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**COMPLEMENTARY REMARKS AND IMPROVEMENTS
TO A LAGRANGEAN HEURISTIC FOR
CAPACITATED PLANT LOCATION PROBLEMS**

Jaume BARCELÓ
Josep CASANOVAS

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J.Barceló and J.Casanovas
Departament d'Investigació Operativa, Facultat d'Informàtica,
Universitat Politècnica de Catalunya.
Jordi Girona Salgado, 31. 08034 Barcelona (Spain).

ABSTRACT

In a former paper, [1], a heuristic using multipliers from a lagrangean relaxation was proposed for getting feasible solutions to a class of pure integer capacitated plant location problems. The heuristic consisted of three steps, the last one being a plant interchange step. Further computational experience has shown that the proposed interchange procedure could fail. In this paper we investigate the computational behaviour of the heuristic without interchange, we propose an alternative plant interchange procedure, and we give the result of our computational experience.

RESUMEN

En un trabajo anterior, [1], proponíamos una heurística para obtener soluciones posibles para problemas de localización de plantas con restricciones de capacidad formulados como problemas enteros puros. La heurística utilizaba multiplicadores de una relajación lagrangiana del problema y operaba en tres etapas, la última de las cuales consistía en un procedimiento de intercambio. Experiencia computacional adicional ha demostrado que en algunos casos el procedimiento de intercambio puede fallar. En este trabajo investigamos el comportamiento de la heurística sin intercambio, proponemos un procedimiento alternativo de intercambio y damos los resultados de experiencias computacionales adicionales.

- [1] J.Barceló and J.Casanovas, A Heuristic Lagrangean Algorithm for the capacitated plant Location Problem, European Journal of Operational Research, 15, no.2, February 1984, pp.212-226.

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In a former paper, [1], a heuristic using multipliers from a lagrangean relaxation was proposed for getting feasible solutions to a class of pure integer capacitated plant location problems. The heuristic consisted of three steps, the last one being a plant interchange step. Further computational experience has shown that the proposed interchange procedure could fail. In this paper we investigate the computational behaviour of the heuristic without interchange, we propose an alternative plant interchange procedure, and we give the result of our computational experience.

1. INTRODUCTION

In a former paper, [1], we proposed a heuristic derived from a lagrangean relaxation, for getting feasible solutions for capacitated plant location problems formulated as:

$$\min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j \quad (1)$$

$$\text{s.t.} \quad (2)$$

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i \in I \quad (3)$$

$$(P) \quad \sum_{j \in J} y_j \leq K \quad (4)$$

$$\sum_{i \in I} d_i x_{ij} \leq b_j y_j \quad \forall j \in J \quad (5)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in I, \quad \forall j \in J \quad (6)$$

$$y_j \in \{0, 1\}, \quad \forall j \in J$$

where as usual I is the set of centers with demands d_i to be supplied from plants j , with capacities b_j , J is the set of potential locations for plants, c_{ij} are the costs of supplying the total demand of center i from plant j , and f_j are the fixed costs of opening a plant at potential location j . The problem is formulated as a pure integer one, meaning that each center has to be supplied from only one plant, so the decision variables are: $x_{ij} = 1$, if center i is supplied from plant j , and $x_{ij} = 0$ otherwise, $y_j = 1$ when plant j is open and $y_j = 0$ when it is closed.

The proposed heuristic worked out in three phases. The first phase was a plant selection procedure operating of the following way: define the values of lagrangean multipliers of constraints (2) as:

$$\bar{u}_i = \min_{j \in J} \left\{ c_{ij} + \frac{d_i f_j}{b_j} \right\}, \quad \forall i \in I \quad (6)$$

compute the amounts,

$$p_j = \sum_{i \in I} \left(c_{ij} \frac{d_i f_j}{b_j} - \bar{u}_i \right), \quad \forall j \in J \quad (7)$$

which could be interpreted as a measure of the interest of including plant j in the solution, and then solve the knapsack problem with an additional constraint,

$$\begin{aligned} \min \quad & \sum_{j \in J} p_j y_j \\ \text{s.t.} \quad & \sum_{j \in J} Y_j \leq K \\ (K1) \quad & \sum_{j \in J} b_j y_j \geq D, \text{ being } D = \sum_{i \in I} d_i \\ & y_j \in \{0,1\}, \quad \forall j \in J \end{aligned}$$

whose solution $J^* = \{j \in J \mid y_j = 1\}$, gives a set of plants of cardinality $|J^*| \leq K$, able to satisfy the total demand D .

The second phase was an heuristic for getting a feasible solution for the remaining generalized assignment subproblem:

$$\begin{aligned}
 & \min \sum_{i \in J} \sum_{j \in J^*} c_{ij} x_{ij} \\
 & \text{s.t. } \sum_{j \in J^*} x_{ij} = 1, \quad \forall i \in I \\
 & \text{(GA)} \quad \sum_{i \in I} d_i x_{ij} \leq b_j, \quad \forall j \in J^* \\
 & \quad \quad x_{ij} \in \{0, 1\}, \quad \forall i \in I, \quad \forall j \in J
 \end{aligned}$$

this heuristic was a modification of the starting procedure of the Ross and Soland algorithm, [2].

The third and last phase proposed an attempt of improving the choice of plants through an interchange based on an estimate of the contribution of each plant to the duality gap. The basic assumption behind this approach was that the amounts $s_j v_j$ (where $s_j, j \in J^*$, was the slack of plant j in (4) and v_j its dual variable) provided such an estimate. So, if a plant $k \in J \setminus J^*$ existed such that its capacity b_k was less than b_j , and $b_j - b_k \leq \sum_{j \in J^*} s_j$, and substitution of b_j by b_k in J^* could still satisfy the total demand D , then it was supposed that this substitution improved the solution if $|p_j - p_k| \leq s_j \cdot v_j$, because the contribution of this variable to the duality gap was greater or equal than the estimate of the increment in the objective function value.

The computational experience done in our earlier work did not detected any inconsistency with these assumptions, however, further computational experience with new test problems suggested to us by L.N. Van Wassenhove, has shown that :

- a) When problems are very tight, that is when $\sum_{j \in J^*} b_j$, total capacity of plants included in the solution, is very close to the total demand D , then the generalized assignment subproblem (GA) can not have feasible solutions. That was implicitly recognized in our work but it raises the question of: what to do then, and how this affects the heuristic performance.
- b) The interchange proposed by comparison between estimators $|p_j - p_k|$ and $s_j v_j$ fails in some cases providing worse solutions.

2. ALTERNATIVE HEURISTICS

A further insight into our argument shows that the proposed estimate is a bad one because the amounts involved can not allways be compared. The reasons for that are that if vector \bar{u} , is a feasible vector of multipliers for the lagrangean of (P) with respect to constraints (2):

$$L(\bar{u}, x) = \min \left\{ \sum_{j \in J} \left[\sum_{i \in I} (c_{ij} - \bar{u}_i) x_{ij} + f_j y_j \right] + \sum_{i \in I} \bar{u}_i \right\}$$

(L) s.t.

(3), (4), (5) and (6).

then the vector (x, y) to be used in computing if (x, y, u) satisfies the optimality conditions or not, should be the optimal solution to problem (L), and only in some cases this vector x will be also feasible for problem (GA) and reciprocally a vector x feasible for problem (GA) can be not optimal for problem (L), and consequently estimator $\sum_j S_j \cdot V_j$, computed from the feasible solution to (GA) could be meaningless.

Thus we must redefine the heuristic reducing it to only two phases: the plant selection phase and the phase of assignment of centers to plants; eliminating the interchange. But doing that does not answer the question of what to do when assignment is infeasible. To deal with these situation we propose the following interchange:

- a) Identify the plant with the greatest p .
- b) Penalize the plant identified increasing its p , coefficient by an arbitrarily large amount and solve again knapsach problem (K1).

However, when problems are so tight that assignments could be infeasible it can happen also that the simpler assignment heuristic proposed in, 1, eventually fait or it tooks a very large number of iterations for getting a feasible solution, in such circumstances a complementary assignment procedure has proven its efficiency in reducing the time required.

With these changes the proposed heuristic works as follows:

Heuristic 1

Step 1: Compute multipliers \bar{u}_i as in (6), for each center $i \in I$.
 Compute amounts p_j , as in (7), for each plant $j \in J$

Step 2: (Plant selection)
 Solve knapsack problem (K1). Let $J^* \subseteq J$ be the set of selected plants.

Step 3: (Assignment Procedure)

3.1 Direct assignment

$\forall i \in I$ define: $l_k = \min_{j \in J^*} \{c_{ij}\}$

Set: $x_{ik} = 1$

Define: $I_j = \{i \in I \mid x_{ij} = 1\}$, $\forall j \in J^*$

Calculate:

$$s_j = b_j - \sum_{i \in I_j} d_i, \quad \forall j \in J^* \quad (8)$$

If $s_j \geq 0$, $\forall j \in J^*$, STOP (the assignment is feasible and optimal) otherwise go to 3.2.

3.2 Reassignment

Define $V = \{j \in J^* \mid s_j < 0\}$ (set of plants whose capacity is violated by the previous assignment).

Define the penalties of reassigning the centers from violated plants to other plants:

For $k \in V$ and $i \in I_k$ while $V \neq \emptyset$.

Define $p_{ih} = \min_{h \in J^* \setminus V} \{c_{ih} - c_{ik} \geq 0, \text{ and } d_i \leq s_h\}$

$p_{ih} = \infty$ otherwise

Solve

$$\min \sum_{i \in I_k} p_{ih} z_i$$

$$\sum_{i \in I_k} d_i z_i \geq -s_k$$

$$z_i \in \{0, 1\}, \quad \forall i \in I_k$$

Reassign step by step

If $z_i=1$ and $s_h-d_i \geq 0$ (s_h can have been modified by a previous reassignment), then:

$$I_k = I_k - \{i\}, \quad I_h = I_h \cup \{i\}$$

Otherwise do not reassign.

After reassignments:

Compute again $s_j, \forall j \in J^*$ as in (8)

Redefine $V = \{j \in J^* \mid s_j < 0\}$

If $V = \emptyset$, STOP, a feasible assignments has been obtained.

Otherwise repeat, or after repetitions go to 3.3.

3.3 If after r repetitions of 3.2 still $V \neq \emptyset$, then,

3.3.1 For $k \in V$ such that $s_k < 0$ and $l \in J^* \setminus V$ such that $s_l \geq 0$.

search for

$$\text{MAX}_{i \in I_k, j \in I_l} \{d_i - d_j > 0 \text{ and } d_i - d_j < s_l\}$$

Then:

$$I_k = (I_k - \{i\}) \cup \{j\}$$

$$I_l = (I_l - \{j\}) \cup \{i\}$$

3.3.2 For $k \in V$ such that $s_k < 0$ and $h \in J \setminus V$ such that $s_h \geq 0$.

Search for:

$$\text{MIN}_{i \in I_k, j \in I_n} \{c_{ih} - c_{ik} + c_{jk} - c_{jh} \mid s_h + d_j - d_i \geq 0, s_k + d_i - d_j < s_k\}$$

Then:

$$I_k = (I_k - \{i\}) \cup \{j\}$$

$$I_h = (I_h - \{j\}) \cup \{i\}$$

3.3.3 Compute again $s_j, \forall j \in J^*$ as in (8).

Redefina $V = \{j \in J^* \mid s_j < 0\}$

If $V = \emptyset$ STOP a feasible assignment has been found.

Otherwise either try with an exact branch and band algorithm to verify that the generalised assignment subproblem is infeasible or, consider it as infeasible and go to step 4.

Step 4: Alternative.

Search for the plant $k \in J^*$, such that $s_k \neq 0$, with the greatest p_k .

Penalise this plant:

set $p_k = p_k + M$ (M an arbitrarily large amount) and go to step 2 to solve again the knapsack problem (K1) and find a new set J^* of open plants.

The following example illustrates how the heuristic 1, works. Let the problem defined by

	c_{ij}				d_i	\bar{u}_i
	2	3	10	12	5	67.7894
	3	2	12	16	7	93.3043
	4	5	15	20	13	174.5652
	1	3	16	18	9	119.4210
	2	1	14	17	8	105.3478
b_j	19	23	20	25	$\Sigma d_i =$	42
f_j	250	300	450	500		
p_j	4.2036	1.3982	451.5288	362.5723		

The corresponding knapsack problem (K1) is

$$\min 4.2036y_1 + 1.3982y_2 + 451.5288y_3 + 362.5723y_4$$

$$y_1 + y_2 + y_3 + y_4 \leq 2$$

$$19y_1 + 23y_2 + 20y_3 + 25y_4 \geq 42$$

$$y_i \in \{0, 1\}, \quad i = 1, 2, 3, 4$$

and has the solution $y_1 = y_2 = 1, y_3 = y_4 = 0$, but the corresponding assignment subproblem is infeasible. The proposed heuristic gives:

$$I_1 = \{1, 3\}, \quad s_1 = 1$$

$$I_2 = \{2, 4, 5\}, \quad s_2 = -1$$

and $z = 843$ (objective function value)

plant 1 is the one with the greatest p , so penalizing p_1 and solving again the knapsack one obtains the solution $y_2 = y_4 = 1, y_1 = y_3 = 0$ which yields a feasible generalized assignment subproblem.

However by inspection it is possible to find that the choice $y_2 = y_3 = 1, y_1 = y_4 = 0$ gives a better solution. This fact arises the questions of whether or not is possible make a better choice of plants improving the first selection.

Trying to answer these questions one can think that the way of calculating the amounts p_j could be not very realistic in some cases, given that the computation of the \bar{u}_i is based on the LP relaxation of problem (P) consisting on the substitution of constraints (5) and (6) by

$$x_{ij} \geq 0, \quad \forall i \in I, \forall j \in J, \quad \text{and } y_j \geq 0, \quad \forall j \in J$$

and then (7) computes p_j as if the problem were an uncapacitated problem and that represents an overrelaxation. This suggests that perhaps taking into account the fact that plants have limited capacities in calculating p_j could provide a more realistic approach. Consequently to include the capacities in the computation of amounts p_j , we substitute (7) by:

$$p_{mj} = \min \sum_{i \in I} \left(c_{ij} + \frac{d_i f_j}{b_j} - \bar{u}_i \right) z_i$$

s.t.

(K2)

$$\sum_{i \in I} d_i z_i \geq b_j$$

$$z_i \in \{0, 1\}, \quad \forall i \in I$$

and then we define heuristic 2 as follows:

Heuristic 2

Step 1: Compute multipliers \bar{u}_i , $\forall i \in I$, as in (6)
 Compute amounts p_{mj} by solving (K2) for each $j \in J$

Step 2: Solve knapsack problem (K1) using p_{mj} instead of p_j .

Steps 3 and 4 remain unchanged

3. COMPUTATIONAL EXPERIENCE WITH HEURISTIC 1 AND HEURISTIC 2

A serie of 20 problems of sizes 20 x 10 each (20 centers and 10 plants) 10 problems of 30x10, and 5 problems of 50 x20 have been solved using both heuristics and computational results are shown in Table 1. All this problems were generated using a random test problem generator with the following features. Given the parameters DMIN, minimum value of the demand, DPROM approximate average value of the demands, CMIN, minimum transportation cost value, CPROM, approximate average transportation cost, and DCOEF average ratio between the capacity of K plants and the total demand D; problem data are generated in the following way: K, limit member of plants to be open, is generated in the interval (0.1n, n), being n the number of centers; demands d_i are generated as randomly distributed in an interval centered on CPROM which has CMIN as lower limit; once the demands are generated BPROM, average plant capacities, is computed as,

TOTAL DEMAND/ DCOEF * K, and then capacities b_j are generated as randomly distributed in the interval (BMIN, 2*BPROM-BMIN), where BMIN is fixed to DMIN. Transportation costs are computed as $c_{ij} = a_{ij} + r_{ij} d_i$, assuming that they have two components, the first one a_{ij} a fixed component depending on the plant, determined randomly, and the second $r_{ij} d_i$, proportional to the demand. Fixed costs f_j are computed assuming also that they are proportional to capacities plus a random component in such a way that they satisfy some proportionality with transportation costs.

Test problems have been generated in such a way that ratio D/B, between total demand D and $B = \sum_{j \in J} b_j$, total capacity offered by open plants, were as high as possible, greater than 0.9 in general, in order to get problems very tight, with capacities closer to total demand, expecting to generate infeasible assignment subproblems, however that happened only in two cases.

Test Problem Number	Size n x m	status	Heuristic 1		Heuristic 2	
			Ratio D/B	$\Delta = \frac{Z_{heu} - Z_{best}}{Z_{best}} \%$	Ratio D/B	$\Delta = \frac{Z_{heu} - Z_{best}}{Z_{best}} \%$
1	20x10	a	0.9506	0.45	0.9506	0.45
2	20x10	b	0.9469	2.94	0.9768	0.0
3	20x10	c	0.9857	1.38	0.9741	1.46
4	20x10	c, e	0.9761	1.87	0.9609	1.72
5	20x10	c, e	0.9747	1.54	0.9658	1.55
6	20x10	c	0.8729	11.02	0.9584	0.83
7	20x10	b	0.9786	1.14	0.9838	1.12
8	20x10	-	0.9729	2.10	0.9875	0.75
9	20x10	c, e	0.9621	2.63	0.9151	6.58
10	20x10	c	0.9294	7.74	0.9526	3.70
11	20x10	c	0.9835	0.42	0.9693	1.93
12	20x10	d, c	0.9705	0.29	0.9365	6.29
13	20x10	e	0.9781	1.42	0.9623	2.63
14	20x10	e	0.7679	27.70	0.9801	0.34
15	20x10	a	0.9887	0.0	0.9313	5.87
16	20x10	d	0.9666	1.98	0.9812	1.12
17	20x10	b, e	0.8776	9.94	0.9714	0.0
18	20x10	c	0.9679	1.11	0.9132	3.71
19	20x10	c, e	0.9107	7.89	0.9344	4.92
20	20x10	-	0.9712	1.87	0.9986	1.86
21	30x10	a, e	0.9932	0.0	0.9809	2.99
22	30x10	a, c	0.9987	0.0	0.9058	3.77
23	30x10	-	0.9583	2.18	0.9681	0.90
24	30x10	-	0.8053	7.33	0.9953	8.27
25	30x10	c, a	0.9495	0.0	0.9630	(*)
26	30x10	c, a	0.9988	0.64	0.9859	1.36
27	30x10	a	0.9884	0.30	0.9633	3.75
28	30x10	c, a	0.9586	0.0	0.9820	0.0
29	30x10	c	0.8920	7.40	0.9564	(*)
30	30x10	c	0.9778	1.60	0.9712	1.06
31	50x20	c	0.9915	5.56	0.9975	4.94
32	50x20	c	0.9621	4.55	0.9798	2.01
33	50x20	-	0.9897	0.54	0.9913	0.51
34	50x20	-	0.9942	8.43	0.9695	10.76
35	50x20	c, a	0.9994	0.31	0.9867	(*)

TABLE 1

Problem status a in Table 1, means that heuristic 1 gives the best solution, of at least the best selection of plants (the small differences in problems 1, 27 and 35 are due to the assignments heuristic), while problem status b means that is heuristic 2 who provides the best solution. To get the best solutions problems have been solved by a special branch and bound code using lagrangean relaxation and subgradient optimization. Problem status e means that optimality was not proved by the branch and bound code, but it was not able to improve the best solution in less than 10 CPU minutes of a VAX 11/780.

Solutions provided by heuristic 1 are on average within the 3,54% of the best solution, and only in two cases (problems 6 and 14) presented a big deviation, but in these cases heuristic 2 gave an almost optimal solution. However in 19 cases out of 35, heuristic 2 gave either a solution as good as that of heuristic 1 or a better one only allowing the violation of constraint (3) on the limit number of plants, (problem status c in Table 1). This situation occurs mainly when there are many small plants able to be included in the set of open plants, in that case all they present an attractive value of p_m . In problems marked with (*) solution of heuristic 2 was better than the optimal but only because of violation of constraint (3).

Two cases out of 35, presented unfeasible assignments subproblems, these were problems 12 and 16 identified by status d in Table 1, in both of them heuristic 1 gave an almost optimal solution.

4. AN ALTERNATIVE INTERCHANGE HEURISTIC.

Computational experience with heuristic 2 disappointed its systematic use, nevertheless we can not forget that it gives in some cases better results than heuristic 1, specially when small plants can satisfy the demand requirements. The analysis of these situations suggests that when a plant k , not included in the solution of heuristic 1, $k \notin J^*$, has a capacity b_k such that b_k is smaller than b_j , capacity of plant j included in the solution of heuristic 1, $j \in J^*$, then if substitution of plant j , by plant k in the solution gives a new feasible solution then if $p_{mk}^{(*)}$ the new solution can be better than the former one. The modified amounts p_m computed by (K2) become in that way the basis of a 1-interchange procedure that substitutes a plant j in the solution by a plant k not in the solution.

(*) is smaller than p_{mj}

This analysis from computational results can be also extended to a 2 interchange procedure, that means a procedure which substitutes plant j in the solution by plants k and l , not in the solution in the following way: if the sum of the capacities $b_k + b_l$ of plants $k, l \notin J^*$, is smaller than capacity of plant b_j , $j \in J^*$, and replacing j by k and l , provides a feasible solution, then if $p_{mk} + p_{ml}$, are smaller than p_{mj} the new solution can be better than the former one.

As consequence of this analysis we propose heuristic 3:

Heuristic 3

Step 1: Compute multipliers \bar{u}_i as in (6), for each center $i \in I$.
Compute amounts p_j , as in (7), for each plant $j \in J$

Step 2: (Plant Selection).
Solve knapsack problem (K1) with p_j as cost coefficients. Let $J^* \subseteq J$ be the set of open plants.

Step 3: (Assignment Procedure).
The same as heuristic 1, except that:
a) when a feasible assignment is detected then go to step 5.
b) if infeasibility is due to a solution provided by a 1-interchange or a 2-interchange then goto step 6.

Step 4: As a heuristic 1.

Step 5: (Interchange Procedure).

5-1 (1-Interchange)

Search for a candidate to enter the solution.

For $s \in J^*$ search for a plant $e \in J \setminus J^*$, such that:

$$D - \left(\sum_{j \in J^*} b_j - b_s \right) < b_e < b_s$$

If such a plant exists then compute p_{me} and p_{ms} , solving the knapsack (K2) for plants e and s .

If $p_{me} < p_{ms}$, then interchange s and e :

$$J^* = (J^* - \{s\}) \cup \{e\}$$

Go to step 3.

When all plants in J^* have been examined then proceed to

5-2 (2-Interchange).

If $|J^*| = K$ then STOP (a 2-interchange would violate the constraint on the maximum number of plants to be open) otherwise:

For $s \in J^*$, search for plants e_1
and e_2 such that,

$$D - \left(\sum_{j \in J^*} b_j - b_s \right) < b_{e_1} + b_{e_2} < b_s$$

If such a plant exists then compute p_{me_1} ,
and p_{me_2} , and p_{ms} solving the knapsack
(K2) for plants e_1 , e_2 and s .

If $p_{me_1} + p_{me_2} < p_{ms}$, then interchange s and
 e_1 , e_2 :

$$J^* = (J^* - \{s\}) \cup \{e_1\} \cup \{e_2\}$$

Go to step 3.

When all the plants in J^* have been examined then STOP.

Step 6: (Infeasible assignments from solutions provided by interchanges).

Infeasibility from a 1-Interchange: eliminate plant e from further considerations for
1 - I n t e r c h a n g e .

Go to step 5.1.

Infeasibility from a 2-Interchange:

If $b_{e_1} < b_{e_2}$, eliminate plant e_1 , from
further considerations for 2-Interchange,
otherwise eliminate plant e_2 , go to step
5.2.

Heuristic 3 applied to the former example gives the same first solution but in this case the 1-Interchange step (step 5.1) gives the following results:

$s=2$ $D - \left(\sum_{j \in J^*} b_j - b_s \right) = 19$ so plant 2 could be
interchanged with plants such that $19 < b_e < 23$,
plant 3 is the only plant filling these
conditions but $p_{m2} = 0$ and $p_{m3} = 209.1305$ and do
not proceed to interchange.

$s=4$ $D - (\sum_{j \in J^*} b_j - b_s) = 17$, and then plants 1 and 3 are candidate to interchange, $P_{m1} = 0.4874$, $P_{m4} = 212.3561$, so interchange of 4 by 1 is proposed, but $J^* = \{1, 2\}$, gives an infeasible assignment, eliminating 1 by step 6, as $P_{m3} < P_{m4}$ interchange of 4 by 3 is proposed, and $J^* = \{2, 3\}$ gives:

$$I_2 = \{1, 4, 5\}$$

$$z^* = 784$$

$$I_3 = \{2, 3\}$$

and no more interchanges are possible.

5. COMPUTATIONAL EXPERIENCE WITH HEURISTIC 3.

A series of 45 test problems, covering a wide variety of situations, were solved using heuristic 3. The first 35 were the ones used for testing heuristic 1 and 2, the remaining 10, of sizes 40×15 and 40×20 , were generated with the same generator and with the goal of covering the intermediate cases. As before problems were generated as tight as possible, that is with a ratio D/B (where B is the amount of the capacity offered by the last solution) as high as possible. In this second series of computational experiences we have found that a second parameter could give some complementary information that helps to explain the behaviour of the heuristic. Such a parameter is the average ratio between the fixed cost f_j and the sum of transportation cost

$$\frac{\sum_{i \in I} c_{ij}}{f_j}, \text{ for each plant.}$$

Computational results are shown in Table 2. For each problem the Table includes the following information: size, average ratio $f_j / \sum_{i \in I} c_{ij}$, the total number of interchanges attempted by the heuristic, the number of feasible interchanges (the remaining ones giving infeasible assignment subproblems), the type of interchanges (1 for 1-interchanges, 2 for 2-interchanges), the improvement provided by the interchange, the final status of the problem (a for those problems for which the heuristic provided the best solution, where, as before, best solution means the optimal in certain cases or at least a solution which could not be improved in less than 10 CPU minutes of a VAX 11/780 using a special branch and bound code, status b means that best solution and heuristic solution include different plants by their objective function values differ in less than 1%, d means, as before that first heuristic solutions, before proceeding to interchange, gave infeasible assignment subproblems, and status c is for the special cases when interchanges start improving the solution but after a certain number of them the solution becomes worse but always within a certain range

Test Problem Number	Size $n \times m$	Average Ratio $f_j / \sum_{i \in S} c_{ij}$	Total Number of Interchanges Proposed by Heuristic	Number of Feasible Interchanges	Type of Interchanges	Improvement $\frac{Z_{heur} - Z_{best}}{Z_{heur}}$ %	status	$\frac{Z_{best} - Z_{opt}}{Z_{opt}}$ %	Final Ratio D/B
1	20x10	1.4019	0	-	-	-	a	0.45	0.9506
2	20x10	1.4019	1	1	1	2.94	a	0.0	0.9768
3	20x10	5.4046	1	1	1	0.68	b	0.23	0.9880
4	20x10	3.1812	2	2	1,2	1.87	a	0.0	0.9946
5	20x10	7.2625	2	2	1,2	1.54	a	0.0	0.9883
6	20x10	6.9880	2	2	1,2	11.07	b	0.0	0.9758
7	20x10	3.2320	1	0	2	-	-	1.14	0.9786
8	20x10	4.7740	1	1	2	1.33	-	0.75	0.9875
9	20x10	4.5851	1	1	2	1.91	a	0.0	0.9971
10	20x10	6.5680	1	1	2	1.21	-	6.45	0.9339
11	20x10	11.5798	1	1	2	0.22	b	0.20	0.9844
12	20x10	10.2858	0	-	-	-	d	2.96	0.9705
13	20x10	16.6891	2	2	2	1.42	a	0.0	0.9935
14	20x10	14.5346	4	3	1,1,1,2	27.70	a	0.0	0.9923
15	20x10	11.6812	1	0	2	-	a	0.0	0.9887
16	20x10	2.8893	0	-	-	-	d	1.98	0.9666
17	20x10	6.9488	1	1	1	9.94	a	0.0	0.9794
18	20x10	11.8246	0	-	-	-	-	1.11	0.9546
19	20x10	6.6907	1	1	1	7.89	a	0.0	0.9856
20	20x10	4.7450	2	1	2	0.90	-	0.96	0.9802
21	30x10	0.5762	0	-	-	-	a	0.0	0.9932
22	30x10	0.9593	0	-	-	-	a	0.08	0.9987
23	30x10	0.1107	4	4	1,2,2,1	2.18/-14.06	a,c	0.0	0.9918
24	30x10	0.0865	2	2	1,1	5.31	-	1.91	0.9837
25	30x10	0.0946	2	2	2,1	-3.94	a,c	0.0	0.9955

TABLE 2

26	30x10	0.7698	0	-	-	-	-	-	0.64	a	0.9988
27	30x10	0.6969	0	-	-	-	-	-	0.30	a	0.9884
28	30x10	0.1125	1	1	1	1	-6.81	-	0.0	c, a	0.9982
29	30x10	4.4860	2	2, 1	2, 1	2, 1	7.40	-	0.0	a	0.9598
30	30x10	2.5352	1	1	1	1	1.60	-	0.0	a	0.9950
31	50x20	2.4614	4	2, 1, 2, 1	2, 1, 2, 1	2, 1, 2, 1	0.55	-	4.98	-	1.0000
32	50x20	2.8837	4	2, 1, 2, 1	2, 1, 2, 1	2, 1, 2, 1	2.59	-	1.91	-	0.9868
33	50x20	1.9343	4	1, 2, 2, 2	1, 2, 2, 2	1, 2, 2, 2	0.54	-	0.0	a	0.9978
34	50x20	1.3920	7	2, 2, 2, 2, 2, 2	2, 2, 2, 2, 2, 2	2, 2, 2, 2, 2, 2	0.0	-	8.43	-	0.9953
35	50x20	0.8081	2	2, 2	2, 2	2, 2	0.0	-	0.31	b	0.9994
36	40x15	0.9980	1	2	2	2	1.5	-	0.0	a	0.9843
37	40x15	0.2588	6	1, 1, 1, 1, 1, 2	1, 1, 1, 1, 1, 2	1, 1, 1, 1, 1, 2	2.16/-6.88	-	0.0	a, c	0.9957
38	40x15	0.0971	1	1	1	1	-	-	0.0	d, a	0.9913
39	40x15	0.7940	0	-	-	-	-	-	0.0	a	0.9957
40	40x15	1.2731	3	2, 1, 1	2, 1, 1	2, 1, 1	2.01/-1.31	-	0.0	c, a	0.9979
41	40x20	1.9940	5	2, 2, 2, 1, 1	2, 2, 2, 1, 1	2, 2, 2, 1, 1	-	-	0.49	b	0.9988
42	40x20	2.0080	7	2, 1, 1, 1, 2, 2, 2	2, 1, 1, 1, 2, 2, 2	2, 1, 1, 1, 2, 2, 2	0.61/-0.19	-	0.74	c, b	0.9976
43	40x20	2.2382	1	2	2	2	-	-	6.62	-	0.9990
44	40x20	0.9938	3	2, 1, 2	2, 1, 2	2, 1, 2	-	-	2.64	-	0.9925
45	40x20	0.2724	2	2, 2	2, 2	2, 2	-1.00	-	0.0	c, a	0.9880

TABLE 2 (cont.)

about the best one), Table 2 also includes the difference with respect to the best solution and the final ratio D/B, that is the ratio for the last of the heuristic solutions.

In 32 cases out of 45 the heuristic provided either the best solution (status a), or an equivalent solution (status b), with an average difference of 3.21% with respect to the best solution in the remaining cases. In 7 cases the heuristic produced a serie of interchanges with the following behaviour (status c): it starts improving the solution and after a series of improvements it produces worse solutions. Analysing these cases one can notice that with exception of problems 40 and 42 all the remaining cases correspond to problems with a very low ratio $f_j / \sum_{i \in I} c_{ij}$, that means problems for which the sum of transportation cost are very much higher than the fixed costs. In problem 23, for existance, the one with the highest difference, this ratio is of 0.1107 meaning that on average transportation costs for each plant are ten times the fixed cost of opening the plant. However, in despite of these ratios in the remaining cases the behaviour of the heuristic could be interpreted as a search on a bounded neighborhood of the best solution. In some other cases, as in 42, the small difference is due to the effect of the assignment heuristic (in some of them later manual adjustments eliminate the difference). Many of these effects could be interpreted taking into account the way in which were defined amounts p_m (knapsack problem K2), and looking at what happens when $\sum_{i \in I} c_{ij} \gg f_j$ and also c_{ij} present wide dispersions among them, but in all the remaining cases, when $f_j > \sum_{i \in I} c_{ij}$, and c_{ij} do not present so wide dispersions heuristic 3 seems to produce systematic good results as computational experience shows.

6. REFERENCES

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