsubishi and Fuji Heavy Industries. These are primarily utilized by Japanese cars such as the Nissan Leaf and Mitsubishi i-MiEV. Currently the CHAdeMO chargers have a max power output of 50 kW.

The SAE developed its own level 3 DC fast charger termed the SAE Combo Charging System (CCS) which is the preferred type for German and US automobile manufacturers. The BMW i3 and VW e-Golf use these type of charger connections. The CCS allows for slow and fast charging from a single charging inlet as opposed to the CHAdeMO which required separate inlets for slow and fast charging. The current max power output level for SAE CCS chargers is also 50 kW.

Before moving on to the next section, we briefly look at the different techniques that can be used to charge an EV battery. These are important as they can help understand the load profile over the entire charge duration of an EV.

There are several charging methods that can be used to charge the battery of an EV. Some charging techniques studied in the academia and used conventionally are constant current (CC), constant voltage (CV), constant power (CP), taper charging and trickle charging [5]. Additionally, advanced charging techniques include combination of one or more of the above methods resulting in techniques such as constant current/constant voltage (CC/CV). Some other advanced techniques include Pulse-charging and negative pulse-charging which are considered good modes of operation for fast charging of EV batteries [5].

CC uses constant charging current flow to the battery till the battery attains a certain voltage level where as CV applies a constant voltage across the battery terminals while constantly adapting the charging current till it falls to almost zero[6] . CP as the name suggests uses constant power while taper charging is done via an unregulated constant voltage source and there is no control over the drop of charging current as cell voltage of the charge builds up[6]. This can tend to damage the battery in case an overcharge happens. Trickle charging uses small currents to account for battery self-discharge. CC/CV charging is the preferred mode of operation for fast charging of lithium-ion batteries. This kind of charging uses constant current up to a certain predefined voltage for the battery post which it switches to constant voltage. So. while majority of the charge is done at constant current, the remaining time constant voltage is used with reduced charging current to top-off the battery. The CC/CV charge profile is illustrated in the Fig 8.

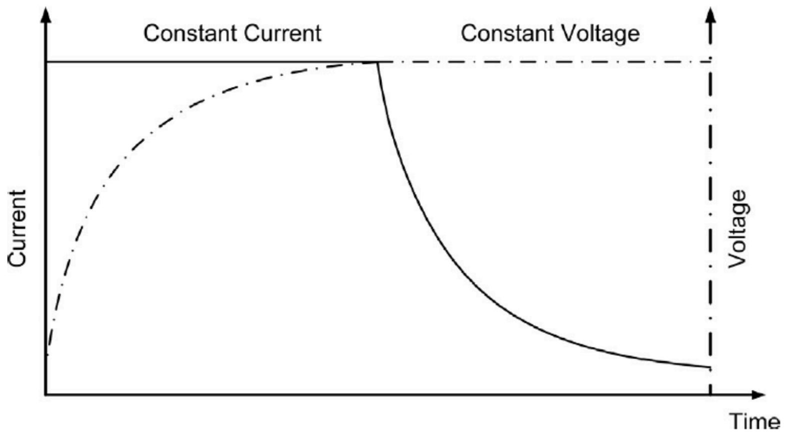

Figure 7: CC/CV charging profile [6]

Pulse-charging profile uses a pulse based charge current to charge the EV battery. This utilizes a short rest period between pulses which can help stabilize battery chemical actions [5]. This rest period is supposed to allow the chemical processes in the battery to keep up with the charging process thereby avoiding gas formation at the electrodes[6]. Negative pulse charging follows an opposite profile to pulse charging in addition to the pulse charging profile by applying a short discharge pulse during the rest period of the pulse charging profile. This is done to depolarize the battery thus clearing gas bubbles which might have formed on the electrode during pulse-charging[5]. This kind of charging is said to improve the efficiency of the overall charging process and prolong battery life. Fig 9. offers demonstrates pulse and negative pulse charging techniques.

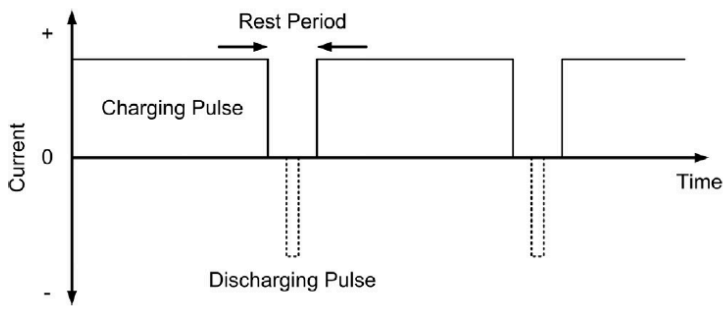

Figure 8: Pulse charging and negative pulse charging

2.1.5. EV regulation and policy

An EV offers much lower running costs when compared to traditional ICE. It is estimated in the Global EV Outlook 2016 [2] that a 100 km trip in an EV would cost about 1/4th to 1/5th of the cost of using an ICE powered vehicle. Over a period of five years, if these savings are aggregated, fuel savings exceeding USD 3000 may be achieved. Even in the light of such savings there are potential obstacles in the way of wide scale EV deployment including but not limited to high cost of battery technology, access to EV infrastructure, installation and costs of such infrastructure as well as general awareness and interest in this kind of technology. However, the identified benefits in terms of pollution reduction, higher integration of renewables, climate change mitigation as well as reduced cost of the technology over the past decade trigger support mechanisms based on policy and regulation to increase the penetration of EVs while overcoming the aforementioned obstacles. In this section, we will go over some of these policy support mechanisms.

These policy support mechanisms can broadly be classified into 3 categories [2]:

- Regulatory measures: These include regulations on vehicle emissions regulations and fuel economy requirements and homologation, which may include credits in favor of electric vehicles
- Financial Levers: These include differentiated vehicle taxation which maybe based on fuel economy or greenhouse gas(GHG) emissions per kilometer
- Other measures: These can include waivers on parking fees and tolls, as well as lifting off access restrictions (e.g. access to EVs on bus, taxi or high-occupancy vehicle [HOV] lanes).

In the following sections, we will look at some examples of EV purchase incentives, EV use and circulation incentives, lifting of access restrictions and emission standards and examine how different policies are aimed at increasing the adoption of EVs.

EV Purchase Incentives: As mentioned in [7], purchase incentives are one of the most motivating of incentives to produce a shift for consumers from conventional ICE based vehicles to EVs. In 2013, France started offering purchase incentives of EUR 6300 for BEVs (defined as emitting less than 20 grams of CO2 per kilometer and EUR 1000 for PHEVs (defined as emitting between 20 gram CO2/km and 60 gram CO2/km). Since 2016 the Netherlands, exempt cars emitting zero CO2 at the tailpipe from registration tax. For other vehicles and EVs, they implemented a sectionalized taxation scheme with five levels of CO2 emissions while progressively increasing taxation per gram CO2/km. For example, PHEVs which qualify for the first level (below 80 g CO2/km) pay EUR 6 per gram CO2/km. Compared to traditional ICE based vehicles this offer a significant rebate to BEVs and PHEVs, as vehicles with ICE have emissions ratings above 106 g CO2/km. In Sweden, vehicles with emissions levels lower than 50 gram CO2/km are granted 40000 kronor as rebate. Fig 10. demonstrates the scale of monetary incentives offered in different countries for different types of EVs.

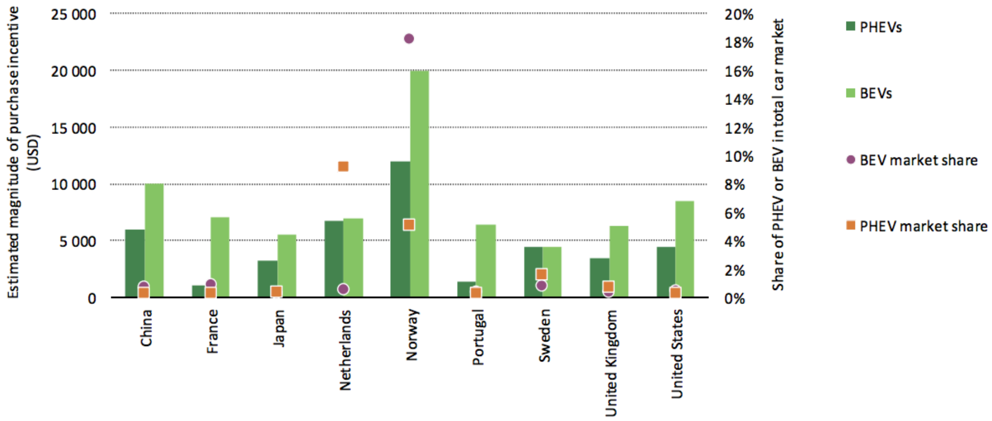

Figure 9: Scale of Purchase incentives for EVs [2]

EV Use and Circulation Incentives: Let us now look at some examples of incentives based on the use and circulation of EVs. BEVs and PHEVs in Germany are exempt from circulation tax for a period of ten years from the date of their first registration. In the Netherlands, zero-emission cars do not have to pay any road taxes. Japan has implemented exemptions from annual tonnage tax and reductions for automobile tax for EVs. In Sweden, EVs are exempt from road tax based on CO2 emissions.

Access Restrictions Waiver: Some examples of this are access to EVs to bus lanes in Ontario, in HOV lanes in Spain, and also in some cities in France, the United Kingdom and Norway. In China, there have been trials with restricting license plates and giving preferential allotment to EVs.

Emissions: The deployment of EVs is favored by increasingly stringent fuel economy requirements and tailpipe carbon dioxide (CO2) emission standards as well on the emission of other local pollutants. BEVs which have zero tailpipe emissions and very good energy efficiency, and PHEVs which have reduced emissions, benefit from these regulations in a big way. In terms of climate change mitigation, EVs can deliver only if there are net CO2 emissions when considering the electricity generation used to charge the vehicle which is challenge for countries which are primarily dependent on fossil fuel based energy sources for their electricity production.

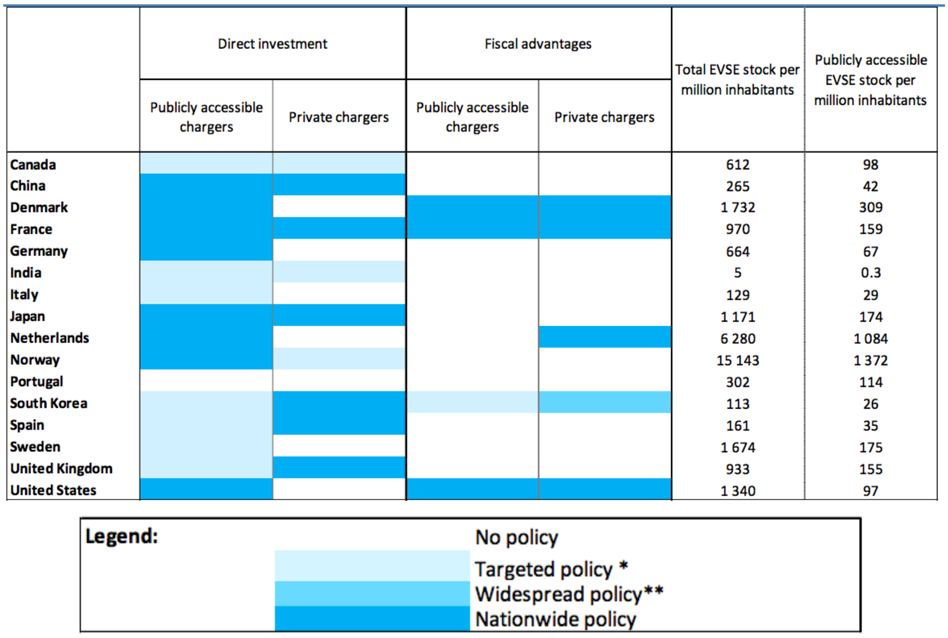

Figure 10: Summary of policy support mechanisms for EVSE deployment [2]

2.1.6. EVs in Spain

The following table shows the EVs and list out their energy related technical parameters. It is to be noted that autonomy here refers to the range of the listed vehicles and has no reference to autonomous electric vehicles.

								Type of recharge		
Brand	Model	Type	Consumption (Wh/Km)	Autonomy	Capacity (kWh)	Type of battery	Power (kW)	Slow	Semi rapid	Rapid
BMW	i3	EV	125	150	19	–	125	✓	✓	✓
BYD	E6	EV	160	250	45	Litium-ion	75	✓	✓	x
Citroën	C-Zero	EV	126	150	16	Litium-ion	49	✓	x	✓
Mitsubishi	i-Miev	EV	135	150	16	Litium-ion	35	✓	x	✓
Nissan	LEAF	EV	173	199	24	Litium-ion	90	✓	x	✓
Peogeot	I.ON	EV	126	150	14,5	Litium-ion	49	✓	x	✓
Renault	ZOE	EV	146	210	22	Litium-ion	65	✓	✓	x
Renault	Fluence Z.E	EV	140	185	22	Litium-ion	70	✓	x	x
Renault	Kangoo Z.E	EV	155	170	22	Litium-ion	44	✓	x	x
Smart	fortwo	EV	151	145	17,6	Litium-ion	35	✓	✓	x
Think	City	EV	144	200	23	Litium-ion	33	✓	x	x
Volkswagen	e-UP!	EV	117	150	18,7	Litium-ion	40	✓	✓	✓

Figure 11: Technical data (energy) for EVs sold in Spain [8]

According to the Global EV Outlook 2016, Spain has an EV stock of 6000 vehicles as of 2015, with a 2020 target of 200,000 EVs. Of the current 6000 EV stock, around 4500 are BEVs and 1500 are PHEVs. Let us now look at some incentives and policy related information. Spain's national government formulated the "Integral Plan for the Promotion of Electric Vehicles", which comprised of the "Integrated Strategy for EVs 2010–2014" initiative in Spain that included the target of 1 million hybrid and electric vehicles on road in Spain by the year 2014 [9]. The following section summarizes some of the provisions under this initiative to promote EVs.

According to [10], the maximum limit for total amount of monetary grants for vehicles that are driven by batteries which may be fully or partially charged by electricity from the grid and whose maximum price does not exceed 32,000 euros, is specified as follows:

- 2,700 euros for vehicles with range not exceeding 40 km and but greater than 15 km.
- 3,700 euros for vehicles with range greater than 40 km and less than or equal to 90 km.
- 5,500 euros for vehicles with range of more than 90 km.

It is to be noted that range above means electric range if the vehicle is a hybrid.

In the case of charging points for electric vehicles in publicly accessible areas, the maximum amount of aid can be 40% of the total eligible cost with the following limits in place:

- 15,000 euros per quick recharge point installed.
- 2,000 euros per semi-fast refueling point installed.

After a brief overview of electric vehicle technology and some insights on regulation and policy, as well as looking at the EV scene in Spain, we will now explore the field of Game Theory, the basic concepts and its applicability to the purpose of our studies.

2.2. Introduction to Game Theory

Game theory provides mathematical frameworks to analyze situations of 'conflict and cooperation' (as described by Roger B. Myerson in his publication *Game Theory: Analysis of Conflict)* between players who can operate on strategies which may or may not influence the strategies of other players. It is essential to note here that the terms 'players' and 'strategies' are used in this regard to indicate a model or a scenario and not recreational or sports activities.

Game theory over the years has seen several classifications: co-operative and non-cooperative; symmetrical and asymmetrical; zero-sum and non-zero-sum; to name a few. For the purposes of this study the classification of importance is co-operative and non-co-operative game theory. As mentioned in [11], Non-cooperative game theory is of importance to analyze the strategic decision making processes of independent players who have conflicting interests over the result of a decision making process which is influenced by their actions. It is to be noted that the term non-cooperative does not essentially imply that the players do not co-operate, but it means that, any co-operation is observed arises from self-interest without any co-ordination and communication of strategies between different players. Thus, it can be said that non-cooperative game theory may be used to model a distributed process to optimize an overall goal which is a result of player decisions without any communication and co-ordination between the strategies of individual players.

Co-operative game theory on the other hand consider incentives for individual players to collaborate. There are two major frame works which form co-operative game theory: Nash bargaining and coalitional game theory. Nash bargaining employs agreement based on terms and conditions between individual players while coalitional game theory takes into account formation of groups or coalitions.

Let us now proceed to look at some of the mathematical basics of Game Theory. To follow a consistency of notation and representation we will be following the style from [12] in order to have a coherent presentation of the concepts.

[12] describes strategic games as a model of interacting decision makers. It further goes on to define strategic games as one which consists the following:

- a set of players
- a set of actions for each individual player
- a preference profile over the set of action profiles

A set A is assumed to be consisting of all the actions that, under certain conditions, are available to a player. In any specific condition the player is faced with a subset of A and chooses a single element therein. The next element which constitutes a strategic game model is the notion of the player preferences. Player preferences are represented in the form of payoff functions which associates a number to the outcome of each action in a way such that actions with higher numbers are more favorable to the player and are hence preferred. Mathematically for any two actions a and b in the action set A, and u(a) is said to represent the payoff function, then u(a) > u(b) implies that the player prefers action a over b. It must be noted that the payoff function only conveys ordinal information [12]. This means that the payoff function can only suggest if an action is preferred over another and not the intensity with which it maybe preferred.

Before proceeding further with the notion of strategic games, it is essential to mention here the concept of the theory of rational choice which as described in [12] states that the action taken by a decision-maker or player in a specific situation is at least as good, according to the player's preferences, as every other available action. This theory is very essential to define why a player would choose a certain action in a certain condition and thus enhances the understanding of any situation.

Moving on from the theory of rational choice we must examine what actions will be taken by a player in a strategic game. As the theory of rational choice implies a player would choose the best available action. However, in a game the best available action will depend on the actions of other players. Thus, a player must form an opinion about the actions of other players and then base his/her action on the same. In basic game theory, it is assumed that each player's opinion of other players' actions is derived from their past experience of playing the game. Furthermore, it is assumed that this experience is sufficiently extensive that he/she knows is sure of how their will act. In[12], it is suggested to view of this scenario in the following idealized manner. Each player in the game is faced with a population of different players who may, on any occasion, by means of rotations take that player's role. For every play of the game, players are picked from each population randomly. Therefore, each player participates in the game against an ever-changing pool of opponents. Their experience helps them form an opinion about a typical set of opponents, not any specific set of opponents. Based on this background we will now proceed to understand the concept of Nash equilibrium which is an essential concept in understanding strategic games. We will borrow the definition from [12] which states that:

*"A Nash equilibrium is an action profile $\boldsymbol{a}^*$ with the property that no player **i** can do better by choosing an action different from $\boldsymbol{a}_i^*$, given that every other player **j** adheres to $\boldsymbol{a}_j^*$"*

What this essentially implies is that for any given play of the game in which the players are randomly drawn from a collection of populations, the Nash equilibrium corresponds to a steady state. In simple terms if a game is played at a certain point of time with an action profile corresponding to the Nash equilibrium profile a^*, then no player has a reason to choose any action outside their component in a^*. This concept will be further demonstrated as it will form the backbone of forming the solution of our given case. The discussion on the same will be taken up in Chapter 4.

The above concept of a steady state is essential in studying strategic games as we can apply to real life scenarios where actions can follow rational choice and it can be assumed that players are more or less sure of the actions of other players and will act only in self-interest. Testing out the applicability of Nash equilibrium in real life situations takes up a significant part of game theory applied to real life problems and the notion of equilibrium has now been expanded to different forms of game. When the modelling of electric vehicle charging demand is attempted, this will also be taken into consideration.

In the following section, we examine some cases where in Game Theory was employed in related domains and study what approaches and methodologies were used therein. This will be helpful in identifying techniques which can be useful in carrying out this project.

2.3. Game Theory Applications in Related Fields

In this section, will look at how researchers and mathematicians have tried to apply game theoretic concepts and formulations to the electric power sector especially looking at smart grids, decentralized electricity markets and electric vehicles integration and optimizations to understand better the possibility and scope of applying similar concepts and formulations to determine the net electric vehicle charging demand of privately owned EVs.

In [11] the authors envision the future smart grid to be a scaled up cyber-physical system with built in state-of-the art power, control, communications and computing technology. The paper analyses the potential of applying game theoretic solutions to address the challenge of integrating these technologies into the Smart Grid. The authors explore three emerging technology areas in the Smart grid namely micro-grid systems, demand-side management, and advanced communications systems and study the contributions of different mathematical game theory modelling systems can have in simulating the respective behavior of these technologies. The authors discuss on how game theory can help in processing and optimizing the various parameters in each of these technologies and suggest further applications of the same.

Application	Game Theoretic Technique	Main Future Extensions
Subsection III-B: Cooperative energy exchange between micro-grids (such as in [22]).	Coalitional games	• Use matching games or auctions for assigning sellers to buyers. • Propose new equilibrium concepts for cooperative games with auctions. • Include communication overhead and market prices. • Study dynamic models and include storage capabilities.
Subsection III-C: Distributed control of loads and sources in a small-scale power system (such as in [23]).	Noncooperative Nash games	• Study the impact of variations in generation rates on the system. • Develop algorithms for finding equilibria in multi-source, multi-load games. • Study evolutionary game models that include notions of information and time. • Develop heterogeneous games which comprise, beyond sources and loads, additional smart grid components as players with different strategies.
Controlling the usage of stored micro-grid energy (such as in [15]).	Noncooperative Nash games, the Potluck problem, and auction theory	• Introduce a stochastic game model. • Develop learning algorithms for multi-player storage control in micro-grids.
Other future game theoretic applications in micro-grids could involve several types of games such as facility location games, Stackelberg games, advanced Nash games, and others.		

Figure 12: Game Theoretic techniques for Micro Grid Applications [11]

The main technical challenges in each of these technology areas are identified and then it is discussed how specific game theory approaches can be applied to mitigate these challenges. The authors also suggest future directions, such as implementing more robust and fool proof strategies amongst other measures, to ensure that the gap between theoretical simulations and practical implementation of Smart Grids is reduced. This is illustrated in the above table from [11] which delineates the above mentioned analysis for the micro grids technology area.

It is also noted that most current and past work is focused on static non-cooperative games and it is suggested that these be also analyzed from a dynamic perspective as a lot of parameters related to the grid such as generation and demand are time variable in nature. There is also mention of Bayesian games which is a type of non-cooperative game in which different players have very limited knowledge of the actions and strategies of of other players. The authors say that given the large-scale nature of the grid it can be interesting to see how Bayesian games can overcome the technical difficulties in estimating the exact strategies of a large number of players.

In [6], the authors undertake an analysis of the economic aspects of the integration of EVs in the smart grid by developing a mean field game model. They develop a framework which enabled an analysis of the variation of electricity price, of the hourly demand, and the possibility of energy reserves in the Smart Grid when EV owners choose to buy/sell energy based on their selfish but rational interests aimed at maximizing their benefits under the restraint of different electricity pricing. The authors go on to say that since the number of players is large and alike, the pricing policy becomes a consequence of the action of all the players, and thus the problem was the formulated as a mean field game and the fundamental differential equations for which was solved to obtain conclusions. From this paper, it is interesting to observe the use of a mean field game analysis. Unlike traditional N player games where the objective is to follow the state of each player, in a mean field game analysis the objective is to obtain the optimal distribution for all players to be at a certain state X at an instant of time t. Thus, in such a case it allows the simulation to follow the state of all users at the same time. A detailed description of the notion of Nash equilibrium in mean field games, which is termed as mean field equilibrium is outside of the scope of this report, and the original publication cited here should be referred for detailed understanding. However, in essence the concept of the equilibrium representing a steady state is similar to that of a Nash Equilibrium as has been discussed in the section explaining the basics of game theory.

In [13], the authors formulate an energy management game which exploits the potential of electric vehicles as the most shiftable load to achieve residential demand side management in the future smart gird. The utilize game theory to come up with an autonomous energy management system for residential users who want to sell energy back to the grid by discharging the battery of their EVs. In this case the players of the game are the residential users and their strategies are their profiles of daily usage of their household appliances. The further demonstrate that the Nash equilibrium of their game theory implementation results in optimization of energy costs even including the depreciation cost and adverse effects on the life of the battery as a result of frequent discharging and selling energy back to the grid. The application of game theory to their energy management model results in reduction of total energy costs and individual utility bills. They do conclude in the end by saying that considering the depreciation costs of the battery the utility company might need to provide incentivized special prices to promote residential users to store and sell energy back to the grid at appropriate times.

Another novel approach to our study of game theory implementation can be the approach used by [14] using a non-cooperative Stackelberg game which is a type of non-cooperative game that works with a multi-tiered strategy based decision making process involving number of independent decision makers or players (called followers) in response to the strategy of a main leading player (the leader). They model the smart grid as the leading player which decides its pricing by striking a balance between optimizing revenue and encouraging participation of EVs. The EVs on the other hand decide on charging strategies to optimize the balance of charging the battery and the cost incurred to do so. The authors further find an equilibrium for the game which in this case is called the Stackelberg equilibrium in which for an optimum grid pricing strategy there are EVs with preferred equilibrium strategies. They further formulate a distributed algorithm to achieve this equilibrium and run simulations on the same. Their model is further expanded to time variable model which can take into account slowly varying conditions. Their simulations demonstrate improved performance gains in terms of utility per EV compared to other optimization techniques.

The above approach may be applied to our attempt to model charging demand of privately owned electric vehicles in a case where there is dynamic pricing. In such a case, the pricing strategy can be assumed to be a leading strategy while the charging strategies of the EV user will result as the followers. This is a suggestion at this point and may or may not lead to delivering optimum results. Overall the above approaches to employing game theory in fields related to our domain of study has helped identify methodologies and approaches which might be possible to implement in trying to estimate the net charging demand from privately owned electric vehicles.

3. Literature Review and Technical Background

In this chapter, we will build background on relevant aspects of this project. In the first section, we will cover electric vehicles and their features which will later be used for this project. Thereafter, we will take a look at certain urban mobility concepts, followed by a study of demand models. In the final section of this chapter we will take a look at concepts essential to game theory modeling which will be employed in this work.

3.1. Electric Vehicles

In this section, we will cover the electric vehicles which were considered to be applicable to this project. These include two of the highest selling electric vehicles in the world, the Tesla Model S and Nissan Leaf.

The Tesla Model S was first introduced by Tesla Inc., in 2012. The vehicle is most renowned for having extensive range of up to 539 kms for the 2017 top end model which comes equipped with 100 kWh battery pack. The top end Model S (performance) is powered by a 3 phase four pole AC induction rear mounted motor with 310 kW of power and 600 N.m of torque. The base model which is considered for this study uses a motor which produces 270 kW and 440 N.m of torque. The battery contains lithium-ion battery cells in modules which are wired in series. It is guaranteed for 8 years or 200,000 kms for the base model. The standard European charger accepts single phase 230 V at 7.6 kW and 3 phase 230V or 400 V at up to 11 kW.

The Nissan leaf is a five-door hatchback electric car manufacture by Nissan and was introduced for the first time in the Japan and United States in 2010. This was followed by its introduction in the European market as well as Canada in the year 2011. The 2016 model year LEAF with the 30 kWh battery is expected to have a range of 172 km while there is a lower spec model with a 24 kWh battery which is expected to give a range of 135 kms. The LEAF uses a front mounted synchronous electric motor which can deliver 90 kW of power and 280 N.m of torque. Models are usually equipped with an on-board 3.6 kW charger that can be fully charged in around 8 hours from a 220/240 V 30 A supply.

3.2. Demand Models

[1] uses a bottom up approach utilizing process data with stochastic variables and them implementing repeated random sampling using the Monte Carlo technique to emulate the parameters. This model is therefore termed as a *'Probabilistic Agent-Based Model'*. The model includes the concept of an EV agent which comprises of the driver plus vehicle. The agent has associated variables such as type of vehicle, battery power, energy consumption, autonomy of operation etc. The EV types uses a probability distribution function to define the probability of different types of passenger cars EVs used in Spain and a similar model will be used to for this paper. The authors further go on to define

mobility pattern for the agents based on trips per day, distance per trip, destination, time and day, and velocity. Social variables are such as GDP and population density are taken into account to determine the likelihood of a number of agents to be charging at a location. The agent based model employs a time loop which updates the environment and inside there is an agent loop which updates the state of the agent. This update is based on the stochastic variables thus relating to a certain probability.

[15] presents a spatial and temporal model of EV charging demand for a single charging station located very close to a highway exit. While this is different from the goals of our study, it is interesting note the approach and certain specific findings. The authors suggest a mathematical model of electric vehicle charging demand for a single rapid charging station. They base the model on traffic flow based on a fluid dynamics model and the M/M/s queueing theory. The traffic model is utilized to determine an arrival rate for EVs in need of charging to the charging station and then the queueing theory is added to forecast a charging demand for the given station enabling distributors and operators to plan for the same. The run a simulation using a numerical example through which they claim that the model captures the spatial and temporal dynamics of a highway charging station.

[16] applies probabilistic power flow (PPF) to analyze the impact of Plug-in Hybrid Electric Vehicles (PHEVs) on the electricity grid. The authors assert that since the charging patters of PHEVs is determined by several uncertain parameters, PPF is a good approach to study the same. They propose a methodology which starts by employing a single PHEV charging demand model and thereafter employs queueing theory to model the behavior of multiple vehicles. The further apply this model to compute the net charging demand at an EV charging station as well as from a residential community. The results obtained from the model are then put on a test case by using IEEE 30 bus test system and the results of the PPF were compared against Monte Carlo simulations. The authors mention that while their methodology yields good results, in future it would be imperative to take into account a scenario with controlled charging of EVs including measures such smart charging.

In [17], the authors try to model and analyze the load demand from an EV battery charging in a typical U.K. distribution system. Their approach is to create a stochastic formulation which takes into account the randomly distributed nature of the battery charging times of EV users and the initial SOC of each battery. They further formulate four EV charging scenarios taking into account future trends in electricity prices in the market and regulations pertaining to EV battery charging and do comparative analysis between the four. The time-based charging load for the EV battery is considered for the most common battery used, the Li-ion battery. The paper further comes to a conclusion that a 10% deployment of EVs in terms of market share in the distribution system under study can result in an increase of 17.9% in the daily peak demand and consequently a 20% EV penetration may result in an increase in daily peak demand of around 36% for a scenario without any external controls imposed on the charging of EVs. Scenarios such as off-peak charging demonstrate an increased demand only during night with no effect on the daily peak demand. The paper also suggests that the distribution of the start times of EV charging can have significant impact on the load. As a consequence, the authors suggest smart charging scenario wherein the cheapest hour of electricity prices is selected to begin charging. However simultaneous start of multiple EV charging may lead to significant increase in off-peak loads which may lead to the creation of a new peak in off-peak demand profile. The authors finally suggest that the load from EVs to be analyzed in detail must be segregated into residential, industrial and commercial to asses correctly the impact of EVs on the demand load profile. This is advice which is taken into consideration for our further studies thus restricting the study to estimating the energy demand from charging only privately owned EVs.

3.3. Game Theory Modelling

When talking about game theory and situations to which it can be applied, we often talk about the participants of that game in terms of players or agents. Please note that in this text both these terms, players and agents, have been used interchangeably. To consider solving problems using game theory it is helpful to think of games in the following basic classification:

1) Simultaneous play games or normal form games:
   The payoffs for a game where each player plays simultaneously without knowledge of previous historical moves and of other players moves i.e. games of the normal form, can be and are usually represented in a matrix form. For a 2-player game it is can be indicated in a 2D matrix while multidimensional matrices may be required to form the play off matrix of for multiple player games.

2) Sequential play games or extensive games:
   For a sequential play scheme where every player is aware of its previous historical moves and has arrived at a state based on previous decisions and strategies, i.e. an extensive form game is usually represented in a tree form where each node in the tree indicates a certain players state.

In the following sections, we will go through certain representations of games and methods of solving them which are termed as solution concepts. These were essential in implementing the game theory modelling in this project. The representation presented herein is inspired by the format in [18].

Representation of Normal form or Simultaneous games – Matrix Representation

A game with finite number of players also called a n-person game can be represented in the following manner. The set N is used to represent all agents or players. Thus Agent 1, Agent 2, Agent 3 and so on up till Agent n are represented by the following set:

$$N = \{1,2, \dots \dots, n\} \tag{1}$$

Now for every player or agent i there exists a finite set of all possible actions A_i. The set of all possible actions which can be taken by a player or agent is called an action profile and is represented as:

$$(a_1, a_2 \dots \dots, a_n) \in A_1 \times A_2 \times \dots \dots \times A_n \tag{2}$$

An action profile is in essence a choice of action or decision for each agent. Now, for each Agent i, we can create a real valued function which will assign utility payoffs based on the action selected by the agent.

$$u_1 : A_1 \times A_2 \times \dots \dots \times A_n \rightarrow \mathbb{R} \tag{3}$$

A natural way to represent a normal-form game is with an n-dimensional payoff (or utility) matrix that shows every agent's utility for every action profile. Each cell in the matrix becomes the position of a utility for a certain action profile. For the famous game theory example problem, the prisoner's dilemma which is a two person comply/defect game, this can be represented as follows:

Table 1: Prisoner's Dilemma payoff matrix

1/2	**C**	**D**
C	-1, -1	-4, 0
D	0, -4	-3, -3

The above representation is explained as follows. The first row corresponds to player 2's strategies, and the first column corresponds to the strategies of player 1. The first number in each cell is the payoff obtained by player 1 for that mix of players' strategies and the second is the payoff available to player 2. In the above table, C stands for Comply and D stands for Defect. These strategies will be further discussed at the end of this section when the above game is solved.

Representation of Extensive form or Sequential games

Extensive form can be converted to normal form and thus basic nomenclature regarding representation remains similar. However, there is an inherent temporal structure in a sequential game and thus the representation is not possible in a matrix form. A tree structure is used to represent sequential games. This is demonstrated in the following image:

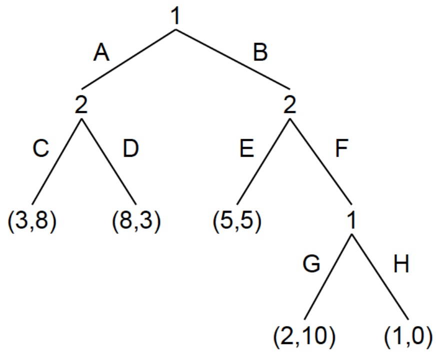

Figure 13: Representation of Sequential games [18]

It is essential to note here that all sequential games can be reduced to normal form or simultaneous games and that at each node in the above tree representation corresponds to a simultaneous move or normal form game.

Before proceeding with the discussion of different kinds of strategies as applicable to games, we will briefly examine a categorization of games which might be useful in our study. This categorization is based on the sum of payoffs obtained for each action profile. If for all action profiles, the sum of the payoff is the same and is a constant, it is called a constant sum game.

$$u_1(a_1, a_2 \dots \dots, a_n) + \quad \dots \dots + u_n(a_1, a_2 \dots \dots, a_n) = C \tag{4}$$

If the sum of payoffs is a constant, the game can be transformed into what is commonly known as a zero-sum game by subtracting C/n from each pay off. These games are purely competitive (win/lose) in nature. This is illustrated in the following payoff matrix in the game of matching pennies.

Table 2: Matching Pennies payoff matrix

1/2	**Heads**	**Tails**
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

On the other hand if the sum $u_1(a_1, a_2 \dots \dots, a_n) + \dots \dots + u_n(a_1, a_2 \dots \dots, a_n)$ is not a constant or zero and is different for different action profiles, the game is called a non-zero sum game. The unique feature of these games is that they can feature co-ordination and co-operation.

Strategies in Games

In games, there can be in essence two kinds of strategies which players can adopt and these are classified based on surety or probability of the players taking a certain action. While in one set of strategies each action is fully certain, in the other certain probability is associated with each action profile. These two kinds of strategies are Pure Strategy and Mixed Strategy.

Pure strategy is defined as a single action that a player or agent can take in a game. It comprises of a single action on part of each of the agents or players. Each row and column of a payoff matrix represent a pure strategy. A set of all such strategies is termed as pure strategy action profile.

A mixed strategy is one which has a certain probability attached to each of the actions that an agent can take. A mixed strategy when represented as s_i, implies:

$s_i(a_i)$= probability that action a_i will be played under mixed strategy s_i (5)

Both these kinds of strategies will be better explained in the section on solving games, where with the help of simple games, examples would be provided of these kinds of strategies.

Since a payoff matrix or game tree represents only payoffs or utility obtained from pure strategy profiles, there is need to introduce a concept regarding the utility for mixed strategy profiles.

Expected Utility

In a payoff matrix, each row and column represents a pure strategy and each cell gives the payoff for a certain strategy based on the actions of all the players. However, when the case is generalized to include mixed strategies, we have to introduce the concept of expected utility. The key here is to calculate the probability of each outcome based on the strategies of all agents and then calculate the average payoff for agent *i* weighted by the probabilities. For a strategy profile $(s_1, s_2 \dots \dots, s_n)$ the expected utility is

$$u_i(s_1, s_2 \dots \dots, s_n) = \sum_{(a_1, a_2 \dots\dots, a_n) \in A} u_i(a_1, a_2 \dots \dots, a_n) \prod_{j=1}^{n} s_j(a_j) \qquad (6)$$

This will be demonstrated numerically in the latter part of the next section.

Solving games:
When we are dealing with a single agent the optimal strategy can be based on maximizing the payoff in the field the decision theory is being applied to. However, with multiple agents, the best strategy can and will usually depend on other agents' choices. This is solved by trying to identify certain logical outcomes defined by different game theory texts and literature and are called solution concepts.

In the following section, we will briefly go through the most important game theory solution concepts and see in detail through examples the solution concept we will be applying to the problem at hand, the Nash Equilibrium.

Dominance
A strategy s_i is said to dominate strategy s_i', if the former gives the agent a better payoff than the latter for every strategy profile $s_{-i}$of other agent. Thus, this solution concept is based on Iterated elimination of strictly dominated strategies (IESD) or other ways of find strategies dominated by other strategies as by the theory of rational choice and to maximize payoff no agent would ever play a dominated strategy. This method is based on iterating repeatedly to check if a certain strategy dominates other for a certain agent.

Pareto optimality
A strategy profile *S* is said to Pareto dominate strategy profile *S'* if no agent *i* gets a payoff which is worse by playing with profile *S* over profile S' for all *i*. While this implies that the payoffs maybe equal, there is an additional clause which states that at least one agent should have a better payoff with strategy *S* than with *S'*.

The concept of Pareto optimality is based on the above definition of Pareto dominance. A strategy profile S is said to be Pareto Optimal if there is no profile S' which Pareto dominates S. It is also known that every game must have at least one Pareto Optimal profile and there always exists one Pareto Optimal profile wherein all strategies are pure.[18]

Nash equilibrium
While the concept of Nash Equilibrium was briefly mentioned in the introduction to game theory in chapter 2, here we will delve deeper into the concept. It is essential to state here that most solution concepts are interrelated in one way or the other and this particular solution concept was chosen for this project as a Nash equilibrium represents a steady and stable state for a given system where no agent has an incentive to shift from his actions. As for a multi agent system at which this project was originally aimed the goal is to find the final state of the system which is stable this concept was chosen for solving the problem at hand using game theory.

To understand the concept of Nash equilibrium it is essential to know first of best response. A best response for a given player is an action profile where in the player cannot gain more utility by shifting to another action profile. In essence drawing from the definition in chapter 2, it can be said when each player is playing best responses to other players best responses, the system is said to be in Nash Equilibrium.

Mathematically this can be explained as follows. Let S_{-i}be the set of all strategies without the strategy of agent *i*.

$$S_{-i} = (s_1, s_2 \dots s_{i-1} s_{i+1} \dots, s_n) \tag{7}$$

Thus S_{-i} is the strategy profile *S* with the strategy of agent *i*. Let s_i be by strategy for agent *i*. Then,

$$(s_i, S_{-i}) = S \tag{8}$$

Let s_i' be the best response to S_{-i}, then for all strategy s_i available to agent *i*:

$$u_i(s_i', S_{-i}) \geq u_i(s_{\square}, S_{-i}) \tag{9}$$

Now coming back to Nash Equilibrium, a strategy profile $s = (s_1, s_2, \dots \dots, s_n)$ is a Nash equilibrium if for every *i*, s_i is the best response to S_{-i}, that is no agent or player can benefit from deviating from his strategy.

A Nash Equilibrium is said to be strict if s_i is the only best response to S_{-i}, that is any deviation from the equilibrium strategy will result in the player doing worse. If there are multiple best responses to S_{-i}, then each of them will form weak Nash equilibrium. Pure strategy nash equilibria can be both wear or strict where as mixed strategy nash equilibria are always weak. The reason for mixed strategy Nash equilibria being weak is because if there are more than 2 pure strategies that are best responses to S_{-i}, then any mixture of them is also a best response.

If a strictly dominant strategy exists for one player in a game, that player will play that strategy in each of the game's Nash equilibria. If both players have a strictly dominant strategy, the game has only one unique Nash equilibrium. However, that Nash equilibrium is not necessarily Pareto optimal, meaning that there may be non-equilibrium outcomes of the game that would be better for both players.

Please note here, that solving games is not being discussed extensively with respect to sequential games, as for our study the scenario reduces to multiple charge/not charge simultaneous games at every time step of the day which is essentially a sequential game. And the method to solve a sequential game often involves reducing it to normal form and then proceeding with a solution concept. While talking of solving sequential games without converting to normal form, two concepts are often discussed sub-game perfect equilibrium and backward induction. Subgame perfect equilibrium deals with an equilibrium situation in a certain branch of the game where a certain agent has no incentive to follow a certain path based on the outcomes. Backward induction is a method of solving games by starting at the end of tree branch retracing steps to see the logical solution of the game often identifying subgame equilibria as well.

After discussing the above strategies, we will now proceed to see some numerical examples where it will be demonstrated on how Nash Equilibria are found in different cases. Let us first look at the prisoner's dilemma game to understand solving for pure strategy Nash equilibrium.

Let us set the premise of the game first. There are two prisoners in police custody in two separate rooms. For each prisoner, the police are trying to get them to testify against the other prisoner and in return for testifying against the other prisoner, the prisoner who testifies will be offered a reduced sentence. The scenario boils down to the fact that if they both testify against each other i.e. defect/betray with the other prisoner, they both get sentence of 3 years each. If they on the other hand both refuse to testify against each other, thereby cooperating with each other and remain silent, the police has less evidence and can only put them both away for 1 year. However, if one betrays(defect) and other remains silent(co-operate), the one who betrays goes free while the other is sentenced to 4 years of prison time. These jail times are represented as their negatives to form the payoff matrix in Table 1 so that higher jail time is a lower payoff.

If we analyze the game, let us see the outcomes when player 1 either defects or co-operates. If player 1 co-operates but player 2 defects, player 1 gets a jail time of 4 years and player 2 goes free. However, if player 2 co-operates both get a jail time for 1 year. So, if player 1, goes with co-operation its always in player 2's interest to defect. Now, if player 1 choses to defect, player 2 will still prefer to defect as 3 years is less jail time than 4. And this will hold true the other way around. Thus, the defect, defect strategy profile becomes a strict Nash equilibrium. Here it can be noticed that defection always results in a better outcome and hence it is the dominant strategy and co-operation is the dominated strategy and hence can be eliminated. Any unilateral deviation from the equilibrium is worse for each player and this is core of the algorithm which will be used to determine the existence of pure strategy Nash equilibrium in our project.

Let us now explore another game called the Battle of the Sexes to demonstrate how to find mixed strategy Nash equilibrium. The premise of the game is as follows. A man and a woman in a couple want to go out one evening for entertainment but they have no means of communication and co-ordination. The man wants to go to watch a fight while the woman prefers to go to the ballet. Moreover, they both prefer being together than ending up alone at their preferred form of entertainment. The payoff matrix for this game is demonstrated in the following table.

Table 3: Battle of the sexes payoff matrix

Man/Woman	Ballet	Fight
Ballet	1,2	0,0
Fight	0,0	2,1

As can be observed the woman has a higher payoff of 2 over the man's 1, if they both attend the ballet while the man has the higher payoff if they both attend the fight. Incase, they attend any form of entertainment alone both receive no payoffs. It is easy to observe that ballet, ballet and fight, fight are both pure strategy Nash equilibria as there is no unilateral deviation for either player in either case to be able to obtain a better outcome. Let us now move on to explore the method of finding mixed strategy Nash equilibrium. The way to do that is to employ a mixed strategy algorithm for each player. We begin here by applying it for player 1, the man in this case. Automatically player 2 is the woman. The concept is to equate expected utilities for player 2, when player 2 plays either ballet or fight, based on what player 1 plays. As mentioned earlier the expected utility is a function of the players mixed strategy probability. Let us denote this as follows:

$$u_{2B} = f(\sigma_{1B}) = \sigma_{1B}(2) + (1 - \sigma_{1B}) * 0 \tag{10}$$
$$u_{2F} = f(\sigma_{1B}) = \sigma_{1B}(0) + (1 - \sigma_{1B}) * 1 \tag{11}$$
$$u_{2B} = u_{2F} \tag{12}$$

The above equations numerically illustrate the expected utilities of player 2 playing ballet, when player 1 plays ballet with a probability σ_{1B}. Thus in eq. 10 when player 1 plays ballet with a probability σ_{1B}, player 2 gets a payoff of 2 for playing ballet while the rest of the time player 1 plays fight with a probability $1 - \sigma_{1B}$and then player 2 receives 0 payoff. Eq. 11 does the same for player 2 playing fight. When these are equated in eq. 12 and solved for σ_{1B}, we get $\sigma_{1B} = 1/3$.

Thus player 1's mixed strategy Nash equilibrium, is (Ballet =1/3|Fight=2/3). However, this is not sufficient representation and we need to find the corresponding mixed strategy for player 2. On solving similarly as above, we obtain $\sigma_{2B} = 2/3$. Thus player 2's mixed strategy component for these equilibria is (Ballet =2/3|Fight=1/3). Thus (Ballet =1/3|Fight=2/3, Ballet =2/3|Fight=1/3) is the mixed strategy Nash equilibrium for this game.

To calculate the payoff for each player when playing the above mixed strategy Nash equilibrium, we multiply the individual probabilities of the players for a certain outcome and then in turn multiply with each players payoff and the sum of all these numbers is the individual's playoff for playing the mixed strategy equilibrium. This is demonstrated through the table below.

Table 4: Mixed strategy Nash Equilibrium: Battle of the Sexes

Man/Woman	**Ballet (2/3)**	**Fight (1/3)**
Ballet (1/3)	1, 2 (2/9)	0, 0 (1/9)
Fight (2/3)	0, 0 (4/9)	2, 1 (2/9)

Thus player 1's utility can be calculated as $1\times\frac{2}{9}+0\times\frac{1}{9}+2\times\frac{2}{9}+0\times\frac{4}{9}=\frac{6}{9}=\frac{2}{3}$. When calculated for player 2, it comes to the same. Thus, it is interesting to note that for both player the pure strategy Nash equilibria mentioned earlier offer higher payoffs than the mixed strategy ones.

After having covered the required topic for carrying out game theory modeling for our project, we will now proceed to make an outline of the process followed to achieve the results.

3.4. Proposed Methodology

As has previously been mentioned, the proposed outline for the project was supposed to be based on the utilization of the algorithm in [1] and thereafter making the agents compete amongst themselves using game theory to formulate a scenario where all the charging strategies which were formulated by the author are chosen based on competition and to check if the system had any equilibria. However, as the work progressed, due to lack of expertise in the domain of game theory modeling it was agreed and the complexity of modeling an extensive n person game, it was decided to narrow the scope of the project. Through discussions with the supervisor, it was concluded, that in the interest of time and owing to the lack of expert guidance on the subject matter, it was best to reduce the given problem to a smaller scope and apply game theory to it to better understand the theory and observe its performance on a simple system before attempting to employ it in a large system. Thus, this project was restricted to a two-step undertaking; in the first part, it was attempted to employ game theory to a multi-agent system but due to the complexity of the system, it was reduced to a form of selecting strategies based on weightage assigned to a certain strategy for a single agent which in turn was based on a payoff assigned. The second step employs game theory to a smaller scenario at hand of a household with 2 electric vehicles with different mobility patterns and to see if these electric vehicles were made to compete using game theory, what kind of load profile was to be observed for the electric vehicle charging. A flow diagram indicating the process flow for the second part is demonstrated below:

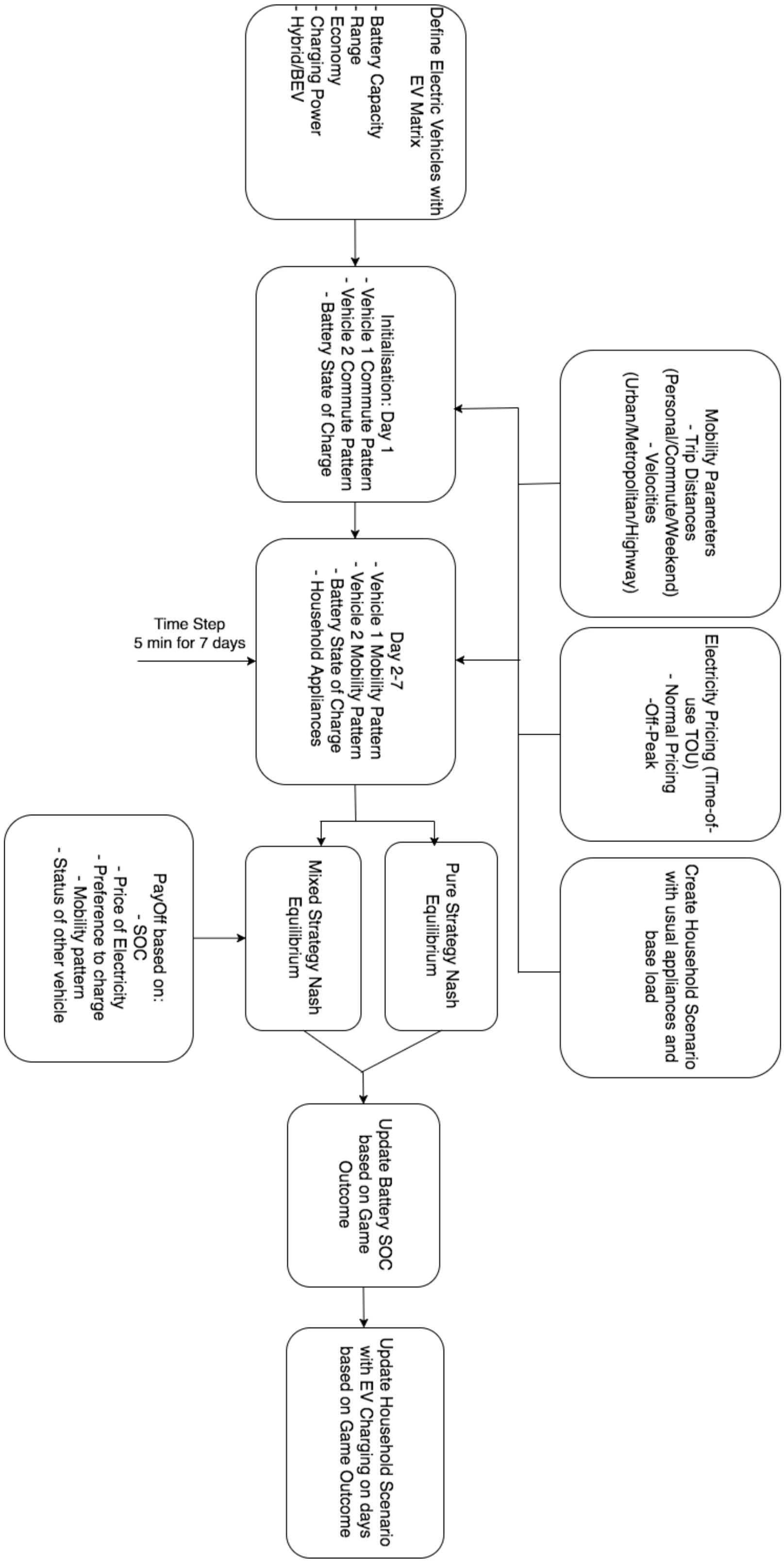

Figure 14: Flowchart illustrating algorithm and interaction of data

4. Methodology

As mentioned in the previous chapter, the central thought behind this undertaking was to employ the existing setup from [1] and employ game theory to the selection of charging strategies by users. In the simplest of terms, the procedure will allow for payoffs to be associated with each agent's charging needs based on the time of day, electricity price and charging price and an agent is more likely to charge his electric vehicle if higher payoff is associated at a certain point of time. Here, it becomes difficult to consider the influence of the remaining agents on the behavior of the said agent. It can be assumed that an agent's behavior will be completely driven by his own selfish intent and as there is no reasonable way of the agent to know of the actions of other agents, he will act solely with the purpose of maximizing his pay off.

While at the outset it was planned to employ game theory to a previous work, the complexity of solving multi-agent n player games was found to be substantial and the necessary expertise in game theory modeling was not available and it was deemed that the required expertise could not be acquired within the time frame of the master thesis.

As such it was decided to undertake this study in the following two parts:

1. To use the existing work and allot weightage to different charging strategies based on SOC, price of electricity and time of day and evaluate how the load profile from electric vehicles would change as a result that

2. To stay true to the initial aim of employing game theory, it was decided to reduce the problem to the scope of a single household and simulate a game of two players in the same household- a scenario was built where a family household had two distinct EVs for different purposes and thus different mobility patterns and they would compete amongst themselves to charge or not charge at a given time in the day. The game could be treated as multiple simultaneous move games at every time step of the day or one single sequential move game to determine optimum equilibrium strategy for each player.

In the following sections, we will go through the steps carried out to implement the above parts.

4.1. Part 1: Modification of existing EV demand prediction model- All charging strategies

For this part it is essential to understand the methodology applied in [1]. A detailed review of the publication in [1] is necessary to fully understand the approach applied and the steps taken to estimate the electric vehicle charging demand. In this section, we will briefly go through the approach and the modifications made to obtain a different charging demand at the 37-node system as considered in the original publication. A short description of the model is mentioned in Section 3.2. Hereafter we will list certain necessary details of the model which are required to understand the modifications carried out in this study:

1. Agent based modelling details: Six groups of agents were defined in this study considering mobility and their residence. Mobility reasons were based on personal or professional functions. Three different areas of residence were identified and formed the basis of start and end of trips and additionally need for charging within the network being examined. These 6 agent groups are enlisted here:

   – Group 1: Residents of the network
   – Group 2: Non-Residents but will charge once their trip is over and will stop within network
   – Group 3: Private individuals from the metropolitan
   – Group 4: Professionals - resident of the network
   – Group 5: Professionals - non-residents of the network
   – Group 6: Metropolitan area professionals

These different groups have an inclination to charge at different times for examples a resident of the network prefers to charge at the completion of his trips where as someone travelling from urban/metropolitan areas can charge between consecutive displacements. The number of these groups is sourced from open data and considers that 38% of all vehicles [19] in Barcelona are driven every day. Additionally, an EV penetration factor of 10% - 40% can be applied to evaluate different results. For the sake of this study we have used an EV penetration factor of 10% which implies that 10% of all vehicles being driven in Barcelona

2. The test network is a 37-node IEEE test feeder MV network which is adapted to Barcelona network characteristics of 25 kV MV and the geographic distribution is adapted to Barcelona's mobility data. High, medium and low inhabitants per house and vehicles per inhabitant are identified and a distribution of branches and nodes is carried out. More details on the same are available in [1].

3. The author in [1] has created 4 charging scenarios between which we will be mixing in this study to obtain a modified charging demand. These are described in brief here:
   – Scenario A- Intensive Charge – User charges as soon as possible whenever possible
   – Scenario B- Plug-And-Play – User charges at home when SOC is less than 20%
   – Scenario C- Off-Peak Tariff – User has a Time-of-Use (TOU) tariff specially for EVs based on Spanish Regulation with the cheapest hour of electricity pricing beginning at 1:00 AM

– Scenario D- Smart Charging- Realized by the aggregator who manages all EVs to consume minimum power at a certain transformer

Based on this back ground we will now proceed to understand the modifications required to the model to implement a scenario in which agents can charge using all of the first 3 charging scenarios.

The methodology and steps followed here were the following:

1) Based on the electricity tariffs on a certain, a savings potential was identified for each agent in every agent group for the different charging scenarios.
2) The savings potential along with electric vehicle state of charge and energy requirements for next trip formed the basis of allotting a payoff/weightage to each agent.
3) Based on the above weightage assigned to the user, the user decides to charge in scenario A, B or C
4) These electric charging demands are aggregated at each node throughout the day and the difference is plotted and inferences dotted therein.
5) Additionally, it is determined what percentage of users in a specific agent group charge using a certain scenario.
6) Care is also taken to take into account that certain user in an agent group might not be eligible for Scenario B and C which are applicable for users who only charge at home and/or have access to TOU tariffs.
   – Group 1: Residents of the network: This group is eligible for all three charging scenarios considered
   – Group 2: Non-Residents but will charge once their trip is over and will stop within network: As these are non-residents they will not be able to avail Scenario B and Scenario C
   – Group 3: Private from the metropolitan: These group of Agents are outside the urban limits and hence cannot avail Scenario B and C either.
   – Group 4: Professional resident of the network: This group is eligible for all three charging scenarios considered
   – Group 5: Professional non-resident of the network: As these are non-residents they will not be able to avail Scenario B and Scenario C
   – Group 6: Metropolitan area professionals: This group since outside the network is not eligible for either of the three charging scenarios considered

4.2. Part 2: Two player game in a single household

In this part, a scenario was constructed considering an affluent family household with a single charging point with typical mobility patterns of commute to work, dropping kids at school, visiting supermarkets, gym and running errands. This was done to simplify the need to model multiple users in order to reduce the complexity of the game. This will form a simple exercise in employing game theory to evaluate how agents within the same household and access to one charging port will compete and collaborate amongst themselves to ensure an optimum charging pattern keeping in mind their mobility patters. Additionally, it will also be interesting to note the effect of electricity prices and off-peak tariffs on their charging decisions. In the following section the major aspects of this scenario will be highlighted.

4.2.1. Two player household game scenario description

Certain salient features of the household used to create this scenario are:

1) Number of Occupants: It is assumed that in the household reside two adults who form a couple and two children aged between 0-17 years; This will be taken into account while considering the energy consumption of the household irrespective of the electric vehicles.

2) Major Electric Loads: The major electric loads of the house outside the electric vehicle charging are considered to be laundry, dishwashing, refrigeration and water/space heating. Typical values applicable for a family will be taken into account to generate a typical load profile for a family. Additionally, a randomly distributed consumption over a mean consumption with certain deviation will be used to account for other smaller electric loads such as lighting, television and computers etc.

3) Electric Vehicles: since two electric vehicles were to be selected, two of the most common and popular EV's were selected. One with very high range and other with a slightly reduced range. These EVs are Tesla Model S and Nissan Leaf respectively. These are hereafter referred to as EV1 and EV2 respectively.

4) Mobility Pattern: Here it is assumed that EV1 is used for commute to work and back by one of the adults in the household and thereafter another trip is made to the gym or market or errands. Similarly, EV2 is used by the other adult to pick-up and drop the children to school/university and run an additional errand a few times a week. This coupled with a longer weekend trip twice a month in EV1 is assumed to be the typical mobility pattern for the household in scenario.

   The size of these trips (commute to work, personal and weekend trip) are estimated from publications which have predicted average commute distances in different countries and distances for personal trips. In the following section, the numerical data used to build the scenario will be enlisted.

4.2.2. Data used to build the scenario

The data used in the scenario is sourced from various open public data platforms, publications related to power consumption and mobility, and manufacturer websites. All sources referenced to estimate the required numbers for the scenario are listed after the data sourced from there is presented in this report. The characteristics of the major loads considered in creating the household scenario are listed in the following table:

Table 5: Household Loads considered for Load Profile

	Dishwasher	Laundry with dryer	Refrigerator	Heating
No. of cycles per day	1	2	Cont.	Cont.
No. of time per week	7	4	Cont.	Cont.
Consumption per cycle (kWh)	1	6.12	4.8 /day	4.38 /day
Cycle Duration (hours)	2	2	Cont.	Cont.

The above-mentioned data on multiple occasions is sourced from and estimated based on data from countries where a scenario for two electric vehicles is more probable especially the Scandinavian countries of Norway, and Sweden. [20][21][22][23]

The characteristics of these vehicles relevant to the modeling of this scenario are listed in the following table:

Table 6: EV Characteristics

	Nissan Leaf (EV2)	Tesla Model S (EV1)
Battery Size (kWh)	30	70
Range (km)	420	172
Economy (Wh/km)	160	170
Max. Onboard charging power (kW)	3.6	10
Hybrid/EV	EV	EV

The above specifications have been sourced from manufacturer websites and product information brochures. [24][25]

The mobility data used to define the length of trips are listed in the following table:

Table 7: Mobility Data

Average Personal trip distance (km)	16.6
Average Professional commute trip distance (km)	10
Average weekend trip distance (km)	90
Number of trips/day (Personal)	2 per vehicle (incl. return)
Number of trips/day (Professional)	2 (incl. return)
Velocity urban (km/hr)	22.2
Velocity metropolitan (km/hr)	59.3

The above data is sourced and estimated from typical data for metropolitan cities. [19][26][27]

Electricity prices as used for Time-of-use tariffs is not listed here due to volume of the data, however a sample of the data used will be presented in Annexure A. They are sourced from Red Eléctrica de España as provided by ENDESA to its customers. [28]

4.2.3. Methodology implementation in the Scenario

In this segment, we discuss the concepts from segment 3.3 and how they are used in solving the given two-player game scenario. Firstly, the scenario at hand is modelled in the form of daily power consumption of a household over a fixed period of time like week, month or year. Thereafter each day is divided in to a 5-min time-step and a two-player game is charging/not-charging game is played between the two EV's in our scenario.

A load profile for each day is created based on the household appliance data listed in the previous section. This is coupled with a base load and additional factors accounting for occupant presence in the household and for sleep/night hours. The base load from lighting and passive electricity usage and accounting for no occupants at home and/or sleep hours is added to the continuous loads from refrigeration and heating. The base load is modelled based on data from [22]. A normal distribution with a mean of 0.2 kW and small standard deviation of 0.05 thus form the base load on which all other loads and factors are superimposed to create a typical load profile for the scenario.

It is then attempted to consider the game as a simultaneous move game represented as a normal form game at every single time step during the day and analyze where it makes sense for the EV user to charge their vehicle. This means in a day there are 288 games and for a valid game there exists a Nash Equilibrium (pure strategy or mixed strategy equilibria) which will ensure that it is the best response for each EV to the conditions of the scenario as well as to the strategy of each EV.

The payoffs attached to each time step for each user are based on the vehicle SOC, price of electricity at that hour, whether or not the other vehicle is charging and how close it is to fulfilling its max charge. These factors determine the inclination of any EV to charge and are used to ensure a logical pattern to the needs of charging.

Thereafter, an algorithm for iterated elimination of strictly dominated strategies is used to arrive at pure strategy Nash equilibria if existing for every time step in the day. Post that an algorithm to find mixed strategy Nash Equilibria is constructed based on the probability of a certain player to play a certain strategy between charge and not charge and matching his expected utilities. From these equilibria, the one with higher utility will be picked as the player's move and the scenario will be updated accordingly.

As mentioned in the section explaining the background on the game theory modelling, the scenario although treated as a simultaneous game at each time step, it is essentially a reduction of a sequential game to be solved as a simultaneous or normal form game.

5. Results and Discussion

In this section, we will present and discuss the results obtained from the two parts of this project. In the following section, we will first present the results obtained from employing weighted strategies based on price of electricity, SOC and mobility on the algorithm from [1], and discuss the same while mentioning certain important observations therein. Thereafter, we will proceed to present the findings from the two-player household scenario game and the results obtained from solving that game.

5.1. Results and Discussion: Part 1

In Fig. 16 the original demand at each node is plotted which is then followed by the modified demand obtained by giving weightage to strategies as described in the previous section. While overall load distribution across the nodes is similar there is a reduction in peak load in some nodes that is observed. This can be attributed to time based preferences in the weightage that is attributed to different strategies.

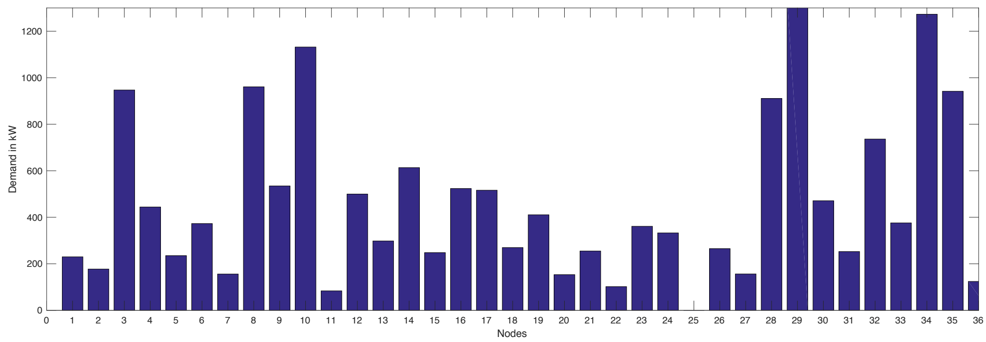

Figure 15: Original Demand at each node

The peak demand seen in the previous figure at nodes 29 and 34 is above 1200 kW however as observed in the following figure the peak demand at those nodes is now well below 1200 kW.

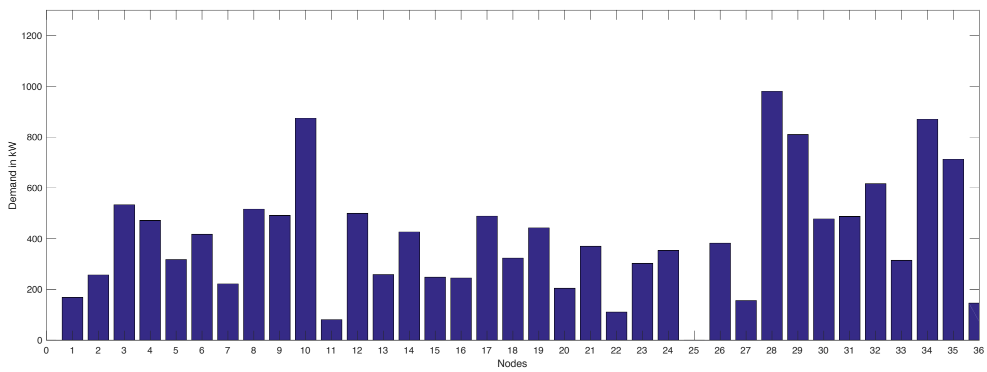

Figure 16: Demand at each node with weighted strategies

Let us now look at a single agent group and how their charging patterns are affected during each time step of the day with the modified strategies. Fig. 18 shows the original demand from agent group 1 at every 5-minute time-step of the day. The 5-minute time-step allows for 288 total time steps in 24 hours thus allowing for easier visualization of the day on a higher resolution.

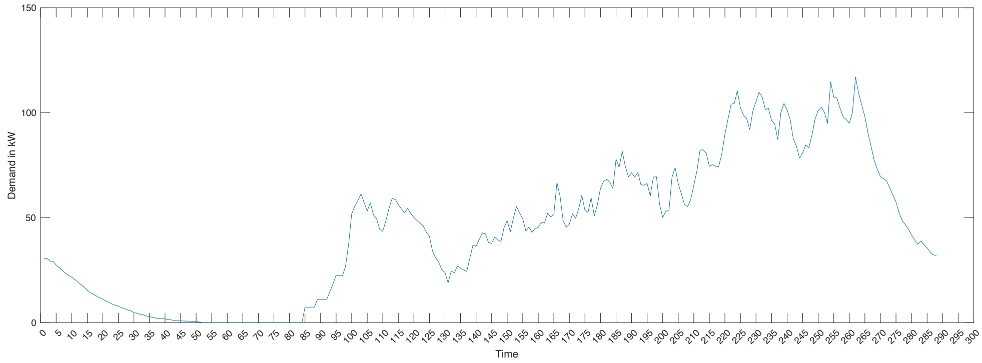

Figure 17: Original demand for every time step for Agent Group 1

In Fig. 19, we observe the modified energy demand from agents of group 1. A spike in demand is observed during low electricity price hours during the early hours of the day and a net reduction in peak demand during peak hours is also noticed. This affirms that a weighted strategy for the scenarios and giving users payoff based on the time they charge amongst other factors can result in a reduction of peak hour demand and can be to study peak shifting and valley filling phenomenon. Although, it was initially aimed to achieve results using game theory and competition, we have only been able to demonstrate certain patterns which can be expected to be observed if this exercise is extended to the implementation of game theory.

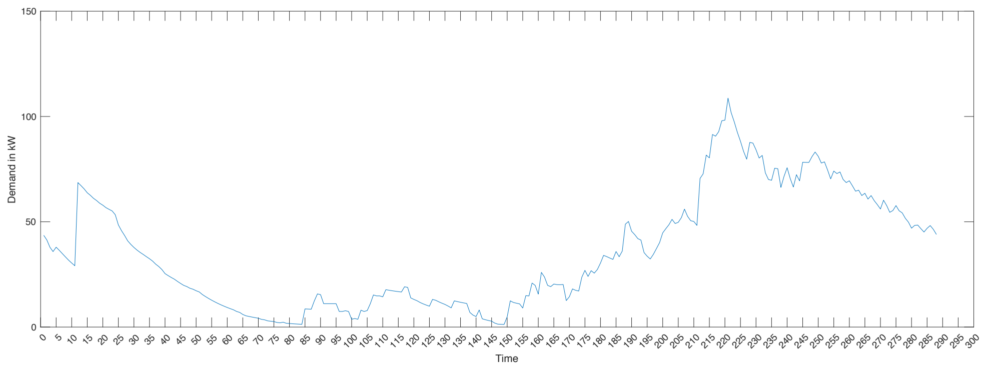

Figure 18: Modified demand with weighted strategies for Agent Group 1

Additionally, as mentioned in the previous section, it was also determined as to what percentage of users in a certain agent group are charging in a particular scenario. The findings from the same are tabulated in the following table.

Table 8: Percentage of agents charging in a scenario

	Percentage of user charging in each Scenario %		
	A	**B**	**C**
Agent Group 1	76	15	9
Agent Group 4	75	25	0

As can be observed, the majority of the agents do continue to charge in the most convenient intensive charge scenario which allows for charging whenever possible, however a shift towards scenarios B and C are also involved. As this part of the study was done by a simple weighted scheme instead of a game theoretic optimization approach technique, it is difficult to draw any strong conclusions from the above results.

Let us now explore in detail the results obtained from Part 2 of this study.

5.2. Results & Discussion: Part 2

In this section, we will examine the results from the second part of the study which involves a specially created scenario of a household with two electric vehicles.

As a first step, we examined the results from the scenario for one day to check if the algorithm was behaving as expected and if the findings seemed logical. It is interesting to observe the equilibria we have obtained when the game is solved as simultaneous game at each time step. As expected the game shows no equilibria for the initial durations of the day and eventually when both EVs are available at the residence and the price of electricity is simultaneously low, both players have a pure strategy Nash equilibrium with the charge strategy with preference going to the player with lower SOC and while the vehicle with preference charges there can be no game until the player's SOC requirement for subsequent trips is met. Thereafter, the second EV start charging and again there is no game involved till the second EV attains a desirable SOC. While this seems logical, there are other iterations which can be examined herein where player two believes he has higher utility by charging when player 1 is not available to charge even though the price of electricity is higher, thus essentially not competing in the game. This is based on subjective utility given to convenience and factors and results in a shift in equilibrium to outside what is observed earlier in the off-peak hours. After observing this on a singular day the game was expanded to run for a week with additional trips.

Just to demonstrate what a typical 2x2 matrix for a time step looks like, in the following table we will see the first equilibrium reached where EV2 has a low SOC and decides to start charging.

Table 9: Typical Payoff matrix as obtained for Charge/Charge equilibrium scenario

EV1/EV2	Charge	Not Charge
Charge	3, 6	1, 1
Not Charge	1, 1	1, 1

In the above table, it is a charge/charge equilibrium but in our scenario only one EV can charge at a time and it makes sense to charge the one having a higher payoff which incidentally and because of the design on the algorithm has a higher payoff which in this case is EV2. It is also essential to observe that the resulting game is not a zero-sum game and thus is not purely competitive and thus there can be scope for co-operation and co-ordination.

In the following section, we will see how the load profile looks in comparison to prices and during what time this equilibrium occurs and whether the player also benefits from low tariffs at that point and how the progression of SOC takes place thus indicating till what time the EV charges. There are efficiency factors which are applied to the EV charging as well as to battery depletion to ensure that all processes are not occurring perfectly ideally and are instead more realistic.

The first equilibrium is observed on day 4, when EV2 has a higher payoff to charge during a low-price time on day 4 of the week. This can be observed in Fig. 19 where a rise in household load is observed around 0900 hrs. This is after EV2 has completed its first trip of dropping the children at school and is back at the household until the next trip which is scheduled to pick up the children from school. The game results in EV2 charging in this duration up till a point where the normal prices are higher than the off-peak prices. This is ensured by a correct construction of the payoff allotting function in the code and is verified by obtaining this result. It is also interesting to note that the game decides when the EV should charge based on it mobility needs. It is seen in Fig. 19 that EV2 engages in charging for the first time in the week on day 4 based on the knowledge of the mobility pattern for the week. Additional equilibriums with EV2 having higher payoffs are observed during the weekend depending on the price of electricity and are contributed to the mobility pattern for weekends being different, that is there are fewer or no trips, the weekend charging allows for charging up to 100% SOC for EV2 on day 6 thus being prepared for the mobility requirements of the coming week.

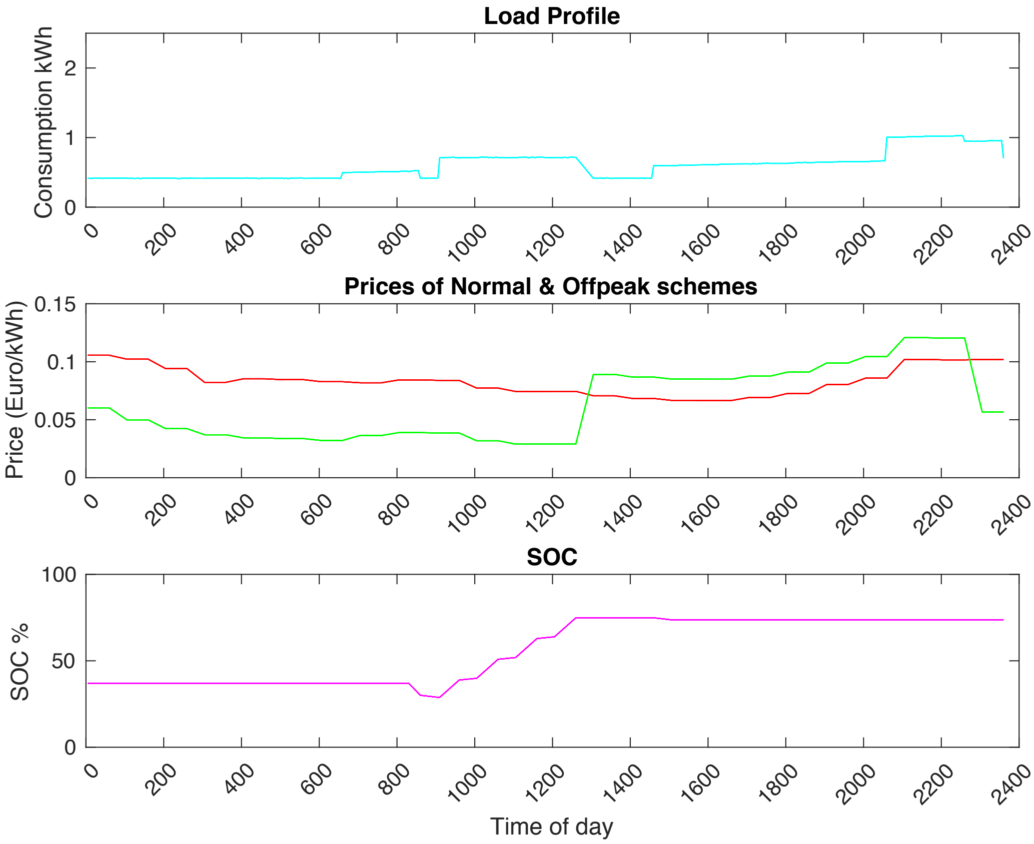

Figure 19: First Equilibrium with EV2 charging on Day 4

Post this the next equilibrium, that is observed is for EV1 with higher utility being observed for EV1 during the early hours of day 5. This is illustrated in Fig. 20. Here, again the game decides that the vehicle needs to charge taking into account user preferences for SOC, which is in turn based on plans to use EV1 over the weekend. While doing so it also takes into account payoff related to SOC and electricity prices to charge the vehicle in low-price hours for the off peak TOU tariff. EV1 owing to the time when system equilibrium is identified charges up to 100 % SOC and is thus prepared for any weekend trip or additional duties over the weekend. It's again interesting to note that the charging here takes place when the vehicle SOC has dropped to below 50% on day 5 of the week. A further equilibrium favorable to EV1 is observed on the final day 7 during off-peak low tariff hours in case there has been a weekend trip which depletes the battery. This equilibrium again ensures that the vehicle has an SOC of 100% before the next week commences. It can be observed in the plot that post the charging in the early hours of day 5 the vehicle continues with it normal mobility pattern for day 5, and the same is seen in the SOC depletion observed during the day hours. The effect observed on the demand profile for charging at the rated max high power for EV1 shows that the charging duration is short but the peak load is higher as compared to EV1 which has a lower max rated charging power. This assumes that both vehicles are able to charge at their max rated onboard charging power.

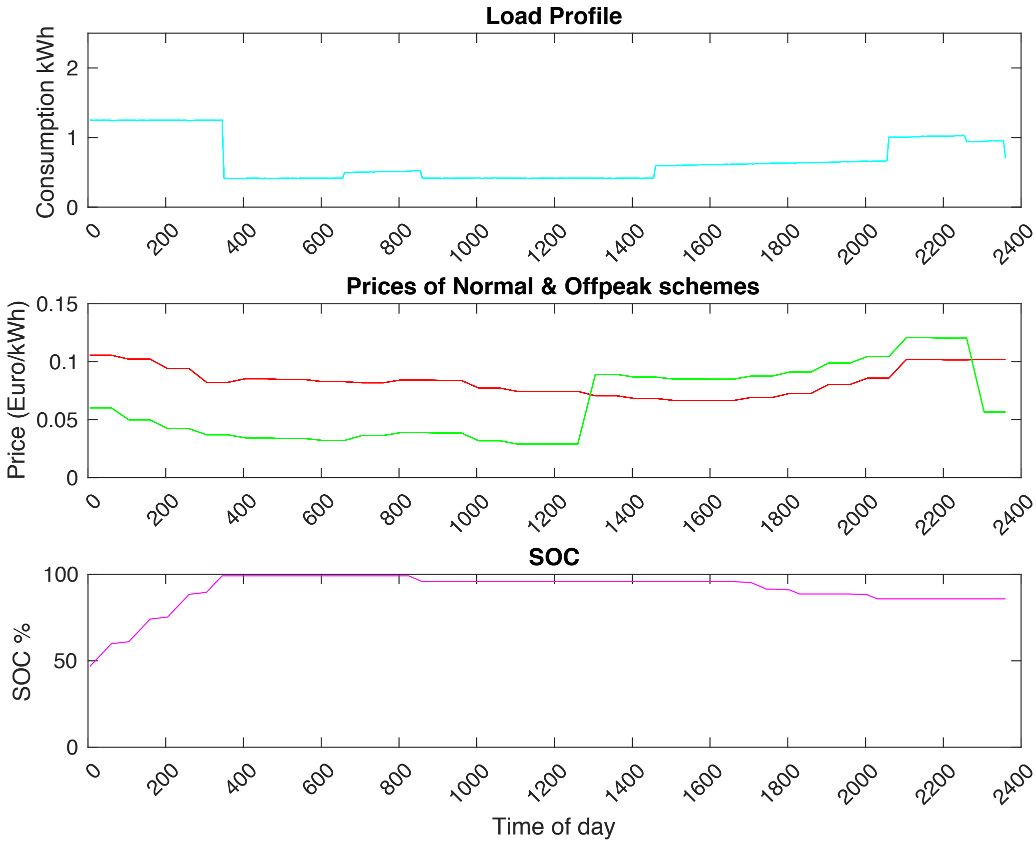

Figure 20: Second Equilibrium with EV1 charging on Day 5

Some other observations which result in a closer examination of the results are that only pure strategy Nash equilibria are observed and that all equilibria which are used to evaluate the charging demand from EV's are strong or strict equilibria. Weak equilibriums are also observed when there is no incentive for either vehicle to charge and thus there is no utility assigned at any action profile and thus all profiles end up being weak equilibriums. No mixed strategy equilibria are observed and this attributed to the fact that there is no uncertainty in the mobility patterns of the EVs. In case, there was a factor which was applied to assign probabilities of taking certain trips to each EV and as a result the SOC at any point of time on any given day of the week would be stochastic, it is expected that in such case mixed strategy Nash equilibria might have also been observed. As has been mentioned previously, the games here non zero sum game and thus allow for co-ordination and co-operation. This has not fully been explored in this project, however there is scope to apply co-ordination and co-operation mechanism to explore the findings therein.

Before proceeding to conclude, we will now take a quick look at the monetary savings which a household could expect to see from the game theoretic management of the electric vehicles charging using off-peak TOU tariff over normal tariffs. This is illustrated in Table 9.

Table 10: Savings from game theoretic electric vehicle charging using off-peak tariffs

	Saving in Euros
Yearly Savings from EV1	154.3
Yearly Savings from EV2	45.9
Net Yearly Savings for household	~ 200.0

Thus, it can be observed that through the game theoretic charging of the electric vehicles using an off-peak TOU tariff, net yearly saving of almost 200 Euros can be achieved. While this is not substantial for a household, it does clearly demonstrate how demand response can have a monetary impact on the spending of a household.

6. Conclusions

In conclusion, the findings from this work are summarized as follows. From including multiple charging scenarios such as intensive charging, plug and play, and off –peak it is observed that demand from electric vehicles is modified. Important trends such as peaks being reduced and certain amount of demand shifting is observed and thus highlights the importance of the carrying out an exercise as in part one of this work along with an optimization technique such that stronger conclusions can be drawn from the work and be utilized to understand the impact on the grid.

From part two of this work, it is possible to conclude that game theoretic methods can be applied to optimization problems in electric vehicle charging scenarios as well household energy management schemes including demand response. The results show that game theory can be a versatile tool to ensure that optimum results are obtained while taking into account preferences from agents or players. The results showed that it was optimum to charge the vehicles 1-2 times a week, even at times not up to 100% SOC while making use of off-peak tariffs to generate monetary savings for the household. The results also demonstrated certain aspects from a game theoretic point of view. These involved the lack of mixed strategy Nash Equilibria illustrating that the user's actions were fully defined and there was no probability involved based on other factors which could be used to attain additional equilibria. Another important conclusion was the observation, that the game at each time step was a non-zero sum game thus demonstrating that co-operation and co-ordination be a factor in such scenarios.

6.1. Scope for Additional Work

It is interesting to note from the above sections, that the results seem very logical when being predicted even without game theory, however this study reinforces the logicality of concepts in load management such as demand side management and demand response where it is observable that the tendency to shift to off peak hours for monetary benefit is the major observable trend while considering that it optimizes the charging behavior in a single household. Additionally, it indicates the optimum pattern in which heavy loads of electric vehicles can be shifted to allow for maximizing their utility, while maintain user convenience. Further extension of this study can involve extending the shifting of loads to all household appliances. Alternately, the game theoretic approach can be extended to an entire neighborhood or district and then the impact on the grid can be observed and potential for mechanisms such as peak shifting and valley filling can be evaluated. Also, as mentioned towards the end of the last section, there is scope to implement co-operation and co-ordination and that can be an interesting extension of this work.

With the higher integration of EVs it also opens up opportunities to utilize EVs and EV charging infrastructure in alternate ways such as the Vehicle-to-Grid(V2G) concept to supplement the grid energy and in turn facilitate higher integration of renewables. In essence, the power grid has negligible storage, as a result of which it is a constant effort to match transmission and generation to match the end user consumption. This is usually achieved by turning on/off facilities in a power plant, and ramping up and down [29]. EVs as in the case of other vehicles are designed to deal with fluctuations in power requirement as per road profile and driving behavior. This in EVs is achieved by hybrid drive trains, or battery technology which are forms of energy storage and thus can proved energy when the vehicles are parked, and with the required connections to the grid can feed power into the grid when required. This is what is termed as the Vehicle-to-Grid concept or V2G. This is additionally aided by driving patterns of personal EVs wherein it can be observed that they are used for only 5-10% per cent of all time for transportation, and 90-95% time are parked, making them available for a 'secondary function' such as V2G [29]. Thus, if a model for charging demand of EVs can be successfully built, it can be further utilized to calculate the potential of privately owned EVs to supply energy back to the grid. This requires the study of additional parameters and optimization of those parameters to achieve a successful implementation which is beyond the scope of this work. The concept has been briefly mentioned here in order to demonstrate possibilities of additional work.

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Annexure A: Electricity Prices

The following table demonstrates sample Time-of-use electricity prices used in this project. The below data is for the 30th of April 2017.

	Off Peak (eur/kWh)	Normal ((eur/kWh))
0h	0.06027	0.10561
1h	0.04992	0.10225
2h	0.0423	0.09407
3h	0.03703	0.08227
4h	0.03437	0.08532
5h	0.03373	0.08456
6h	0.03213	0.08279
7h	0.03645	0.08186
8h	0.03893	0.08434
9h	0.03855	0.08397
10h	0.03188	0.07729
11h	0.02899	0.07439
12h	0.02908	0.07448
13h	0.08909	0.07067
14h	0.08678	0.06836
15h	0.08514	0.06673
16h	0.08497	0.06657
17h	0.08761	0.06918
18h	0.09115	0.0727
19h	0.09891	0.08037
20h	0.10449	0.08589
21h	0.12075	0.10201
22h	0.12037	0.10164
23h	0.0567	0.10198

Annexure B: MATLAB Code

```
%% 2-player game for a houshold with 2 EVs and other loads
EV_matrix=[160 420 70 10 1; 170 172 30 3.6 1];

EV_SOC=[100 100];

[Price_5min_OffPeak]=xlsread('Data_Master File','Price 288
resolution','B2:B289');
[Price_5min_Normal]=xlsread('Data_Master File','Price 288
resolution','C2:C289');

vel_urb= 22.2; % city region velocity 22.2 km/h
vel_met= 59.3; % metropolitan region velocity 59.3 km/h

Avg_Trip_Dist_commute = 16.6;
Avg_Trip_Dist_personal = 10;
Avg_Trip_Dist_weekend =90;

time_step=zeros(288,7); %each column is a separate day of the week

for k=1:1:7
    for i=1:1:24
        for j=1:1:12
            time_step((i-1)*12+j,k)=(i-1)*100+5*(j);
        end
    end
end

%% intiaalise for day 1
pattern_EV1=zeros(2016, 3); %[energy_consumed SOC C=1/NC=0...]
pattern_EV1(1,2)=100; %initial SOC

EV1_commute_time=floor((Avg_Trip_Dist_commute/vel_urb)*60);
EV1_commute_energy=((Avg_Trip_Dist_commute*EV_matrix(1,1))/EV1_commute_time
)*5;

EV1_personal_time=floor((Avg_Trip_Dist_personal/vel_urb)*60);
EV1_personal_energy=((Avg_Trip_Dist_personal*EV_matrix(1,1))/EV1_personal_t
ime)*5;

pattern_EV2=zeros(2016, 3);
pattern_EV2(1,2)=100; %initial SOC

EV2_personal_time=floor((Avg_Trip_Dist_personal/vel_urb)*60);
EV2_personal_energy=((Avg_Trip_Dist_personal*EV_matrix(2,1))/EV2_personal_t
ime)*5;

PayOff_EV1= ones(2,2,2016);
PayOff_EV2= ones(2,2,2016);

PS_NE=zeros(2016,4);

day_of_week=0; j=1;

for i=1:1:288
    %for EV1
```

```
    if(time_step(i,j)>=830 && time_step(i,j)<=830+EV1_commute_time+5)
        temp=0;
        while(temp<=round(EV1_commute_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_commute_energy;
            temp=temp+1;
        end
    end

    if(time_step(i,j)>=1700 && time_step(i,j)<=1700+EV1_commute_time+5)
        temp=0;
        while(temp<=round(EV1_commute_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_commute_energy;
            temp=temp+1;
        end
    end

    if(time_step(i,j)>=1800 && time_step(i,j)<=1800+EV1_personal_time+5)
        temp=0;
        while(temp<=round(EV1_personal_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_personal_energy;
            temp=temp+1;
        end
    end
    if(time_step(i,j)>=2000 && time_step(i,j)<=2000+EV1_personal_time+5)
        temp=0;
        while(temp<=round(EV1_personal_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_personal_energy;
            temp=temp+1;
        end
    end
    pattern_EV1(day_of_week*288+i+1,2)=pattern_EV1(day_of_week*288+i,2)-
((pattern_EV1(day_of_week*288+i,1)/1000)/(0.9*EV_matrix(1,3))*100);

    if(pattern_EV1(day_of_week*288+i,2)<60 &&
pattern_EV1(day_of_week*288+i,1)==0)
      pattern_EV1(day_of_week*288+i,3)=1;
    else
      pattern_EV1(day_of_week*288+i,3)=0;
    end

    % for EV2
    if(time_step(i,j)>=830 && time_step(i,j)<=830+2*EV2_personal_time+5)
        temp=0;
        while(temp<=floor((2*EV2_personal_time)/5))
            pattern_EV2(day_of_week*288+i,1)=EV2_personal_energy;
            temp=temp+1;
        end
    end
    if(time_step(i,j)>=1500 &&
time_step(i,j)<=1500+2*EV2_personal_time+5)
        temp=0;
        while(temp<=floor((2*EV2_personal_time)/5))
            pattern_EV2(day_of_week*288+i,1)=EV2_personal_energy;
            temp=temp+1;
        end
    end
    pattern_EV2(day_of_week*288+i+1,2)=pattern_EV2(day_of_week*288+i,2)-
((pattern_EV2(day_of_week*288+i,1)/1000)/(0.9*EV_matrix(2,3))*100);
```

```
    if(pattern_EV2(day_of_week*288+i,2)<60 &&
pattern_EV2(day_of_week*288+i,1)==0)
      pattern_EV2(day_of_week*288+i,3)=1;
    else
      pattern_EV2(day_of_week*288+i,3)=0;
    end

end

[PayOff_EV1,PayOff_EV2]=assign_2P_payoff_1W(PayOff_EV1,PayOff_EV2,pattern_E
V1,pattern_EV2,Price_5min_Normal,Price_5min_OffPeak,j-1);
[PS_NE]=PS_NE_1W(PayOff_EV1,PayOff_EV2,j-1,PS_NE);


day_of_week=day_of_week+1;


%% Mobility pattern and energy consumption of EV1 & EV2

% movement pattern of EVs
for j=2:1:7
for i=1:1:288
    %for EV1
    if(time_step(i,j)>=830 && time_step(i,j)<=830+EV1_commute_time+5)
        temp=0;
        while(temp<=round(EV1_commute_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_commute_energy;
            temp=temp+1;
        end
    end

    if(time_step(i,j)>=1700 && time_step(i,j)<=1700+EV1_commute_time+5)
        temp=0;
        while(temp<=round(EV1_commute_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_commute_energy;
            temp=temp+1;
        end
    end

    if(time_step(i,j)>=1800 && time_step(i,j)<=1800+EV1_personal_time+5)
        temp=0;
        while(temp<=round(EV1_personal_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_personal_energy;
            temp=temp+1;
        end
    end
    if(time_step(i,j)>=2000 && time_step(i,j)<=2000+EV1_personal_time+5)
        temp=0;
        while(temp<=round(EV1_personal_time/5))
            pattern_EV1(day_of_week*288+i,1)=EV1_personal_energy;
            temp=temp+1;
        end
    end
    pattern_EV1(day_of_week*288+i+1,2)=pattern_EV1(day_of_week*288+i,2)-
((pattern_EV1(day_of_week*288+i,1)/1000)/(0.9*EV_matrix(1,3))*100);

    if(pattern_EV1(day_of_week*288+i,2)<60 &&
pattern_EV1(day_of_week*288+i,1)==0)
      pattern_EV1(day_of_week*288+i,3)=1;
```

```
    else
      pattern_EV1(day_of_week*288+i,3)=0;
    end

    % for EV2
    if(time_step(i,j)>=830 && time_step(i,j)<=830+2*EV2_personal_time+5)
        temp=0;
        while(temp<=floor((2*EV2_personal_time)/5))
            pattern_EV2(day_of_week*288+i,1)=EV2_personal_energy;
            temp=temp+1;
        end
    end
    if(time_step(i,j)>=1500 &&
time_step(i,j)<=1500+2*EV2_personal_time+5)
        temp=0;
        while(temp<=floor((2*EV2_personal_time)/5))
            pattern_EV2(day_of_week*288+i,1)=EV2_personal_energy;
            temp=temp+1;
        end
    end
    pattern_EV2(day_of_week*288+i+1,2)=pattern_EV2(day_of_week*288+i,2)-
((pattern_EV2(day_of_week*288+i,1)/1000)/(0.9*EV_matrix(2,3))*100);


    if(pattern_EV2(day_of_week*288+i,2)<60 &&
pattern_EV2(day_of_week*288+i,1)==0)
      pattern_EV2(day_of_week*288+i,3)=1;
    else
      pattern_EV2(day_of_week*288+i,3)=0;
    end



end

[PayOff_EV1,PayOff_EV2]=assign_2P_payoff_1W(PayOff_EV1,PayOff_EV2,pattern_E
V1,pattern_EV2,Price_5min_Normal,Price_5min_OffPeak,j-1);
[PS_NE]=PS_NE_1W(PayOff_EV1,PayOff_EV2,j-1,PS_NE);


day_of_week=day_of_week+1;
end



%plotday 4 EV2 charging
plot_day_4(4,time_step,EV_matrix,Price_5min_Normal,Price_5min_OffPeak,patte
rn_EV2,pattern_EV1,EV2_personal_energy);

%plotday 5 EV1 charging
plot_day_5(5,time_step,EV_matrix,Price_5min_Normal,Price_5min_OffPeak,patte
rn_EV2,pattern_EV1,EV1_personal_energy,EV1_commute_energy);
```

Code for Load profile:

```
base_load=normrnd(0.2,0.0025,288,1); %0.2 kw with std deviation of 0.05
x=[1:1:288];

load_profile=base_load;
ctr=0;
```

```
for i=1:1:288

    load_profile(i)=load_profile(i)+((2.19*2)/(24*12)+0.2);

    if floor(i/12)>ctr
       ctr=ctr+1;
    end

    if ctr>=7 && ctr<=8
        load_profile(i)=load_profile(i)+i/1000;
    end

    if ctr>=15
        load_profile(i)=load_profile(i)+i/1000;
    end

    if ctr>=21 && ctr <23
        load_profile(i)=load_profile(i)+1/12;
    end

    if ctr>=21 && ctr <24
        load_profile(i)=load_profile(i)+3.06/12;
    end

end
```

Function for Pure Strategy Nash Equilibrium:

```
function [EQM]=PS_NE_1W(PayOff_EV1,PayOff_EV2,day_week,EQM)
%EQM =[C,C C,NC NC,NC NC,C] clockwise across the 2x2 grid
    for i=1:1:288 %decision block every 5 mins

            if PayOff_EV2(1,1,day_week*288+i) >
PayOff_EV2(1,2,day_week*288+i) && PayOff_EV2(1,1,day_week*288+i) >
PayOff_EV2(2,1,day_week*288+i)
                if PayOff_EV1(1,1,day_week*288+i) >
PayOff_EV1(1,2,day_week*288+i) && PayOff_EV1(1,1,day_week*288+i) >
PayOff_EV1(2,1,day_week*288+i)
                %if both above statements become true then C,C is a PS_NE
                EQM(day_week*288+i,1)=1;
                end
            elseif PayOff_EV2(2,2,day_week*288+i) >
PayOff_EV2(1,2,day_week*288+i) && PayOff_EV2(2,2,day_week*288+i) >
PayOff_EV2(2,1,day_week*288+i)
                if PayOff_EV1(2,2,day_week*288+i) >
PayOff_EV1(1,2,day_week*288+i) && PayOff_EV1(2,2,day_week*288+i) >
PayOff_EV1(2,1,day_week*288+i)
                    %if both above statements become true then NC, NC is a
PS_NE
                EQM(day_week*288+i,3)=1;
                end
            elseif PayOff_EV2(1,2,day_week*288+i) >
PayOff_EV2(1,1,day_week*288+i) && PayOff_EV2(1,2,day_week*288+i) >
PayOff_EV2(2,2,day_week*288+i)
                if PayOff_EV1(1,2,day_week*288+i) >
PayOff_EV1(1,1,day_week*288+i) && PayOff_EV1(1,2,day_week*288+i) >
PayOff_EV1(2,2,day_week*288+i)
```

```
                    %if both above statements become true then C, NC is a
PS_NE
                EQM(day_week*288+i,2)=1;
                end
            elseif PayOff_EV2(2,1,day_week*288+i) >
PayOff_EV2(1,1,day_week*288+i) && PayOff_EV2(2,1,day_week*288+i) >
PayOff_EV2(2,2,day_week*288+i)
                if PayOff_EV1(2,1,day_week*288+i) >
PayOff_EV1(1,1,day_week*288+i) && PayOff_EV1(2,1,day_week*288+i) >
PayOff_EV1(2,2,day_week*288+i)
                    %if both above statements become true then NC, C is a
PS_NE
                EQM(day_week*288+i,4)=1;
                end
            end

    end
end
```

Function for Mixed Strategy Nash Equilibrium:

```
function [Pat_EV1,Pat_EV2]=MS_NE(PayOff_EV1,PayOff_EV2,Pat_EV1,Pat_EV2)


    for i=1:1:288 %decision block every 5 mins

            syms sig_u exp_util_l exp_util_r
            syms sig_l exp_util_u exp_util_d
            sig_P1=0;
            sig_P2=0;

            %player 1 mixed strategy
            eqn1=exp_util_l==sig_u*PayOff_EV2(1,1,i)+(1-
sig_u)*PayOff_EV2(1,2,i);
            eqn2=exp_util_r==sig_u*PayOff_EV2(2,1,i)+(1-
sig_u)*PayOff_EV2(2,2,i);
            eqn3=exp_util_l==exp_util_r;

            [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [sig_u,
exp_util_l, exp_util_r]);
            sol_1 = solve([eqn1, eqn2, eqn3], [sig_u, exp_util_l,
exp_util_r]);
            sig_P1(i,1)=sol_1.sig_u;
            if sig_P1>0 & sig_P1<1
               Pat_EV1(i,5)=sig_P1;
            end


            %player 2 mixed strategy
            eqn4=exp_util_u==sig_l*PayOff_EV1(1,1,i)+(1-
sig_l)*PayOff_EV1(1,2,i);
            eqn5=exp_util_d==sig_l*PayOff_EV1(2,1,i)+(1-
sig_l)*PayOff_EV1(2,2,i);
            eqn6=exp_util_u==exp_util_d;

            [C,D] = equationsToMatrix([eqn4, eqn5, eqn6], [sig_l,
exp_util_u, exp_util_d]);
            sol_2 = solve([eqn4, eqn5, eqn6], [sig_l, exp_util_u,
exp_util_d]);
```

```
            sig_P2=sol_2.sig_l;
            if sig_P2>0 & sig_P2<1
                Pat_EV2(i,5)=sig_P2;
            end


    end
end
```