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Cug 1207 Alu

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M-REPORT/370

**LOCAL AND PARTIAL CORRESPONDENCE  
ANALYSIS APPLICATION TO THE  
ANALYSIS OF ELECTORAL DATA**

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**RR85/11**

**Juliol 1985**

## RESUM

En anàlisi de dades sovint hom analitza conjunts de dades les observacions dels quals venen lligades mitjançant un graf. Aquest és el cas per dades de tipus electoral, on les unitats electorals corresponen a àrees geogràfiques ben delimitades. Aleshores pot ésser interessant analitzar el mateix fenòmen fixant la relació a priori definida pel graf.

D'antuvi presentem el "rationale" d'aquests mètodes. L'objecte de l'anàlisi local és eliminar l'efecte de la posició geogràfica dels individus, definida mitjançant un graf de contigüitat, en una anàlisi factorial exploratòria de dades parcials. Serà interessant també, analitzar els resultats electorals mantenint constant l'efecte de la posició socio-econòmica, mitjançant un graf de similitud. Això és anomenat anàlisi parcial, car és basat en la mateixa idea que l'anàlisi de correlacions parcials o de variables instrumentals d'en Rao.

En la segona part de l'article, apliquem aquesta metodologia a la matriu de dades formada per 1059 seccions censals de Barcelona, donant per cadascuna els resultats electorals en les últimes eleccions autonòmiques de 1984. A més a més també serà interessant definir zones de Barcelona amb un comportament electoral homogeni, això es aconseguit mitjançant un algorisme de classificació amb restricció de contigüitat.

LOCAL AND PARTIAL CORRESPONDENCE ANALYSIS  
APPLICATION TO THE ANALYSIS OF ELECTORAL DATA

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SUMMARY

In data analysis we must often analyze data sets whose observations are related by a graph structure. This is the case for electoral data, where the electoral units correspond to a definite geographical areas. In this case can be interesting to analyze the same phenomenon fixing some *a priori* relation.

First part we are going to present the rationale of these methods. The local analysis aims to eliminate the effect of geographical position of individuals, represented by a contiguity graph, in an exploratory factorial analysis of spatial data. It will be proved interesting to analyze the electoral results keeping the socio-economic position constant, by means of a similarity graph. This is called partial analysis because it is based on the same idea of instrumental variables of Rao and partial correlation analysis.

In the second part of the article, this methodology is applied to the data matrix formed by 1059 electoral units, called sections, giving the electoral results in the last autonomous election of 1984 in Barcelona. Moreover, it will be interesting to define regions of units with homogeneous electoral behaviour, obtained by an algorithm of clustering with contiguity constraint.

**Keywords:** Correspondence analysis, local analysis, partial analysis, local covariance matrix, Geary coefficient.

1 Introduction

Descriptive data analysis consists of the analysis of large numerical arrays by means of visual displays and some clustering methods, without presupposing any probabilistic hypotheses about the data. Very often, however, it is useful

to focus our attention upon a partial aspect of data. For example in electoral data, it may be worthwhile studying the observed phenomenon separately from the geographical position of the electoral units, or to study it by keeping fixed the so-called instrumental variables like the socio-economic profile of the electoral units; in such cases we use an a priori relation between individuals, which is classically represented by means of a non-oriented graph structure. Lebart (1969,73,84) was the first to introduce this analysis for a contiguity graph of spatial data, and for this reason, he called it "local factorial analysis". Later it was generalized for similarity graphs of individuals, (Aluja, Lebart (1985)), and it was then called "partial factorial analysis", because it is based on the same idea of instrumental variables of Rao (1965), or on classical partial correlation analysis, without, however, having the strong hypotheses implied by these latter analyses. It has been also generalized for temporal graphs by Carlier (1985).

## 2 Symbols

Let  $E$  be a set of  $n$  individuals related by a graph  $G(E,T)$ , over which we have measured  $p$  variables. Let  $X$  be the data matrix  $(n,p)$  formed. Let  $D$  be the metric matrix  $(p,p)$ , and  $p_i$  the weight of each individual.

Let  $S$  be a diagonal matrix  $(p,p)$  of standard deviations of variables.

Let  $M$  be the symmetric matrix  $(n,n)$  associated with the graph  $G(E,T)$ ,  $m_{ij} = 1$  if  $i$  and  $j$  are joined by an edge,  $m_{ij} = 0$  otherwise.

Let  $N$  be the diagonal matrix  $(n,n)$  of degrees of vertices,  $n_{ii} = \sum_j m_{ij}$ .

Let  $m$  double the number of edges,  $m = \sum_i n_{ii}$ .

Let  $T$  be the matrix  $(n(n-1)/2, n)$  crossing the edges with

the vertices. An edge joining vertices  $i$  and  $j$  is coded by a sequence 000000100-1000, with 1 and -1 in the  $i$  and  $j$  position.

Let  $U$  be the matrix with unity in all its terms,  $u_{ij} = 1 \forall i, \forall j$ . Thus  $U$  is the associated matrix for a complete graph.

Let  $I$  be the identity matrix  $(n,n)$ , Thus  $nI$  is the degrees matrix for a complete graph.

Let  $B$  be the matrix  $(n(n-1)/2, n)$  crossing the edges with vertices for a complete graph.

The following straightforward relation exists between these matrices:  $N - M = T'T = B'T = T'B$ .

Let  $L$  be a diagonal matrix  $(n(n-1)/2, n(n-1)/2)$  with weight of edges  $l_k = p_i p_{i'}$ , (if edge  $k$  is formed by vertices  $i$  and  $i'$ ).

Finally  $\sum_k l_k$  represent the double sum over the edges of the graph  $G(E,T)$ .

### 3.1 Factorial analysis upon a graph. Analysis on $R^p$

Let be a set of  $n$  points with the rows of  $X$  as coordinates and  $p_i$  weight, defined in a space  $R^p$  of metric  $D$ , and related by a graph  $G(E,T)$ . The quantity to optimize is now written as:

$$(1) \quad \text{Max } \left[ \sum_{i,i'} l_k p_i p_{i'}, d_H^2(i,i') \right]$$

where  $d_H^2(i,i')$  is the squared distance between vertices  $i$  and  $i'$  projected upon a subspace  $H$ . Taking first a subspace of dimension one defined by a unitary vector  $w$ , the projection of points over it will be written:  $X D w$ . Thus quantity (1) becomes:

$$\text{Max } [w'D X'T'L T X D w] = \lambda$$

with constraint  $w'D w = 1$

This maximum is obtained for the first eigenvalue of matrix  $X'T'L T X D$ ,  $w$  being its associated eigenvector.

$$(2) \quad X'T'L T X D w = \lambda w$$

In order to diagonalize a symmetric matrix, we pre-multiply (2) by  $D^{1/2}$ .

$$(3) \quad A u = D^{1/2} X'T'L T X D^{1/2} u = \lambda u$$

with constraint  $u'u = 1$

where  $u = D^{1/2} w$

Likewise, the subspace of dimension two would be obtained by the first two eigenvectors of matrix  $A$ . Thus, the subspace of dimension  $q$  which maximizes criterion (1) is defined by the first  $q$  eigenvectors associated with the  $q$  largest eigenvalues of matrix  $A$ .

Note that matrix  $A$  coincides with the local covariance matrix taking  $p_i = 1/\sqrt{m}$  and  $D = I$  as metric, (Lebart, 1969).

$$V_1 = 1/m X'(N - M) X = 1/m X'T'T X$$

It coincides with the contiguity matrix if  $D = S^{-2}$ , and with the local correlation matrix if  $D = S_1^{-2}$ . Note also that the local covariance matrix coincides with the classical covariance matrix of variables for a complete graph:

$$V_1 = (1/n^2) X'(nI - U) X = (1/n) X'X = V_q$$

whereas the contiguity matrix and local correlation matrix coincide with the empirical correlation matrix for a complete graph.

Looking at relation (3) we see that the factorial analysis upon a graph is equivalent to a normal (=global) analysis of matrix  $TX$  with weight matrix  $L$ . We call the columns of  $TX$  local variables, where each component represents an edge; whereas we call the columns of  $X$  (or the columns of  $BX$ ) global variables.

The projection of individuals (=vertices) upon the local factorial axis will be obtained by the relation:

$$(4) \quad \Psi_\alpha = X D^{1/2} u_\alpha$$

and the coordinates of edges:  $\xi_\alpha = T \Psi_\alpha$ .

The local inertia is:

$$In = \text{tr}(A) = \sum_\alpha \lambda_\alpha = \sum_{ij} p_i p_j (x_{ij} - x_{i+j})^2 = v_j^1.$$

Thus the influence of one variable to the local analysis will be greater if the values of that variable are very different over the edges of the graph.

### 3.2 Induced analysis on $R^n$

The induced analysis on  $R^n$ , is done by taking the symmetric of relation (3), that is:

$$(5) \quad S v = L^{1/2} T X D X' T' L^{1/2} v = \lambda v$$

with constraint  $v' v = 1$

This means taking the columns of matrix  $T X D^{1/2}$  as coordinates of variables in  $R^n$ , with  $L$  metric. The sub-space of dimension  $r$  which maximizes the projection of local variables is defined by the  $r$  eigenvectors of matrix  $B$ .

associated with its  $r$  largest eigenvalues.

It is easy to find the following relations between the eigenvectors of both analyses:

$$(6) \quad u_\alpha = 1/\sqrt{\lambda_\alpha} D^{1/2} X' T' L^{1/2} v_\alpha$$

$$(7) \quad v_\alpha = 1/\sqrt{\lambda_\alpha} L^{1/2} T X D^{1/2} u_\alpha$$

We can obtain the graphic representation of active variables according to the classical relation:

$$(8) \quad \Psi_\alpha = D^{1/2} X' T' L^{1/2} v_\alpha = \sqrt{\lambda_\alpha} u_\alpha$$

and the illustrative ones by:

$$(9) \quad \Psi_\alpha^+ = 1/\sqrt{\lambda_\alpha} D^{1/2} X'_+ T' L^+ T \Psi_\alpha$$

The contribution of one edge can be computed easily by relation:

$$\text{cont}(i, i') = (p_i p_{i'}) \sum_j 1/d_j (x_{ij} - x_{i'j})^2$$

### 3.3 Relation between the global and local analysis

From Lebart (1973,84) we know that local analysis leads to a particular partial operator (supposing graph G to be planar and regular), which means projecting the global variables upon a space orthogonal to M. On the other hand, provided that the number of edges is greater than the number of variables ( $m/2 > p$ ), the dimension of local space is  $p$ .

We evaluate the strength of the relation between both types of variables by means of its covariance matrix, computed as:

$$V_{q1} = (1/n\sqrt{m}) X' B' T X = (1/n\sqrt{m}) X' (N - M) X = (\sqrt{m}/n) V_1$$

and the correlation matrix:  $C_{q1} = S_q^{-1} V_{q1} S_1^{-1}$

$$\text{thus: } \text{cor}(j_g, j'_1) = \text{cov}(j_g, j'_1) / \sqrt{v(j_g) v(j'_1)} = \\ = (\sqrt{m}/n) \text{cov}(j_1, j'_1) / \sqrt{v(j_g) v(j'_1)}$$

It is worthwhile looking at the correlation between a global variable and its corresponding local one:

$$(10) \quad \text{cor}(j_g, j_1) = (\sqrt{m}/n) \sqrt{v(j_1)/v(j_g)} = \\ = \sqrt{\sum_i^L (x_{ij} - \bar{x}_{i'j})^2 / \sum_i^L (x_{ij} - \bar{x}_{i'j})^2}$$

Consequently, although the correlation between  $j_g$  and  $j_1$  does not depend on  $m$ , it tends to the unity when we operate with larger graphs. Generally the number of edges of a contiguity or similarity graph is far lower compared with the complete graph, (around three per cent). It implies that the local variable would be very much shorter than the global variable. In order to make the comparison plausible we weigh each individual by  $1/\sqrt{m}$  and not by the classical  $1/n$ . This leads to multiplying the local variable by an  $n^2/m$  coefficient.

Moreover, we can visualise the change in variables when we switch from a global to a local level, projecting both types of variable on the same basis:

$$\Psi_\alpha^l = D^{1/2} X' T' L^{1/2} v_{\alpha} = 1/\sqrt{\lambda_\alpha} D^{1/2} X' T' L B X D^{1/2} u_{\alpha}$$

where  $u_\alpha$  and  $v_\alpha$  are the eigenvectors issuing from the classical global analysis on  $R^p$  and  $R^n$ .

#### 4 Correspondence analysis upon a graph

Data matrices very often adopt the form of contingency tables. In such cases correspondence analysis is the appropriate description technique. Let  $F$  be the relative frequency table,  $D_n$  and  $D_p$  their diagonal matrices of marginal relative frequencies. Thus we take as individual coordinates their profile defined by the rows of  $D_n^{-1} F$  matrix, with their marginal relative frequency  $D_n^{-1}$  as weight, and a Khi-square metric, defined by the  $D_p^{-1}$  matrix. Equation (3) is now written:

$$(11) \quad D_p^{-1/2} F' D_n^{-1} T' L T' D_n^{-1} F' D_p^{-1/2} u = \lambda u$$

with  $u'u = 1$

The projection of individuals on the factorial axis will be computed:

$$(12) \quad \Psi_\alpha = D_n^{-1} F' D_p^{-1/2} u_\alpha$$

and the projection of edges:  $\xi_\alpha = T \Psi_\alpha$ .

The analysis on  $R^n$  induced by relation (12) is written:

$$(13) \quad L^{1/2} T' D_n^{-1} F' D_p^{-1/2} F' D_n^{-1} T' L^{1/2} v = \lambda v$$

with  $v'v = 1$

that is, we take the columns of  $D_n^{-1} F' D_p^{-1/2}$  matrix as variable coordinates, with weight matrix  $D_p^{-1}$  and metric  $L$ . As

can be pointed out, taking account of the graph relation means breaking up the symmetry between rows and columns that exists in classical correspondence analysis. The relation between both analyses is now written:

$$(14) \quad v_\alpha = 1/\sqrt{\lambda_\alpha} L^{1/2} T D_n^{-1} F D_p^{-1/2} u_\alpha$$

$$(15) \quad u_\alpha = 1/\sqrt{\lambda_\alpha} D_p^{-1/2} F' D_n^{-1} T' L^{1/2} v_\alpha$$

The graphic displays of active local variables on the local factorial axis are obtained:

$$(16) \quad \Psi_\alpha = D_p^{-1} F' D_n^{-1} T' L^{1/2} v_\alpha = \sqrt{\lambda_\alpha} D_p^{-1/2} u_\alpha$$

and the supplementary variables:

$$(17) \quad \Psi_\alpha^+ = 1/\sqrt{\lambda_\alpha} D_p^{-1} F' D_n^{-1} T' L T \Psi_\alpha$$

These techniques have been programmed in modular steps compatible with the software SPAD, developped by the CESIA.

## 5 Application to the analysis of electoral data

We have performed a local correspondence analysis of the electoral data emerging from the last autonomous elections in Barcelona. It is argued that electoral behaviour depends on geographical localization, that is, neighbouring electoral units vote in a similar way one to another, and the electoral distribution of votes will be very homogeneous and even all over the city. On the other hand, it is also argued that electoral behaviour depends on socio-economic position. We can show here how local correspondence analysis allows us to go further into answering these questions.

The data matrix formed contains the number of votes for each political party in each one of the 1059 electoral units. Moreover, we have recorded the distribution of inhabitants into 15 socio-economic categories; finally the used variables were:

Political parties	PCC	(Communist pro-Soviet)
	AP	(Conservative)
	PSC	(Socialist)
	PSUC	(Euro-communist)
	CIU	(Centre-right, moderate Nationalist)
	ERC	(Centre, Nationalist)
	EMES	(extreme Left)
	EEC	(Left, strongly Nationalist)
	VERT	(ecologist)
	ABST	(non voters)
Socio-economic categories	DIRE	(Managers)
	DICO	(Tradesmen)
	PLIB	(Professionals)
	TESU	(Executives)
	TEMI	(Middle order executives)
	ALTE	(Lower order executives)
	QUIN	(Foremen)
	ADMI	(Office workers)
	VENE	(Salesmen)
	SERV	(Service workers)
	DETR	(Transport workers)
	OBIN	(Industrial workers)
	OBCO	(Building workers)

In order to analyse the behaviour of electoral units, we performed a correspondence analysis taking the political parties as active variables, first fixing the geographical position of the electoral units by means of a contiguity graph. Later, we repeated the same analysis, but this time fixing the socio-economic position of the electoral units.

### 5.1 Analysis fixing the geographical position of electoral units

#### 5.1.1 Global analysis :

It gives two main factors (with 76.28 and 10.81 per cent of total inertia), the former revealing the classical "left-right" opposition, whereas the latter opposes the two brands of nationalism "Catalan against Spanish". We can see from the projection of supplementary variables that upper classes tend to be Conservative and Spanish-nationalist, the middle classes, clerical and service workers relate to the centre and Catalan Nationalism, and working class categories vote for the Left. We can also see that non-voters are related in this election with the Left and the lower classes.

#### 5.1.2 Relation between the global and local levels

In order to compare both types of variables, (local variables are identified by an asterisk in the fourth position), we can see their Geary coefficients, computed as the ratio between the global and local inertia of each variable, which are also related with their correlation, (see relation 10). Thus the Geary coefficient is a good measurement of the change of variables when we switch from the global to the local level. We can see for example from fig. 2, that all political parties have positive autocorrelation over the contiguity graph, but majority parties have strong autocorrelation rather than minority ones. With regard to the socio-economic categories, worker categories have the strong autocorrelation, followed by the upper classes, with middle classes in last position. Notice, however, that "salesmen" and "tradesmen" have negative autocorrelation. By looking at fig. 1 we can see the local variables projected on the first global factorial plan, as supplementary, contracted to the origin, although the opposition stated by the second axis seems to stand out more clearly than the opposition of the first one.

#### 5.1.3 Local analysis

Local analysis reveals, as before two main factors (41.35 and 21.64 of the local inertia = 0.035); their graphic display shows almost the same pattern we have seen in global analysis. Roughly speaking, it allows us to conclude that Barcelona displays quite heterogeneous behaviour from an electoral point of view.

### 5.2 Analysis fixing the socio-economic profile of the electoral units

The graph is now formed by edges joining electoral units with a similar distribution of socio-economic categories. The coefficients of Geary (fig. 4), now give us a measurement of the inter-classism of the political parties. For example, we can see that the ecologist receive votes from a wide range of socio-economic categories.

In fig. 5 we see the graphic display of global and local variables all together on the first global factorial plan. We can see there the different behaviour of local variables, for example, AP and PSC seem to approach each other, whereas CiU approaches the other two Nationalist parties.

Local analysis (fig. 6), gives an inertia of 0.055 with two main factors (with 59.74 and 11.93), the Nationalist parties clearly opposing the remaining parties, whereas the second factor breaks these latter parties depending on whether they are of the right or the left. Consequently, within electoral units with similar socio-economic position, the main factor of opposition is Nationalism rather than the classical "Right-Left".

### 5.3 Clustering with contiguity constraint

Although the territory of Barcelona does not appear homogeneous from an electoral point of view, this does not mean it is impossible to define regions with similar voting trends. In fact, all variables have positive autocorrelation. Thus, we have obtained a partition of Barcelona into eight zones from the dendrogram of an algorithm of hierarchical classification with contiguity constraint (see figure in fig. 8 on the first global factorial plan) the position and the number of electoral units of each one of these clusters.

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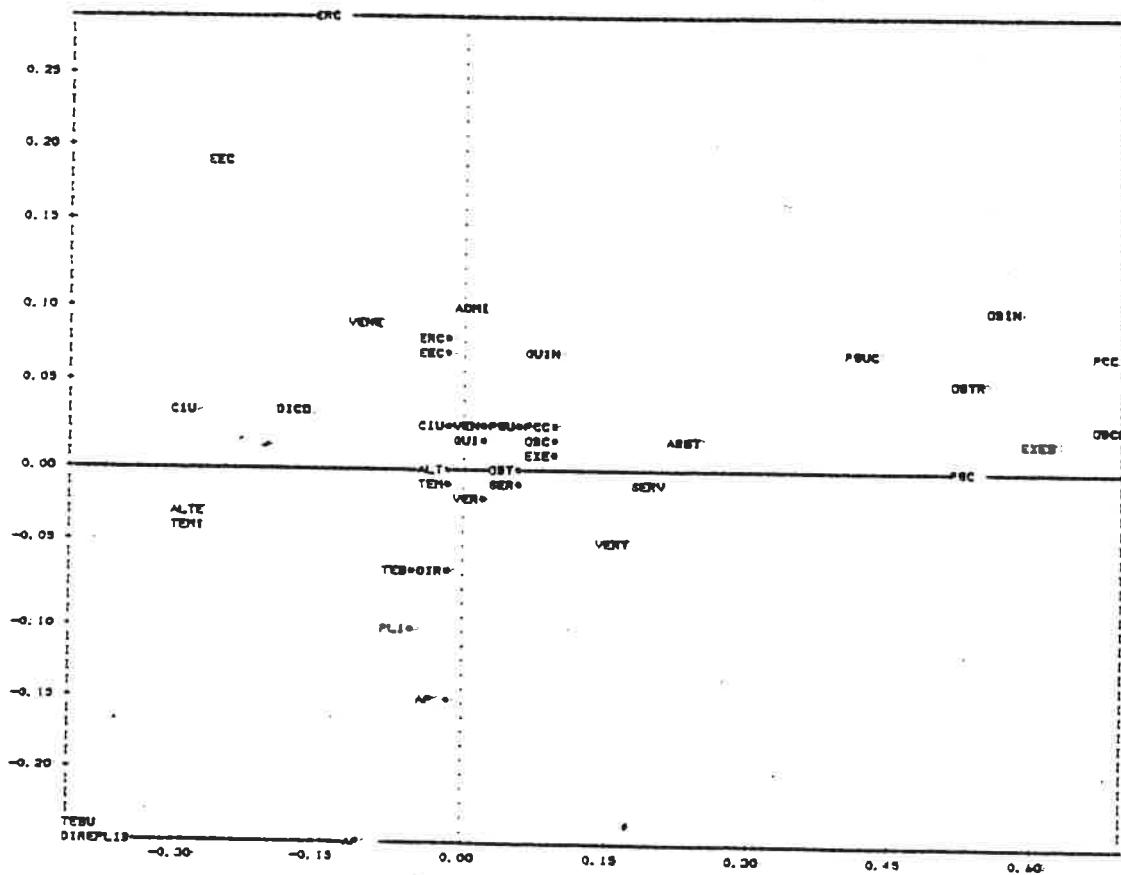


Fig. 1 First global plan. Display of local & global vars.

COEFFICIENTS DE CONTIGUITÉ DE GEARY												
ÉCART-TYPE = 0.0214												
VARIABLE C. GEARY	PROBA=	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EEC*	0.2917	0.0000		—	—	—	—	—	—	—	—	—
AP*	0.3813	0.0000		—	—	—	—	—	—	—	—	—
PSC*	0.1149	0.0000		—	—	—	—	—	—	—	—	—
PSU*	0.2479	0.0000		—	—	—	—	—	—	—	—	—
CIU*	0.1247	0.0000		—	—	—	—	—	—	—	—	—
ERC*	0.3292	0.0000		—	—	—	—	—	—	—	—	—
EXE*	0.3021	0.0000		—	—	—	—	—	—	—	—	—
EEC*	0.3101	0.0000		—	—	—	—	—	—	—	—	—
VERN	0.7039	0.0000		—	—	—	—	—	—	—	—	—
VARIABLES SUPPLÉMENTAIRES												
ABST	0.4724	0.0000		—	—	—	—	—	—	—	—	—
DIRE	0.3336	0.0000		—	—	—	—	—	—	—	—	—
DICO	1.1494	0.0000		—	—	—	—	—	—	—	—	—
PLJ	0.1877	0.0000		—	—	—	—	—	—	—	—	—
TEBU	0.2333	0.0000		—	—	—	—	—	—	—	—	—
TEM	0.4431	0.0000		—	—	—	—	—	—	—	—	—
ALTE	0.4386	0.0000		—	—	—	—	—	—	—	—	—
GUIN	0.3326	0.0000		—	—	—	—	—	—	—	—	—
ADM	0.4020	0.0000		—	—	—	—	—	—	—	—	—
VERN	1.1343	0.0000		—	—	—	—	—	—	—	—	—
SER	0.3036	0.0000		—	—	—	—	—	—	—	—	—
OBT	0.2203	0.0000		—	—	—	—	—	—	—	—	—
OBG	0.1312	0.0000		—	—	—	—	—	—	—	—	—
OSC	0.2002	0.0000		—	—	—	—	—	—	—	—	—

Fig. 2 Geary coefficients (contiguity graph).

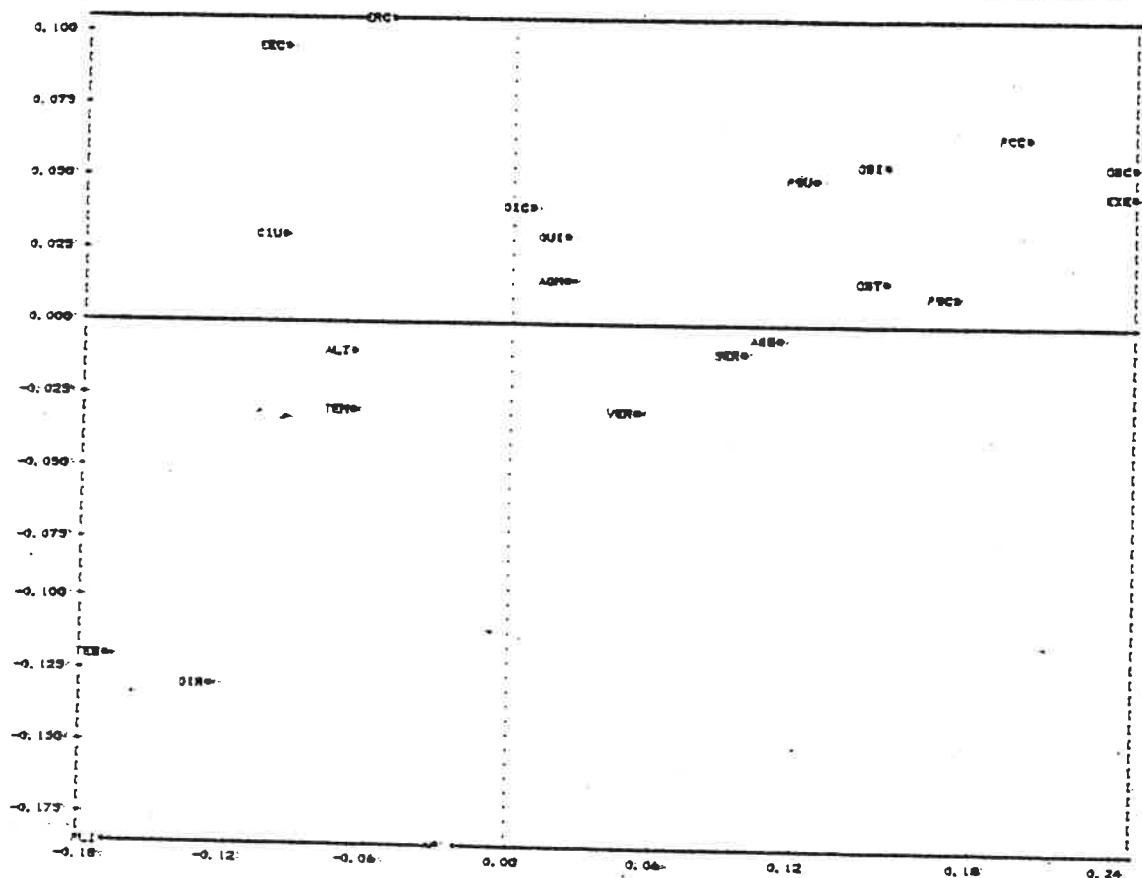


Fig. 3 First local plan

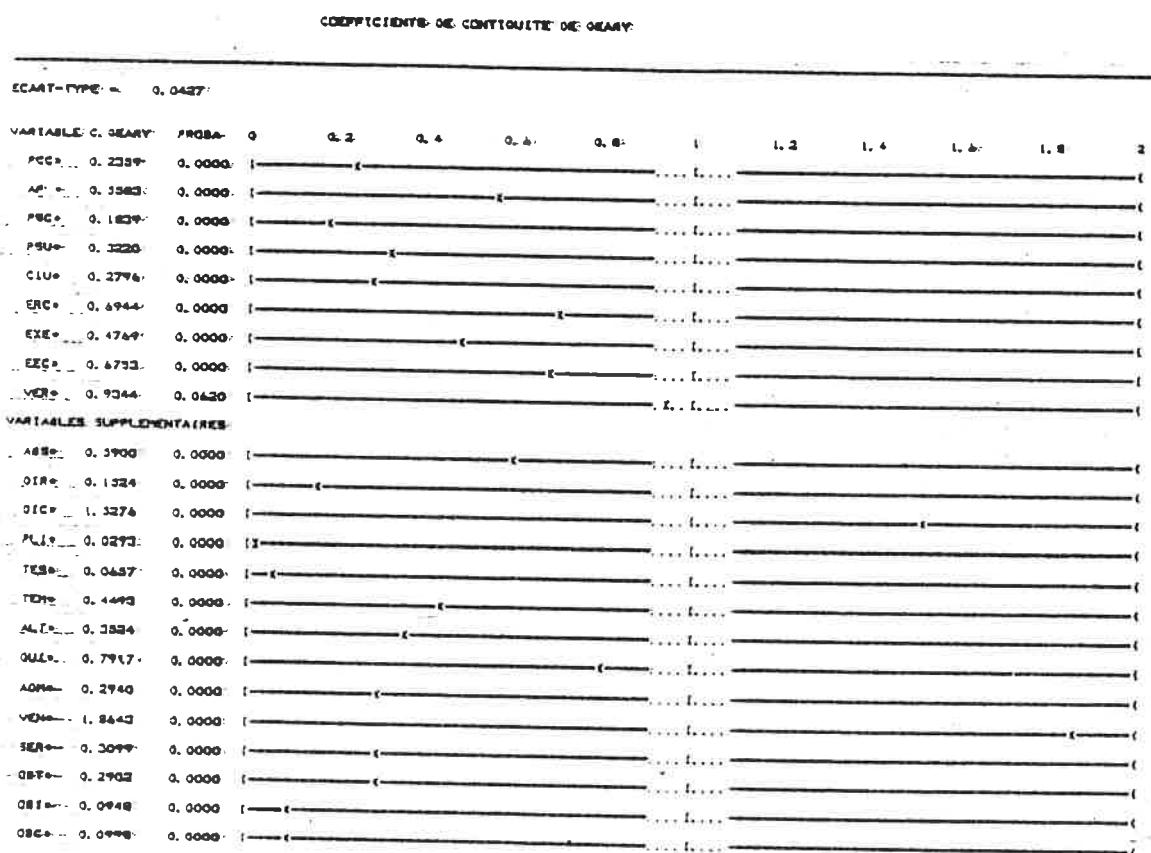


Fig. 4 Geary coefficients (similarity graph).

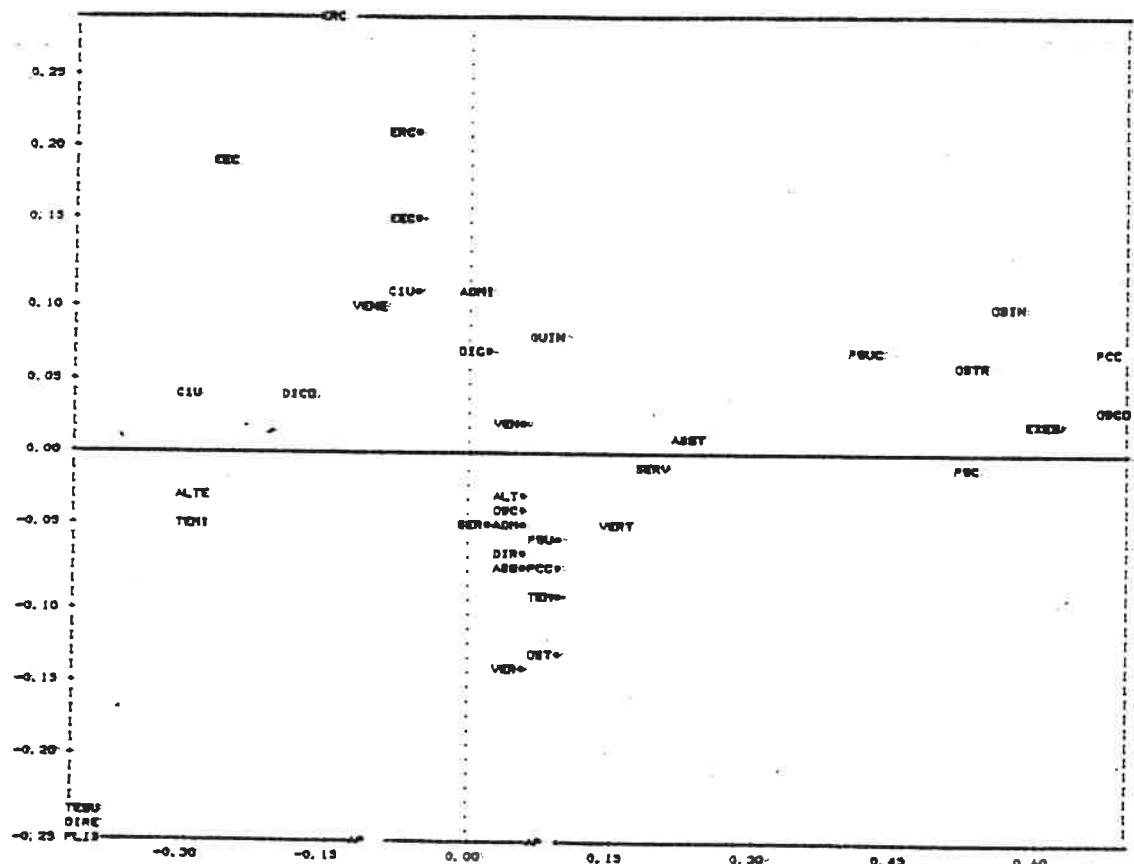


Fig. 5 First global plan. Display of global & "partial" vars.

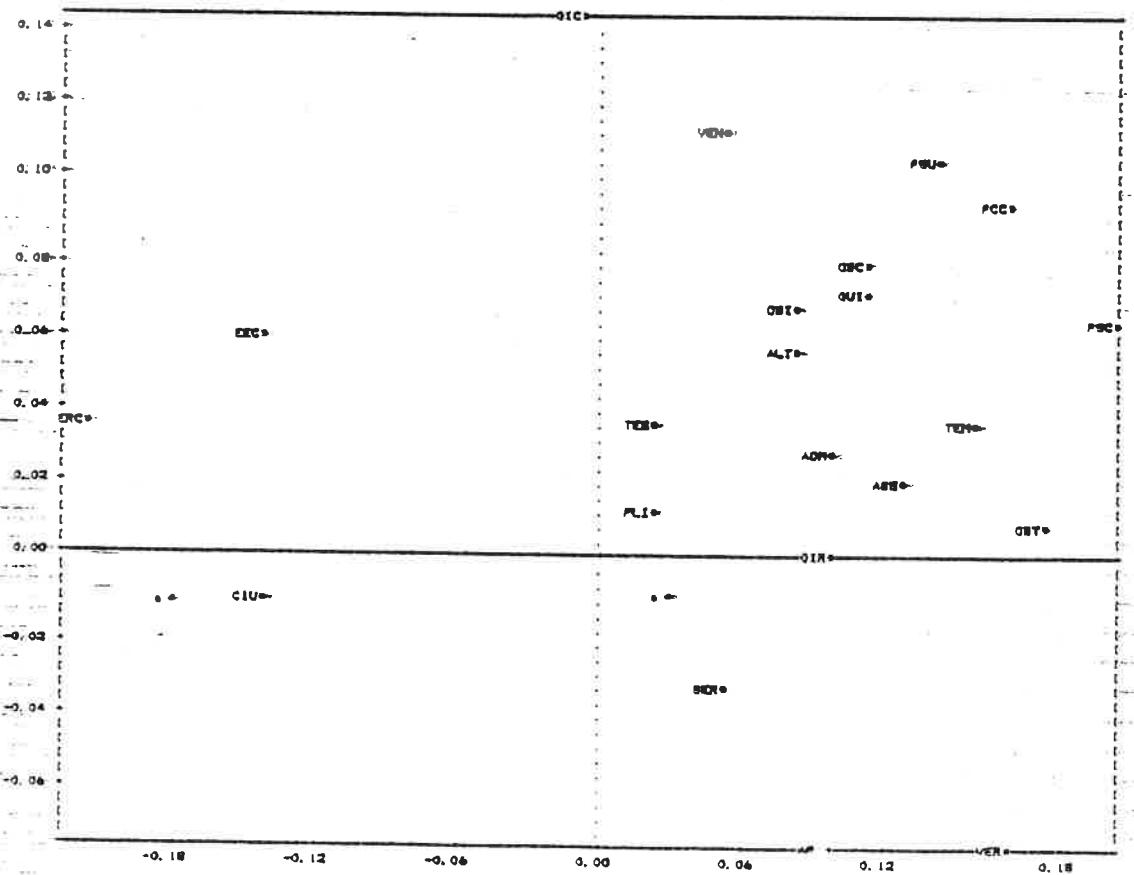


Fig. 6 First "partial" plan.

CLASSIFICATION ASCENDANTE HIERARCHIQUE  
AVEC CONTRAINTE DE CONTIUITÉ ET RESTRICTION (PARIS = 0) SUR LES EFFECTIFS DES CLASSES

DESCRIPTION DES 30 NOEUDS DE LA HIERARCHIE D'INDICES LES PLUS ELEVES

NOEUD	INDICE	ATME	REMARQUE	EFFECTIF	POIDS	HISTOGRAMME DES INDICES DE NIVEAU
2089	0. 00033	2083		122	115232	1 •
2087	0. 00034	2082		83	84049	1 •
2070	0. 00042	2072		113	81104	1 •
2091	0. 00026	2080		137	106453	1 •
2072	0. 00044	1879		87	88546	1 •
2092	0. 00044	2052		71	70321	1 •
2094	0. 00048	1059		70	89433	1 •
2095	0. 00031	2046		149	135944	1 •
2096	0. 00034	2070		177	131194	1 •
2097	0. 00049	2030		231	147549	1 •
2079	0. 00045	742		37	35018	1 •
2079	0. 00046	1763		27	23744	1 •
2100	0. 00073	2066		241	174854	1 •
2101	0. 00077	2026		10	10140	1 •
2102	0. 00080	1799		2101	11051	1 •
2103	0. 00079	123		2102	11511	1 •
2104	0. 00082	2036		83	85698	1 •
2105	0. 00084	2019		147	152016	1 •
2106	0. 00078	2097		398	201543	1 •
2107	0. 00103	2087		106	80443	1 •
2108	0. 00107	2046		286	229804	1 •
2109	0. 00123	333		2106	220077	1 •
2110	0. 00132	2010		104	104232	1 •
2111	0. 00137	2076		143	116281	1 •••••
2112	0. 00160	2104		302	405797	1 •••••
2113	0. 00163	2103		317	417308	1 •••••
2114	0. 00174	2104		372	315779	1 •••••
2115	0. 01018	2079		344	443052	1 •••••
2116	0. 02702	2111		467	354333	1 •••••
2117	0. 04631	2114		1059	573108	1 •••••

Fig. 7 30 last clusters of hierarchical algorithm with continuity constraint.

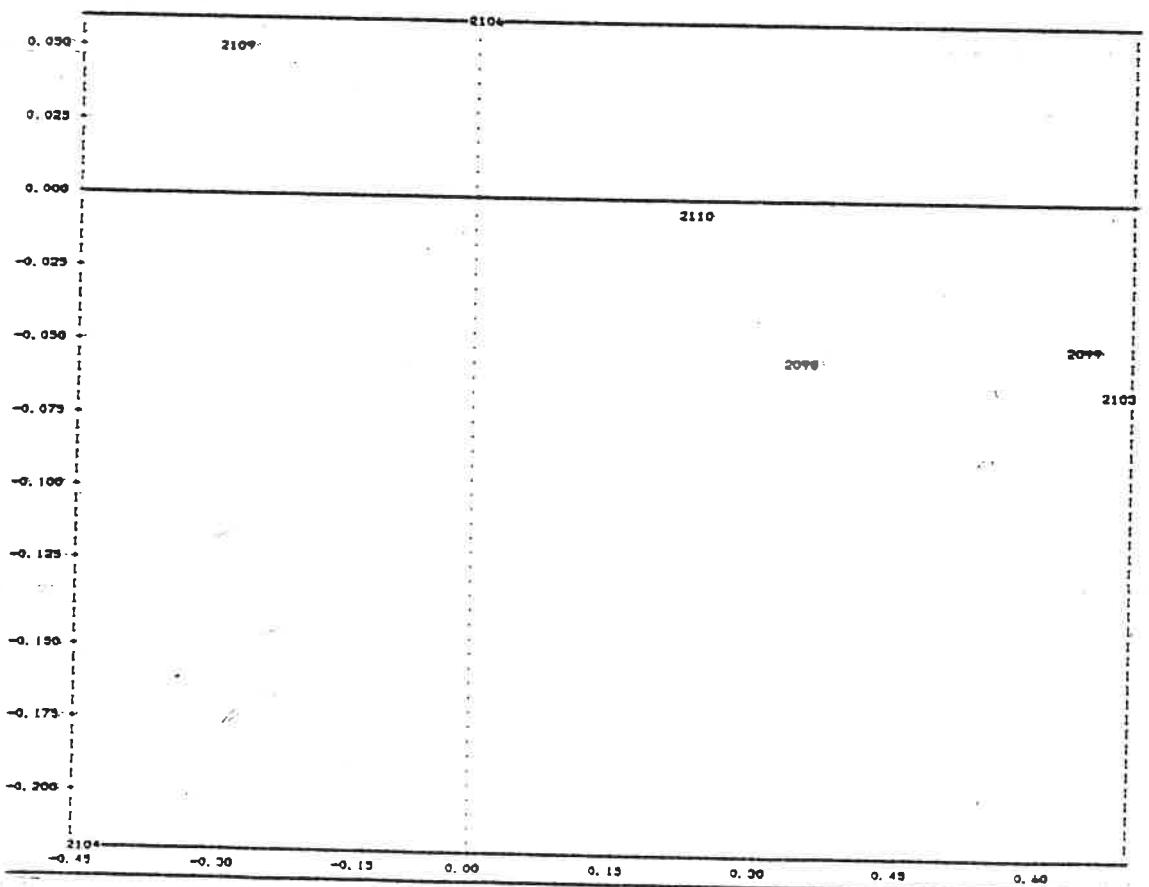


Fig. 8 Partition of eight clusters = contiguous zones.